4 Two-Dimensional Kinematics

When you hear the word "projectile," you probably think of an artillery shell or perhaps a home run into the upper deck. But as we'll see in this chapter, the term applies to any object moving under the influence of gravity alone. For example, each of these juggling balls undergoes projectile motion as it moves from one hand to the other. In this chapter we will explore the physical laws that govern such motion, and will learn—among other things—that these balls follow a parabolic path.

We now extend our study of kinematics to motion in two dimensions. This allows us to consider a much wider range of physical phenomena observed in everyday life. Of particular interest is **projectile motion,** the motion of objects that are initially launched—or "projected"—and that then continue moving under the influence of gravity alone. Examples of projectile motion include balls thrown from one person to another, water spraying from a hose, salmon leaping over rapids, and divers jumping from the cliffs of Acapulco.

The main idea of this chapter is quite simple: Horizontal and vertical motions are independent. That's it. For example, a ball thrown horizontally with a speed v continues to move with the same speed v in the horizontal direction, even as it falls with an increasing speed in the vertical direction. Similarly, the time of fall is the same whether a ball is dropped from rest straight down, or thrown horizontally. Simply put, each motion continues as if the other motion were not present.

This chapter develops and applies the idea of independence of motion to many common physical systems.

4–1 Motion in Two Dimensions

In this section we develop equations of motion to describe objects moving in two dimensions. First, we consider motion with constant velocity, determining x and y as functions of time. Next, we investigate motion with constant acceleration. We show that the one-dimensional kinematic equations of Chapter 2 can be extended in a straightforward way to apply to two dimensions.

Constant Velocity

To begin, consider the simple situation shown in **Figure 4–1**. A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26\ \mathrm{m/s}$ in a direction 25° above the x axis. How far has the turtle moved in the x and y directions after 5.0 seconds?

First, note that the turtle moves in a straight line a distance

$$d = v_0 t = (0.26\ \mathrm{m/s})(5.0\ \mathrm{s}) = 1.3\ \mathrm{m}$$

as indicated in **Figure 4–1(a)**. From the definitions of sine and cosine given in the previous chapter, we see that

$$x = d\cos 25° = 1.2\ \mathrm{m}$$
$$y = d\sin 25° = 0.55\ \mathrm{m}$$

An alternative way to approach this problem is to treat the x and y motions separately. First, we determine the speed of the turtle in each direction. Referring to **Figure 4–1(b)**, we see that the x component of velocity is

$$v_{0x} = v_0 \cos 25° = 0.24\ \mathrm{m/s}$$

and the y component is

$$v_{0y} = v_0 \sin 25° = 0.11\ \mathrm{m/s}$$

Next, we find the distance traveled by the turtle in the x and y directions by multiplying the speed in each direction by the time:

$$x = v_{0x} t = (0.24\ \mathrm{m/s})(5.0\ \mathrm{s}) = 1.2\ \mathrm{m}$$

and

$$y = v_{0y} t = (0.11\ \mathrm{m/s})(5.0\ \mathrm{s}) = 0.55\ \mathrm{m}$$

This is in agreement with our previous results. To summarize, we can think of the turtle's actual motion as a combination of separate x and y motions.

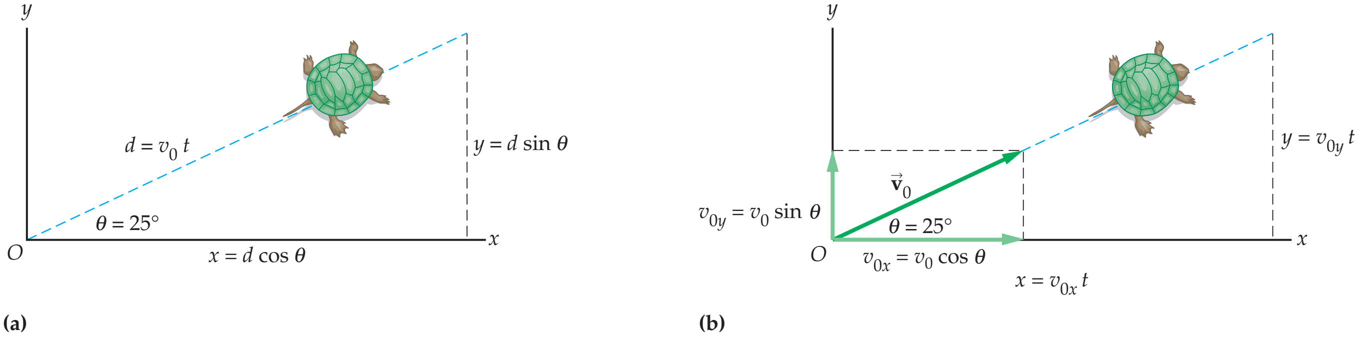
In general, the turtle might start at a position $x = x_0$ and $y = y_0$ at time $t = 0$. In this case, we have

$$x = x_0 + v_{0x} t \qquad 4\text{–}1$$

and

$$y = y_0 + v_{0y} t \qquad 4\text{–}2$$

as the x and y equations of motion.



▲ **FIGURE 4–1 Constant velocity**

A turtle walks from the origin with a speed of $v_0 = 0.26\ \mathrm{m/s}$. **(a)** In a time t the turtle moves through a straight-line distance of $d = v_0 t$; thus the x and y displacements are $x = d\cos\theta$, $y = d\sin\theta$. **(b)** Equivalently, the turtle's x and y components of velocity are $v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$; hence $x = v_{0x}t$ and $y = v_{0y}t$.

Compare these equations with Equation 2–11, $x = x_0 + v_0 t + \frac{1}{2}at^2$, which gives position as a function of time in one dimension. When acceleration is zero, as it is for the turtle, Equation 2–11 reduces to $x = x_0 + v_0 t$. Replacing v_0 with the x component of the velocity, v_{0x}, yields Equation 4–1. Similarly, replacing each x in Equation 4–1 with y converts it to Equation 4–2, the y equation of motion.

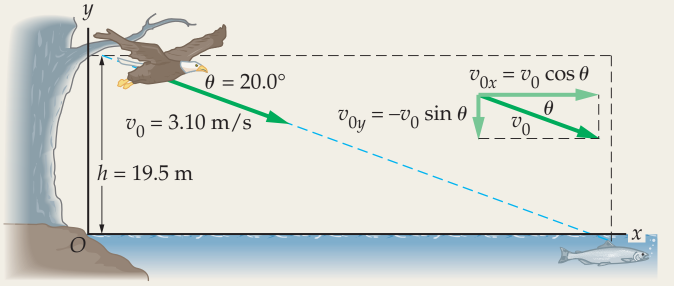
A situation illustrating the use of Equations 4–1 and 4–2 is given in Example 4–1.

EXAMPLE 4–1 THE EAGLE DESCENDS

An eagle perched on a tree limb 19.5 m above the water spots a fish swimming near the surface. The eagle pushes off from the branch and descends toward the water. By adjusting its body in flight, the eagle maintains a constant speed of 3.10 m/s at an angle of 20.0° below the horizontal. **(a)** How long does it take for the eagle to reach the water? **(b)** How far has the eagle traveled in the horizontal direction when it reaches the water?

PICTURE THE PROBLEM

We set up our coordinate system so that the eagle starts at $x_0 = 0$ and $y_0 = h = 19.5$ m. The water level is $y = 0$. As indicated in our sketch, $v_{0x} = v_0 \cos\theta$ and $v_{0y} = -v_0 \sin\theta$, where $v_0 = 3.10$ m/s and $\theta = 20.0°$. Notice that both components of the eagle's velocity are constant, and therefore the equations of motion given in Equations 4–1 and 4–2 apply.



STRATEGY

As usual in such problems, it is best to treat the eagle's flight as a combination of separate x and y motions. Since we are given the constant speed of the eagle, and the angle at which it descends, we can find the x and y components of its velocity. We then use the y equation of motion, $y = y_0 + v_{0y}t$, to find the time t when the eagle reaches the water. Finally, we use this value of t in the x equation of motion, $x = x_0 + v_{0x}t$, to find the horizontal distance the bird travels.

SOLUTION

Part (a)

1. Begin by determining v_{0x} and v_{0y}:

$$v_{0x} = v_0 \cos\theta = (3.10\ \text{m/s}) \cos 20.0° = 2.91\ \text{m/s}$$
$$v_{0y} = -v_0 \sin\theta = -(3.10\ \text{m/s}) \sin 20.0° = -1.06\ \text{m/s}$$

2. Now, set $y = 0$ in $y = y_0 + v_{0y}t$ and solve for t:

$$y = y_0 + v_{0y}t = h + v_{0y}t = 0$$
$$t = -\frac{h}{v_{0y}} = -\frac{19.5\ \text{m}}{(-1.06\ \text{m/s})} = 18.4\ \text{s}$$

Part (b)

3. Substitute $t = 18.4$ s into $x = x_0 + v_{0x}t$ to find x:

$$x = x_0 + v_{0x}t = 0 + (2.91\ \text{m/s})(18.4\ \text{s}) = 53.5\ \text{m}$$

INSIGHT

Notice how the two minus signs in Step 2 combine to give a positive time. One minus sign comes from setting $y = 0$, the other from the fact that v_{0y} is negative. No matter where we choose the origin, or what direction we choose to be positive, the time will always have the same value.

As mentioned in the problem statement, the eagle cannot travel in a straight line by simply dropping from the tree limb—it has to adjust its wings and tail to produce enough lift to balance the force of gravity. Airplanes do the same thing when they adjust their flight surfaces to make a smooth landing.

PRACTICE PROBLEM

What is the location of the eagle 2.00 s after it takes flight? [**Answer:** $x = 5.82$ m, $y = 17.4$ m]

Some related homework problems: Problem 2, Problem 3

Constant Acceleration

To study motion with constant acceleration in two dimensions we repeat what was done in one dimension in Chapter 2, but with separate equations for both x and y. For example, to obtain x as a function of time we start with $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Equation 2–11), and replace both v_0 and a with the corresponding x components, v_{0x} and a_x. This gives

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \qquad \text{4–3(a)}$$

To obtain y as a function of time, we write y in place of x in Equation 4–3(a):

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad \text{4–3(b)}$$

These are the position-versus-time equations of motion for two dimensions. (In three dimensions we introduce a third coordinate direction and label it z. We would then simply replace x with z in Equation 4–3(a) to obtain z as a function of time.)

The same approach gives velocity as a function of time. Start with Equation 2–7, $v = v_0 + at$, and write it in terms of x and y components. This yields

$$v_x = v_{0x} + a_x t \qquad \text{4–4(a)}$$

$$v_y = v_{0y} + a_y t \qquad \text{4–4(b)}$$

Note that we simply repeat everything we did for one dimension, only now with separate equations for the x and y components.

Finally, we can write $v^2 = v_0^{\,2} + 2a\Delta x$ in terms of components as well:

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x\Delta x \qquad \text{4–5(a)}$$

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y\Delta y \qquad \text{4–5(b)}$$

The following table summarizes our results:

Table 4–1 Constant-Acceleration Equations of Motion

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_x^{\,2} = v_{0x}^{\,2} + 2a_x\Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y = v_{0y} + a_y t$	$v_y^{\,2} = v_{0y}^{\,2} + 2a_y\Delta y$

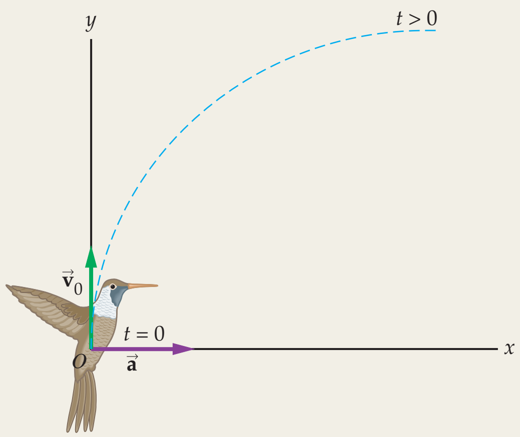
These are the fundamental equations that will be used to obtain *all* of the results presented throughout the rest of this chapter. Though it may appear sometimes that we are writing new sets of equations for different special cases, the equations aren't new—what we are actually doing is simply writing these equations again, but with specific values substituted for the constants that appear in them.

EXAMPLE 4–2 A HUMMER ACCELERATES

A hummingbird is flying in such a way that it is initially moving vertically with a speed of 4.6 m/s and accelerating horizontally at 11 m/s^2. Assuming the bird's acceleration remains constant for the time interval of interest, find **(a)** the horizontal and vertical distances through which it moves in 0.55 s and **(b)** its x and y velocity components at $t = 0.55$ s.

PICTURE THE PROBLEM

In our sketch we have placed the origin of a two-dimensional coordinate system at the location of the hummingbird at the initial time, $t = 0$. In addition, we have chosen the initial direction of motion to be in the positive y direction, and the direction of acceleration to be in the positive x direction. As a result, it follows that $x_0 = y_0 = 0$, $v_{0x} = 0$, $v_{0y} = 4.6$ m/s, $a_x = 11$ m/s^2, and $a_y = 0$. As the hummingbird moves upward, its x component of velocity increases, resulting in a curved path, as shown.



STRATEGY

(a) Since we want to relate position and time, we find the horizontal position of the hummingbird using $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, and the vertical position using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$. **(b)** The velocity components as a function of time can be found using $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$.

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SOLUTION

Part (a)

1. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$ to find x at $t = 0.55$ s: $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 = 0 + 0 + \frac{1}{2}(11\text{ m/s}^2)(0.55\text{ s})^2 = 1.7\text{ m}$
2. Use $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ to find y at $t = 0.55$ s: $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 = 0 + (4.6\text{ m/s})(0.55\text{ s}) + 0 = 2.5\text{ m}$

Part (b)

3. Use $v_x = v_{0x} + a_xt$ to find v_x at $t = 0.55$ s: $v_x = v_{0x} + a_xt = 0 + (11\text{ m/s}^2)(0.55\text{ s}) = 6.1\text{ m/s}$
4. Use $v_y = v_{0y} + a_yt$ to find v_y at $t = 0.55$ s: $v_y = v_{0y} + a_yt = 4.6\text{ m/s} + (0)(0.55\text{ s}) = 4.6\text{ m/s}$

INSIGHT

In 0.55 s the hummingbird moves 1.7 m horizontally and 2.5 m vertically. The horizontal position of the bird will eventually increase more rapidly with time than the vertical position, due to the t^2 dependence of x as compared with the t dependence of y. This results in a curved, parabolic path for the hummingbird, as shown in our sketch. The bird's velocity at 0.55 s is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.1\text{ m/s})^2 + (4.6\text{ m/s})^2} = 7.6\text{ m/s}$ at an angle of $\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(4.6\text{ m/s})/(6.1\text{ m/s})] = 37°$ above the x axis. It's clear the angle of flight must be less than 45° at this time, since the x component of velocity is greater than the y component.

PRACTICE PROBLEM

How much time is required for the hummingbird to move 2.0 m horizontally from its initial position? [**Answer:** $t = 0.60$ s]

Some related homework problems: Problem 4, Problem 5, Problem 62

4–2 Projectile Motion: Basic Equations

We now apply the independence of horizontal and vertical motions to projectiles. Just what do we mean by a projectile? Well, a **projectile** is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity. As you might expect, this covers a wide variety of physical systems.

In studying projectile motion we make the following assumptions:

- air resistance is ignored
- the acceleration due to gravity is constant, downward, and has a magnitude equal to $g = 9.81\text{ m/s}^2$
- the Earth's rotation is ignored

Air resistance can be significant when a projectile moves with relatively high speed or if it encounters a strong wind. In many everyday situations, however, like tossing a ball to a friend or dropping a book, air resistance is relatively insignificant. As for the acceleration due to gravity, $g = 9.81\text{ m/s}^2$, this value varies slightly from place to place on the Earth's surface and decreases with increasing altitude. In addition, the rotation of the Earth can be significant when considering projectiles that cover great distances. Little error is made in ignoring the variation of g or the rotation of the Earth, however, in the examples of projectile motion considered in this chapter.

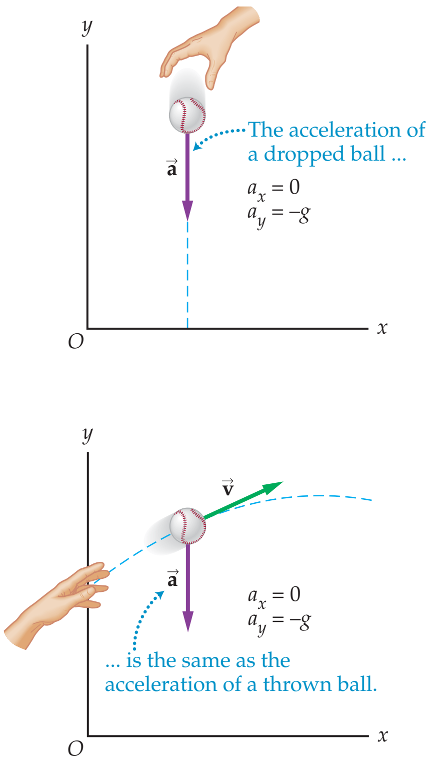
Let's incorporate these assumptions into the equations of motion given in the previous section. Suppose, as in Figure 4–2, that the x axis is horizontal and the y axis is vertical, with the positive direction upward. Since downward is the negative direction, it follows that

$$a_y = -9.81\text{ m/s}^2 = -g$$

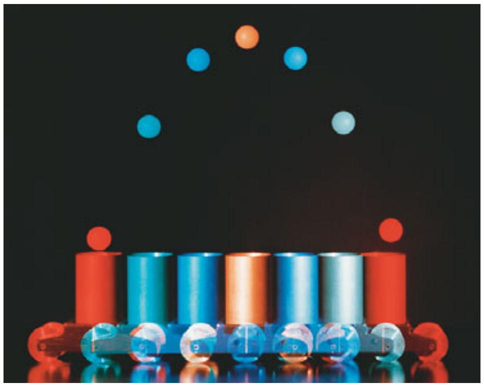
Gravity causes no acceleration in the x direction. Thus, the x component of acceleration is zero:

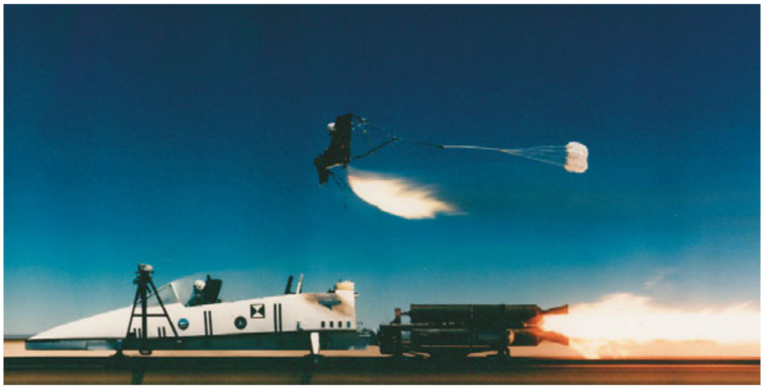
$$a_x = 0$$

With these acceleration components substituted into the fundamental constant-acceleration equations of motion (Table 4–1) we find:



▲ **FIGURE 4–2 Acceleration in free fall**
All objects in free fall have acceleration components $a_x = 0$ and $a_y = -g$ when the coordinate system is chosen as shown here. This is true regardless of whether the object is dropped, thrown, kicked, or otherwise set into motion.





▲ In the multiple-exposure photo at left, a ball is projected upward from a moving cart. The ball retains its initial horizontal velocity; as a result, it follows a parabolic path and remains directly above the cart at all times. When the ball lands, it falls back into the cart, just as it would if the cart had been at rest. (In this sequence, the exposures were made at equal time intervals with light of different colors, making it easier to follow the relative motion of the ball and the cart.) In the photo at right, the pilot ejection seat of a jet fighter is being ground-tested. Here too the horizontal and vertical motions are independent; thus, the test dummy is still almost directly above the cockpit from which it was ejected. Note, however, that air resistance is beginning to reduce the dummy's horizontal velocity. Eventually, it will fall far behind the speeding plane.

Projectile Motion ($a_x = 0, a_y = -g$)

$$x = x_0 + v_{0x}t \qquad v_x = v_{0x} \qquad v_x^2 = v_{0x}^2$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad v_y = v_{0y} - gt \qquad v_y^2 = v_{0y}^2 - 2g\Delta y \qquad \text{4–6}$$

PROBLEM-SOLVING NOTE

Acceleration of a Projectile

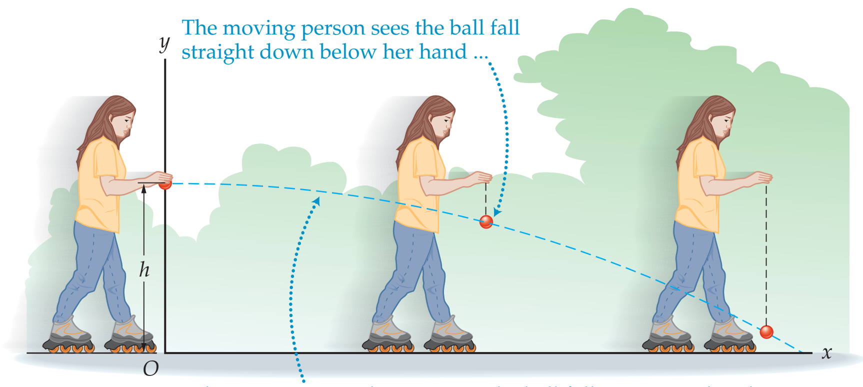
When the x axis is chosen to be horizontal and the y axis points vertically upward, it follows that the acceleration of an ideal projectile is $a_x = 0$ and $a_y = -g$.

Note that in these expressions the positive y direction is upward and the quantity g is positive. All of our studies of *projectile motion* will use Equations 4–6 as our fundamental equations—again, special cases will simply correspond to substituting specific values for the constants.

A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Note that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second.

Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully. To you, its motion looks the same as before: It goes straight down, lands near your feet, bounces straight back up, and returns in about one second. This is illustrated in **Figure 4–3**. The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions were independent.

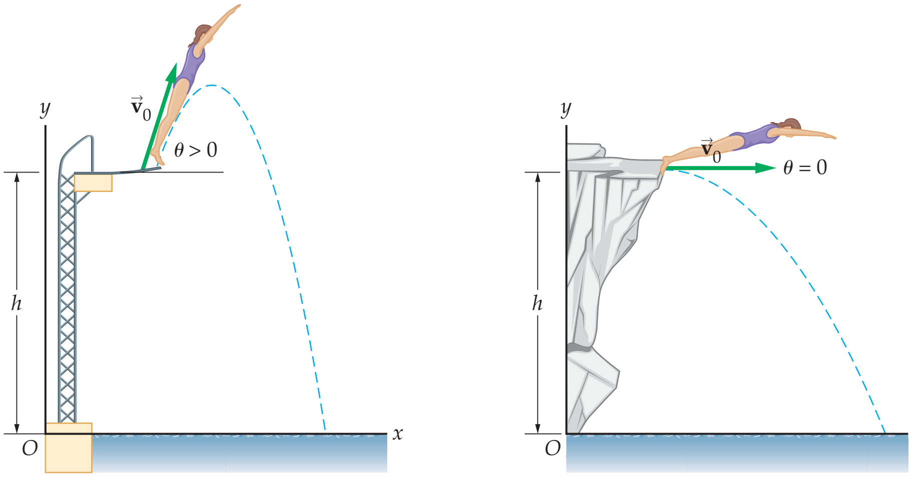
To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path is determined in the next section.



