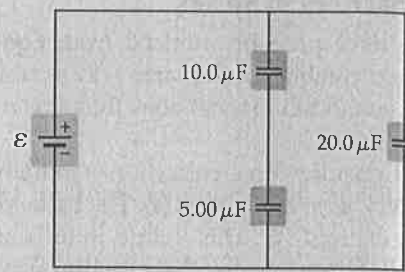
ACTIVE EXAMPLE 21–3 FIND THE EQUIVALENT CAPACITANCE AND THE STORED ENERGY

Consider the electric circuit shown here, consisting of a 12.0-V battery and three capacitors connected partly in series and partly in parallel. Find **(a)** the equivalent capacitance of this circuit and **(b)** the total energy stored in the capacitors.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Find the equivalent capacitance of a 10.0-μF capacitor in series with a 5.00-μF capacitor: — 3.33 μF
2. Find the equivalent capacitance of a 3.33-μF capacitor in parallel with a 20.0-μF capacitor: — $C_{eq} = 23.3\ \mu$F

Part (b)

3. Calculate the stored energy using $U = \frac{1}{2}C_{eq}V^2$: — $U = 1.68 \times 10^{-3}$ J

INSIGHT

Notice that the 10.0-μF capacitor and the 5.00-μF capacitor are connected in series. As you might expect, one of these capacitors stores twice as much energy as the other. Which is it? Check the Your Turn question for the answer.

YOUR TURN

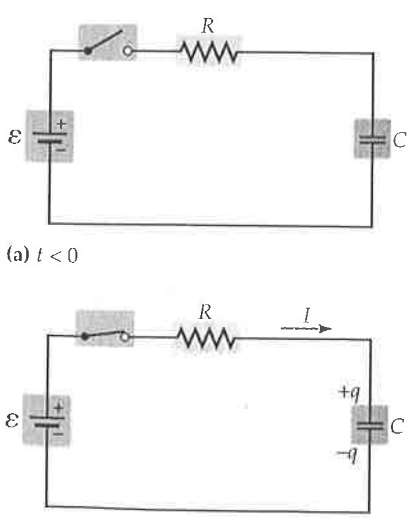
Is the energy stored in the 10.0-μF capacitor greater than or less than the energy stored in the 5.0-μF capacitor? Explain. Check your answer by calculating the energy stored in each of the capacitors.

(Answers to **Your Turn** *problems are given in the back of the book.)*

21–7 *RC* Circuits

When the switch is closed on a circuit containing only batteries and capacitors, the charge on the capacitor plates appears almost instantaneously—essentially at the speed of light. This is not the case, however, in circuits that also contain resistors. In these situations, the resistors limit the rate at which charge can flow, and an appreciable amount of time may be required before the capacitors acquire a significant charge. A useful analogy is the amount of time needed to fill a bucket with water. If you use a fire hose, which has little resistance to the flow of water, the bucket fills almost instantly. If you use a garden hose, which presents a much greater resistance to the water, filling the bucket may take a minute or more.

The simplest example of such a circuit, a so-called ***RC* circuit**, is shown in **Figure 21–19**. Initially (before $t = 0$) the switch is open, and there is no current in the resistor or charge on the capacitor. At $t = 0$ the switch is closed and current begins to flow. If the resistor was not present, the capacitor would immediately take on the charge $Q = C\mathcal{E}$. The effect of the resistor, however, is to slow the charging process—in fact, the larger the resistance, the longer it takes for the capacitor to charge. One way to think of this is to note that as long as a current flows in the circuit, as in Figure 21–19 (b), there is a potential drop across the resistor; hence, the potential difference between the plates of the capacitor is less than the emf of the battery. With less voltage across the capacitor there will be less charge on its plates compared with the charge that would result if the plates were connected directly to the battery.



(a) $t < 0$

(b) $t > 0$

▲ **FIGURE 21–19 A typical *RC* circuit**
(a) Before the switch is closed ($t < 0$) there is no current in the circuit and no charge on the capacitor. **(b)** After the switch is closed ($t > 0$), current flows and the charge on the capacitor builds up over a finite time. As $t \to \infty$ the charge on the capacitor approaches $Q = C\mathcal{E}$.

