Unit 3: Kinematics in Two Dimensions; Vectors

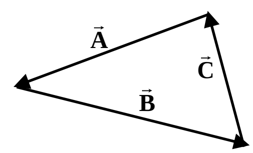
Answers to Questions

1. Their velocities are NOT equal, because the two velocities have different directions.

2. The displacement can be thought of as the "straight line" path from the initial location to the final location. The length of path will always be greater than or equal to the displacement, because the displacement is the shortest distance between the two locations. Thus the displacement can never be longer than the length of path, but it can be less. For any path that is not a single straight line segment, the length of path will be longer than the displacement.

3. The magnitude of the vector sum need not be larger than the magnitude of either contributing vector. For example, if the two vectors being added are the exact opposite of each other, the vector sum will have a magnitude of 0. The magnitude of the sum is determined by the angle between the two contributing vectors.

4. Two vectors of unequal magnitude can never add to give the zero vector. However, three vectors of unequal magnitude can add to give the zero vector. If their geometric sum using the tail-to-tip method gives a closed triangle, then the vector sum will be zero. See the diagram, in which $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=0$



5. A particle with constant speed can be accelerating, if its direction is changing. Driving on a curved roadway at constant speed would be an example. However, a particle with constant velocity cannot be accelerating – its acceleration must be zero. It has both constant speed and constant direction.

6. Assume that the bullet was fired from behind and below the airplane. As the bullet rose in the air, its vertical speed would be slowed by both gravity and air resistance, and its horizontal speed would be slowed by air resistance. If the altitude of the airplane was slightly below the maximum height of the bullet, then at the altitude of the airplane, the bullet would be moving quite slowly in the vertical direction. If the bullet's horizontal speed had also slowed enough to approximately match the speed of the airplane, then the bullet's velocity relative to the airplane would be small. With the bullet moving slowly, it could safely be caught by hand.

7. Your reference frame is that of the train you are riding. If you are traveling with a relatively constant velocity (not over a hill or around a curve or drastically changing speed), then you will interpret your reference frame as being at rest. Since you are moving forward faster than the other train, the other train is moving backwards relative to you. Seeing the other train go past your window from front to rear makes it look like the other train is going backwards. This is similar to passing a semi truck on the interstate – out of a passenger window, it looks like the truck is going backwards.

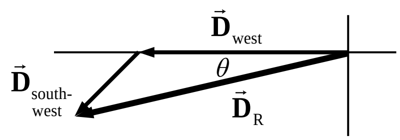
8. (*a*) The ball lands at the same point from which it was thrown inside the train car – back in the thrower's hand.
   (*b*) If the car accelerates, the ball will land behind the point from which it was thrown.
   (*c*) If the car decelerates, the ball will land in front of the point from which it was thrown.
   (*d*) If the car rounds a curve (assume it curves to the right), then the ball will land to the left of the point from which it was thrown.
   (*e*) The ball will be slowed by air resistance, and so will land behind the point from which it was thrown.

9. The baseball is hit and caught at approximately the same height, and so the range formula of $R = v_0^2 \sin 2\theta_0 / g$ is particularly applicable. Thus the baseball player is judging the initial speed of the ball and the initial angle at which the ball was hit.

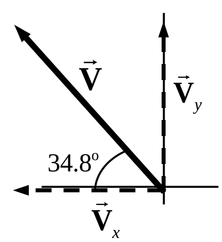
10. The horizontal component of the velocity stays constant in projectile motion, assuming that air resistance is negligible. Thus the horizontal component of velocity 1.0 seconds after launch will be the same as the horizontal component of velocity 2.0 seconds after launch. In both cases the horizontal velocity will be given by $v_x = v_0 \cos\theta = (30\,\text{m/s})(\cos 30^\circ) = 26\,\text{m/s}$.

Solutions to Problems

1. The resultant vector displacement of the car is given by $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_{\text{west}} + \vec{\mathbf{D}}_{\text{south-west}}$. The westward displacement is $215 + 85\cos 45^\circ = 275.1$ km and the south displacement is $85\sin 45^\circ = 60.1$ km. The resultant displacement has a magnitude of $\sqrt{275.1^2 + 60.1^2} = 281.6$ km $\approx \boxed{282\text{ km}}$. The direction is $\theta = \tan^{-1} 60.1/275.1 = 12.3^\circ \approx \boxed{12^\circ\text{ south of west}}$.



