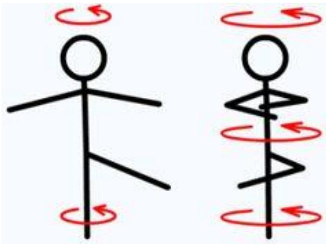
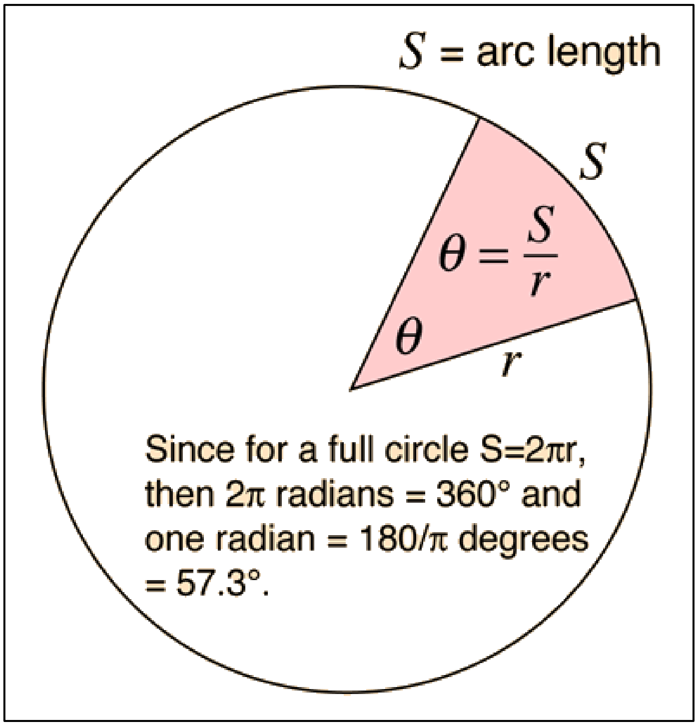
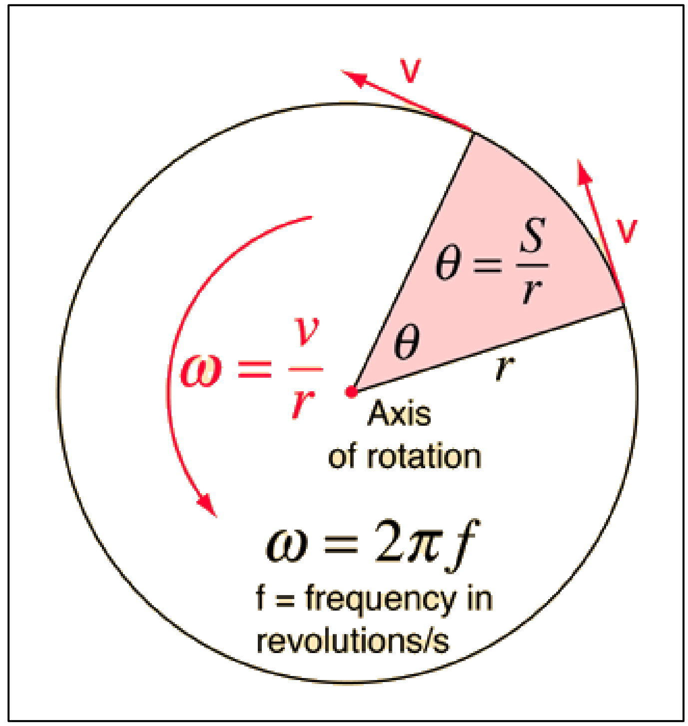


Rotary Motion Quick Notes



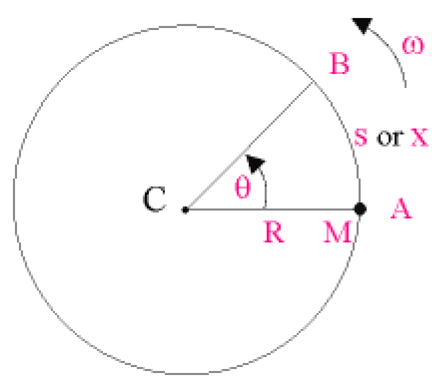


When mass M moves on the circle from A to B, the radius R sweeps angle θ that is called the "angular displacement" of mass M.

The linear displacement of mass M is arc S or X given by $S = R\theta$ as was discussed in Chapter 5.

ω is the change in θ over time (t).

$\omega = \frac{\Delta\theta}{\Delta t}$ preferably in rd/s.