The methods of calculus can be used to show that the charge on the capacitor in Figure 21–19 varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$

In this expression, e is Euler's number ($e = 2.718\ldots$) or, more precisely, the base of natural logarithms (see Appendix A). The quantity τ is referred to as the **time constant** of the circuit. The time constant is related to the resistance and capacitance of a circuit by the following simple relation: $\tau = RC$. As we shall see, τ can be thought of as a characteristic time for the behavior of an *RC* circuit.

For example, at time $t = 0$ the exponential term is $e^{-0/\tau} = e^0 = 1$; therefore, the charge on the capacitor is zero at $t = 0$, as expected:

$$q(0) = C\mathcal{E}(1 - 1) = 0$$

In the opposite limit, $t \rightarrow \infty$, the exponential vanishes: $e^{-\infty/\tau} = 0$. Thus the charge in this limit is $C\mathcal{E}$:

$$q(t \rightarrow \infty) = C\mathcal{E}(1 - 0) = C\mathcal{E}$$

This is just the charge Q the capacitor would have had from $t = 0$ on if there had been no resistor in the circuit. Finally, at time $t = \tau$ the charge on the capacitor is $q = C\mathcal{E}(1 - e^{-1}) = C\mathcal{E}(1 - 0.368) = 0.632C\mathcal{E}$, which is 63.2% of its final charge. The charge on the capacitor as a function of time is plotted in **Figure 21–20**.

Before we continue, let's check to see that the quantity $\tau = RC$ is in fact a time. Suppose, for example, that the resistor and capacitor in an RC circuit have the values $R = 120\ \Omega$ and $C = 3.5\ \mu\text{F}$, respectively. Multiplying R and C we find

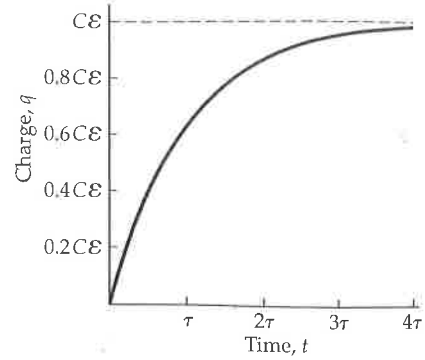
$$\tau = RC = (120\ \text{ohm})(3.5 \times 10^{-6}\ \text{farad})$$
$$= \left(\frac{120\ \text{volt}}{\text{coulomb/second}}\right)\left(\frac{3.5 \times 10^{-6}\ \text{coulomb}}{\text{volt}}\right) = 4.2 \times 10^{-4}\ \text{second}$$

The tick marks on the horizontal axis in Figure 21–20 indicate the times τ, 2τ, 3τ, and 4τ. Notice that the capacitor is almost completely charged by the time $t = 4\tau$.

Figure 21–20 also shows that the charge on the capacitor increases rapidly initially, indicating a large current in the circuit. Eventually, the charging slows down, because the greater the charge on the capacitor, the harder it is to transfer additional charge against the electrical repulsive force. Later, the charge barely changes with time, which means that the current is essentially zero. In fact, the mathematical expression for the current—again derived from calculus—is the following:

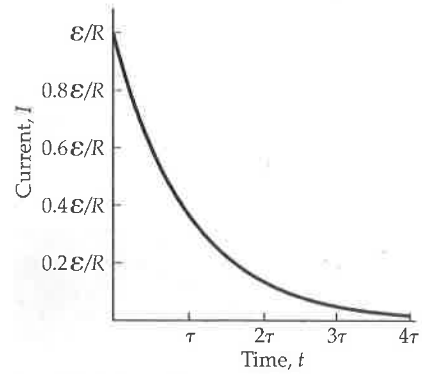
$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad \text{21–19}$$

This expression is plotted in **Figure 21–21**, where we see that significant variation in the current occurs over times ranging from $t = 0$ to $t \sim 4\tau$. At time $t = 0$ the current is $I(0) = \mathcal{E}/R$, which is the value it would have if the capacitor were replaced by an ideal wire. As $t \rightarrow \infty$, the current approaches zero, as expected: $I(t \rightarrow \infty) \rightarrow 0$. In this limit, the capacitor is essentially fully charged, so that no more charge can flow onto its plates. Thus, in this limit, the capacitor behaves like an open switch.