▲ **FIGURE 4–3 Independence of vertical and horizontal motions**

When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.

▲ This rollerblader may not be thinking about independence of motion, but the ball she released illustrates the concept perfectly; it continues to move horizontally with constant speed—even though she's no longer touching it—at the same time that it accelerates vertically downward.



(a) **(b)**

FIGURE 4–4 Launch angle of a projectile
(a) A projectile launched at an angle above the horizontal, $\theta > 0$. A launch below the horizontal would correspond to $\theta < 0$. **(b)** A projectile launched horizontally, $\theta = 0$. In this section we consider $\theta = 0$. The next section deals with $\theta \neq 0$.

4–3 Zero Launch Angle

A special case of some interest is a projectile launched horizontally, so that the angle between the initial velocity and the horizontal is $\theta = 0$. We devote this section to a brief look at this type of motion.

Equations of Motion

Suppose you are walking with a speed v_0 when you release a ball from a height h, as discussed in the previous section. If we choose ground level to be $y = 0$ and the release point to be directly above the origin, the initial position of the ball is given by

$$x_0 = 0$$

and

$$y_0 = h$$

This is illustrated in Figure 4–3.

The initial velocity is horizontal, corresponding to $\theta = 0$ in **Figure 4–4**. As a result, the x component of the initial velocity is simply the initial speed:

$$v_{0x} = v_0 \cos 0° = v_0$$

and the y component of the initial velocity is zero:

$$v_{0y} = v_0 \sin 0° = 0$$

Substituting these specific values into our fundamental equations for projectile motion (Equations 4–6) gives the following simplified results for zero launch angle ($\theta = 0$):

$$\begin{aligned} x &= v_0 t & v_x &= v_0 = \text{constant} & v_x^2 &= v_0^2 = \text{constant} \\ y &= h - \tfrac{1}{2}gt^2 & v_y &= -gt & v_y^2 &= -2g\Delta y \end{aligned} \qquad 4\text{–}7$$

Note that the x component of velocity remains the same for all time and that the y component steadily decreases with time. As a result, x increases linearly with time, and y decreases with a t^2 dependence. Snapshots of this motion at equal time intervals are shown in **Figure 4–5**.

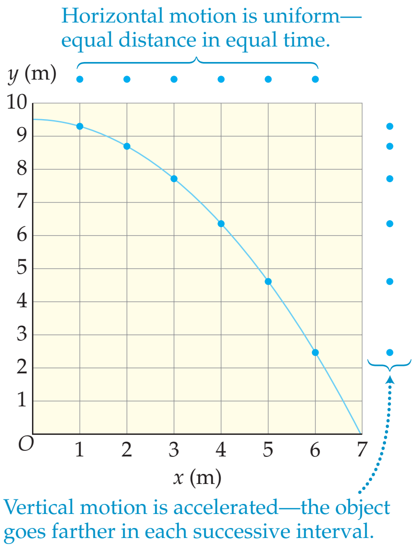


FIGURE 4–5 Trajectory of a projectile launched horizontally
In this plot, the projectile was launched from a height of 9.5 m with an initial speed of 5.0 m/s. The positions shown in the plot correspond to the times $t = 0.20$ s, 0.40 s, 0.60 s, ... Note the uniform motion in the x direction, and the accelerated motion in the y direction.

PROBLEM-SOLVING NOTE

Identify Initial Conditions

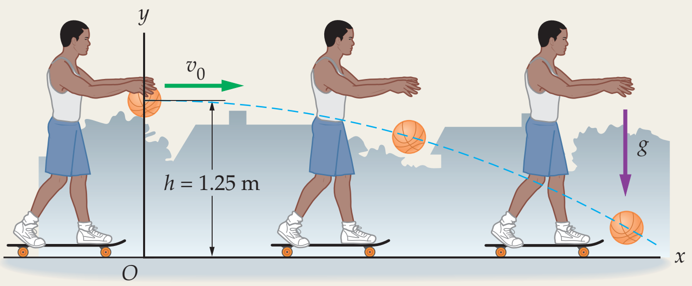
The launch point of a projectile determines x_0 and y_0. The initial velocity of a projectile determines v_{0x} and v_{0y}.

EXAMPLE 4–3 DROPPING A BALL

A person skateboarding with a constant speed of 1.30 m/s releases a ball from a height of 1.25 m above the ground. Given that $x_0 = 0$ and $y_0 = h = 1.25$ m, find x and y for **(a)** $t = 0.250$ s and **(b)** $t = 0.500$ s. **(c)** Find the velocity, speed, and direction of motion of the ball at $t = 0.500$ s.

PICTURE THE PROBLEM

The ball starts at $x_0 = 0$ and $y_0 = h = 1.25$ m. Its initial velocity is horizontal, therefore $v_{0x} = v_0 = 1.30$ m/s and $v_{0y} = 0$. In addition, it accelerates with the acceleration due to gravity in the negative y direction, $a_y = -g$, and moves with constant speed in the x direction, $a_x = 0$.



STRATEGY

The x and y positions are given by $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, respectively. We simply substitute time into these expressions. Similarly, the velocity components are $v_x = v_0$ and $v_y = -gt$.

SOLUTION

Part (a)

1. Substitute $t = 0.250$ s into the x and y equations of motion:

$$x = v_0 t = (1.30\ \text{m/s})(0.250\ \text{s}) = 0.325\ \text{m}$$
$$y = h - \tfrac{1}{2}gt^2$$
$$= 1.25\ \text{m} - \tfrac{1}{2}(9.81\ \text{m/s}^2)(0.250\ \text{s})^2 = 0.943\ \text{m}$$

Part (b)

2. Substitute $t = 0.500$ s into the x and y equations of motion: Note that the ball is only about an inch above the ground at this time:

$$x = v_0 t = (1.30\ \text{m/s})(0.500\ \text{s}) = 0.650\ \text{m}$$
$$y = h - \tfrac{1}{2}gt^2$$
$$= 1.25\ \text{m} - \tfrac{1}{2}(9.81\ \text{m/s}^2)(0.500\ \text{s})^2 = 0.0238\ \text{m}$$

Part (c)

3. First, calculate the x and y components of the velocity at $t = 0.500$ s using $v_x = v_0$ and $v_y = -gt$:

$$v_x = v_0 = 1.30\ \text{m/s}$$
$$v_y = -gt = -(9.81\ \text{m/s}^2)(0.500\ \text{s}) = -4.91\ \text{m/s}$$

4. Use these components to determine $\vec{\mathbf{v}}$, v, and θ:

$$\vec{\mathbf{v}} = (1.30\ \text{m/s})\hat{\mathbf{x}} + (-4.91\ \text{m/s})\hat{\mathbf{y}}$$
$$v = \sqrt{v_x{}^2 + v_y{}^2}$$
$$= \sqrt{(1.30\ \text{m/s})^2 + (-4.91\ \text{m/s})^2} = 5.08\ \text{m/s}$$
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{(-4.91\ \text{m/s})}{1.30\ \text{m/s}} = -75.2°$$

INSIGHT

Note that the x position of the ball does not depend on the acceleration of gravity, g, and that its y position does not depend on the initial horizontal speed of the ball, v_0. For example, if the person is running when he drops the ball, the ball is moving faster in the horizontal direction, and it keeps up with the person when it is dropped. Its vertical motion doesn't change at all, however; it drops to the ground in exactly the same time and bounces back to the same height as before.

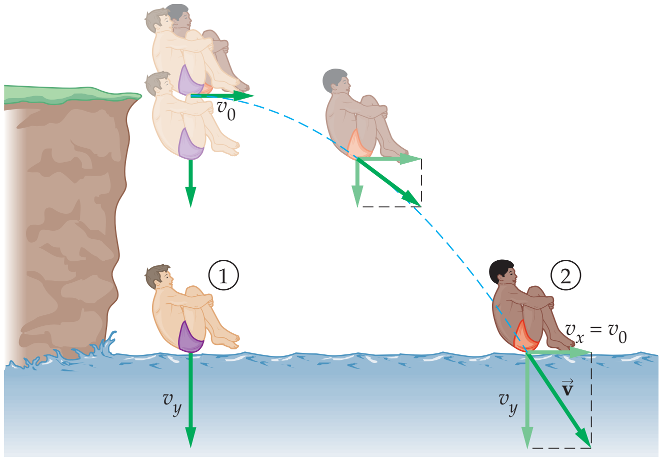
PRACTICE PROBLEM

How long does it take for the ball to land? [**Answer:** Referring to the results of part (b), it is clear that the time of landing is slightly greater than 0.500 s. Setting $y = 0$ gives a precise answer; $t = \sqrt{2h/g} = 0.505$ s.]

Some related homework problems: Problem 15, Problem 16, Problem 20

CONCEPTUAL CHECKPOINT 4–1 COMPARE SPLASHDOWN SPEEDS

Two youngsters dive off an overhang into a lake. Diver 1 drops straight down, diver 2 runs off the cliff with an initial horizontal speed v_0. Is the splashdown speed of diver 2 **(a)** greater than, **(b)** less than, or **(c)** equal to the splashdown speed of diver 1?



REASONING AND DISCUSSION

Note that neither diver has an initial y component of velocity, and that they both fall with the same vertical acceleration—the acceleration due to gravity. Therefore, the two divers fall for the same amount of time, and their y components of velocity are the same at splashdown. Since diver 2 also has a nonzero x component of velocity, unlike diver 1, the speed of diver 2 is greater.

ANSWER

(a) The speed of diver 2 is greater than that of diver 1.

REAL-WORLD PHYSICS

The parabolic trajectory of projectiles

Parabolic Path

Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = x/v_0$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \tfrac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0^2}\right)x^2 \qquad \text{4–8}$$

Note that y has the form

$$y = a + bx^2$$

where $a = h =$ constant and $b = -g/2v_0^2 =$ constant. This is the equation of a parabola that curves downward, a characteristic shape in projectile motion.

▲ Lava bombs (top) and fountain jets (bottom) trace out parabolic paths, as is typical in projectile motion. The trajectories are only slightly altered by air resistance.

Landing Site

Where does a projectile land if it is launched horizontally with a speed v_0 from a height h?

The most direct way to answer this question is to set $y = 0$ in Equation 4–8, since $y = 0$ corresponds to ground level. This gives

$$0 = h - \left(\frac{g}{2v_0^2}\right)x^2$$

Solving for x yields the landing site:

$$x = v_0\sqrt{\frac{2h}{g}} \qquad \text{4–9}$$

Note that we have chosen the positive sign for the square root since the projectile was launched in the positive x direction, and hence lands at a positive value of x.

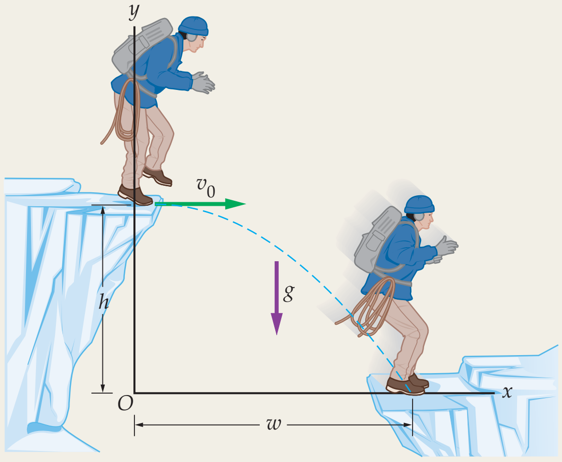
A useful alternative approach is to find the time of landing with the kinematic relations given in Equation 4–7, and then substitute this time into $x = v_0 t$. This approach is illustrated in the next Example.

EXAMPLE 4–4 JUMPING A CREVASSE

A mountain climber encounters a crevasse in an ice field. The opposite side of the crevasse is 2.75 m lower, and is separated horizontally by a distance of 4.10 m. To cross the crevasse, the climber gets a running start and jumps in the horizontal direction. **(a)** What is the minimum speed needed by the climber to safely cross the crevasse? If, instead, the climber's speed is 6.00 m/s, **(b)** where does the climber land, and **(c)** what is the climber's speed on landing?

PICTURE THE PROBLEM

The mountain climber jumps from $x_0 = 0$ and $y_0 = h = 2.75$ m. The landing site for part (a) is $x = w = 4.10$ m and $y = 0$. Note that the y position of the climber *decreases* by h, and therefore $\Delta y = -h = -2.75$ m. As for the initial velocity, we are given that $v_{0x} = v_0$ and $v_{0y} = 0$. Finally, with our choice of coordinates it follows that $a_x = 0$ and $a_y = -g$.



STRATEGY

We can model the climber as a projectile, and apply our equations for projectile motion with a horizontal launch.

a. From Equations 4–7 we have that $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$. Setting $y = 0$ determines the time of landing. Using this time in the x equation gives the horizontal landing position in terms of the initial speed.

b. We can now use the relation from part (a) to find x in terms of $v_0 = 6.00$ m/s.

c. We already know v_x, since it remains constant, and we can calculate v_y using $v_y^2 = -2g\Delta y$ (Equations 4–7). With the velocity components known, we can use the Pythagorean theorem to find the speed.

SOLUTION

Part (a)

1. Set $y = h - \frac{1}{2}gt^2$ equal to zero (landing condition) and solve for the corresponding time t:

$$y = h - \tfrac{1}{2}gt^2 = 0$$
$$t = \sqrt{\frac{2h}{g}}$$

2. Substitute this expression for t into the x equation of motion, $x = v_0 t$, and solve for the speed, v_0:

$$x = v_0 t = v_0\sqrt{\frac{2h}{g}} \quad \text{or} \quad v_0 = x\sqrt{\frac{g}{2h}}$$

3. Substitute numerical values in this expression:

$$v_0 = x\sqrt{\frac{g}{2h}} = (4.10\text{ m})\sqrt{\frac{9.81\text{ m/s}^2}{2(2.75\text{ m})}} = 5.48\text{ m/s}$$

Part (b)

4. Substitute $v_0 = 6.00$ m/s into the expression for x obtained in Step 2, $x = v_0\sqrt{2h/g}$:

$$x = v_0\sqrt{\frac{2h}{g}} = (6.00\text{ m/s})\sqrt{\frac{2(2.75\text{ m})}{9.81\text{ m/s}^2}} = 4.49\text{ m}$$

Part (c)

5. Use the fact that the x component of velocity does not change to determine v_x, and use $v_y^2 = -2g\Delta y$ to determine v_y. For v_y, note that we choose the minus sign for the square root because the climber is moving downward:

$$v_x = v_0 = 6.00\text{ m/s}$$
$$v_y = \pm\sqrt{-2g\Delta y}$$
$$= -\sqrt{-2(9.81\text{ m/s}^2)(-2.75\text{ m})} = -7.35\text{ m/s}$$

6. Use the Pythagorean theorem to determine the speed:

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(6.00\text{ m/s})^2 + (-7.35\text{ m/s})^2} = 9.49\text{ m/s}$$

INSIGHT

The minimum speed needed to safely cross the crevasse is 5.48 m/s. If the initial horizontal speed is 6.00 m/s, the climber will land 4.49 m − 4.10 m = 0.39 m beyond the edge of the crevasse with a speed of 9.49 m/s.

PRACTICE PROBLEM

(a) When the climber's speed is the minimum needed to cross the crevasse, $v_0 = 5.48$ m/s, how long is the climber in the air? **(b)** How long is the climber in the air when $v_0 = 6.00$ m/s? [**Answer: (a)** $t = x/v_0 = (4.10\text{ m})/(5.48\text{ m/s}) = 0.748$ s. **(b)** $t = x/v_0 = (4.49\text{ m})/(6.00\text{ m/s}) = 0.748$ s. The times are the same! The answer to both parts is simply the time needed to fall through a height h; $t = \sqrt{2h/g} = 0.748$ s.]

Some related homework problems: Problem 11, Problem 12, Problem 17

PROBLEM-SOLVING NOTE

Use Independence of Motion

Projectile problems can be solved by breaking the problem into its x and y components, and then solving for the motion of each component separately.

CONCEPTUAL CHECKPOINT 4–2 MINIMUM SPEED

If the height h is increased in the previous example but the width w remains the same, does the minimum speed needed to cross the crevasse **(a)** increase, **(b)** decrease, or **(c)** stay the same?

REASONING AND DISCUSSION

If the height is greater, the time of fall is also greater. Since the climber is in the air for a greater time, the horizontal distance covered for a given initial speed is also greater. Thus, if the width of the crevasse is the same, a lower initial speed allows for a safe crossing.

ANSWER

(b) The minimum speed decreases.

4–4 General Launch Angle

We now consider the more general case of a projectile launched at an arbitrary angle with respect to the horizontal. This means we can no longer use the simplifications associated with zero launch angle. As always, we return to our basic equations for projectile motion (Equations 4–6), and this time we simply let θ be nonzero.

Figure 4–6 (a) shows a projectile launched with an initial speed v_0 at an angle θ above the horizontal. Since the projectile starts at the origin, the initial x and y positions are zero:

$$x_0 = y_0 = 0$$

The components of the initial velocity are determined as indicated in **Figure 4–6 (b)**:

$$v_{0x} = v_0 \cos\theta$$

and

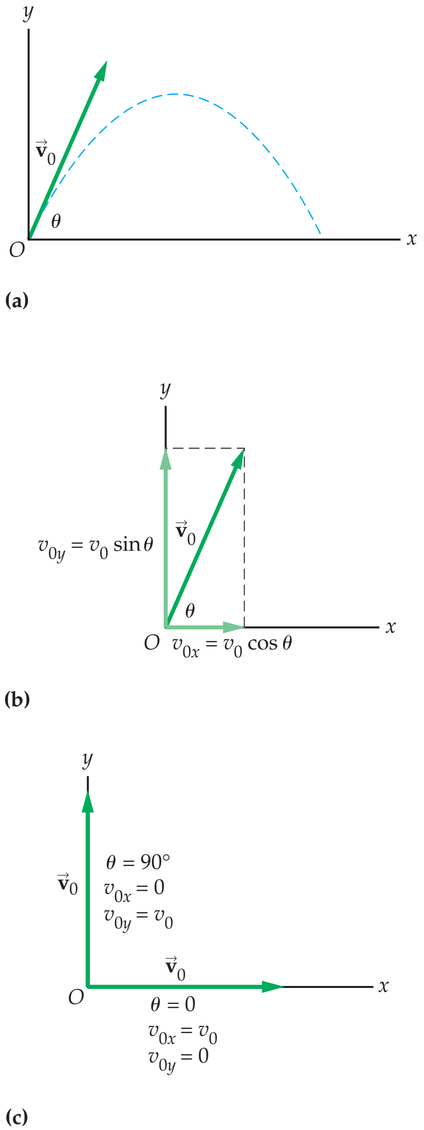
$$v_{0y} = v_0 \sin\theta$$

As a quick check, note that if $\theta = 0$, then $v_{0x} = v_0$ and $v_{0y} = 0$. Similarly, if $\theta = 90°$ we find $v_{0x} = 0$ and $v_{0y} = v_0$. These checks are depicted in **Figure 4–6 (c)**.

Substituting these results into the basic equations for projectile motion yields the following results for a general launch angle:

$$\begin{array}{lll} x = (v_0 \cos\theta)t & v_x = v_0\cos\theta & v_x{}^2 = v_0{}^2\cos^2\theta \\ y = (v_0\sin\theta)t - \frac{1}{2}gt^2 & v_y = v_0\sin\theta - gt & v_y{}^2 = v_0{}^2\sin^2\theta - 2g\Delta y \end{array} \qquad \text{4–10}$$

Note that these equations, which are valid for any launch angle, reduce to the simpler Equations 4–7 when we set $\theta = 0$ and $y_0 = h$. In the next two Exercises, we use Equations 4–10 to calculate a projectile's position and velocity for three equally spaced times.



▲ **FIGURE 4–6 Projectile with an arbitrary launch angle**

(a) A projectile launched from the origin at an angle θ above the horizontal. **(b)** The x and y components of the initial velocity. **(c)** Velocity components in the limits $\theta = 0$ and $\theta = 90°$.

EXERCISE 4–1

A projectile is launched from the origin with an initial speed of 20.0 m/s at an angle of 35.0° above the horizontal. Find the x and y positions of the projectile at times **(a)** $t = 0.500$ s, **(b)** $t = 1.00$ s, and **(c)** $t = 1.50$ s.

SOLUTION

a. $x = 8.19$ m, $y = 4.51$ m,

b. $x = 16.4$ m, $y = 6.57$ m,

c. $x = 24.6$ m, $y = 6.17$ m. Note that x increases steadily; y increases, then decreases.

EXERCISE 4–2

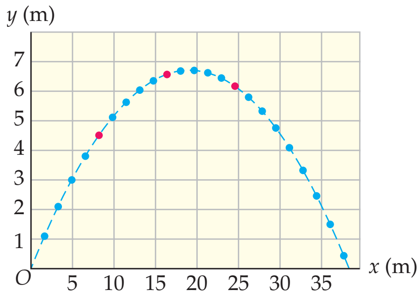
Referring to Exercise 4–1, find the velocity of the projectile at times **(a)** $t = 0.500$ s, **(b)** $t = 1.00$ s, and **(c)** $t = 1.50$ s.

SOLUTION

a. $\vec{\mathbf{v}} = (16.4\ \mathrm{m/s})\hat{\mathbf{x}} + (6.57\ \mathrm{m/s})\hat{\mathbf{y}}$,

b $\vec{\mathbf{v}} = (16.4\ \mathrm{m/s})\hat{\mathbf{x}} + (1.66\ \mathrm{m/s})\hat{\mathbf{y}}$,

c. $\vec{\mathbf{v}} = (16.4\ \mathrm{m/s})\hat{\mathbf{x}} + (-3.24\ \mathrm{m/s})\hat{\mathbf{y}}$.



▲ **FIGURE 4–7 Snapshots of a trajectory**
This plot shows a projectile launched from the origin with an initial speed of 20.0 m/s at an angle of 35.0° above the horizontal. The positions shown in the plot correspond to the times $t = 0.1$ s, 0.2 s, 0.3 s, ... Red dots mark the positions considered in Exercises 4–1 and 4–2.

Figure 4–7 shows the projectile referred to in the previous Exercises for a series of times spaced by 0.10 s. Note that the points in Figure 4–7 are not evenly spaced in terms of position, even though they are evenly spaced in time. In fact, the points bunch closer together at the top of the trajectory, showing that a comparatively large fraction of the flight time is spent near the highest point. This is why it seems that a basketball player soaring toward a slam dunk, or a ballerina performing a grand jeté, is "hanging" in air.