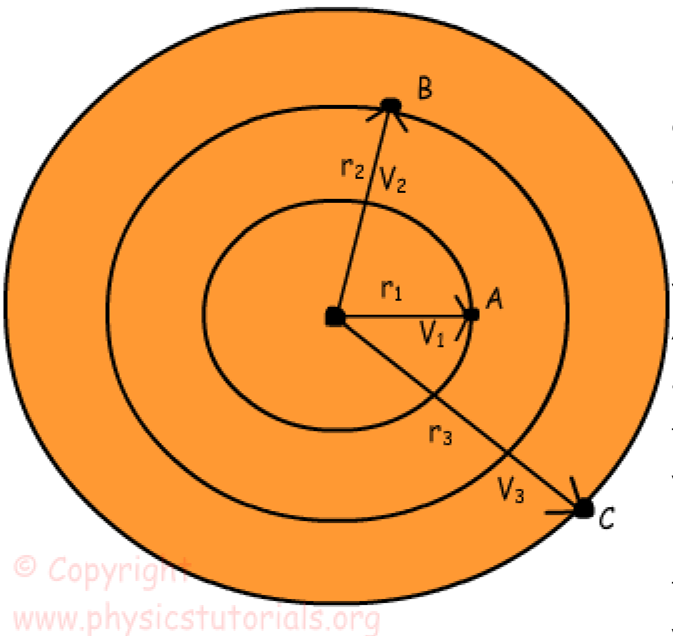
1. $\theta = \bar{\omega} t$ $\qquad \bar{\omega} = \frac{\omega_0 + \omega}{2}$
2. $\omega = \omega_0 + \alpha t$
3. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
4. $\omega^2 = \omega_0^2 + 2\alpha\theta$

Equations for constant angular acceleration.

Tangential Speed Velocity with Examples

Linear Speed (Tangential Speed):

Linear speed and tangential speed gives the same meaning for circular motion. In one dimension motion we define speed as the distance taken in a unit of time. In this case we use again same definition. However, in this case the direction of motion is always tangent to the path of the object. Thus, it can also be called as tangential speed, distance taken in a given time. Look at the given picture and try to sequence the velocities of the points larger to smaller.



In a given period of time all points on this rotating object have same revolutions. In other words, if A completes one revolution, then B and C also have one revolution in a same time. The formula of the speed in linear motion is;

Speed=distance/time

As I said before, speed in circular motion is also defined as the distance taken in a given time. Thus, speeds of the points given in the picture below are;

V =Distance/time If the object has one complete revolution then distance traveled becomes; **2πr** which is the circumference of the circle object.

V_A= 2πr / time

Period: Time passing for one revolution is called ***period***. The unit of period is **second**. **T** is the representation of period. The equation of tangential speed becomes;

V_A=2πr/T

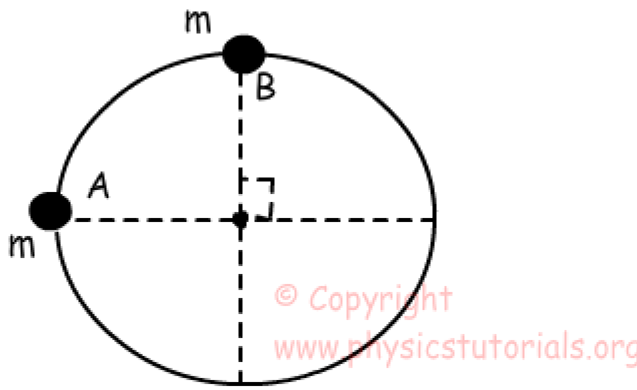
Frequency: Number of revolutions per one second. The unit of frequency is **1/second**. We show frequency with letter **f**. The relation of **f** and **T** is; **f=1/T**

Now; with the help of the information given above lets' sequence the velocities of the points on given picture. Since the velocity or speed of the points on rotating object is linearly proportional to the radius **$r_3>r_2>r_1$**;

$V_3>V_2>V_1$

To sum up, we can say that tangential speed of the object is linearly proportional to the distance from the center. Increase in the distance results in the increase in the amount of speed. As we move to the center speed decreases, and at the center speed becomes zero. We use the same unit for tangential speed as linear motion which is "m/s".

Example A particle having mass m travels from point A to B in a circular path having radius R in 4 seconds. Find the period of this particle.