▲ **FIGURE 21–20 Charge versus time for the *RC* circuit in Figure 21–19**

The horizontal axis shows time in units of the characteristic time, $\tau = RC$. The vertical axis shows the magnitude of the charge on the capacitor in units of $C\mathcal{E}$.



▲ **FIGURE 21–21 Current versus time for the *RC* circuit in Figure 21–19**

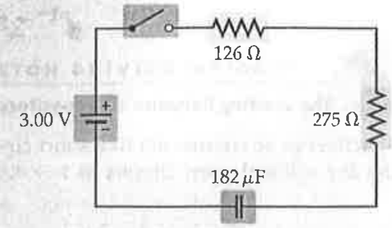
Initially the current is $\mathcal{E}/R$, the same as if the capacitor were not present. The current approaches zero after a period equal to several time constants, $\tau = RC$.

EXAMPLE 21–9 CHARGING A CAPACITOR

A circuit consists of a 126-Ω resistor, a 275-Ω resistor, a 182-μF capacitor, a switch, and a 3.00-V battery all connected in series. Initially the capacitor is uncharged and the switch is open. At time $t = 0$ the switch is closed. **(a)** What charge will the capacitor have a long time after the switch is closed? **(b)** At what time will the charge on the capacitor be 80.0% of the value found in part (a)?

PICTURE THE PROBLEM

The circuit described in the problem statement is shown with the switch in the open position. Once the switch is closed at $t = 0$, current will flow in the circuit and charge will begin to accumulate on the capacitor plates.



INTERACTIVE FIGURE

STRATEGY

a. A long time after the switch is closed, the current stops and the capacitor is fully charged. At this point, the voltage across the capacitor is equal to the emf of the battery. Therefore, the charge on the capacitor is $Q = C\mathcal{E}$.

b. To find the time when the charge will be 80.0% of the full charge, $Q = C\mathcal{E}$, we can set $q(t) = C\mathcal{E}(1 - e^{-t/\tau}) = 0.800C\mathcal{E}$ and solve for the desired time, t.

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SOLUTION

Part (a)

1. Evaluate $Q = C\mathcal{E}$ for this circuit: $Q = C\mathcal{E} = (182\ \mu\text{F})(3.00\ \text{V}) = 546\ \mu\text{C}$

Part (b)

2. Set $q(t) = 0.800C\mathcal{E}$ in $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and cancel $C\mathcal{E}$:
$$q(t) = 0.800C\mathcal{E} = C\mathcal{E}(1 - e^{-t/\tau})$$
$$0.800 = 1 - e^{-t/\tau}$$

3. Solve for t in terms of the time constant τ:
$$e^{-t/\tau} = 1 - 0.800 = 0.200$$
$$t = -\tau \ln(0.200)$$

4. Calculate τ and use the result to find the time t:
$$\tau = RC = (126\ \Omega + 275\ \Omega)(182\ \mu F) = 73.0\ \text{ms}$$
$$t = -(73.0\ \text{ms})\ln(0.200)$$
$$= -(73.0\ \text{ms})(-1.61) = 118\ \text{ms}$$