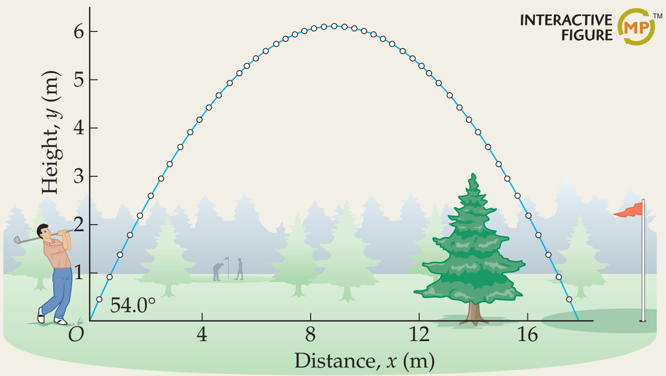
▲ "Hanging" in air near the peak of a jump requires no special knack—in fact, it's an unavoidable consequence of the laws of physics. This phenomenon, which makes big leapers (such as deer and dancers) look particularly graceful, can also make life more dangerous for salmon fighting their way upstream to spawn.

EXAMPLE 4–5 A ROUGH SHOT

Chipping from the rough, a golfer sends the ball over a 3.00-m-high tree that is 14.0 m away. The ball lands at the same level from which it was struck after traveling a horizontal distance of 17.8 m—on the green, of course. **(a)** If the ball left the club 54.0° above the horizontal and landed on the green 2.24 s later, what was its initial speed? **(b)** How high was the ball when it passed over the tree?

PICTURE THE PROBLEM

Our sketch shows the ball taking flight from the origin, $x_0 = y_0 = 0$, with a launch angle of 54.0°, and arcing over the tree. The individual points along the parabolic trajectory correspond to equal time intervals.



STRATEGY

a. Since the projectile moves with constant speed in the x direction, the x component of velocity is simply horizontal distance divided by time. Knowing v_x and θ, we can find v_0 from $v_x = v_0 \cos\theta$.

b. We can use $x = (v_0 \cos\theta)t$ to find the time when the ball is at $x = 14.0$ m. Substituting this time into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ gives the height.

SOLUTION

Part (a)

1. Divide the horizontal distance, d, by the time of flight, t, to obtain v_x:

$$v_x = \frac{d}{t} = \frac{17.8\ \mathrm{m}}{2.24\ \mathrm{s}} = 7.95\ \mathrm{m/s}$$

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2. Use $v_x = v_0 \cos\theta$ to find v_0, the initial speed:

$$v_x = v_0 \cos\theta \quad \text{or} \quad v_0 = \frac{v_x}{\cos\theta} = \frac{7.95\ \text{m/s}}{\cos 54.0°} = 13.5\ \text{m/s}$$

Part (b)

3. Use $x = (v_0 \cos\theta)t$ to find the time when $x = 14.0$ m. Recall that $x_0 = 0$:

$$x = (v_0 \cos\theta)t \quad \text{or} \quad t = \frac{x}{v_0 \cos\theta} = \frac{14.0\ \text{m}}{7.95\ \text{m/s}} = 1.76\ \text{s}$$

4. Evaluate $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ at the time found in Step 3. Recall that $y_0 = 0$:

$$\begin{aligned} y &= (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \\ &= [(13.5\ \text{m/s}) \sin 54.0°](1.76\ \text{s}) - \tfrac{1}{2}(9.81\ \text{m/s}^2)(1.76\ \text{s})^2 \\ &= 4.03\ \text{m} \end{aligned}$$

INSIGHT

The ball clears the top of the tree by 1.03 m and lands on the green 0.48 s later. When it lands, its speed (in the absence of air resistance) is again 13.5 m/s—the same as when it was launched. This result will be verified in the next section.

PRACTICE PROBLEM

What are the speed and direction of the ball when it passes over the tree? [**Answer:** To find the ball's speed and direction, note that $v_x = 7.95$ m/s and $v_y = v_0 \sin\theta - gt = -6.34$ m/s. It follows that $v = \sqrt{v_x^2 + v_y^2} = 10.2$ m/s and $\theta = \tan^{-1}(v_y/v_x) = -38.6°$.]

Some related homework problems: Problem 31, Problem 39

ACTIVE EXAMPLE 4–1 AN ELEVATED GREEN

A golfer hits a ball from the origin with an initial speed of 30.0 m/s at an angle of 50.0° above the horizontal. The ball lands on a green that is 5.00 m above the level where the ball was struck.

a. How long is the ball in the air?

b. How far has the ball traveled in the horizontal direction when it lands?

c. What are the speed and direction of motion of the ball just before it lands?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Let $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 5.00$ m and solve for t: $t = 0.229$ s, 4.46 s

2. When $t = 0.229$ s, the ball is moving upward; when $t = 4.46$ s, the ball is on the way down. Choose the later time: $t = 4.46$ s

Part (b)

3. Substitute $t = 4.46$ s into $x = (v_0 \cos\theta)t$: $x = 86.0$ m

Part (c)

4. Use $v_x = v_0 \cos\theta$ to calculate v_x: $v_x = 19.3$ m/s

5. Substitute $t = 4.46$ s into $v_y = v_0 \sin\theta - gt$ to find v_y: $v_y = -20.8$ m/s

6. Calculate v and θ: $v = 28.4$ m/s, $\theta = -47.1°$

YOUR TURN

How long is the ball in the air if the green is 5.00 m *below* the level where the ball was struck?

(Answers to **Your Turn** *problems are given in the back of the book.)*

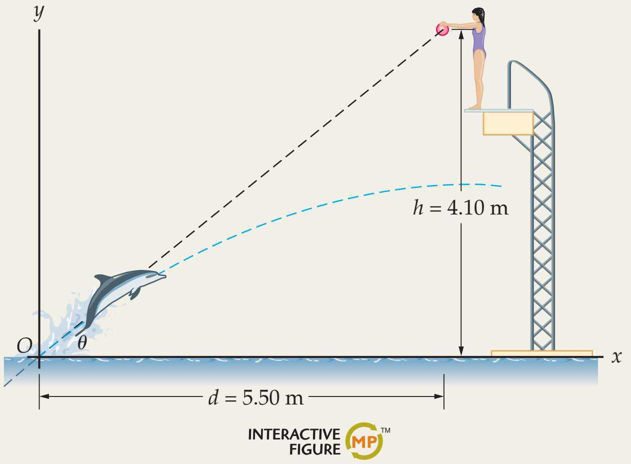
The next Example presents a classic situation in which two projectiles collide. One projectile is launched from the origin, and thus its equations of motion are given by Equations 4–10. The second projectile is simply dropped from a height, which is a special case of the equations of motion in Equations 4–7 with $v_0 = 0$.

EXAMPLE 4–6 A LEAP OF FAITH

A trained dolphin leaps from the water with an initial speed of 12.0 m/s. It jumps directly toward a ball held by the trainer a horizontal distance of 5.50 m away and a vertical distance of 4.10 m above the water. In the absence of gravity the dolphin would move in a straight line to the ball and catch it, but because of gravity the dolphin follows a parabolic path well below the ball's initial position, as shown in the sketch. If the trainer releases the ball the instant the dolphin leaves the water, show that the dolphin and the falling ball meet.

PICTURE THE PROBLEM

In our sketch we have the dolphin leaping from the water at the origin $x_0 = y_0 = 0$ with an angle above the horizontal given by $\theta = \tan^{-1}(h/d)$. The initial position of the ball is $x_0 = d = 5.50$ m and $y_0 = h = 4.10$ m, and its initial velocity is zero. The ball drops straight down with the acceleration of gravity, $a_y = -g$.



STRATEGY

We want to show that when the dolphin is at $x = d$, its height above the water is the same as the height of the ball above the water. To do this we first find the time when the dolphin is at $x = d$, then calculate y for the dolphin at this time. Next, we calculate y of the ball at the same time and then check to see if they are equal.

Since the ball drops from rest from a height h, its y equation of motion is $y = h - \frac{1}{2}gt^2$, as in Equations 4–7 in Section 4–3.

SOLUTION

1. Calculate the angle at which the dolphin leaves the water:

$$\theta = \tan^{-1}\left(\frac{h}{d}\right) = \tan^{-1}\left(\frac{4.10\text{ m}}{5.50\text{ m}}\right) = 36.7°$$

2. Use this angle and the initial speed to find the time t when the x position of the dolphin, x_d, is equal to 5.50 m. The x equation of motion is $x_d = (v_0 \cos\theta)t$:

$$x_d = (v_0 \cos\theta)t = [(12.0\text{ m/s})\cos 36.7°]t = (9.62\text{ m/s})t = 5.50\text{ m}$$

$$t = \frac{5.50\text{ m}}{9.62\text{ m/s}} = 0.572\text{ s}$$

3. Evaluate the y position of the dolphin, y_d, at $t = 0.572$ s. The y equation of motion is $y_d = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y_d = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$
$$= [(12.0\text{ m/s})\sin 36.7°](0.572\text{ s}) - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2$$
$$= 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

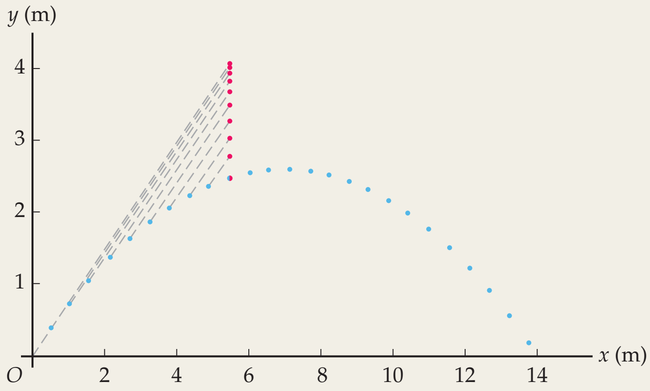
4. Finally, evaluate the y position of the ball, y_b, at $t = 0.572$ s. The ball's equation of motion is $y_b = h - \frac{1}{2}gt^2$:

$$y_b = h - \tfrac{1}{2}gt^2 = 4.10\text{ m} - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2$$
$$= 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

INSIGHT

In the absence of gravity, both the dolphin and the ball would be at $x = 5.50$ m and $y = 4.10$ m at $t = 0.572$ s. Because of gravity, however, the dolphin and the ball fall below their zero-gravity positions—and by the same amount, 1.60 m. In fact, from the point of view of the dolphin, the ball is always at the same angle of 36.7° above the horizontal until it is caught.

This is shown in the accompanying plot, where the red dots show the position of the ball at ten equally spaced times, and the blue dots show the position of the dolphin at the corresponding times. In addition, the dashed lines from the dolphin to the ball all make the same angle with the horizontal, 36.7°.



PRACTICE PROBLEM

At what height does the dolphin catch the ball if it leaves the water with an initial speed of 8.00 m/s? [**Answer:** $y_d = y_b = 0.493$ m. If the dolphin's initial speed is less than 7.50 m/s, it reenters the water before catching the ball.]

Some related homework problems: Problem 31, Problem 40

4–5 Projectile Motion: Key Characteristics

We conclude this chapter with a brief look at some additional characteristics of projectile motion that are both interesting and useful. In all cases our results follow as a direct consequence of the fundamental kinematic equations (Equations 4–10) describing projectile motion.

Range

The **range**, R, of a projectile is the horizontal distance it travels before landing. We consider the case shown in **Figure 4–8**, where the initial and final elevations are the same ($y = 0$). One way to obtain the range, then, is as follows: (i) Find the time when the projectile lands by setting $y = 0$ in the expression $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$; (ii) Substitute the time found in (i) into the x equation of motion.

Carrying out the first part of the calculation yields the following:

$$(v_0 \sin\theta)t - \tfrac{1}{2}gt^2 = 0 \quad \text{or} \quad (v_0 \sin\theta)t = \tfrac{1}{2}gt^2$$

Clearly, $t = 0$ is a solution to this equation—corresponding to the initial condition—but the solution we seek is a time that is greater than zero. We can find the desired time by dividing both sides of the equation by t. This gives

$$(v_0 \sin\theta) = \tfrac{1}{2}gt \quad \text{or} \quad t = \left(\frac{2v_0}{g}\right)\sin\theta \qquad \text{4–11}$$

This is the time when the projectile lands—also known as the time of flight.

Now, substitute this time into $x = (v_0 \cos\theta)t$ to find the value of x when the projectile lands:

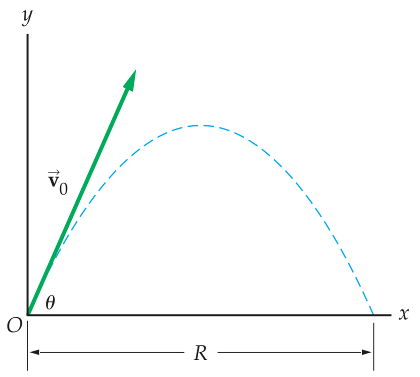
$$x = (v_0 \cos\theta)t = (v_0 \cos\theta)\left(\frac{2v_0}{g}\right)\sin\theta = \left(\frac{2v_0^2}{g}\right)\sin\theta\cos\theta$$

This value of x is the range, R, thus

$$R = \left(\frac{2v_0^2}{g}\right)\sin\theta\cos\theta$$

Using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, as given in Appendix A, we can write this more compactly as follows:

$$R = \left(\frac{v_0^2}{g}\right)\sin 2\theta \quad \textit{(same initial and final elevation)} \qquad \text{4–12}$$



▲ **FIGURE 4–8 Range of a projectile**
The range R of a projectile is the horizontal distance it travels between its takeoff and landing positions.

PROBLEM-SOLVING NOTE

Use the Same Math Regardless of the Initial Conditions

Once an object is launched, its trajectory follows the kinematic equations of motion, regardless of specific differences in the initial conditions. Thus, our equations of motion can be used to derive any desired characteristic of projectile motion, including range, symmetry, and maximum height.

ACTIVE EXAMPLE 4–2 FIND THE INITIAL SPEED

A football game begins with a kickoff in which the ball travels a horizontal distance of 45 yd and lands on the ground. If the ball was kicked at an angle of 40.0° above the horizontal, what was its initial speed?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 4–12 for the initial speed v_0:	$v_0 = \sqrt{gR/\sin 2\theta}$
2. Convert the range to meters:	$R = 41$ m
3. Substitute numerical values:	$v_0 = 20$ m/s

INSIGHT

Note that we choose the positive square root in Step 1 because we are interested only in the *speed* of the ball, which is always positive.

YOUR TURN

Suppose the initial speed of the ball is increased by 10%, to 22 m/s. By what percentage does the range increase?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note that R depends inversely on the acceleration of gravity, g—thus the smaller g, the larger the range. For example, a projectile launched on the Moon, where the acceleration of gravity is only about 1/6 that on Earth, travels about six times as far as it would on Earth. It was for this reason that astronaut Alan Shepard simply couldn't resist the temptation of bringing a golf club and ball with him on the third lunar landing mission in 1971. He ambled out onto the Fra Mauro Highlands and became the first person to hit a tee shot on the Moon. His distance was undoubtedly respectable—unfortunately, his ball landed in a sand trap.

REAL-WORLD PHYSICS

Golf on the Moon

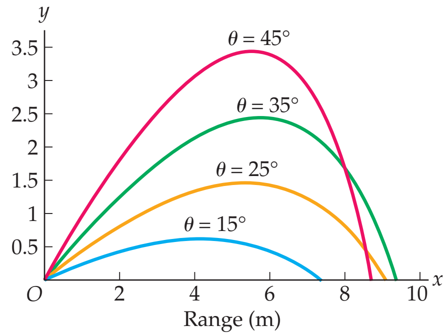
Now, what launch angle gives the greatest range? From Equation 4–12 we see that R varies with angle as $\sin 2\theta$; thus R is largest when $\sin 2\theta$ is largest—that is, when $\sin 2\theta = 1$. Since $\sin 90° = 1$, it follows that $\theta = 45°$ gives the maximum range. Thus

$$R_{\text{max}} = \frac{v_0^2}{g} \tag{4–13}$$

As expected, the range (Equation 4–12) and maximum range (Equation 4–13) depend strongly on the initial speed of the projectile—they are both proportional to v_0^2.

Note that these results are specifically for the case where a projectile lands at the same level from which it was launched. If a projectile lands at a higher level, for example, the launch angle that gives maximum range is greater than 45°, and if it lands at a lower level, the angle for maximum range is less than 45°.

Finally, the range given here applies only to the ideal case of no air resistance. In cases where air resistance is significant, as in the flight of a rapidly moving golf ball, for example, the overall range of the ball is reduced. In addition, the maximum range occurs for a launch angle less than 45° (**Figure 4–9**). The reason is that with a smaller launch angle the golf ball is in the air for less time, giving air resistance less time to affect its flight.



▲ **FIGURE 4–9 Projectiles with air resistance**

Projectiles with the same initial speed but different launch angles showing the effects of air resistance. Notice that the maximum range occurs for a launch angle less than 45°, and that the projectiles return to the ground at a steeper angle than the launch angle.

Symmetry in Projectile Motion

There are many striking symmetries in projectile motion, beginning with the graceful symmetry of the parabola itself. As a first example, recall that earlier in this section, in Equation 4–11, we found the time when a projectile lands:

$$t = \left(\frac{2v_0}{g}\right)\sin\theta$$

Now, by symmetry, the time it takes a projectile to reach its highest point (in the absence of air resistance) should be just half this time. After all, the projectile moves in the x direction with constant speed, and the highest point—by symmetry—occurs at $x = \frac{1}{2}R$.

This all seems reasonable, but is there another way to check? Well, at the highest point the projectile is moving horizontally, thus its y component of velocity is zero. Let's find the time when $v_y = 0$ and compare with the time to land:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$

$$t = \left(\frac{v_0}{g}\right)\sin\theta \tag{4–14}$$

As expected from symmetry, the time at the highest point is one-half the time at landing.

There is another interesting symmetry concerning speed. Recall that when a projectile is launched, its y component of velocity is $v_y = v_0 \sin\theta$. When the projectile lands, at time $t = (2v_0/g)\sin\theta$, its y component of velocity is

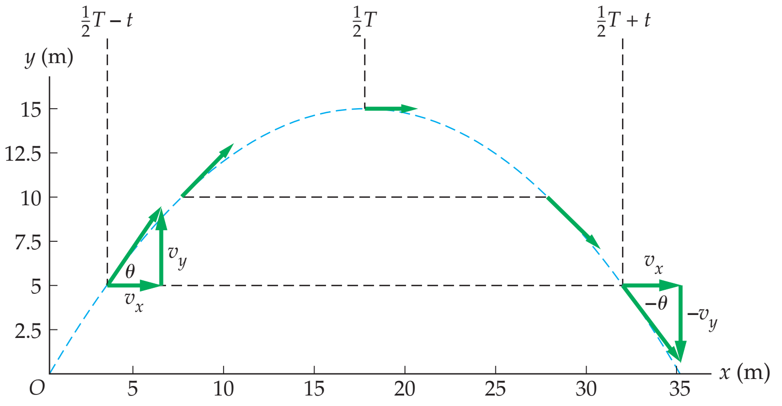
$$v_y = v_0 \sin\theta - gt = v_0 \sin\theta - g\left(\frac{2v_0}{g}\right)\sin\theta = -v_0 \sin\theta$$

▲ To be successful, a juggler must master the behavior of projectile motion. Physicist Richard Feynman shows that just knowing the appropriate equations is not enough; one must also practice. In this sense, learning to juggle is similar to learning to solve physics problems.

This is exactly the opposite of the y component of the velocity when it was launched. Since the x component of velocity is always the same, it follows that when the projectile lands, its speed, $v = \sqrt{v_x{}^2 + v_y{}^2}$, is the same as when it was launched—as one might expect from symmetry.

The velocities are different, however, since the direction of motion is different at launch and landing. Even so, there is still a symmetry—the initial velocity is *above* the horizontal by the angle θ; the landing velocity is *below* the horizontal by the same angle θ.

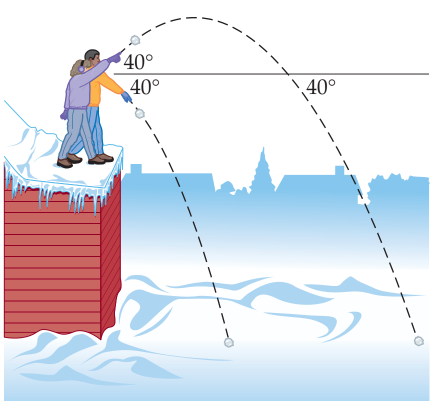
So far, these results have referred to launching and landing, which both occur at $y = 0$. The same symmetry extends to any level, though. That is, at a given height the speed of a projectile is the same on the way up as on the way down. In addition, the angle of the velocity above the horizontal on the way up is the same as the angle below the horizontal on the way down. This is illustrated in **Figure 4–10** and in the next Conceptual Checkpoint.



▲ **FIGURE 4–10 Velocity vectors for a projectile launched at the origin**
At a given height the speed (length of velocity vector) is the same on the way up as on the way down. The direction of motion on the way up is above the horizontal by the same amount that it is below the horizontal on the way down. In this case, the total time of flight is T, and the greatest height is reached at the time $T/2$. Notice that the speed is the same at the time $(T/2) - t$ as it is at the time $(T/2) + t$.

CONCEPTUAL CHECKPOINT 4–3 COMPARE LANDING SPEEDS

You and a friend stand on a snow-covered roof. You both throw snowballs with the same initial speed, but in different directions. You throw your snowball downward, at 40° *below* the horizontal; your friend throws her snowball upward, at 40° *above* the horizontal. When the snowballs land on the ground, is the speed of your snowball **(a)** greater than, **(b)** less than, or **(c)** the same as the speed of your friend's snowball?