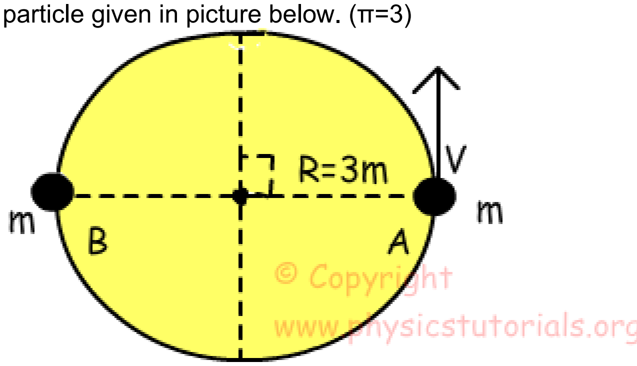


Particle travels one fourth of the circle in 4 seconds. Period is the time necessary for one revolution. So,

T/4=4s

T=16s.

Example: If the particle having mass m travels from point A to B in 4 seconds find the tangential velocity of that particle given in picture below. (π=3)



We first find the period of the motion. If the particle travels half of the circle in 4 seconds;

T/2=4s

T=8s

v=2 π R/T

v=2.3.3m/8s=9/4 m/s tangential speed of the particle

Tangential Speed

Tangential speed is a measure of linear speed of a particle which is under a circular motion. This can be denoted as Vt and the formula is given by

$$V_t = r\omega$$

Where $\mathbf{V_t}$ is the tangential speed **r** is the radius of the circular path **ω** is the angular velocity

Tangential Acceleration Units

The SI unit of tangential acceleration (at) is m/s2, which is same as that of normal acceleration. But the unit of radius (r) is m and angular acceleration (α) is rad/s2.

Tangential Component of Acceleration

Before explaining about the tangential acceleration, we should know about the direction along tangent. Consider a circle in which a particle is moving in circular direction. In this circular motion, two directions are possible for a particle. One is tangential and the other is radial direction. Radial direction is directed away from the center and the tangential direction is perpendicular to the radial direction. So, the acceleration of a particle in circular motion has both tangential as well as radial components. Radial component is also known as the normal component. The direction of the tangential component is same as that of the direction of the tangent itself. The tangential component of acceleration denoted by a_t.

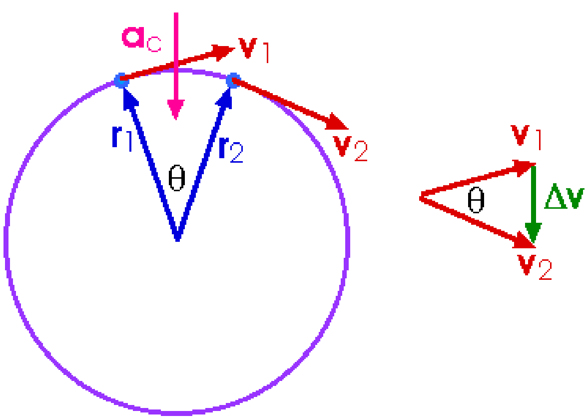
Tangential and Radial Acceleration

Just because an object moves in a circle, it has a **centripetal acceleration a_c,** directed **toward the center.** We know this centripetal acceleration is given by

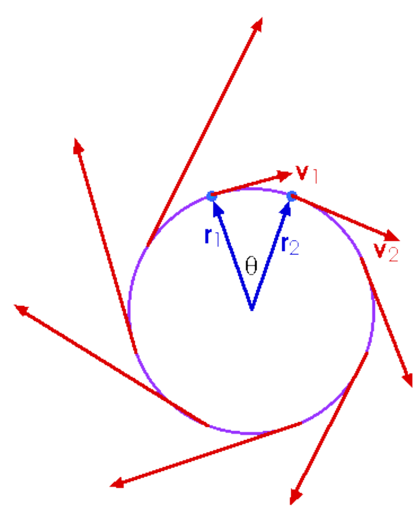
$$a_c = v^2 / r$$

This centripetal acceleration is directed along a radius so it may also be called the **radial acceleration a_r**.

If the **speed** is not constant, then there is also a **tangential acceleration a_t.**



The **tangential acceleration** is, indeed, **tangent** to the path of the particle's motion.