2. (*a*) See the accompanying diagram
   (*b*) $V_x = -14.3\cos 34.8^\circ = \boxed{-11.7\text{ units}}$ $V_y = 14.3\sin 34.8^\circ = \boxed{8.16\text{ units}}$
   (*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-11.7)^2 + (8.16)^2} = \boxed{14.3\text{ units}}$
   $\theta = \tan^{-1}\dfrac{8.16}{11.7} = \boxed{34.8^\circ\text{ above the } -x\text{ axis}}$



3. (*a*) $v_{\text{north}} = (735\,\text{km/h})(\cos 41.5^\circ) = \boxed{550\,\text{km/h}}$ $\quad v_{\text{west}} = (735\,\text{km/h})(\sin 41.5^\circ) = \boxed{487\,\text{km/h}}$

(*b*) $\Delta d_{\text{north}} = v_{\text{north}} t = (550\ \text{km/h})(3.00\ \text{h}) = \boxed{1650\ \text{km}}$

$\Delta d_{\text{west}} = v_{\text{west}} t = (487\ \text{km/h})(3.00\ \text{h}) = \boxed{1460\ \text{km}}$

4. The x component is negative and the y component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive x axis would be 122.4°. Thus the components are found to be

$$x = -4580 \sin 32.4^\circ = -2454\ \text{m} \qquad y = 4580 \cos 32.4^\circ = 3867\ \text{m} \qquad z = 2450\ \text{m}$$

$$\boxed{\vec{\mathbf{r}} = (-2450\ \text{m}, 3870\ \text{m}, 2450\ \text{m})} \qquad |\vec{\mathbf{r}}| = \sqrt{(-2454)^2 + (4580)^2 + (2450)^2} = \boxed{5190\ \text{m}}$$

5. Choose downward to be the positive y direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.5\,\text{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the final location $y = 6.5\,\text{m}$. The time for the tiger to reach the ground is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad 6.5\text{m} = 0 + 0 + \tfrac{1}{2}(9.8\,\text{m/s}^2) t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(6.5\text{m})}{9.8\,\text{m/s}^2}} = 1.15\ \text{sec}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = (3.5\,\text{m/s})(1.15\ \text{sec}) = \boxed{4.0\ \text{m}}$$

6. Choose downward to be the positive y direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the displacement is 45.0 m. The time of flight is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad 45.0\ \text{m} = \tfrac{1}{2}(9.80\,\text{m/s}^2) t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(45.0\ \text{m})}{9.80\,\text{m/s}^2}} = 3.03\ \text{sec}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x / t = 24.0\ \text{m}/3.03\ \text{s} = \boxed{7.92\,\text{m/s}}.$$

7. Choose the point at which the football is kicked the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $v_{y0} = (18.0 \sin 35.0^\circ)\,\text{m/s}$, $a_y = -9.80\,\text{m/s}^2$ and the final y velocity will be the opposite of the starting y velocity (reference problem 3-28). Use Eq. 2-11a to find the time of flight.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{(-18.0 \sin 35.0^\circ)\,\text{m/s} - (18.0 \sin 35.0^\circ)\,\text{m/s}}{-9.80\,\text{m/s}^2} = \boxed{2.11\text{ s}}$$

8. Choose downward to be the positive y direction. The origin is the point where the supplies are dropped. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the final position is $y = 160\,\text{m}$. The time of flight is found from applying Eq. 2-11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 160\text{ m} = 0 + 0 + \tfrac{1}{2}(9.80\,\text{m/s}^2)t^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2(160\text{ m})}{9.80\,\text{m/s}^2}} = \boxed{5.71\text{ s}}$$

Note that the speed of the airplane does not enter into this calculation.

9. Choose the origin to be where the projectile is launched, and upwards to be the positive y direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The maximum height is found from Eq. 2-11c, $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$, with $v_y = 0$ at the maximum height.

$$y_{\max} = 0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{-v_0^2 \sin^2\theta_0}{-2g} = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{(65.2\,\text{m/s})^2 \sin^2 34.5^\circ}{2(9.80\,\text{m/s}^2)} = \boxed{69.6\text{ m}}$$

(*b*) The total time in the air is found from Eq. 2-11b, with a total vertical displacement of 0 for the ball to reach the ground.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{2v_0 \sin\theta_0}{g} = \frac{2(65.2\,\text{m/s})\sin 34.5^\circ}{(9.80\,\text{m/s}^2)} = \boxed{7.54\text{ s}} \text{ and } t = 0$$

The time of 0 represents the launching of the ball.

(*c*) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0 \cos\theta_0)t = (65.2\,\text{m/s})(\cos 34.5^\circ)(7.54\text{ s}) = \boxed{405\text{ m}}$$

(*d*) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_0 \cos\theta_0 = (65.2\,\text{m/s})(\cos 34.5^\circ) = 53.7\,\text{m/s}$. The vertical velocity is found from Eq. 2-11a.

$$v_y = v_{y0} + at = v_0 \sin\theta_0 - gt = (65.2\,\text{m/s})\sin 34.5^\circ - (9.80\,\text{m/s}^2)(1.50\,\text{s}) = 22.2\,\text{m/s}$$

Thus the speed of the projectile is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{53.7^2 + 22.2^2} = \boxed{58.1\,\text{m/s}}$.