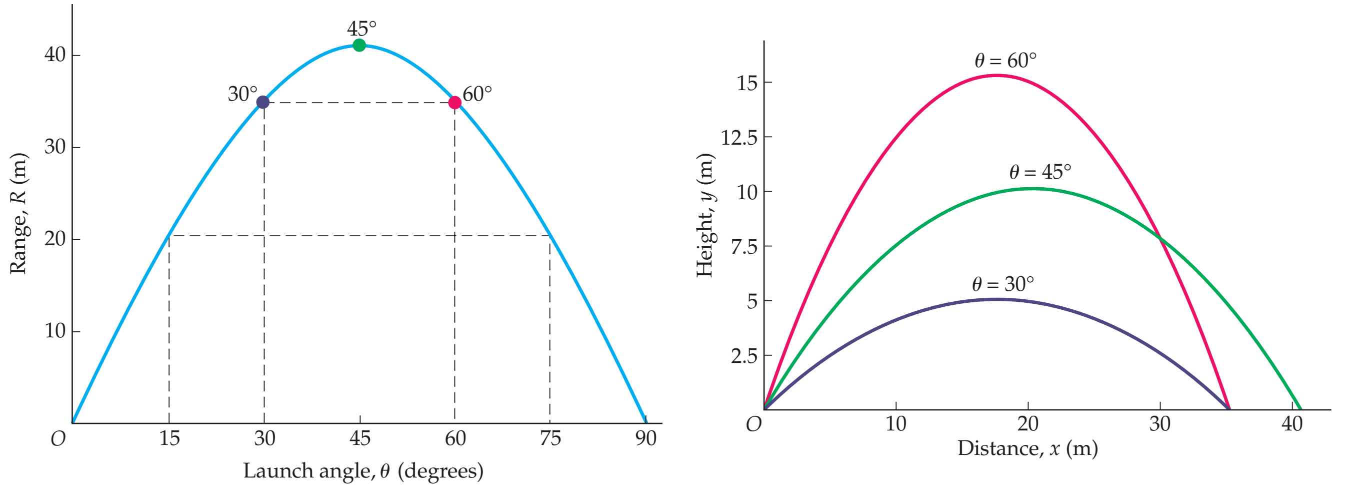
REASONING AND DISCUSSION

One consequence of symmetry in projectile motion is that when your friend's snowball returns to the level of the throw, its speed will be the same as the initial speed. In addition, it will be moving downward, at 40° below the horizontal. From that point on its motion is the same as that of your snowball; thus it lands with the same speed.

What if you throw your snowball horizontally? Or suppose you throw it straight down? In either case, the final speed is unchanged! In fact, for a given initial speed, the speed on landing simply doesn't depend on the direction in which you throw the ball. This is shown in Homework Problems 35 and 76. We return to this point in Chapter 8 when we discuss potential energy and energy conservation.

ANSWER

(c) The snowballs have the same speed.



(a) Launch angles that are greater or less than 45° by the same amount give the same range.

(b) Projectiles with $\theta = 30°$ and $\theta = 60°$ follow different paths but have the same range.

▲ **FIGURE 4–11 Range and launch angle in the absence of air resistance**

(a) A plot of range versus launch angle for a projectile launched with an initial speed of 20 m/s. Note that the maximum range occurs at $\theta = 45°$. Launch angles equally greater than or less than 45°, such as 30° and 60°, give the same range. **(b)** Trajectories of projectiles with initial speeds of 20 m/s and launch angles of 60°, 45°, and 30°. The projectiles with launch angles of 30° and 60° land at the same location.

As our final example of symmetry, consider the range R. A plot of R versus launch angle θ is shown in **Figure 4–11 (a)** for $v_0 = 20$ m/s. Note that in the absence of air resistance, R is greatest at $\theta = 45°$, as pointed out previously. In addition, we can see from the figure that the range for angles equally above or below 45° is the same. For example, if air resistance is negligible, the range for $\theta = 30°$ is the same as the range for $\theta = 60°$, as we can see in both parts (a) and (b) of Figure 4–11.

Symmetries such as these are just some of the many reasons why physicists find physics to be "beautiful" and "aesthetically pleasing." Discovering such patterns and symmetries in nature is really what physics is all about. A physicist does not consider the beauty of projectile motion to be diminished by analyzing it in detail. Just the opposite—detailed analysis reveals deeper, more subtle, and sometimes unexpected levels of beauty.

▲ An archerfish would have trouble procuring its lunch without an instinctive grasp of projectile motion.

Maximum Height

Let's follow up on an observation made earlier in this section, namely, that a projectile is at maximum height when its y component of velocity is zero. In fact, we will use this observation to determine the maximum height of an arbitrary projectile. This can be accomplished with the following two-step calculation: (i) Find the time when $v_y = 0$; (ii) Substitute this time into the y-versus-t equation of motion, $y = (v_0 \sin \theta)t - \frac{1}{2}gt^2$. This calculation is carried out in the next Example.

EXAMPLE 4–7 WHAT A SHOT!

The archerfish hunts by dislodging an unsuspecting insect from its resting place with a stream of water expelled from the fish's mouth. Suppose the archerfish squirts water with an initial speed of 2.30 m/s at an angle of 19.5° above the horizontal. When the stream of water reaches a beetle on a leaf at height h above the water's surface, it is moving horizontally.

a. How much time does the beetle have to react?

b. What is the height h of the beetle?

c. What is the horizontal distance d between the fish and the beetle when the water is launched?

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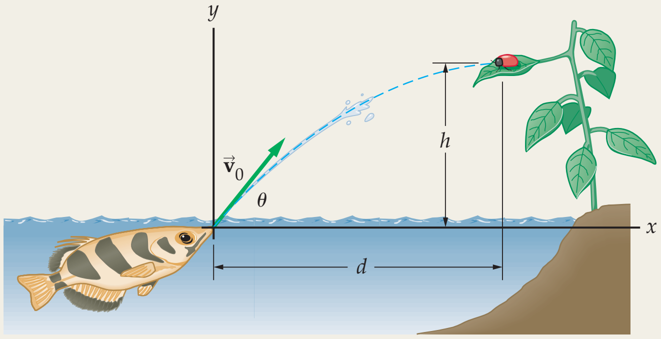
PICTURE THE PROBLEM

Our sketch shows the fish squirting water from the origin, $x_0 = y_0 = 0$, and the beetle at $x = d$, $y = h$. The stream of water starts off with a speed $v_0 = 2.30\ \mathrm{m/s}$ at an angle $\theta = 19.5°$ above the horizontal. Note that the water is moving horizontally when it reaches the beetle.



STRATEGY

a. Because the stream of water is moving horizontally when it reaches the beetle, it is at the top of its parabolic trajectory, as can be seen in Figure 4–10. This means that its y component of velocity is zero. Therefore, we can set $v_y = 0$ in $v_y = v_0 \sin\theta - gt$ and solve for the time t.

b. To find the maximum height of the stream of water, and of the beetle, we substitute the time found in part (a) into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$.

c. Similarly, we can find the horizontal distance d by substituting the time from part (a) into $x = (v_0 \cos\theta)t$.

SOLUTION

Part (a)

1. Set $v_y = v_0 \sin\theta - gt$ equal to zero and solve for the corresponding time t:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$
$$t = \frac{v_0 \sin\theta}{g}$$

2. Substitute numerical values to determine the reaction time:

$$t = \frac{v_0 \sin\theta}{g} = \frac{(2.30\ \mathrm{m/s}) \sin 19.5°}{9.81\ \mathrm{m/s^2}} = 0.0783\ \mathrm{s}$$

Part (b)

3. To calculate the height, we substitute $t = (v_0 \sin\theta)/g$ into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y = (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2 = \frac{(v_0 \sin\theta)^2}{2g}$$

4. Substitute numerical values to find the height h:

$$h = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(2.30\ \mathrm{m/s}) \sin 19.5°]^2}{2(9.81\ \mathrm{m/s^2})} = 0.0300\ \mathrm{m}$$

Part (c)

5. We can find the horizontal distance d using x as a function of time, $x = (v_0 \cos\theta)t$:

$$x = (v_0 \cos\theta)t$$
$$d = [(2.30\ \mathrm{m/s}) \cos 19.5°](0.0783\ \mathrm{s}) = 0.170\ \mathrm{m}$$

INSIGHT

To hit the beetle, the fish aims 19.5° above the horizontal. For comparison, note that the straight-line angle to the beetle is $\tan^{-1}(0.0300/0.170) = 10.0°$. Therefore, the fish cannot aim directly at its prey if it wants a meal.

Finally, note that by working symbolically in Step 3 we have derived a general result for the maximum height of a projectile. In particular, we find $y_{max} = (v_0 \sin\theta)^2/2g$, a result that is valid for any launch speed and angle. As a check of our result, note that if we launch a projectile straight upward ($\theta = 90°$), the maximum height is $y_{max} = v_0{}^2/2g$. Comparing with the one-dimensional kinematics of Chapter 2, if an object is thrown straight upward with an initial speed v_0, and the object accelerates downward with the acceleration of gravity, $a = -g$, it comes to rest ($v = 0$) after covering a vertical distance Δy given by $0 = v_0{}^2 + 2(-g)\Delta y$. Solving for the distance yields $\Delta y = v_0{}^2/2g = y_{max}$. This is an example of the internal consistency that characterizes all of physics.

PRACTICE PROBLEM

How far does the stream of water go if it happens to miss the beetle? [**Answer:** By symmetry, the distance d is half the range. Thus the stream of water travels a distance $R = 2d = 0.340\ \mathrm{m}$.]

Some related homework problems: Problem 81, Problem 82

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

This chapter provides a number of opportunities to use the vector methods developed in Chapter 3. In Section 4–4, for example, we resolve a velocity vector into its x and y components, and then use the components in Equations 4–10.

The equations of one-dimensional kinematics derived in Chapter 2 are used again in this chapter, even though we are now studying kinematics in two dimensions. For example, the equations in Table 4–1 are the same as those used in Chapter 2, only now applied individually to the x and y directions.

LOOKING AHEAD

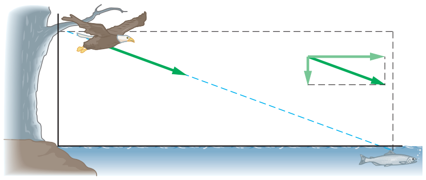
The basic idea behind projectile motion will be used again in Chapter 12, when we consider orbital motion. See, in particular, the illustration presented in Section 12–1.

Two-dimensional kinematics comes up again when we study the motion of charged particles (like electrons) in electric fields. To see the connection, compare Figures 19–41 and 22–10 (a) with the person jumping a crevasse in Example 4–4. The same basic principles apply.

CHAPTER SUMMARY

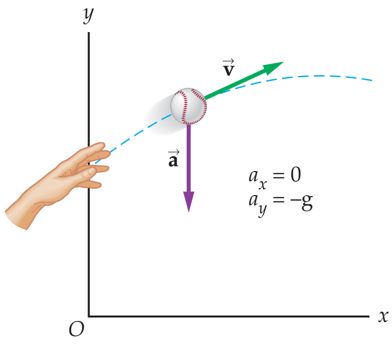
4–1 MOTION IN TWO DIMENSIONS

Independence of Motion
Components of motion in the x and y directions can be treated independently of one another. Thus, two-dimensional motion with constant acceleration is described by the same kinematic equations derived in Chapter 2, only now written in terms of x and y components.



4–2 PROJECTILE MOTION: BASIC EQUATIONS

Projectile motion refers to the path of an object after it is thrown, kicked, batted, or otherwise launched into the air. For the ideal case, we assume no air resistance and a constant downward acceleration of magnitude g.



Acceleration Components
In projectile motion, with the x axis horizontal and the y axis upward, the components of the acceleration of gravity are

$$a_x = 0$$
$$a_y = -g$$

***x* and *y* as Functions of Time**
The x and y equations of motion are

$$x = x_0 + v_{0x}t$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad 4\text{–}6$$

v_x and v_y as Functions of Time
The velocity components vary with time as follows:

$$v_x = v_{0x}$$
$$v_y = v_{0y} - gt \qquad 4\text{–}6$$

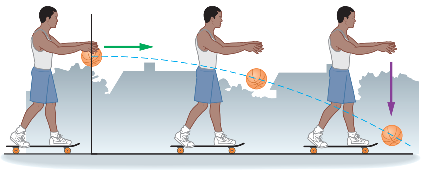
v_x and v_y as Functions of Displacement
v_x and v_y vary with displacement as

$$v_x^2 = v_{0x}^2$$
$$v_y^2 = v_{0y}^2 - 2g\Delta y \qquad 4\text{–}6$$

4–3 ZERO LAUNCH ANGLE

Equations of Motion

A projectile launched horizontally from $x_0 = 0$, $y_0 = h$ with an initial speed v_0 has the following equations of motion:



$$x = v_0 t \qquad v_x = v_0 \qquad v_x^2 = v_0^2$$
$$y = h - \tfrac{1}{2}gt^2 \qquad v_y = -gt \qquad v_y^2 = -2g\Delta y \tag{4–7}$$

Parabolic Path

The path followed by a projectile launched horizontally with an initial speed v_0 is described by

$$y = h - \left(\frac{g}{2v_0^2}\right)x^2 \tag{4–8}$$

This path is a parabola.

Landing Site

The landing site of a projectile launched horizontally is

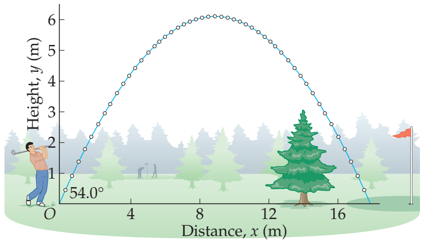
$$x = v_0\sqrt{\frac{2h}{g}} \tag{4–9}$$

In this expression, v_0 is the initial speed and h is the initial height. Note that this result is simply the speed in the x direction multiplied by the time of fall.

4–4 GENERAL LAUNCH ANGLE

Launch from the Origin

The equations of motion for a launch from the origin with an initial speed v_0 at an angle of θ with respect to the horizontal are

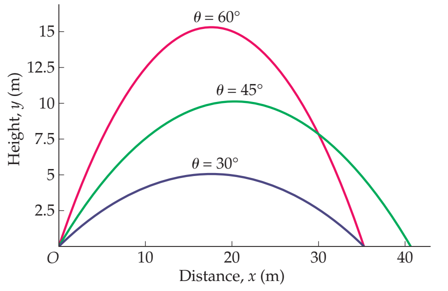


$$x = (v_0 \cos\theta)t \qquad v_x = v_0\cos\theta \qquad v_x^2 = v_0^2\cos^2\theta$$
$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \qquad v_y = v_0\sin\theta - gt \qquad v_y^2 = v_0^2\sin^2\theta - 2g\Delta y \tag{4–10}$$

4–5 PROJECTILE MOTION: KEY CHARACTERISTICS

Range

The range of a projectile launched from the origin with an initial speed v_0 and a launch angle θ is



$$R = \left(\frac{v_0^2}{g}\right)\sin 2\theta \tag{4–12}$$

This expression applies only to projectiles that land at the same level from which they were launched.

Symmetry

Projectile motion exhibits many symmetries. For example, the speed of a projectile depends only on its height and not on whether it is moving upward or downward.

Maximum Height

The maximum height of a projectile above its launch site is

$$y_{max} = \frac{v_0^2\sin^2\theta}{2g}$$

In this equation, v_0 is the initial speed and θ is the launch angle.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Study two-dimensional motion with constant acceleration.	Motion in the x direction is independent of motion in the y direction. This is the basis for the equations of motion given in Table 4–1. Note that these equations are the same as the kinematic equations of Chapter 2, only written in terms of x and y components.	Examples 4–1, 4–2
Find the location and velocity of a projectile launched horizontally.	When a projectile is launched horizontally with a speed v_0 its initial velocity components are $v_{0x} = v_0$ and $v_{0y} = 0$. Make these substitutions in the equations of projectile motion given in Equations 4–6.	Examples 4–3, 4–4 Conceptual Checkpoints 4–1, 4–2
Find the location and velocity of a projectile launched with an arbitrary launch angle.	If a projectile is launched at an angle θ, its initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. Make these substitutions in the equations of projectile motion given in Equations 4–6.	Examples 4–5, 4–6, 4–7 Active Examples 4–1, 4–2

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. What is the acceleration of a projectile when it reaches its highest point? What is its acceleration just before and just after reaching this point?
2. A projectile is launched with an initial speed of v_0 at an angle θ above the horizontal. It lands at the same level from which it was launched. What was its average velocity between launch and landing? Explain.
3. A projectile is launched from level ground. When it lands, its direction of motion has rotated clockwise through 60°. What was the launch angle? Explain.
4. In a game of baseball, a player hits a high fly ball to the outfield. **(a)** Is there a point during the flight of the ball where its velocity is parallel to its acceleration? **(b)** Is there a point where the ball's velocity is perpendicular to its acceleration? Explain in each case.
5. A projectile is launched with an initial velocity of $\vec{\mathbf{v}} = (4\text{ m/s})\hat{\mathbf{x}} + (3\text{ m/s})\hat{\mathbf{y}}$. What is the velocity of the projectile when it reaches its highest point? Explain.
6. A projectile is launched from a level surface with an initial velocity of $\vec{\mathbf{v}} = (2\text{ m/s})\hat{\mathbf{x}} + (4\text{ m/s})\hat{\mathbf{y}}$. What is the velocity of the projectile just before it lands? Explain.
7. Do projectiles for which air resistance is nonnegligible, such as a bullet fired from a rifle, have maximum range when the launch angle is greater than, less than, or equal to 45°? Explain.
8. Two projectiles are launched from the same point at the same angle above the horizontal. Projectile 1 reaches a maximum height twice that of projectile 2. What is the ratio of the initial speed of projectile 1 to the initial speed of projectile 2? Explain.
9. A child rides on a pony walking with constant velocity. The boy leans over to one side and a scoop of ice cream falls from his ice cream cone. Describe the path of the scoop of ice cream as seen by **(a)** the child and **(b)** his parents standing on the ground nearby.
10. Driving down the highway, you find yourself behind a heavily loaded tomato truck. You follow close behind the truck, keeping the same speed. Suddenly a tomato falls from the back of the truck. Will the tomato hit your car or land on the road, assuming you continue moving with the same speed and direction? Explain.
11. A projectile is launched from the origin of a coordinate system where the positive x axis points horizontally to the right and the positive y axis points vertically upward. What was the projectile's launch angle with respect to the x axis if, at its highest point, its direction of motion has rotated **(a)** clockwise through 50° or **(b)** counterclockwise through 30°? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(Air resistance should be ignored in the problems for this chapter, unless specifically stated otherwise.)

SECTION 4–1 MOTION IN TWO DIMENSIONS

1. • **CE Predict/Explain** As you walk briskly down the street, you toss a small ball into the air. **(a)** If you want the ball to land in your hand when it comes back down, should you toss the ball straight upward, in a forward direction, or in a backward direction, relative to your body? **(b)** Choose the *best explanation* from among the following:
   - **I.** If the ball is thrown straight up you will leave it behind.
   - **II.** You have to throw the ball in the direction you are walking.
   - **III.** The ball moves in the forward direction with your walking speed at all times.

2. • A sailboat runs before the wind with a constant speed of 4.2 m/s in a direction 32° north of west. How far **(a)** west and **(b)** north has the sailboat traveled in 25 min?

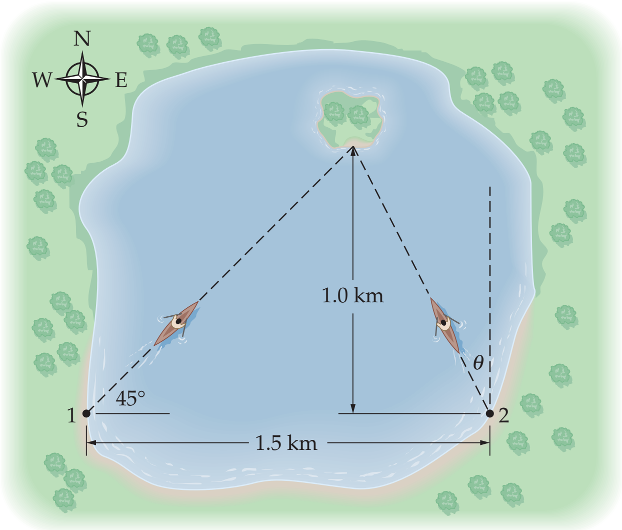
3. • As you walk to class with a constant speed of 1.75 m/s, you are moving in a direction that is 18.0° north of east. How much time does it take to change your displacement by **(a)** 20.0 m east or **(b)** 30.0 m north?

4. • Starting from rest, a car accelerates at $2.0\ \mathrm{m/s^2}$ up a hill that is inclined 5.5° above the horizontal. How far **(a)** horizontally and **(b)** vertically has the car traveled in 12 s?

5. •• **IP** A particle passes through the origin with a velocity of $(6.2\ \mathrm{m/s})\hat{\mathbf{y}}$. If the particle's acceleration is $(-4.4\ \mathrm{m/s^2})\hat{\mathbf{x}}$, **(a)** what are its x and y positions after 5.0 s? **(b)** What are v_x and v_y at this time? **(c)** Does the speed of this particle increase with time, decrease with time, or increase and then decrease? Explain.

6. •• An electron in a cathode-ray tube is traveling horizontally at $2.10 \times 10^9\ \mathrm{cm/s}$ when deflection plates give it an upward acceleration of $5.30 \times 10^{17}\ \mathrm{cm/s^2}$. **(a)** How long does it take for the electron to cover a horizontal distance of 6.20 cm? **(b)** What is its vertical displacement during this time?

7. •• Two canoeists start paddling at the same time and head toward a small island in a lake, as shown in **Figure 4–12**. Canoeist 1 paddles with a speed of 1.35 m/s at an angle of 45° north of east. Canoeist 2 starts on the opposite shore of the lake, a distance of 1.5 km due east of canoeist 1. **(a)** In what direction relative to north must canoeist 2 paddle to reach the island? **(b)** What speed must canoeist 2 have if the two canoes are to arrive at the island at the same time?



▲ **FIGURE 4–12** Problem 7

SECTION 4–3 ZERO LAUNCH ANGLE

8. • **CE Predict/Explain** Two divers run horizontally off the edge of a low cliff. Diver 2 runs with twice the speed of diver 1. **(a)** When the divers hit the water, is the horizontal distance covered by diver 2 twice as much, four times as much, or equal to the horizontal distance covered by diver 1? **(b)** Choose the *best explanation* from among the following:
   I. The drop time is the same for both divers.
   II. Drop distance depends on t^2.
   III. All divers in free fall cover the same distance.

9. • **CE Predict/Explain** Two youngsters dive off an overhang into a lake. Diver 1 drops straight down, and diver 2 runs off the cliff with an initial horizontal speed v_0. **(a)** Is the splashdown speed of diver 2 greater than, less than, or equal to the splashdown speed of diver 1? **(b)** Choose the *best explanation* from among the following:
   I. Both divers are in free fall, and hence they will have the same splashdown speed.
   II. The divers have the same vertical speed at splashdown, but diver 2 has the greater horizontal speed.
   III. The diver who drops straight down gains more speed than the one who moves horizontally.

10. • An archer shoots an arrow horizontally at a target 15 m away. The arrow is aimed directly at the center of the target, but it hits 52 cm lower. What was the initial speed of the arrow?

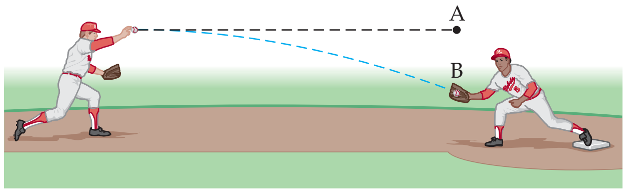
11. • **Victoria Falls** The great, gray-green, greasy Zambezi River flows over Victoria Falls in south central Africa. The falls are approximately 108 m high. If the river is flowing horizontally at 3.60 m/s just before going over the falls, what is the speed of the water when it hits the bottom? Assume the water is in free fall as it drops.