Tangential and Radial Acceleration

The formulas for tangential and radial acceleration is given by:

<u>Tangential acceleration</u>:

$a_t = r\,\alpha$

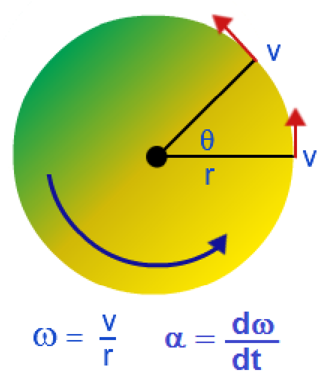
Where a_t is the tangential acceleration
r is the radius
α is the angular acceleration

<u>Radial acceleration:</u>

$a_r = v^2 / r$

Where a_r is the radial acceleration
v is the velocity
r is the radius of curvature

<u>Angular acceleration</u> is the rate of change of**angular** velocity. In SI units, it is measured in radians per second squared (rad/s^2), and is usually denoted by the Greek letter alpha (α).



TORQUE AND ANGULAR ACCELERATION

In this section, we will develop the relationship between torque and angular acceleration. You will need to have a basic understanding of moments of inertia for this section.

Imagine a force **F** acting on some object at a distance **r** from its axis of rotation. We can break up the force into tangential (F_{tan}), radial (F_{rad}) (see Figure 1). (This is assuming a two-dimensional scenario. For three dimensions -- a more realistic, but also more complicated situation -- we have three components of force: the tangential component F_{tan}, the radial component F_{rad} and the z-component F_z. All components of force are mutually perpendicular, or **normal**.)

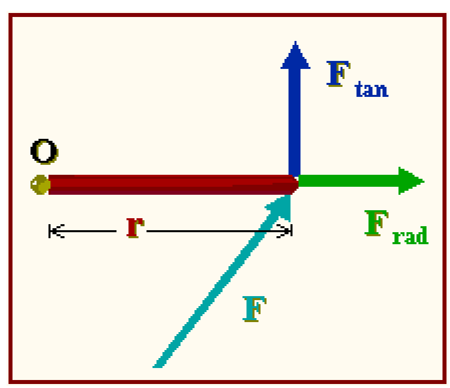


Figure 1 *Radial and Tangential Components of Force, two*

From Newton's Second Law,

$F_{tan} = m\,a_{tan}$

However, we know that angular acceleration, α, and the tangential acceleration a_{tan} are related by:

$$a_{tan} = r\ \alpha$$

Then,

$$F_{tan} = m\, r\, \alpha$$

If we multiply both sides by r (the moment arm), the equation becomes

$$F_{tan}\, r = m\, r^2\, \alpha$$

Note that the radial component of the force goes through the axis of rotation, and so has no contribution to torque. The left hand side of the equation is torque. For a whole object, there may be many torques. So the sum of the torques is equal to the moment of inertia (of a particle mass, which is the assumption in this derivation), $\mathbf{I} = m\,\mathbf{r}^2$ multiplied by the angular acceleration, α.

$$\sum \tau = I \cdot \alpha$$

dimensions

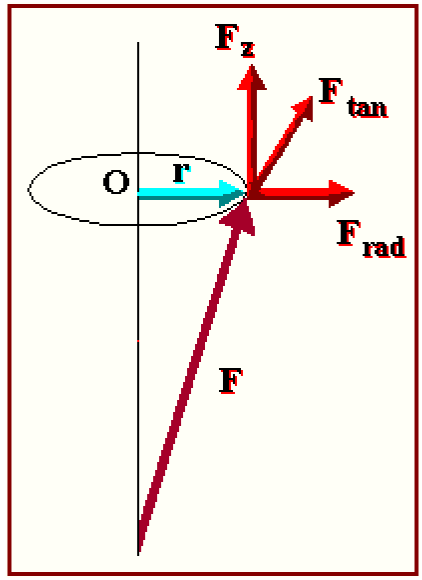
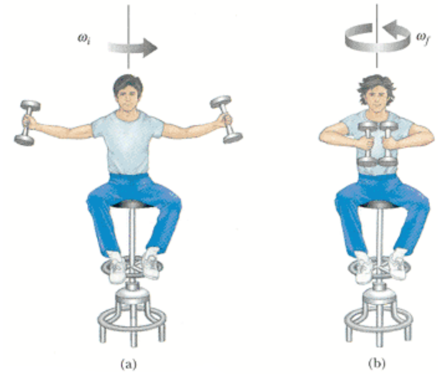


Figure 2 *Radial, Tangential and z-Components of Force, three dimensions*

If we make an analogy between translational and rotational motion, then this relation between torque and angular acceleration is analogous to the Newton's Second Law. Namely, taking torque to be analogous to force, moment of inertia analogous to mass, and angular acceleration analogous to acceleration, then we have an equation very much like the Second Law.