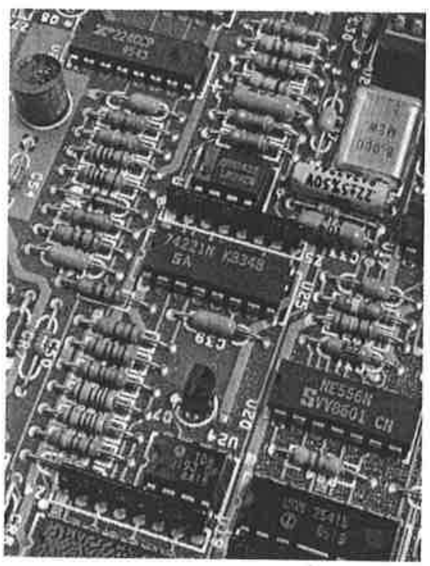
INSIGHT

Note that the time required for the charge on a capacitor to reach 80.0% of its final value is 1.61 time constants. This result is independent of the values of R and C in an RC circuit.

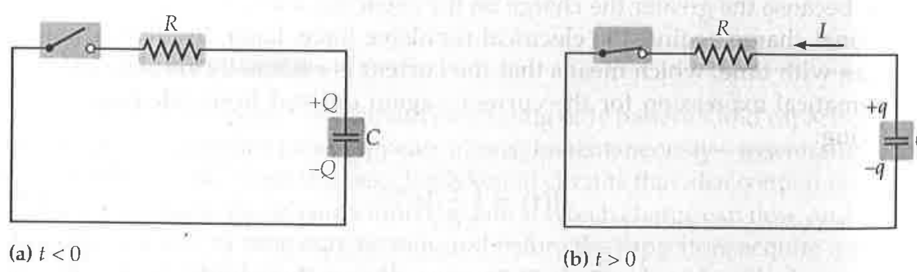
PRACTICE PROBLEM

What is the current in this circuit at the time found in part (b)? [Answer: $I(t) = (\mathcal{E}/R)e^{-t/\tau} = [(3.00\ \text{V})/(126\ \Omega + 275\ \Omega)](0.200) = (7.48\ \text{mA})(0.200) = 1.50\ \text{mA}$]

Some related homework problems: Problem 79, Problem 82



▲ A modern-day circuit board incorporates numerous resistors (cylinders with colored bands) and capacitors (yellow cylinders and metal container).



▲ **FIGURE 21–22 Discharging a capacitor**
(a) A charged capacitor is connected to a resistor. Initially the circuit is open, and no current can flow. **(b)** When the switch is closed, current flows from the + plate of the capacitor to the − plate. The charge remaining on the capacitor approaches zero after several time units, RC.

PROBLEM-SOLVING NOTE

The Limiting Behavior of Capacitors

Capacitors in dc circuits act like short circuits at $t = 0$ and open circuits as $t \to \infty$.

Similar behavior occurs when a charged capacitor is allowed to discharge, as in **Figure 21–22**. In this case, the initial charge on the capacitor is Q. If the switch is closed at $t = 0$, the charge for later times is

$$q(t) = Qe^{-t/\tau} \qquad \text{21–20}$$

Like charging, the discharging of a capacitor occurs with a characteristic time $\tau = RC$.

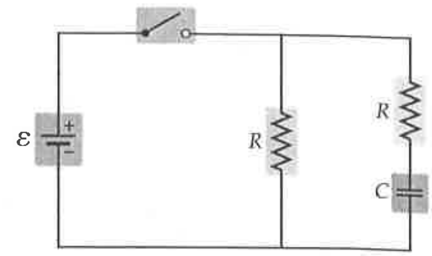
To summarize, circuits with resistors and capacitors have the following general characteristics:

- Charging and discharging occur over a finite, characteristic time given by the time constant, $\tau = RC$.
- At $t = 0$ current flows freely through a capacitor being charged; it behaves like a short circuit.
- As $t \to \infty$ the current flowing into a capacitor approaches zero. In this limit, a capacitor behaves like an open switch.

We explore these features further in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 21–4 CURRENT IN AN *RC* CIRCUIT

What current flows through the battery in this circuit **(a)** immediately after the switch is closed and **(b)** a long time after the switch is closed?