12. • A diver runs horizontally off the end of a diving board with an initial speed of 1.85 m/s. If the diving board is 3.00 m above the water, what is the diver's speed just before she enters the water?

13. • An astronaut on the planet Zircon tosses a rock horizontally with a speed of 6.95 m/s. The rock falls through a vertical distance of 1.40 m and lands a horizontal distance of 8.75 m from the astronaut. What is the acceleration of gravity on Zircon?

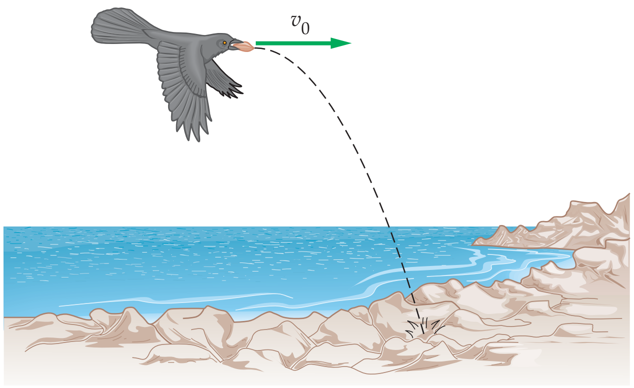
14. •• **IP Pitcher's Mounds** Pitcher's mounds are raised to compensate for the vertical drop of the ball as it travels a horizontal distance of 18 m to the catcher. **(a)** If a pitch is thrown horizontally with an initial speed of 32 m/s, how far does it drop by the time it reaches the catcher? **(b)** If the speed of the pitch is increased, does the drop distance increase, decrease, or stay the same? Explain. **(c)** If this baseball game were to be played on the Moon, would the drop distance increase, decrease, or stay the same? Explain.

15. •• Playing shortstop, you pick up a ground ball and throw it to second base. The ball is thrown horizontally, with a speed of 22 m/s, directly toward point A (**Figure 4–13**). When the ball reaches the second baseman 0.45 s later, it is caught at point B. **(a)** How far were you from the second baseman? **(b)** What is the distance of vertical drop, AB?



▲ **FIGURE 4–13** Problem 15

16. •• **IP** A crow is flying horizontally with a constant speed of 2.70 m/s when it releases a clam from its beak (**Figure 4–14**). The clam lands on the rocky beach 2.10 s later. Just before the clam lands, what is **(a)** its horizontal component of velocity, and **(b)** its vertical component of velocity? **(c)** How would your answers to parts (a) and (b) change if the speed of the crow were increased? Explain.

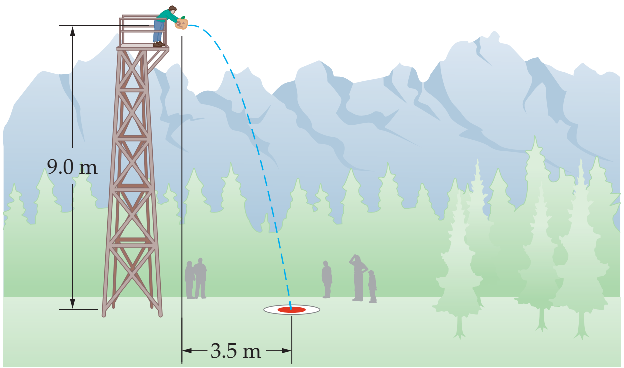


▲ **FIGURE 4–14** Problem 16

17. •• A mountain climber jumps a 2.8-m-wide crevasse by leaping horizontally with a speed of 7.8 m/s. **(a)** If the climber's direction of motion on landing is −45°, what is the height difference between the two sides of the crevasse? **(b)** Where does the climber land?

18. •• **IP** A white-crowned sparrow flying horizontally with a speed of 1.80 m/s folds its wings and begins to drop in free fall. **(a)** How far does the sparrow fall after traveling a horizontal distance of 0.500 m? **(b)** If the sparrow's initial speed is increased, does the distance of fall increase, decrease, or stay the same?

19. •• **Pumpkin Toss** In Denver, children bring their old jack-o-lanterns to the top of a tower and compete for accuracy in hitting a target on the ground (**Figure 4–15**). Suppose that the tower is 9.0 m high and that the bull's-eye is a horizontal distance of 3.5 m from the launch point. If the pumpkin is thrown horizontally, what is the launch speed needed to hit the bull's-eye?

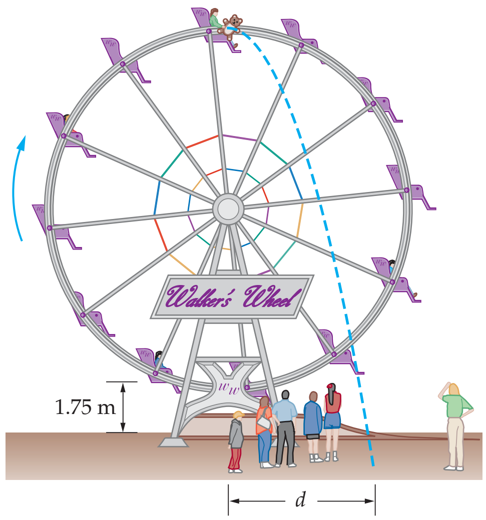


▲ **FIGURE 4–15** Problems 19 and 20

20. •• If, in the previous problem, a jack-o-lantern is given an initial horizontal speed of 3.3 m/s, what are the direction and magnitude of its velocity **(a)** 0.75 s later, and **(b)** just before it lands?

21. •• Fairgoers ride a Ferris wheel with a radius of 5.00 m (**Figure 4–16**). The wheel completes one revolution every 32.0 s. **(a)** What is the average speed of a rider on this Ferris wheel? **(b)** If a rider accidentally drops a stuffed animal at the top of the wheel, where does it land relative to the base of the ride? (Note: The bottom of the wheel is 1.75 m above the ground.)

22. •• **IP** A swimmer runs horizontally off a diving board with a speed of 3.32 m/s and hits the water a horizontal distance of 1.78 m from the end of the board. **(a)** How high above the water was the diving board? **(b)** If the swimmer runs off the board

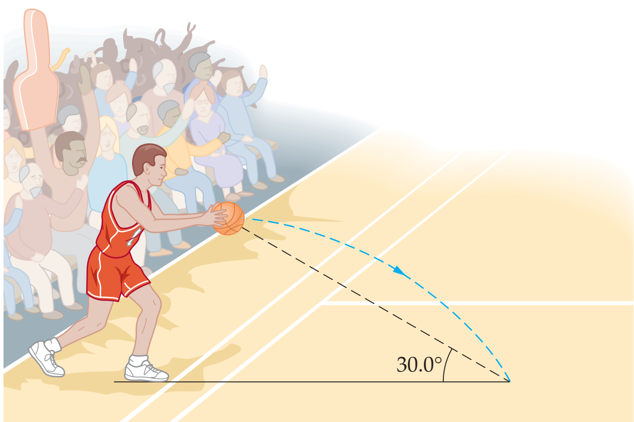


▲ **FIGURE 4–16** Problems 21 and 42

with a reduced speed, does it take more, less, or the same time to reach the water?

23. •• **Baseball and the Washington Monument** On August 25, 1894, Chicago catcher William Schriver caught a baseball thrown from the top of the Washington Monument (555 ft, 898 steps). **(a)** If the ball was thrown horizontally with a speed of 5.00 m/s, where did it land? **(b)** What were the ball's speed and direction of motion when caught?

24. ••• A basketball is thrown horizontally with an initial speed of 4.20 m/s (**Figure 4–17**). A straight line drawn from the release point to the landing point makes an angle of 30.0° with the horizontal. What was the release height?



▲ **FIGURE 4–17** Problem 24

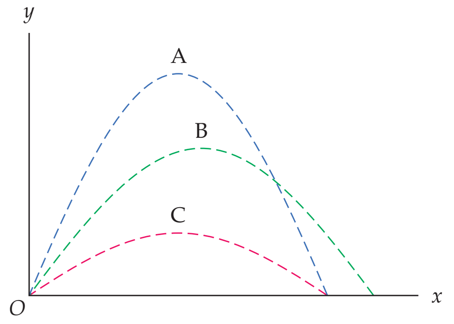
25. ••• **IP** A ball rolls off a table and falls 0.75 m to the floor, landing with a speed of 4.0 m/s. **(a)** What is the acceleration of the ball just before it strikes the ground? **(b)** What was the initial speed of the ball? **(c)** What initial speed must the ball have if it is to land with a speed of 5.0 m/s?

SECTION 4–4 GENERAL LAUNCH ANGLE

26. • **CE** A certain projectile is launched with an initial speed v_0. At its highest point its speed is $\frac{1}{2}v_0$. What was the launch angle of the projectile?

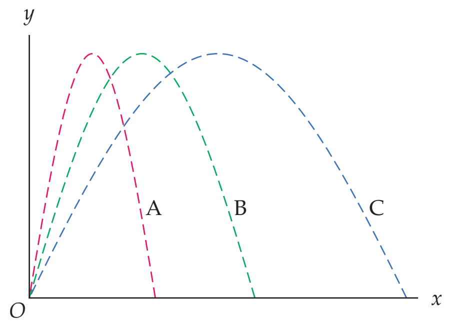
A. 30° **B.** 45° **C.** 60° **D.** 75°

27. • **CE** Three projectiles (A, B, and C) are launched with the same initial speed but with different launch angles, as shown in Figure 4–18. Rank the projectiles in order of increasing **(a)** horizontal component of initial velocity and **(b)** time of flight. Indicate ties where appropriate.



▲ **FIGURE 4–18** Problem 27

28. • **CE** Three projectiles (A, B, and C) are launched with different initial speeds so that they reach the same maximum height, as shown in Figure 4–19. Rank the projectiles in order of increasing **(a)** initial speed and **(b)** time of flight. Indicate ties where appropriate.



▲ **FIGURE 4–19** Problem 28

29. • A second baseman tosses the ball to the first baseman, who catches it at the same level from which it was thrown. The throw is made with an initial speed of 18.0 m/s at an angle of 37.5° above the horizontal. **(a)** What is the horizontal component of the ball's velocity just before it is caught? **(b)** How long is the ball in the air?

30. • Referring to the previous problem, what are the y component of the ball's velocity and its direction of motion just before it is caught?

31. • A cork shoots out of a champagne bottle at an angle of 35.0° above the horizontal. If the cork travels a horizontal distance of 1.30 m in 1.25 s, what was its initial speed?

32. • A soccer ball is kicked with a speed of 9.85 m/s at an angle of 35.0° above the horizontal. If the ball lands at the same level from which it was kicked, how long was it in the air?

33. •• In a game of basketball, a forward makes a bounce pass to the center. The ball is thrown with an initial speed of 4.3 m/s at an angle of 15° below the horizontal. It is released 0.80 m above the floor. What horizontal distance does the ball cover before bouncing?

34. •• Repeat the previous problem for a bounce pass in which the ball is thrown 15° *above* the horizontal.

35. •• **IP** Snowballs are thrown with a speed of 13 m/s from a roof 7.0 m above the ground. Snowball A is thrown straight downward; snowball B is thrown in a direction 25° above the horizontal. **(a)** Is the landing speed of snowball A greater than, less than, or the same as the landing speed of snowball B? Explain. **(b)** Verify your answer to part (a) by calculating the landing speed of both snowballs.

36. •• In the previous problem, find the direction of motion of the two snowballs just before they land.

37. •• A golfer gives a ball a maximum initial speed of 34.4 m/s. **(a)** What is the longest possible hole-in-one for this golfer? Neglect any distance the ball might roll on the green and assume that the tee and the green are at the same level. **(b)** What is the minimum speed of the ball during this hole-in-one shot?

38. •• What is the highest tree the ball in the previous problem could clear on its way to the longest possible hole-in-one?

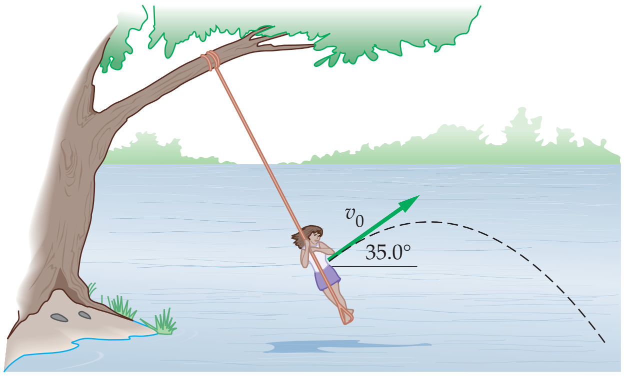
39. •• The "hang time" of a punt is measured to be 4.50 s. If the ball was kicked at an angle of 63.0° above the horizontal and was caught at the same level from which it was kicked, what was its initial speed?

40. •• In a friendly game of handball, you hit the ball essentially at ground level and send it toward the wall with a speed of 18 m/s at an angle of 32° above the horizontal. **(a)** How long does it take for the ball to reach the wall if it is 3.8 m away? **(b)** How high is the ball when it hits the wall?

41. •• **IP** In the previous problem, **(a)** what are the magnitude and direction of the ball's velocity when it strikes the wall? **(b)** Has the ball reached the highest point of its trajectory at this time? Explain.

42. •• A passenger on the Ferris wheel described in Problem 21 drops his keys when he is on the way up and at the 10 o'clock position. Where do the keys land relative to the base of the ride?

43. •• On a hot summer day, a young girl swings on a rope above the local swimming hole (Figure 4–20). When she lets go of the rope her initial velocity is 2.25 m/s at an angle of 35.0° above the horizontal. If she is in flight for 0.616 s, how high above the water was she when she let go of the rope?



▲ **FIGURE 4–20** Problem 43

44. •• A certain projectile is launched with an initial speed v_0. At its highest point its speed is $v_0/4$. What was the launch angle?

SECTION 4–5 PROJECTILE MOTION: KEY CHARACTERISTICS

45. • **Punkin Chunkin** In Sussex County, Delaware, a post-Halloween tradition is "Punkin Chunkin," in which contestants build cannons, catapults, trebuchets, and other devices to launch pumpkins and compete for the greatest distance. Though hard to believe, pumpkins have been projected a distance of 4086 feet in this contest. What is the minimum initial speed needed for such a shot?

46. • A dolphin jumps with an initial velocity of 12.0 m/s at an angle of 40.0° above the horizontal. The dolphin passes through the center of a hoop before returning to the water. If the dolphin is moving horizontally when it goes through the hoop, how high above the water is the center of the hoop?

47. • A player passes a basketball to another player who catches it at the same level from which it was thrown. The initial speed of the ball is 7.1 m/s, and it travels a distance of 4.6 m. What were **(a)** the initial direction of the ball and **(b)** its time of flight?

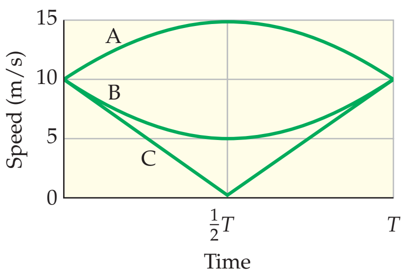
48. • A golf ball is struck with a five iron on level ground. It lands 92.2 m away 4.30 s later. What were **(a)** the direction and **(b)** the magnitude of the initial velocity?

49. • **A Record Toss** Babe Didrikson holds the world record for the longest baseball throw (296 ft) by a woman. For the following questions, assume that the ball was thrown at an angle of 45.0° above the horizontal, that it traveled a horizontal distance of 296 ft, and that it was caught at the same level from which it was thrown. **(a)** What was the ball's initial speed? **(b)** How long was the ball in the air?

50. • In the photograph to the left on page 87, suppose the cart that launches the ball is 11 cm high. Estimate **(a)** the launch speed of the ball and **(b)** the time interval between successive stroboscopic exposures.

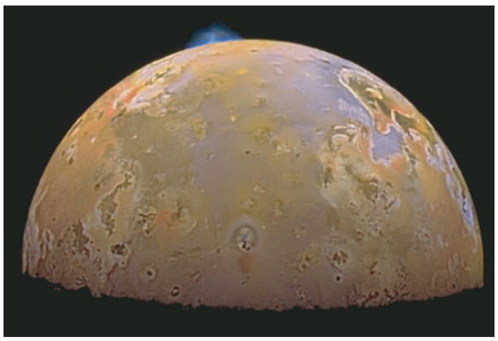
51. •• **CE Predict/Explain** You throw a ball into the air with an initial speed of 10 m/s at an angle of 60° above the horizontal. The ball returns to the level from which it was thrown in the time T. **(a)** Referring to **Figure 4–21**, which of the plots (A, B, or C) best represents the speed of the ball as a function of time? **(b)** Choose the *best explanation* from among the following:

I. Gravity causes the ball's speed to increase during its flight.
II. The ball has zero speed at its highest point.
III. The ball's speed decreases during its flight, but it doesn't go to zero.



▲ **FIGURE 4–21** Problem 51

52. •• **IP Volcanoes on Io** Astronomers have discovered several volcanoes on Io, a moon of Jupiter. One of them, named Loki, ejects lava to a maximum height of 2.00×10^5 m. **(a)** What is the initial speed of the lava? (The acceleration of gravity on Io is $1.80\ \mathrm{m/s^2}$.) **(b)** If this volcano were on Earth, would the maximum height of the ejected lava be greater than, less than, or the same as on Io? Explain.



A volcano on Io, the innermost moon of Jupiter, displays the characteristic features of projectile motion. (Problem 52)

53. •• **IP** A soccer ball is kicked with an initial speed of 10.2 m/s in a direction 25.0° above the horizontal. Find the magnitude and direction of its velocity **(a)** 0.250 s and **(b)** 0.500 s after being kicked. **(c)** Is the ball at its greatest height before or after 0.500 s? Explain.

54. •• A second soccer ball is kicked with the same initial speed as in Problem 53. After 0.750 s it is at its highest point. What was its initial direction of motion?

55. •• **IP** A golfer tees off on level ground, giving the ball an initial speed of 46.5 m/s and an initial direction of 37.5° above the horizontal. **(a)** How far from the golfer does the ball land? **(b)** The next golfer in the group hits a ball with the same initial speed but at an angle above the horizontal that is greater than 45.0°. If the second ball travels the same horizontal distance as the first ball, what was its initial direction of motion? Explain.

56. •• **IP** One of the most popular events at Highland games is the hay toss, where competitors use a pitchfork to throw a bale of hay over a raised bar. Suppose the initial velocity of a bale of hay is $\vec{\mathbf{v}} = (1.12\ \mathrm{m/s})\hat{\mathbf{x}} + (8.85\ \mathrm{m/s})\hat{\mathbf{y}}$. **(a)** After what minimum time is its speed equal to 5.00 m/s? **(b)** How long after the hay is tossed is it moving in a direction that is 45.0° below the horizontal? **(c)** If the bale of hay is tossed with the same initial speed, only this time straight upward, will its time in the air increase, decrease, or stay the same? Explain.

GENERAL PROBLEMS

57. • **CE** Child 1 throws a snowball horizontally from the top of a roof; child 2 throws a snowball straight down. Once in flight, is the acceleration of snowball 2 greater than, less than, or equal to the acceleration of snowball 1?

58. • **CE** The penguin to the left in the accompanying photo is about to land on an ice floe. Just before it lands, is its speed greater than, less than, or equal to its speed when it left the water?

This penguin behaves much like a projectile from the time it leaves the water until it touches down on the ice. (Problem 58)

59. • **CE Predict/Explain** A person flips a coin into the air and it lands on the ground a few feet away. **(a)** If the person were to perform an identical coin flip on an elevator rising with constant speed, would the coin's time of flight be greater than, less than, or equal to its time of flight when the person was at rest? **(b)** Choose the *best explanation* from among the following:

I. The floor of the elevator is moving upward, and hence it catches up with the coin in mid flight.

II. The coin has the same upward speed as the elevator when it is tossed, and the elevator's speed doesn't change during the coin's flight.

III. The coin starts off with a greater upward speed because of the elevator, and hence it reaches a greater height.

60. • **CE Predict/Explain** Suppose the elevator in the previous problem is rising with a constant upward acceleration, rather than constant velocity. **(a)** In this case, would the coin's time of flight be greater than, less than, or equal to its time of flight when the person was at rest? **(b)** Choose the *best explanation* from among the following:

I. The coin has the same acceleration once it is tossed, whether the elevator accelerates or not.

II. The elevator's upward speed increases during the coin's flight, and hence it catches up with the coin at a greater height than before.

III. The coin's downward acceleration is less than before because the elevator's upward acceleration partially cancels it.

61. • A train moving with constant velocity travels 170 m north in 12 s and an undetermined distance to the west. The speed of the train is 32 m/s. **(a)** Find the direction of the train's motion relative to north. **(b)** How far west has the train traveled in this time?

62. • Referring to Example 4–2, find **(a)** the x component and **(b)** the y component of the hummingbird's velocity at the time $t = 0.72$ s. **(c)** What is the bird's direction of travel at this time, relative to the positive x axis?

63. • A racket ball is struck in such a way that it leaves the racket with a speed of 4.87 m/s in the horizontal direction. When the ball hits the court, it is a horizontal distance of 1.95 m from the racket. Find the height of the racket ball when it left the racket.

64. •• **IP** A hot-air balloon rises from the ground with a velocity of $(2.00\ \text{m/s})\hat{\mathbf{y}}$. A champagne bottle is opened to celebrate takeoff, expelling the cork horizontally with a velocity of $(5.00\ \text{m/s})\hat{\mathbf{x}}$ relative to the balloon. When opened, the bottle is 6.00 m above the ground. **(a)** What is the initial velocity of the cork, as seen by an observer on the ground? Give your answer in terms of the x and y unit vectors. **(b)** What are the speed of the cork and its initial direction of motion as seen by the same observer? **(c)** Determine the maximum height above the ground attained by the cork. **(d)** How long does the cork remain in the air?

65. •• Repeat the previous problem, this time assuming that the balloon is *descending* with a speed of 2.00 m/s.

66. •• **IP** A soccer ball is kicked from the ground with an initial speed of 14.0 m/s. After 0.275 s its speed is 12.9 m/s. **(a)** Give a strategy that will allow you to calculate the ball's initial direction of motion. **(b)** Use your strategy to find the initial direction.

67. •• A particle leaves the origin with an initial velocity $\vec{\mathbf{v}} = (2.40\ \text{m/s})\hat{\mathbf{x}}$, and moves with constant acceleration $\vec{\mathbf{a}} = (-1.90\ \text{m/s}^2)\hat{\mathbf{x}} + (3.20\ \text{m/s}^2)\hat{\mathbf{y}}$. **(a)** How far does the particle move in the x direction before turning around? **(b)** What is the particle's velocity at this time? **(c)** Plot the particle's position at $t = 0.500$ s, 1.00 s, 1.50 s, and 2.00 s. Use these results to sketch position versus time for the particle.