The direction above the horizontal is given by $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{22.2}{53.7} = \boxed{22.5^\circ}$

10. Call the direction of the flow of the river the x direction, and the direction the boat is headed the y direction.

(*a*) $v_{\substack{\text{boat rel.}\\\text{shore}}} = \sqrt{v^2_{\substack{\text{water rel.}\\\text{shore}}} + v^2_{\substack{\text{boat rel.}\\\text{water}}}} = \sqrt{1.20^2 + 2.30^2} = \boxed{2.59\,\text{m/s}}$

$$\theta = \tan^{-1}\frac{1.20}{2.30} = 27.6^\circ \ , \ \phi = 90^\circ - \theta = \boxed{62.4^\circ \text{ relative to shore}}$$

(*b*) The position of the boat after 3.00 seconds is given by

$$\Delta d = v_{\substack{\text{boat rel.}\\\text{shore}}} t = \left[(1.20, 2.30)\,\text{m/s}\right](3.00\,\text{sec})$$

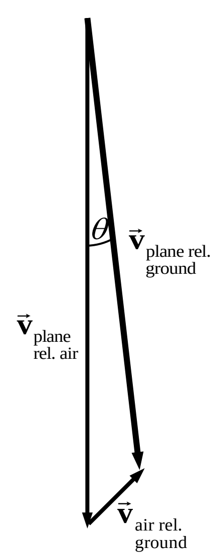
$$= \boxed{(3.60\text{ m downstream}, 6.90\text{ m across the river})}$$

As a magnitude and direction, it would be 7.8 m away from the starting point, at an angle of 62.4° relative to the shore.

11. If each plane has a speed of 785 km/hr, then their relative speed of approach is 1570 km/hr. If the planes are 11 km apart, then the time for evasive action is found from

$$\Delta d = vt \quad \rightarrow \quad t = \frac{\Delta d}{v} = \left(\frac{11.0\text{ km}}{1570\,\text{km/hr}}\right)\left(\frac{3600\text{ sec}}{1\text{ hr}}\right) = \boxed{25.2\text{ s}}$$

12. Call east the positive x direction and north the positive y direction. Then the following vector velocity relationship exists.



(*a*) $\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$

$$= (0, -600)\,\text{km/h} + (100\cos 45.0^\circ, 100\sin 45.0^\circ)\,\text{km/h}$$

$$= (70.7, -529)\,\text{km/h}$$

$$v_{\substack{\text{plane rel.}\\\text{ground}}} = \sqrt{(70.7\,\text{km/h})^2 + (-529\,\text{km/h})^2} = \boxed{540\,\text{km/h}}$$

$$\theta = \tan^{-1}\frac{70.7}{529} = \boxed{7.6^\circ \text{ east of south}}$$

(*b*) The plane is away from its intended position by the distance the air has caused it to move. The wind speed is 100 km/h, so after 10 min (1/6 h), the plane is off course by $\Delta x = v_x t = (100\,\text{km/h})\left(\tfrac{1}{6}\,\text{h}\right) = \boxed{17\text{ km}}$.

13. Take the origin to be the location at which the speeder passes the police car, in the reference frame of the unaccelerated police car. The speeder is traveling at $145\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)=40.28\,\text{m/s}$ relative to the ground, and the policeman is traveling at $95\,\text{km/h}\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)=26.39\,\text{m/s}$ relative to the ground. Relative to the unaccelerated police car, the speeder is traveling at $13.89\,\text{m/s}=v_s$, and the police car is not moving. Do all of the calculations in the frame of reference of the unaccelerated police car.

The position of the speeder in the chosen reference frame is given by $\Delta x_s = v_s t$. The position of the policeman in the chosen reference frame is given by $\Delta x_p = \frac{1}{2}a_p\left(t-1\right)^2, t>1$. The police car overtakes the speeder when these two distances are the same.; i.e., $\Delta x_s = \Delta x_p$.

$$\Delta x_s = \Delta x_p \quad \to \quad v_s t = \tfrac{1}{2}a_p\left(t-1\right)^2 \quad \to \quad \left(13.89\,\text{m/s}\right)t = \tfrac{1}{2}\left(2\,\text{m/s}^2\right)\left(t^2-2t+1\right)=t^2-2t+1$$

$$t^2-15.89t+1=0 \quad \to \quad t=\frac{15.89\pm\sqrt{15.89^2-4}}{2}=0.0632\text{ s}, 15.83\text{ s}$$

Since the police car doesn't accelerate until $t=1.00\text{ s}$, the correct answer is $\boxed{t=15.8\text{ s}}$.