REASONING AND DISCUSSION

a. Immediately after the switch is closed, the capacitor acts like a short circuit; that is, as if the battery were connected to two resistors R in parallel. The equivalent resistance in this case is $R/2$; therefore, the current is $I = \mathcal{E}/(R/2) = 2\mathcal{E}/R$.

b. After current has been flowing in the circuit for a long time, the capacitor acts like an open switch. Now current can flow only through the one resistor, R; hence, the current is $I = \mathcal{E}/R$, half of its initial value.

ANSWER

(a) The current is $2\mathcal{E}/R$; **(b)** the current is $\mathcal{E}/R$.

The fact that *RC* circuits have a characteristic time makes them useful in a variety of different applications. On a rather mundane level, *RC* circuits are used to determine the time delay on windshield wipers. When you adjust the delay knob in your car, you change a resistance or a capacitance, which in turn changes the time constant of the circuit. This results in a greater or a smaller delay. The blinking rate of turn signals is also determined by the time constant of an *RC* circuit.

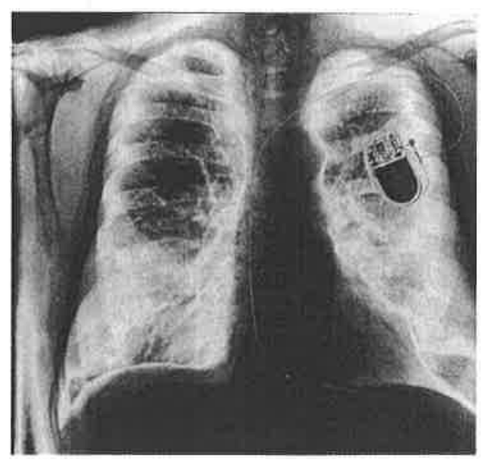
REAL-WORLD PHYSICS

Delay circuits in windshield wipers and turn signals

A more critical application of *RC* circuits is the heart pacemaker. In the simplest case, these devices use an *RC* circuit to deliver precisely timed pulses directly to the heart. The more sophisticated pacemakers available today can even "sense" when a patient's heart rate falls below a predetermined value. The pacemaker then begins sending appropriate pulses to the heart to increase its rate. Many pacemakers can even be reprogrammed after they are surgically implanted to respond to changes in a patient's condition.

REAL-WORLD PHYSICS: BIO

Pacemakers



▲ An X-ray showing a pacemaker installed in a person's chest. The timing of the electrical pulses that keep the heart beating regularly is determined by an *RC* circuit powered by a small, long-lived battery.

Normally, the heart's rate of beating is determined by its own natural pacemaker, the sinoatrial or SA node, located in the upper right chamber of the heart. If the SA node is not functioning properly, it may cause the heart to beat slowly or irregularly. To correct the problem, a pacemaker is implanted just under the collarbone, and an electrode is introduced intravenously via the cephalic vein. The distal end of the electrode is positioned, with the aid of fluoroscopic guidance, in the right ventricular apex. From that point on, the operation of the pacemaker follows the basic principles of electric circuits, as described in this chapter.

*21–8 Ammeters and Voltmeters

Devices for measuring currents and voltages in a circuit are referred to as **ammeters** and **voltmeters**, respectively. In each case, the ideal situation is for the meter to measure the desired quantity without altering the characteristics of the circuit being studied. This is accomplished in different ways for these two types of meters, as we shall see.

First, the ammeter is designed to measure the flow of current through a particular portion of a circuit. For example, we may want to know the current flowing between points A and B in the circuit shown in **Figure 21–23 (a)**. To measure this current, we insert the ammeter into the circuit in such a way that all the current flowing from A to B must also flow through the meter. This is done by connecting the meter "in series" with the other circuit elements between A and B, as indicated in **Figure 21–23 (b)**.

If the ammeter has a finite resistance—which must be the case for real meters—the presence of the meter in the circuit will alter the current it is intended to measure. Thus, an *ideal* ammeter would be one with zero resistance. In practice, if the resistance of the ammeter is much less than the other resistances in the circuit, its reading will be reasonably accurate.