68. •• When the dried-up seed pod of a scotch broom plant bursts open, it shoots out a seed with an initial velocity of 2.62 m/s at an angle of 60.5° above the horizontal. If the seed pod is 0.455 m above the ground, **(a)** how long does it take for the seed to land? **(b)** What horizontal distance does it cover during its flight?

69. •• Referring to Problem 68, a second seed shoots out from the pod with the same speed but with a direction of motion 30.0° below the horizontal. **(a)** How long does it take for the second seed to land? **(b)** What horizontal distance does it cover during its flight?

70. •• A shot-putter throws the shot with an initial speed of 12.2 m/s from a height of 5.15 ft above the ground. What is the range of the shot if the launch angle is **(a)** 20.0°, **(b)** 30.0°, or **(c)** 40.0°?

71. •• **Pararescue Jumpers** Coast Guard pararescue jumpers are trained to leap from helicopters into the sea to save boaters in distress. The rescuers like to step off their helicopter when it is "ten and ten", which means that it is *ten* feet above the water and moving forward horizontally at *ten* knots. What are **(a)** the speed and **(b)** the direction of motion as a pararescuer enters the water following a ten and ten jump?

72. •• A ball thrown straight upward returns to its original level in 2.75 s. A second ball is thrown at an angle of 40.0° above the horizontal. What is the initial speed of the second ball if it also returns to its original level in 2.75 s?

73. •• **IP** To decide who pays for lunch, a passenger on a moving train tosses a coin straight upward with an initial speed of 4.38 m/s and catches it again when it returns to its initial level. From the point of view of the passenger, then, the coin's initial velocity is $(4.38\ \text{m/s})\hat{\mathbf{y}}$. The train's velocity relative to the ground is $(12.1\ \text{m/s})\hat{\mathbf{x}}$. **(a)** What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain. **(b)** Find the initial speed and direction of the coin as seen by an observer on the ground. **(c)** Use the expression for y_{max} derived in Example 4–7 to calculate the maximum height of the coin, as seen by an observer on the ground. **(d)** Calculate the maximum height of the coin from the point of view of the passenger, who sees only one-dimensional motion.

74. •• **IP** A cannon is placed at the bottom of a cliff 61.5 m high. If the cannon is fired straight upward, the cannonball just reaches the top of the cliff. **(a)** What is the initial speed of the cannonball? **(b)** Suppose a second cannon is placed at the top of the cliff. This cannon is fired horizontally, giving its cannonballs the same initial speed found in part (a). Show that the range of this cannon is the same as the maximum range of the cannon at the base of the cliff. (Assume the ground at the base of the cliff is level, though the result is valid even if the ground is not level.)

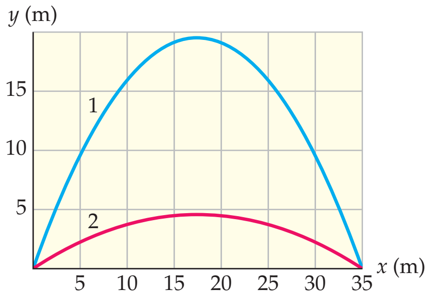
75. •• **Shot Put Record** The men's world record for the shot put, 23.12 m, was set by Randy Barnes of the United States on May 20, 1990. If the shot was launched from 6.00 ft above the ground at an initial angle of 42.0°, what was its initial speed?

76. •• Referring to Conceptual Checkpoint 4–3, suppose the two snowballs are thrown from an elevation of 15 m with an initial speed of 12 m/s. What is the speed of each ball when it is 5.0 m above the ground?

77. •• **IP** A hockey puck just clears the 2.00-m-high boards on its way out of the rink. The base of the boards is 20.2 m from the point where the puck is launched. **(a)** Given the launch angle of the puck, θ, outline a strategy that you can use to find its initial speed, v_0. **(b)** Use your strategy to find v_0 for $\theta = 15.0°$.

78. •• Referring to Active Example 4–2, suppose the ball is punted from an initial height of 0.750 m. What is the initial speed of the ball in this case?

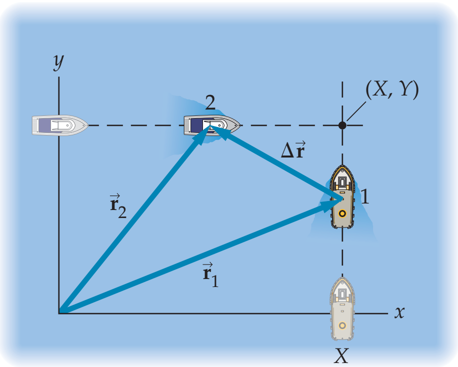
79. •• **A "Lob" Pass Versus a "Bullet"** A quarterback can throw a receiver a high, lazy "lob" pass or a low, quick "bullet" pass. These passes are indicated by curves 1 and 2, respectively, in **Figure 4–22**. **(a)** The lob pass is thrown with an initial speed of 21.5 m/s and its time of flight is 3.97 s. What is its launch angle?



▲ **FIGURE 4–22** Problem 79

(b) The bullet pass is thrown with a launch angle of 25.0°. What is the initial speed of this pass? **(c)** What is the time of flight of the bullet pass?

80. ••• **Collision Course** A useful rule of thumb in boating is that if the heading from your boat to a second boat remains constant, the two boats are on a collision course. Consider the two boats shown in **Figure 4–23**. At time $t = 0$, boat 1 is at the location $(X, 0)$ and moving in the positive y direction; boat 2 is at $(0, Y)$ and moving in the positive x direction. The speed of boat 1 is v_1. **(a)** What speed must boat 2 have if the boats are to collide at the point (X, Y)? **(b)** Assuming boat 2 has the speed found in part (a), calculate the displacement from boat 1 to boat 2, $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$. **(c)** Use your results from part (b) to show that $(\Delta r)_y/(\Delta r)_x = -Y/X$, independent of time. This shows that $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$ maintains a constant direction until the collision, as specified in the rule of thumb.



▲ **FIGURE 4–23** Problem 80

81. ••• As discussed in Example 4–7, the archerfish hunts by dislodging an unsuspecting insect from its resting place with a stream of water expelled from the fish's mouth. Suppose the archerfish squirts water with a speed of 2.15 m/s at an angle of 52.0° above the horizontal, and aims for a beetle on a leaf 3.00 cm above the water's surface. **(a)** At what horizontal distance from the beetle should the archerfish fire if it is to hit its target in the least time? **(b)** How much time will the beetle have to react?

82. ••• **(a)** What is the greatest horizontal distance from which the archerfish can hit the beetle, assuming the same squirt speed and direction as in Problem 81? **(b)** How much time does the beetle have to react in this case?

83. ••• Find the launch angle for which the range and maximum height of a projectile are the same.

84. ••• A mountain climber jumps a crevasse of width W by leaping horizontally with speed v_0. **(a)** If the height difference between the two sides of the crevasse is h, what is the minimum value of v_0 for the climber to land safely on the other side? **(b)** In this case, what is the climber's direction of motion on landing?

85. ••• Prove that the landing speed of a projectile is independent of launch angle for a given height of launch.

86. •• **Maximum Height and Range** Prove that the maximum height of a projectile, H, divided by the range of the projectile, R, satisfies the relation $H/R = \frac{1}{4}\tan\theta$.

87. •• **Landing on a Different Level** A projectile fired from $y = 0$ with initial speed v_0 and initial angle θ lands on a different level, $y = h$. Show that the time of flight of the projectile is

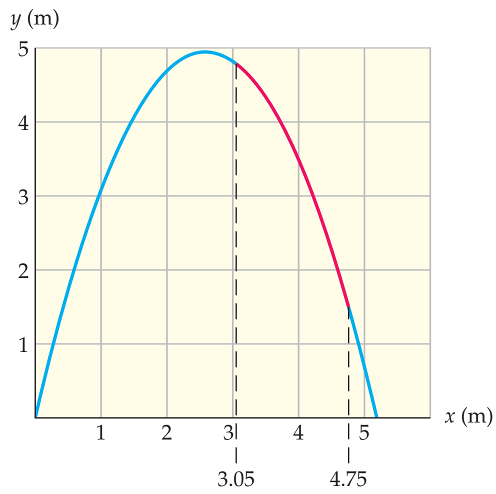
$$T = \tfrac{1}{2}T_0\left(1 + \sqrt{1 - \frac{h}{H}}\right)$$

where T_0 is the time of flight for $h = 0$ and H is the maximum height of the projectile.

88. ••• A mountain climber jumps a crevasse by leaping horizontally with speed v_0. If the climber's direction of motion on landing is θ below the horizontal, what is the height difference h between the two sides of the crevasse?

89. •••**IP** Referring to Problem 73, suppose the initial velocity of the coin tossed by the passenger is $\vec{\mathbf{v}} = (-2.25\text{ m/s})\hat{\mathbf{x}} + (4.38\text{ m/s})\hat{\mathbf{y}}$. The train's velocity relative to the ground is still $(12.1\text{ m/s})\hat{\mathbf{x}}$. **(a)** What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain. **(b)** Find the initial speed and direction of the coin as seen by an observer on the ground. **(c)** Use the expression for y_{max} derived in Example 4–7 to calculate the maximum height of the coin, as seen by an observer on the ground. **(d)** Repeat part (c) from the point of view of the passenger. Verify that both observers calculate the same maximum height.

90. ••• **Projectiles: Coming or Going?** Most projectiles continually move farther from the origin during their flight, but this is not the case if the launch angle is greater than $\cos^{-1}\left(\frac{1}{3}\right) = 70.5°$. For example, the projectile shown in **Figure 4–24** has a launch angle of 75.0° and an initial speed of 10.1 m/s. During the portion of its motion shown in red, it is moving closer to the origin—it is moving away on the blue portions. Calculate the distance from the origin to the projectile **(a)** at the start of the red portion, **(b)** at the end of the red portion, and **(c)** just before the projectile lands. Notice that the distance for part (b) is the smallest of the three.



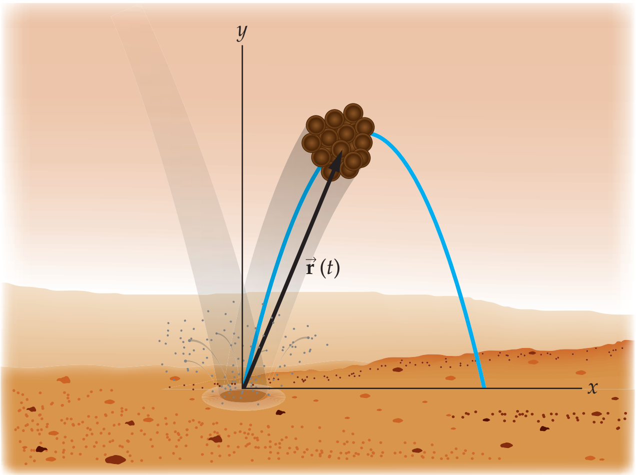
▲ **FIGURE 4–24** Problem 90

PASSAGE PROBLEMS

Landing Rovers on Mars

When the twin Mars exploration rovers, *Spirit* and *Opportunity*, set down on the surface of the red planet in January of 2004, their method of landing was both unique and elaborate. After initial braking with retro rockets, the rovers began their long descent through the thin Martian atmosphere on a parachute until they reached an altitude of about 16.7 m. At that point a system of four air bags with six lobes each were inflated, additional retro rocket blasts brought the craft to a virtual standstill, and the rovers detached from their parachutes. After a period of free fall to the surface, with an acceleration of 3.72 m/s^2, the rovers bounced about a dozen times before coming to rest. They then deflated their air bags, righted themselves, and began to explore the surface.

Figure 4–25 shows a rover with its surrounding cushion of air bags making its first contact with the Martian surface. After a typical first bounce the upward velocity of a rover would be 9.92 m/s at an angle of 75.0° above the horizontal. Assume this is the case for the problems that follow.



▲ **FIGURE 4–25** Problems 91, 92, 93, and 94

91. • What is the maximum height of a rover between its first and second bounces?

A. 2.58 m **B.** 4.68 m

C. 12.3 m **D.** 148 m

92. • How much time elapses between the first and second bounces?

A. 1.38 s **B.** 2.58 s

C. 5.15 s **D.** 5.33 s

93. • How far does a rover travel in the horizontal direction between its first and second bounces?

A. 13.2 m **B.** 49.4 m

C. 51.1 m **D.** 98.7 m

94. •• What is the average velocity of a rover between its first and second bounces?

A. 0

B. 2.57 m/s in the x direction

C. 9.92 m/s at 75.0° above the x axis

D. 9.58 m/s in the y direction

INTERACTIVE PROBLEMS

95. •• **Referring to Example 4–5 (a)** At what launch angle *greater* than 54.0° does the golf ball just barely miss the top of the tree in front of the green? Assume the ball has an initial speed of 13.5 m/s, and that the tree is 3.00 m high and is a horizontal distance of 14.0 m from the launch point. **(b)** Where does the ball land in the case described in part (a)? **(c)** At what launch angle *less* than 54.0° does the golf ball just barely miss the top of the tree in front of the green? **(d)** Where does the ball land in the case described in part (c)?

96. •• **Referring to Example 4–5** Suppose that the golf ball is launched with a speed of 15.0 m/s at an angle of 57.5° above the horizontal, and that it lands on a green 3.50 m above the level where it was struck. **(a)** What horizontal distance does the ball cover during its flight? **(b)** What increase in initial speed would be needed to increase the horizontal distance in part (a) by 7.50 m? Assume everything else remains the same.

97. •• **Referring to Example 4–6** Suppose the ball is dropped at the horizontal distance of 5.50 m, but from a new height of 5.00 m. The dolphin jumps with the same speed of 12.0 m/s. **(a)** What launch angle must the dolphin have if it is to catch the ball? **(b)** At what height does the dolphin catch the ball in this case? **(c)** What is the minimum initial speed the dolphin must have to catch the ball before it hits the water?

98. •• **IP Referring to Example 4–6** Suppose we change the dolphin's launch angle to 45.0°, but everything else remains the same. Thus, the horizontal distance to the ball is 5.50 m, the drop height is 4.10 m, and the dolphin's launch speed is 12.0 m/s. **(a)** What is the vertical distance between the dolphin and the ball when the dolphin reaches the horizontal position of the ball? We refer to this as the "miss distance." **(b)** If the dolphin's launch speed is reduced, will the miss distance increase, decrease, or stay the same? **(c)** Find the miss distance for a launch speed of 10.0 m/s.

5 Newton's Laws of Motion

Bobsledders know that a force is required to accelerate an object. In fact, the greater the force, the greater the acceleration. What they may not realize, however, is that forces always come in pairs that are equal in magnitude but opposite in direction. For example, when these athletes push on the bobsled, it pushes back on them with equal strength. All of these observations follow directly from Newton's three laws of motion, the subject of this chapter.

We are all subject to Newton's laws of motion, whether we know it or not. You can't move your body, drive a car, or toss a ball in a way that violates his rules. In short, our very existence is constrained and regulated by these three fundamental statements concerning matter and its motion.

Yet Newton's laws are surprisingly simple, especially when you consider that they apply equally well to galaxies, planets, comets, and yes, even apples falling from trees. In this chapter we present the three laws of Newton, and we show how they can be applied to everyday situations. Using them, we go beyond a simple description of motion, as in kinematics, to a study of the *causes* of motion, referred to as **dynamics.**

With the advent of Newtonian dynamics in 1687, science finally became quantitative and predictive. Edmund Halley, inspired by Newton's laws, used them to predict the return of the comet that today bears his name. In all of recorded history, no one had ever before predicted the appearance of a comet; in fact, they were generally regarded as supernatural apparitions. Though Halley didn't live to see his comet's return, his correct prediction illustrated the power of Newton's laws in a most dramatic and memorable way.

Today, we still recognize Newton's laws as the indispensable foundation for all of physics. It would be nice to say that these laws are the complete story when it comes to analyzing motion, but that is not the case. In the early part of the last century, physicists discovered that Newton's laws must be modified for objects moving at speeds near that of light and for objects comparable in size to atoms. In the world of everyday experience, however, Newton's laws still reign supreme.

5–1 Force and Mass

A **force,** simply put, is a push or a pull. When you push on a box to slide it across the floor, for example, or pull on the handle of a wagon to give a child a ride, you are exerting a force. Similarly, when you hold this book in your hand, you exert an upward force to oppose the downward pull of gravity. If you set the book on a table, the table exerts the same upward force you exerted a moment before. Forces are truly all around us.

Now, when you push or pull on something, there are two quantities that characterize the force you are exerting. The first is the strength or **magnitude** of your force; the second is the **direction** in which you are pushing or pulling. Because a force is determined by both a magnitude and a direction, it is a vector. We consider the vector properties of forces in more detail in Section 5–5.

In general, an object has several forces acting on it at any given time. In the previous example, a book at rest on a table experiences a downward force due to gravity and an upward force due to the table. If you push the book across the table, it also experiences a horizontal force due to your push. The total, or net, force exerted on the book is the vector sum of the individual forces acting on it.

After the net force acting on an object, the second key ingredient in Newton's laws is the **mass** of an object, which is a measure of how difficult it is to change its velocity—to start an object moving if it is at rest, to bring it to rest if it is moving, or to change its direction of motion. For example, if you throw a baseball or catch one thrown to you, the force required is not too great. But if you want to start a car moving or to stop one that is coming at you, the force involved is much greater. It follows that the mass of a car is greater than the mass of a baseball.

In agreement with everyday usage, mass can also be thought of as a measure of the quantity of matter in an object. Thus, it is clear that the mass of an automobile, for example, is much greater than the mass of a baseball, but much less than the mass of Earth. We measure mass in units of kilograms (kg), where one kilogram is defined as the mass of a standard cylinder of platinum-iridium, as discussed in Chapter 1. A list of typical masses is given in Table 5–1.

TABLE 5–1 Typical Masses in Kilograms (kg)

Earth	5.97×10^{24}
Space shuttle	2,000,000
Blue whale (largest animal on Earth)	178,000
Whale shark (largest fish)	18,000
Elephant (largest land animal)	5400
Automobile	1200
Human (adult)	70
Gallon of milk	3.6
Quart of milk	0.9
Baseball	0.145
Honeybee	0.00015
Bacterium	10^{-15}

These properties of force and mass are developed in detail in the next three sections.

5–2 Newton's First Law of Motion

If you've ever stood in line at an airport, pushing your bags forward a few feet at a time, you know that as soon as you stop pushing the bags, they stop moving. Observations such as this often lead to the erroneous conclusion that a force is required for an object to move. In fact, according to Newton's first law of motion, a force is required only to *change* an object's motion.

What is missing in this analysis is the force of friction between the bags and the floor. When you stop pushing the bags, it is not true that they stop moving because they no longer have a force acting on them. On the contrary, there is a rather large *frictional force* between the bags and the floor. It is this force that causes the bags to come to rest.

To see how motion is affected by reducing friction, imagine that you slide on dirt into second base during a baseball game. You won't slide very far before stopping. On the other hand, if you slide with the same initial speed on a sheet of ice—where the friction is much less than on a ball field—you slide considerably farther. If you could reduce the friction more, you would slide even farther.

In the classroom, air tracks allow us to observe motion with practically no friction. An example of such a device is shown in **Figure 5–1**. Note that air is blown through small holes in the track, creating a cushion of air for a small "cart" to ride on. A cart placed at rest on a level track remains at rest—unless you push on it to get it started.

Once set in motion, the cart glides along with constant velocity—constant speed in a straight line—until it hits a bumper at the end of the track. The bumper

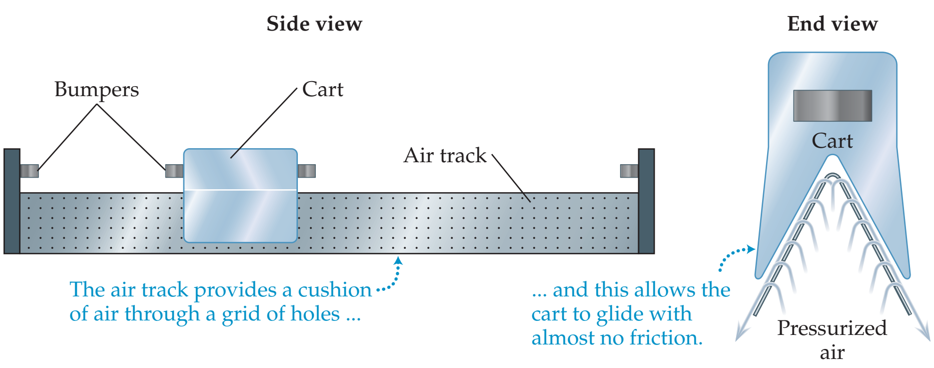


FIGURE 5–1 The air track
An air track provides a cushion of air on which a cart can ride with virtually no friction.

exerts a force on the cart, causing it to change its direction of motion. After bouncing off the bumper, the cart again moves with constant velocity. If the track could be extended to infinite length, and could be made perfectly frictionless, the cart would simply keep moving with constant velocity forever.

Newton's first law of motion summarizes these observations in the following statements:

Newton's First Law

An object at rest remains at rest as long as no net force acts on it.

An object moving with constant velocity continues to move with the same speed and in the same direction as long as no net force acts on it.

An air track provides a nearly frictionless environment for experiments involving linear motion.

Notice the recurring phrase, "no net force," in these statements. It is important to realize that this can mean one of two things: (i) no force acts on the object; or (ii) forces act on the object, but they sum to zero. We shall see examples of the second possibility later in this chapter and again in the next chapter.

Newton's first law, which was first enunciated by Galileo, is also known as the **law of inertia,** which is appropriate since the literal meaning of the word *inertia* is "laziness." Speaking loosely, we can say that matter is "lazy," in that it won't change its motion unless forced to do so. For example, if an object is at rest, it won't start moving on its own. If an object is already moving with constant velocity, it won't alter its speed *or* direction, unless a force causes the change. We call this property of matter its inertia.

According to Newton's first law, being at rest and moving with constant velocity are actually equivalent. To see this, imagine two observers: one is in a train moving with constant velocity; the second is standing next to the tracks, at rest on the ground. The observer in the train places an ice cube on a dinner tray. From that person's point of view—that is, in that person's **frame of reference**—the ice cube has no net force acting on it and it is at rest on the tray. It obeys the first law. In the frame of reference of the observer on the ground, the ice cube has no net force on it and it moves with constant velocity. This also agrees with the first law. Thus Newton's first law holds for both observers: They both see an ice cube with zero net force moving with constant velocity—it's just that for the first observer the constant velocity happens to be zero.