Moment of Inertia

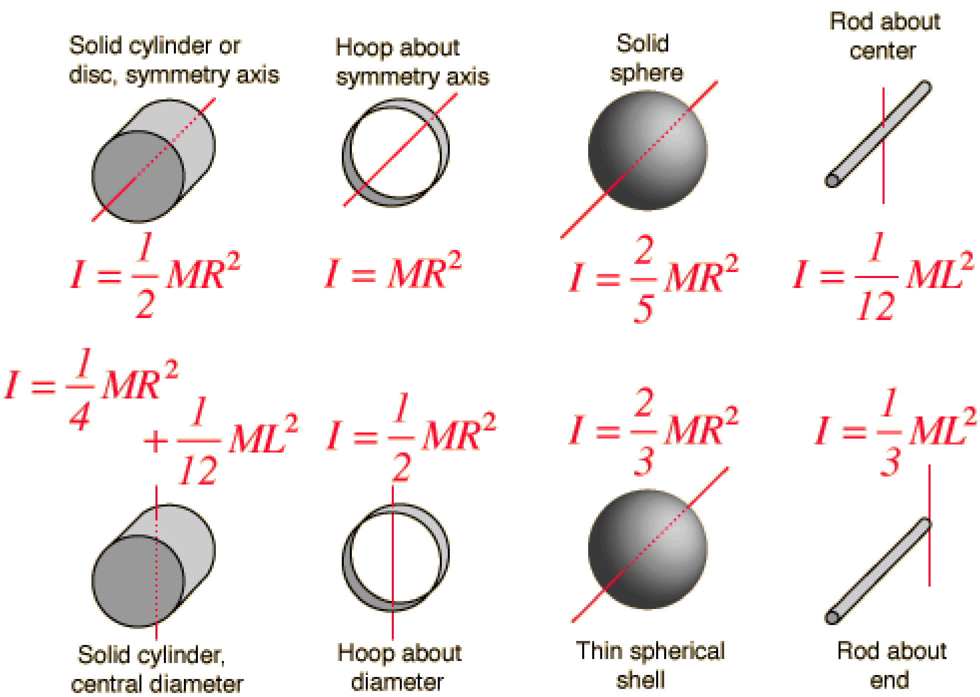


In physics, when you calculate an object's moment of inertia, you need to consider not only the mass of the object but also how the mass is distributed. For example, if two disks have the same mass but one has all the mass around the rim and the other is solid, then the disks would have different moments of inertia.

[Linear Momentum P = m v : Angular Momentum L = I ω] L = I ω L = I ω

Calculating moments of inertia is fairly simple if you only have to examine the orbital motion of small point-like objects, where all the mass is concentrated at one particular point at a given radius r. For instance, for a golf ball you're whirling around on a string, the moment of inertia depends on the radius of the circle the ball is spinning in:

$I = mr^2$ We use this equation as the whirling golf ball resembles a hoop.



Angular momentum,

Property characterizing the rotary inertia of an object or system of objects in motion about an axis that may or may not pass through the object or system. The Earth has orbital angular momentum by reason of its annual revolution about the Sun and spin angular momentum because of its daily rotation about its axis. Angular momentum is a vector quantity, requiring the specification of both a magnitude and a direction for its complete description. The magnitude of the angular momentum of an orbiting object is equal to its linear momentum (product of its mass *m* and linear velocity *v*) times the perpendicular distance *r* from the centre of rotation to a line drawn in the direction of its instantaneous motion and passing through the object's centre of gravity, or simply *mvr.* For a spinning object, on the other hand, the angular momentum must be considered as the summation of the quantity *mvr* for all the particles composing the object. Angular momentum may be formulated equivalently as the product of *I,* the moment of inertia, and *ω,* the angular velocity, of a rotating body or system, or simply *Iω*. The direction of the angular-momentum vector is that of the axis of rotation of the given object and is designated as positive in the direction that a right-hand screw would advance if turned similarly. Appropriate MKS or SI units for angular momentum are kilogram metres squared per second (kg-m^2/sec).

For a given object or system isolated from external forces, the total angular momentum is a constant, a fact that is known as the law of conservation of angular momentum. A rigid spinning object, for example, continues to spin at a constant rate and with a fixed orientation unless influenced by the application of an external torque. (The rate of change of the angular momentum is, in fact, equal to the applied torque.) A figure skater spins faster, or has a greater angular velocity *ω,* when the arms are drawn inward, because this action reduces the moment of inertia *I* while the product *Iω,* the skater's angular momentum, remains constant. Because of the conservation of direction as well as magnitude, a spinning gyrocompass in an airplane remains fixed in its orientation, independent of the motion of the airplane.