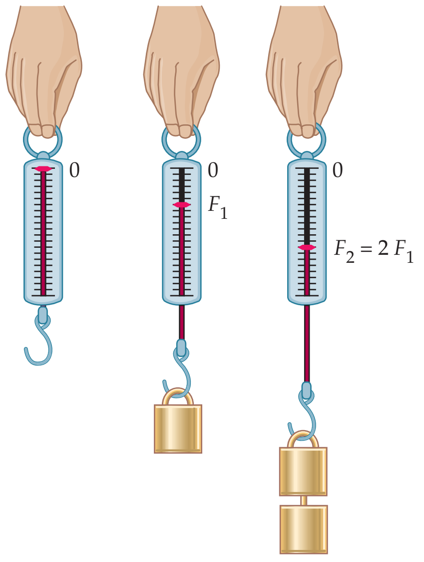
In this example, we say that each observer is in an **inertial frame of reference;** that is, a frame of reference in which the law of inertia holds. In general, if one frame is an inertial frame of reference, then any frame of reference that moves with constant velocity relative to that frame is also an inertial frame of reference. Thus, if an object moves with constant velocity in one inertial frame, it is always possible to find another inertial frame in which the object is at rest. It is in this sense that there really isn't any difference between being at rest and moving with constant velocity. It's all relative—relative to the frame of reference the object is viewed from.

This gives us a more compact statement of the first law:

If the net force on an object is zero, its velocity is constant.

As an example of a frame of reference that is not inertial, imagine that the train carrying the first observer suddenly comes to a halt. From the point of view of that observer, there is still no net force on the ice cube. However, because of the rapid braking, the ice cube flies off the tray. In fact, the ice cube simply continues to move forward with the same constant velocity while the *train* comes to rest. To the observer on the train, it appears that the ice cube has accelerated forward, even though no force acts on it, which is in violation of Newton's first law.

In general, any frame that accelerates relative to an inertial frame is a noninertial frame. The surface of the Earth accelerates slightly, due to its rotational and orbital motions, but since the acceleration is so small, it may be considered an excellent approximation to an inertial frame of reference. Unless specifically stated otherwise, we will always consider the surface of the Earth to be an inertial frame.



▲ **FIGURE 5–2 Calibrating a "force meter"** With two weights, the force exerted by the scale is twice the force exerted when only a single weight is attached.

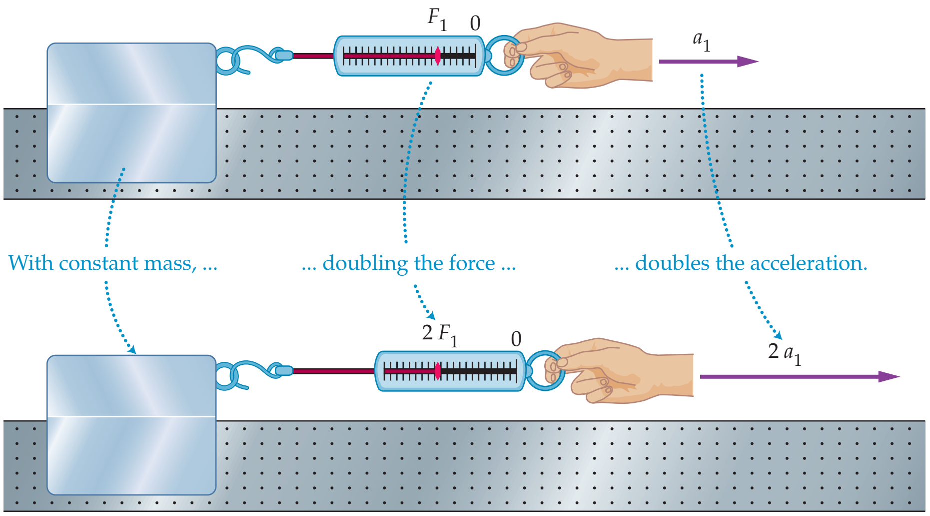
5–3 Newton's Second Law of Motion

To hold an object in your hand, you have to exert an upward force to oppose, or "balance," the force of gravity. If you suddenly remove your hand so that the only force acting on the object is gravity, it accelerates downward, as discussed in Chapter 2. This is one example of Newton's second law, which states, basically, that unbalanced forces cause accelerations.

To explore this in more detail, consider a spring scale of the type used to weigh fish. The scale gives a reading of the force, F, exerted by the spring contained within it. If we hang one weight from the scale, it gives a reading that we will call F_1. If two identical weights are attached, the scale reads $F_2 = 2F_1$, as indicated in **Figure 5–2**. With these two forces marked on the scale, we are ready to perform some force experiments.

First, attach the scale to an air-track cart, as in **Figure 5–3**. If we pull with a force F_1, we observe that the cart accelerates at the rate a_1. If we now pull with a force $F_2 = 2F_1$, the acceleration we observe is $a_2 = 2a_1$. Thus, the acceleration is proportional to the force—the greater the force, the greater the acceleration.

▶ **FIGURE 5–3 Acceleration is proportional to force** The spring calibrated in Figure 5–2 is used to accelerate a mass on a "frictionless" air track. If the force is doubled, the acceleration is also doubled.



Second, instead of doubling the force, let's double the mass of the cart by connecting two together, as in **Figure 5–4**. In this case, if we pull with a force F_1 we find an acceleration equal to $\frac{1}{2}a_1$. Thus, the acceleration is inversely proportional to mass—the greater the mass, the less the acceleration.

Combining these results, we find that in this simple case—with just one force in just one direction—the acceleration is given by

$$a = \frac{F}{m}$$

Rearranging the equation yields the form of Newton's law that is perhaps best known, $F = ma$.

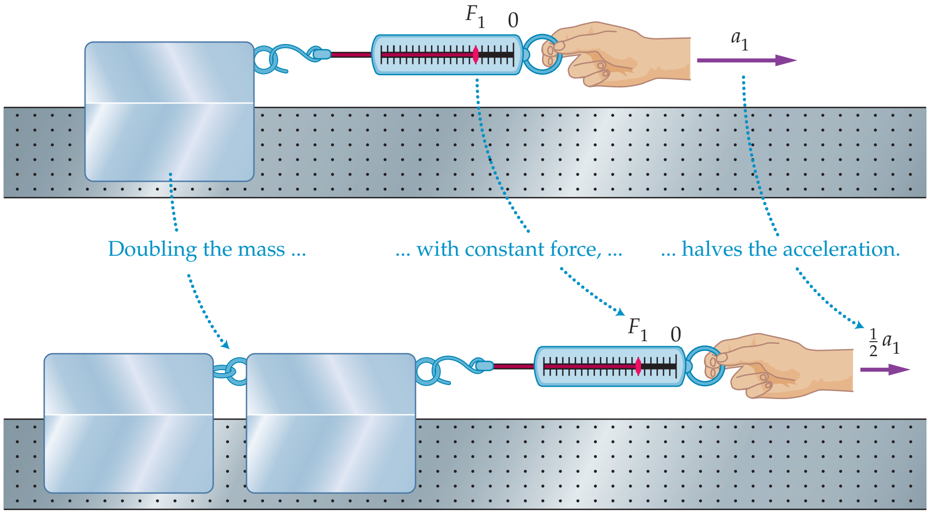


FIGURE 5–4 Acceleration is inversely proportional to mass
If the mass of an object is doubled but the force remains the same, the acceleration is halved.

In general, there may be several forces acting on a given mass, and these forces may be in different directions. Thus, we replace F with the sum of the force vectors acting on a mass:

$$\text{sum of force vectors} = \vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}}$$

The notation, $\Sigma\vec{\mathbf{F}}$, which uses the Greek letter sigma (Σ), is read "sum $\vec{\mathbf{F}}$." Recalling that acceleration is also a vector, we arrive at the formal statement of Newton's second law of motion:

Newton's Second Law

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \quad \text{or} \quad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad 5\text{–}1$$

In words:

If an object of mass m is acted on by a net force $\Sigma\vec{\mathbf{F}}$, it will experience an acceleration $\vec{\mathbf{a}}$ that is equal to the net force divided by the mass. Because the net force is a vector, the acceleration is also a vector. In fact, the direction of an object's acceleration is the *same* as the direction of the net force acting on it.

One should note that Newton's laws cannot be derived from anything more basic. In fact, this is what we mean by a law of nature. The validity of Newton's laws, and all other laws of nature, comes directly from comparisons with experiment.

In terms of vector components, an equivalent statement of the second law is:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \qquad 5\text{–}2$$

Note that Newton's second law holds independently for each coordinate direction. This component form of the second law is particularly useful when solving problems.

Let's pause for a moment to consider an important special case of the second law. Suppose an object has zero net force acting upon it. This may be because no forces act on it at all, or because it is acted on by forces whose vector sum is zero. In either case, we can state this mathematically as:

$$\sum \vec{\mathbf{F}} = 0$$

Now, according to Newton's second law, we conclude that the acceleration of this object must be zero:

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{0}{m} = 0$$

But if an object's acceleration is zero, its velocity must be constant. In other words, if the net force on an object is zero, the object moves with constant velocity. This is

Even though the tugboat exerts a large force on this ship, the ship's acceleration is small. This is because the acceleration of an object is inversely proportional to its mass, and the mass of the ship is enormous. The force exerted on the unfortunate hockey player is much smaller. The resulting acceleration is much larger, however, due to the relatively small mass of the player compared to that of the ship.

Newton's first law. Thus we see that Newton's first and second laws are consistent with one another.

Forces are measured in units called, appropriately enough, the **newton (N).** In particular, one newton is defined as the force required to give one kilogram of mass an acceleration of 1 $\mathrm{m/s^2}$. Thus,

$$1\ \mathrm{N} = (1\ \mathrm{kg})(1\ \mathrm{m/s^2}) = 1\ \mathrm{kg \cdot m/s^2} \qquad 5\text{–}3$$

In everyday terms, a newton is roughly a quarter of a pound. Note that a force in newtons divided by a mass in kilograms has the units of acceleration:

$$\frac{1\ \mathrm{N}}{1\ \mathrm{kg}} = \frac{1\ \mathrm{kg \cdot m/s^2}}{1\ \mathrm{kg}} = 1\ \mathrm{m/s^2} \qquad 5\text{–}4$$

Other common units for force are presented in Table 5–2. Typical forces and their magnitudes in newtons are listed in Table 5–3.

TABLE 5–2 Units of Mass, Acceleration, and Force

System of units	Mass	Acceleration	Force
SI	kilogram (kg)	$\mathrm{m/s^2}$	newton (N)
cgs	gram (g)	$\mathrm{cm/s^2}$	dyne (dyn)
British	slug	$\mathrm{ft/s^2}$	pound (lb)

(*Note:* 1 N = 10^5 dyne = 0.225 lb.)

TABLE 5–3 Typical Forces in Newtons (N)

Main engines of space shuttle	31,000,000
Pulling force of locomotive	250,000
Thrust of jet engine	75,000
Force to accelerate a car	7000
Weight of adult human	700
Weight of an apple	1
Weight of a rose	0.1
Weight of an ant	0.001

EXERCISE 5–1

The net force acting on a Jaguar XK8 has a magnitude of 6800 N. If the car's acceleration is 3.8 $\mathrm{m/s^2}$, what is its mass?

SOLUTION

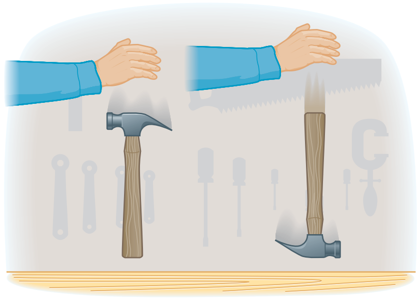
Since the net force and the acceleration are always in the same direction, we can replace the vectors in Equation 5–1 with magnitudes. Solving $\Sigma F = ma$ for the mass yields

$$m = \frac{\sum F}{a} = \frac{6800\ \mathrm{N}}{3.8\ \mathrm{m/s^2}} = 1800\ \mathrm{kg}$$

The following Conceptual Checkpoint presents a situation in which both Newton's first and second laws play an important role.

CONCEPTUAL CHECKPOINT 5–1 TIGHTENING A HAMMER

The metal head of a hammer is loose. To tighten it, you drop the hammer down onto a table. Should you **(a)** drop the hammer with the handle end down, **(b)** drop the hammer with the head end down, or **(c)** do you get the same result either way?



REASONING AND DISCUSSION

It might seem that since the same hammer hits against the same table in either case, there shouldn't be a difference. Actually, there is.

In case (a) the handle of the hammer comes to rest when it hits the table, but the head continues downward until a force acts on it to bring it to rest. The force that acts on it is supplied by the handle, which results in the head being wedged more tightly onto the handle. Since the metal head is heavy, the force wedging it onto the handle is great. In case (b) the head of the hammer comes to rest, but the handle continues to move until a force brings it to rest. The handle is lighter than the head, however; thus the force acting on it is less, resulting in less tightening.

ANSWER

(a) Drop the hammer with the handle end down.

A similar effect occurs when you walk—with each step you take you tamp your head down onto your spine, as when dropping a hammer handle end down.

This causes you to grow shorter during the day! Try it. Measure your height first thing in the morning, then again before going to bed. If you're like many people, you'll find that you have shrunk by an inch or so during the day.

REAL-WORLD PHYSICS: BIO

How walking affects your height

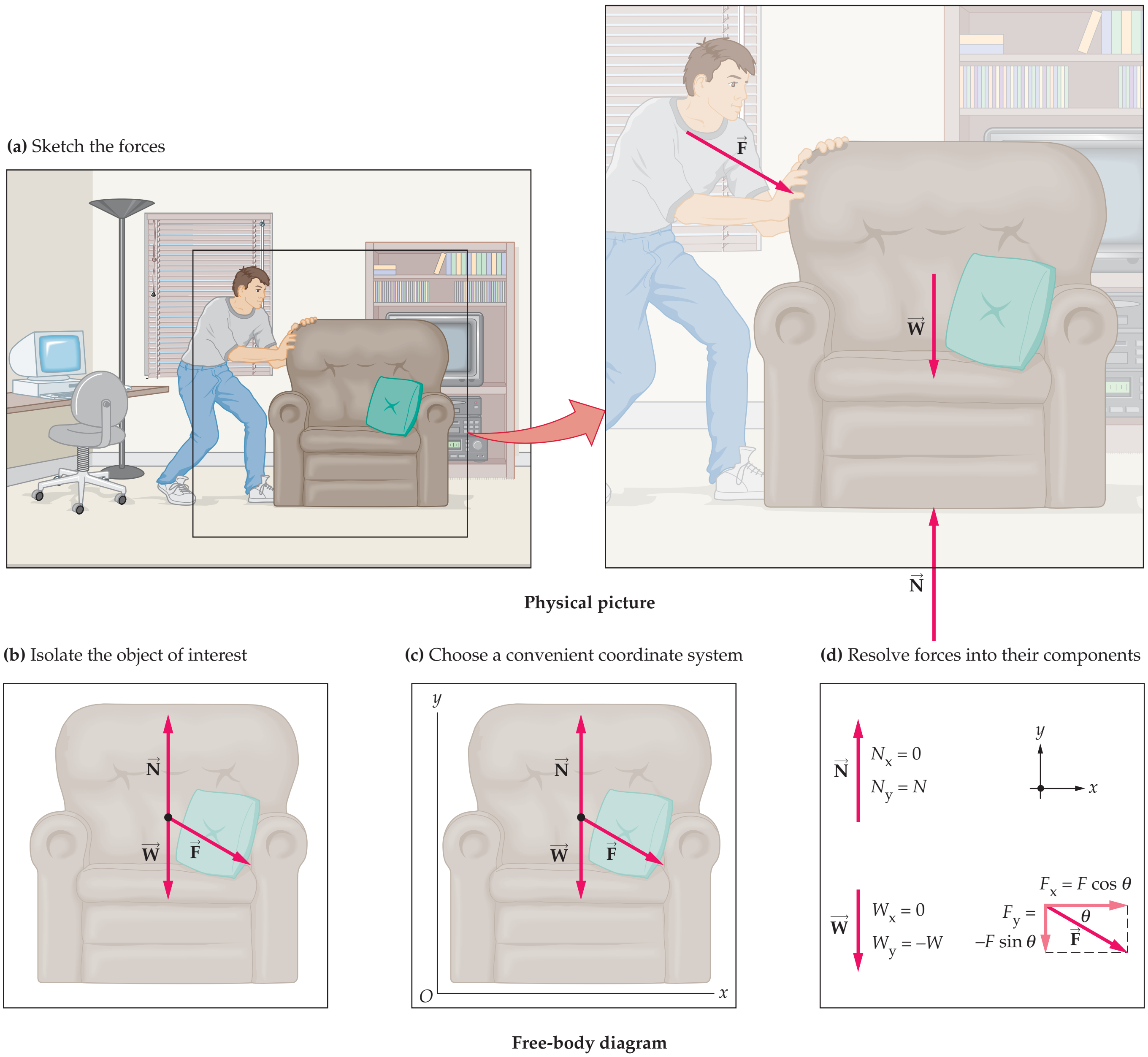
Free-Body Diagrams

When solving problems involving forces and Newton's laws, it is essential to begin by making a sketch that indicates *each and every external force* acting on a given object. This type of sketch is referred to as a **free-body diagram.** If we are concerned only with nonrotational motion, as is the case in this and the next chapter, we treat the object of interest as a point particle and apply each of the forces acting on the object to that point, as **Figure 5–5** shows. Once the forces are drawn, we choose a coordinate system and resolve each force into components. At this point, Newton's second law can be applied to each coordinate direction separately.

PROBLEM-SOLVING NOTE

External Forces

External forces acting on an object fall into two main classes: (i) Forces at the point of contact with another object, and (ii) forces exerted by an external agent, such as gravity.



▲ **FIGURE 5–5 Constructing and using a free-body diagram**

The four basic steps in constructing and using a free-body diagram are illustrated in these sketches. **(a)** Sketch all of the external forces acting on an object of interest. Note that only forces acting *on* the object are shown; none of the forces exerted *by* the object are included. **(b)** Isolate the object and treat it as a point particle. **(c)** Choose a convenient coordinate system. This will often mean aligning a coordinate axis to coincide with the direction of one or more forces in the system. **(d)** Resolve each of the forces into components using the coordinate system of part (c).

For example, in Figure 5–5 there are three external forces acting on the chair. One is the force $\vec{\mathbf{F}}$ exerted by the person. In addition, gravity exerts a downward force, $\vec{\mathbf{W}}$, which is simply the weight of the chair. Finally, the floor exerts an upward force on the chair that prevents it from falling toward the center of the Earth. This force is referred to as the *normal force*, $\vec{\mathbf{N}}$, because it is perpendicular (that is, normal) to the surface of the floor. We will consider the weight and the normal force in greater detail in Sections 5–6 and 5–7, respectively.

We can summarize the steps involved in constructing a free-body diagram as follows:

Sketch the Forces
Identify and sketch all of the external forces acting on an object. Sketching the forces roughly to scale will help in estimating the direction and magnitude of the net force.

Isolate the Object of Interest
Replace the object with a point particle of the same mass. Apply each of the forces acting on the object to that point.

Choose a Convenient Coordinate System
Any coordinate system will work; however, if the object moves in a known direction, it is often convenient to pick that direction for one of the coordinate axes. Otherwise, it is reasonable to choose a coordinate system that aligns with one or more of the forces acting on the object.

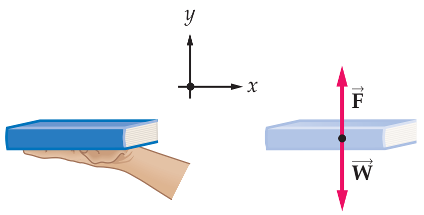
Resolve the Forces into Components
Determine the components of each force in the free-body diagram.

Apply Newton's Second Law to Each Coordinate Direction
Analyze motion in each coordinate direction using the component form of Newton's second law, as given in Equation 5–2.

PROBLEM-SOLVING NOTE

Picture the Problem

In problems involving Newton's laws, it is important to begin with a free-body diagram and to identify all the external forces that act on an object. Once these forces are identified and resolved into their components, Newton's laws can be applied in a straightforward way. It is crucial, however, that only external forces acting on the object be included, and that none of the external forces be omitted.



(a) Physical picture **(b) Free-body diagram**

▲ **FIGURE 5–6 A book supported in a person's hand**

(a) The physical situation. **(b)** The free-body diagram for the book, showing the two external forces acting on it. We also indicate our choice for a coordinate system.

These basic steps are illustrated in Figure 5–5. Note that the figures in this chapter use the labels "Physical picture" to indicate a sketch of the physical situation and "Free-body diagram" to indicate a free-body sketch.

We start by applying this procedure to a simple one-dimensional example, saving two-dimensional systems for Section 5–5. Suppose, for instance, that you hold a book at rest in your hand. What is the magnitude of the upward force that your hand must exert to keep the book at rest? From everyday experience, we expect that the upward force must be equal in magnitude to the weight of the book, but let's see how this result can be obtained directly from Newton's second law.

We begin with a sketch of the physical situation, as shown in **Figure 5–6 (a)**. The corresponding free-body diagram, in **Figure 5–6 (b)**, shows just the book, represented by a point, and the forces acting on it. Note that two forces act on the book: (i) the downward force of gravity, $\vec{\mathbf{W}}$, and (ii) the upward force, $\vec{\mathbf{F}}$, exerted by your hand. Only the forces acting *on* the book are included in the free-body diagram.

Now that the free-body diagram is drawn, we indicate a coordinate system so that the forces can be resolved into components. In this case all the forces are vertical. Thus we draw a y axis in the vertical direction in Figure 5–6 (b). Note that we have chosen upward to be the positive direction. With this choice, the y components of the forces are $F_y = F$ and $W_y = -W$. It follows that

$$\sum F_y = F - W$$

Using the y component of the second law $(\Sigma F_y = ma_y)$ we find

$$F - W = ma_y$$

Since the book remains at rest, its acceleration is zero. Thus, $a_y = 0$, which gives

$$F - W = ma_y = 0 \qquad \text{or} \qquad F = W$$

as expected.

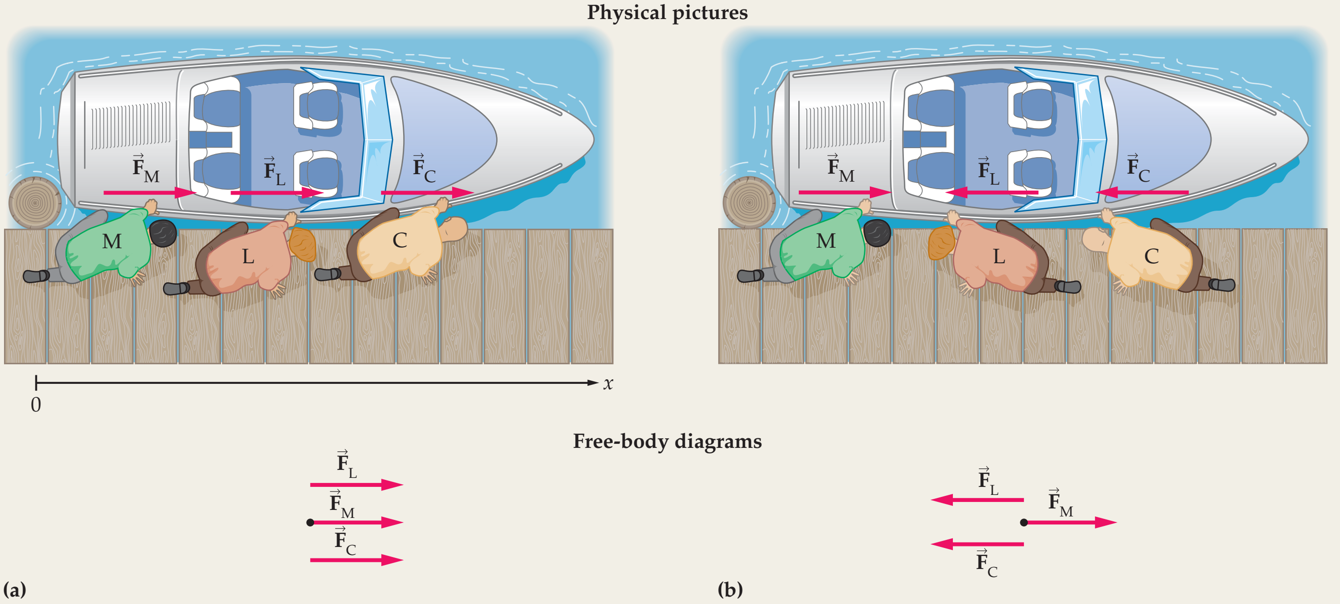
Next, we consider a situation where the net force acting on an object is nonzero, meaning that its acceleration is also nonzero.

EXAMPLE 5–1 THREE FORCES

Moe, Larry, and Curly push on a 752-kg boat that floats next to a dock. They each exert an 80.5-N force parallel to the dock. **(a)** What is the acceleration of the boat if they all push in the same direction? Give both direction and magnitude. **(b)** What are the magnitude and direction of the boat's acceleration if Larry and Curly push in the opposite direction to Moe's push?

PICTURE THE PROBLEM

In our sketch we indicate the three relevant forces acting on the boat: $\vec{\mathbf{F}}_M$, $\vec{\mathbf{F}}_L$, and $\vec{\mathbf{F}}_C$. Note that we have chosen the positive x direction to the right, in the direction that all three push for part (a). Therefore, all three forces have a positive x component in part (a). In part (b), however, the forces exerted by Larry and Curly have negative x components.



STRATEGY

Since we know the mass of the boat and the forces acting on it, we can find the acceleration using $\Sigma F_x = ma_x$. Even though this problem is one-dimensional, it is important to think of it in terms of vector components. For example, when we sum the x components of the forces, we are careful to use the appropriate signs—just as we always do when dealing with vectors.

SOLUTION

Part (a)

1. Write out the x component for each of the three forces: $F_{M,x} = F_{L,x} = F_{C,x} = 80.5\text{ N}$

2. Sum the x components of force and set equal to ma_x: $\sum F_x = F_{M,x} + F_{L,x} + F_{C,x} = 241.5\text{ N} = ma_x$

3. Divide by the mass to find a_x. Since a_x is positive, the acceleration is to the right, as expected: $$a_x = \frac{\sum F_x}{m} = \frac{241.5\text{ N}}{752\text{ kg}} = 0.321\text{ m/s}^2$$

Part (b)

4. Again, start by writing the x component for each force:
$$F_{M,x} = 80.5\text{ N}$$
$$F_{L,x} = F_{C,x} = -80.5\text{ N}$$

5. Sum the x components of force and set equal to ma_x:
$$\sum F_x = F_{M,x} + F_{L,x} + F_{C,x}$$
$$= 80.5\text{ N} - 80.5\text{ N} - 80.5\text{ N} = -80.5\text{ N} = ma_x$$

6. Solve for a_x. In this case a_x is negative, indicating an acceleration to the left: $$a_x = \frac{\sum F_x}{m} = \frac{-80.5\text{ N}}{752\text{ kg}} = -0.107\text{ m/s}^2$$

INSIGHT

The results of this Example are in agreement with everyday experience: three forces in the same direction cause more acceleration than three forces in opposing directions. The method of using vector components and being careful about their signs gives the expected results in a simple situation like this, and also works in more complicated situations where everyday experience may be of little help.

PRACTICE PROBLEM

If Moe, Larry, and Curly all push to the right with 85.0-N forces, and the boat accelerates at 0.530 m/s^2, what is its mass? **[Answer:** 481 kg]

Some related homework problems: Problem 2, Problem 4

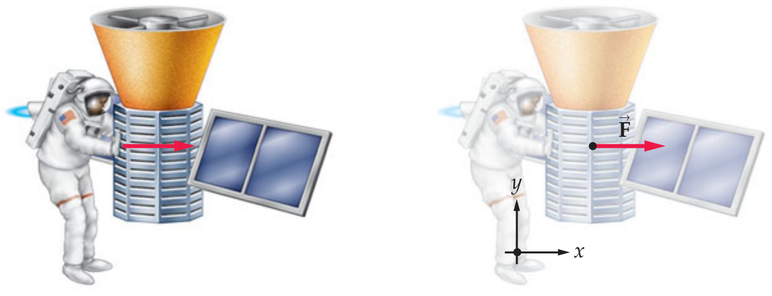
In some problems, we are given information that allows us to calculate an object's acceleration using the kinematic equations of Chapters 2 and 4. Once the acceleration is known, the second law can be used to find the net force that caused the acceleration.

REAL-WORLD PHYSICS

Astronaut jet packs

For example, suppose that an astronaut uses a jet pack to push a satellite toward the space shuttle. These jet packs, which are known to NASA as Manned Maneuvering Units, or MMUs, are basically small "one-person rockets" strapped to the back of an astronaut's spacesuit. An MMU contains pressurized nitrogen gas that can be released through varying combinations of 24 nozzles spaced around the unit, producing a force of about 10 pounds. The MMUs contain enough propellant for a six-hour EVA (extra-vehicular activity).

We show the physical situation in **Figure 5–7 (a)**, where an astronaut pushes on a 655-kg satellite. The corresponding free-body diagram for the satellite is shown in **Figure 5–7 (b)**. Note that we have chosen the x axis to point in the direction of the push. Now, if the satellite starts at rest and moves 0.675 m after 5.00 seconds of pushing, what is the force, F, exerted on it by the astronaut?



(a) Physical picture **(b) Free-body diagram**

▶ **FIGURE 5–7 An astronaut using a jet pack to push a satellite**
(a) The physical situation. **(b)** The free-body diagram for the satellite. Only one force acts on the satellite, and it is in the positive x direction.

▲ A technician inspects the landing gear of an airliner in a test of Foamcrete, a solid paving material that is just soft enough to collapse under the weight of an airliner. A plane that has run off the runway will slow safely to a stop as its wheels plow through the crumbling Foamcrete.

Clearly, we would like to use Newton's second law (basically, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$) to find the force, but we know only the mass of the satellite, not its acceleration. We can find the acceleration, however, by assuming constant acceleration (after all, the force is constant) and using the kinematic equation relating position to time: $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$. We can choose the initial position of the satellite to be $x_0 = 0$, and we are given that it starts at rest, thus $v_{0x} = 0$. Hence,

$$x = \frac{1}{2}a_x t^2$$

Since we know the distance covered in a given time, we can solve for the acceleration:

$$a_x = \frac{2x}{t^2} = \frac{2(0.675\ \mathrm{m})}{(5.00\ \mathrm{s})^2} = 0.0540\ \mathrm{m/s^2}$$

Now that kinematics has provided the acceleration, we use the x component of the second law to find the force. Only one force acts on the satellite, and its x component is F; thus,

$$\sum F_x = F = ma_x$$

$$F = ma_x = (655\ \mathrm{kg})(0.0540\ \mathrm{m/s^2}) = 35.4\ \mathrm{N}$$

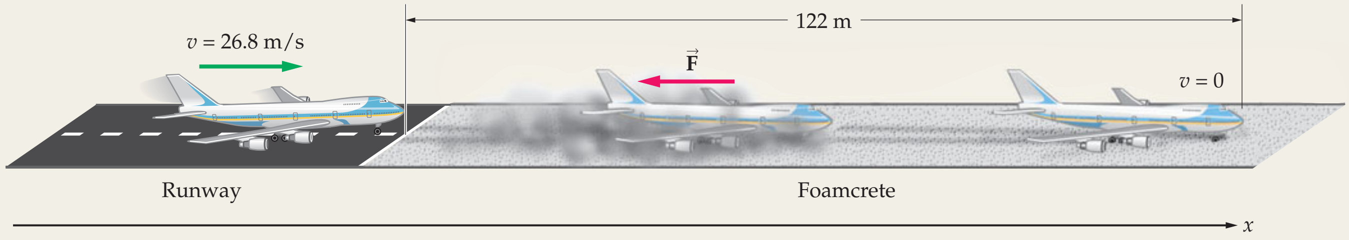
This force corresponds to a push of about 8 lb.

Another problem in which we use kinematics to find the acceleration is presented in the following Active Example.

ACTIVE EXAMPLE 5–1 THE FORCE EXERTED BY FOAMCRETE

REAL-WORLD PHYSICS

Foamcrete is a substance designed to stop an airplane that has run off the end of a runway, without causing injury to passengers. It is solid enough to support a car, but crumbles under the weight of a large airplane. By crumbling, it slows the plane to a safe stop. For example, suppose a 747 jetliner with a mass of 1.75×10^5 kg and an initial speed of 26.8 m/s is slowed to a stop in 122 m. What is the magnitude of the average retarding force $\vec{\mathbf{F}}$ exerted by the Foamcrete on the plane?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use $v^2 = v_0^2 + 2a_x \Delta x$ to find the plane's average acceleration: $a_x = -2.94\ \mathrm{m/s^2}$
2. Sum the forces in the x direction. Let F represent the magnitude of the force $\vec{F}$: $\sum F_x = -F$
3. Set the sum of forces equal to mass times acceleration: $-F = ma_x$
4. Solve for the magnitude of the average force, F: $F = -ma_x = 5.15 \times 10^5\ \mathrm{N}$

INSIGHT

Though the plane moves in the positive direction, its acceleration, and the net force exerted on it, are in the negative direction. As a result, the plane's speed decreases with time.

YOUR TURN

Find the plane's stopping distance if the magnitude of the average force exerted by the Foamcrete is doubled.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note again the care we take with the signs. The plane's acceleration is negative, hence the net force acting on it, $\vec{F}$, is in the negative x direction. On the other hand, the magnitude of the force, F, is positive, as is always the case for magnitudes.

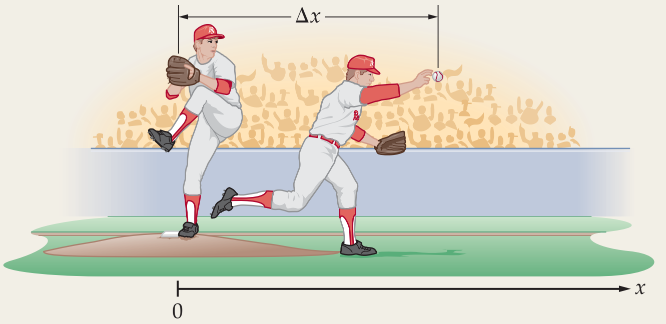
Finally, we end this section with an estimation problem.

EXAMPLE 5–2 PITCH MAN: ESTIMATE THE FORCE ON THE BALL

A pitcher throws a 0.15-kg baseball, accelerating it from rest to a speed of about 90 mi/h. Estimate the force exerted by the pitcher on the ball.

PICTURE THE PROBLEM

We choose the x axis to point in the direction of the pitch. Also indicated in the sketch is the distance over which the pitcher accelerates the ball, Δx. Since we are interested only in the pitch, and not in the subsequent motion of the ball, we ignore the effects of gravity.



STRATEGY

We know the mass, so we can find the force with $F_x = ma_x$ if we can estimate the acceleration. To find the acceleration, we start with the fact that $v_0 = 0$ and $v \approx 90$ mi/h. In addition, we can see from the sketch that a reasonable estimate for Δx is about 2.0 m. Combining these results with the kinematic equation $v^2 = v_0^2 + 2a_x\Delta x$ yields the acceleration, which we then use to find the force.

SOLUTION

1. Starting with the fact that 60 mi/h = 1 mi/min, perform a rough back-of-the-envelope conversion of 90 mi/h to meters per second:
$$v \approx 90\ \mathrm{mi/h} = \frac{1.5\ \mathrm{mi}}{\mathrm{min}} \approx \frac{2400\ \mathrm{m}}{60\ \mathrm{s}} = 40\ \mathrm{m/s}$$
2. Solve $v^2 = v_0^2 + 2a_x \Delta x$ for the acceleration, a_x. Use the estimates $\Delta x \approx 2.0$ m and $v \approx 40$ m/s:
$$a_x = \frac{v^2 - v_0^2}{2\ \Delta x} \approx \frac{(40\ \mathrm{m/s})^2 - 0}{2(2.0\ \mathrm{m})} = 400\ \mathrm{m/s^2}$$
3. Find the corresponding force with $F_x = ma_x$:
$$F_x = ma_x \approx (0.15\ \mathrm{kg})(400\ \mathrm{m/s^2}) = 60\ \mathrm{N} \approx 10\ \mathrm{lb}$$

INSIGHT

On the one hand, this is a sizable force, especially when you consider that the ball itself weighs only about 1/3 lb. Thus, the pitcher exerts a force on the ball that is about 30 times greater than the force exerted by Earth's gravity. It follows that ignoring gravity during the pitch is a reasonable approximation.

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On the other hand, you might say that 10 lb isn't that much force for a person to exert. That's true, but this force is being exerted with an average speed of about 20 m/s, which means that the pitcher is actually generating about 1.5 horsepower—a sizable power output for a person. We will cover power in detail in Chapter 7, and relate it to human capabilities.

PRACTICE PROBLEM

What is the approximate speed of the pitch if the force exerted by the pitcher is $\frac{1}{2}(60\text{ N}) = 30\text{ N}$? **[Answer:** 30 m/s or 60 mi/h]

Some related homework problems: Problem 5, Problem 8

Another way to find the acceleration is to estimate the amount of time it takes to make the pitch. However, since the pitch is delivered so quickly—about 1/10 s—estimating the time would be more difficult than estimating the distance Δx.

5–4 Newton's Third Law of Motion

Nature never produces just one force at a time; *forces always come in pairs*. In addition, the forces in a pair, which always act on *different objects*, are equal in magnitude and opposite in direction. This is Newton's third law of motion.

Newton's Third Law

For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.

In a somewhat more specific form:

If object 1 exerts a force $\vec{\mathbf{F}}$ on object 2, then object 2 exerts a force $-\vec{\mathbf{F}}$ on object 1.

This law, more commonly known by its abbreviated form, "for every action there is an equal and opposite reaction," completes Newton's laws of motion.

Figure 5–8 illustrates some action-reaction pairs. Notice that there is always a reaction force, whether the action force pushes on something hard to move, like a refrigerator, or on something that moves with no friction, like an air-track cart. In some cases, the reaction force tends to be overlooked, as when the Earth exerts a *downward* gravitational force on the space shuttle, and the shuttle exerts an equal and opposite *upward* gravitational force on the Earth. Still, the reaction force always exists.

Another important aspect of the third law is that the action-reaction forces always act on *different* objects. This, again, is illustrated in Figure 5–8. Thus, in drawing a free-body diagram, only one of the action-reaction pair of forces would be drawn for a given object. The other force in the pair would appear in the free-body diagram of a different object. As a result, *the two forces do not cancel*.

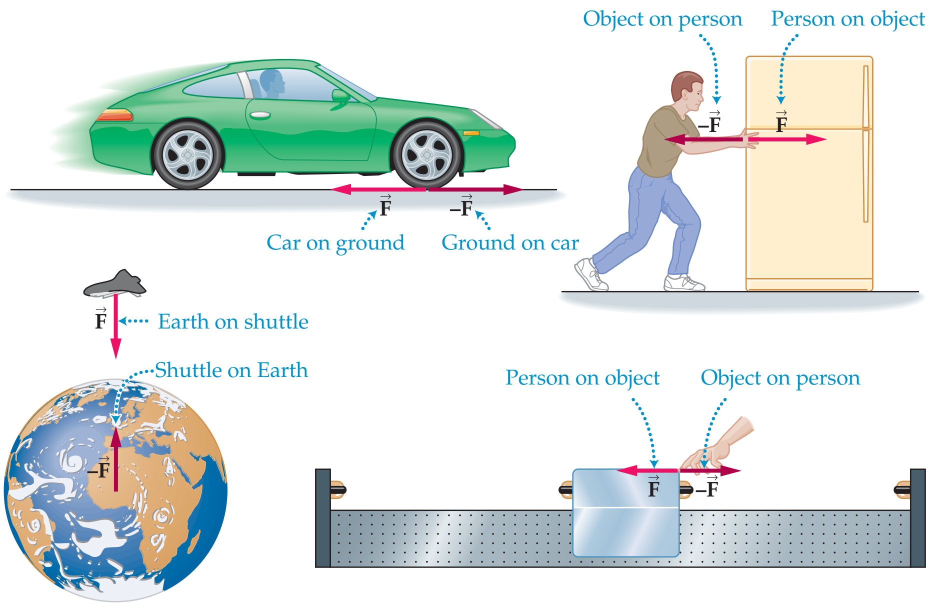


FIGURE 5–8 Examples of action-reaction force pairs

For example, consider a car accelerating from rest, as in Figure 5–8. As the car's engine turns the wheels, the tires exert a force on the road. By the third law, the road exerts an equal and opposite force on the car's tires. It is this second force—which acts on the car through its tires—that propels the car forward. The force exerted by the tires on the road does not accelerate the car.

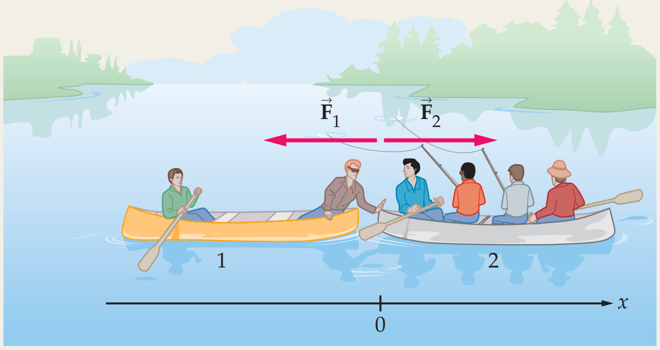
Since the action-reaction forces act on different objects, they generally produce different accelerations. This is the case in the next Example.

EXAMPLE 5–3 TIPPY CANOE

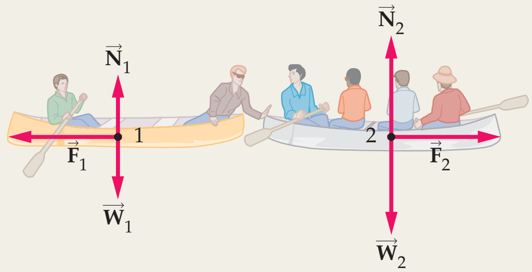
Two groups of canoeists meet in the middle of a lake. After a brief visit, a person in canoe 1 pushes on canoe 2 with a force of 46 N to separate the canoes. If the mass of canoe 1 and its occupants is $m_1 = 150$ kg, and the mass of canoe 2 and its occupants is $m_2 = 250$ kg, **(a)** find the acceleration the push gives to each canoe. **(b)** What is the separation of the canoes after 1.2 s of pushing?

PICTURE THE PROBLEM

We have chosen the positive x direction to point from canoe 1 to canoe 2. With this choice, the force exerted on canoe 2 is $\vec{\mathbf{F}}_2 = (+46\text{ N})\hat{\mathbf{x}}$. By Newton's third law, the force exerted on the person in canoe 1, and thus on canoe 1 itself if the person is firmly seated, is $\vec{\mathbf{F}}_1 = (-46\text{ N})\hat{\mathbf{x}}$. For convenience, we have placed the origin at the point where the canoes touch.



Physical picture



Free-body diagrams

STRATEGY

From Newton's third law, the force on canoe 1 is equal in magnitude to the force on canoe 2—the masses of the canoes are different, however, and therefore their accelerations are different as well. **(a)** We can find the acceleration of each canoe by solving $\Sigma F_x = ma_x$ for a_x. **(b)** The kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, can then be used to find the displacement of each canoe.

SOLUTION

Part (a)

1. Use Newton's second law to find the acceleration of canoe 2:
$$a_{2,x} = \frac{\sum F_{2,x}}{m_2} = \frac{46\text{ N}}{250\text{ kg}} = 0.18\text{ m/s}^2$$

2. Do the same calculation for canoe 1. Note that the acceleration of canoe 1 is in the negative direction:
$$a_{1,x} = \frac{\sum F_{1,x}}{m_1} = \frac{-46\text{ N}}{150\text{ kg}} = -0.31\text{ m/s}^2$$

Part (b)

3. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the position of canoe 2 at $t = 1.2$ s. From the problem statement, we know the canoes start at the origin $(x_0 = 0)$ and at rest $(v_{0x} = 0)$:
$$x_2 = \tfrac{1}{2}a_{2,x}t^2 = \tfrac{1}{2}(0.18\text{ m/s}^2)(1.2\text{ s})^2 = 0.13\text{ m}$$

4. Repeat the calculation for canoe 1:
$$x_1 = \tfrac{1}{2}a_{1,x}t^2 = \tfrac{1}{2}(-0.31\text{ m/s}^2)(1.2\text{ s})^2 = -0.22\text{ m}$$

5. Subtract the two positions to find the separation of the canoes:
$$x_2 - x_1 = 0.13\text{ m} - (-0.22\text{ m}) = 0.35\text{ m}$$

INSIGHT

The same magnitude of force acts on each canoe; hence the lighter one has the greater acceleration and the greater displacement. If the heavier canoe were replaced by a large ship of great mass, both vessels would still accelerate as a result of the push. However, the acceleration of the large ship would be so small as to be practically imperceptible. In this case, it would appear as if only the canoe moved, whereas, in fact, both vessels move.

PRACTICE PROBLEM

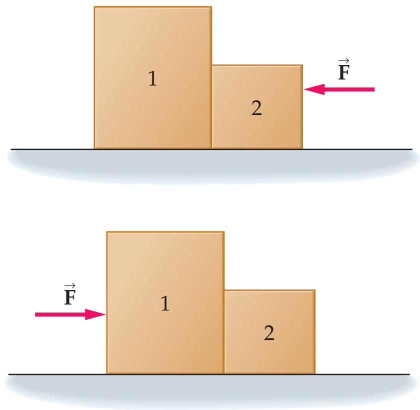
If the mass of canoe 2 is increased, does its acceleration increase, decrease, or stay the same? Check your answer by calculating the acceleration for the case where canoe 2 is replaced by a 25,000-kg ship. [**Answer:** The acceleration will decrease. In this case, $a = 0.0018\text{ m/s}^2$.]

Some related homework problems: Problem 18, Problem 19

When objects are touching one another, the action-reaction forces are often referred to as **contact forces.** The behavior of contact forces is explored in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 5–2 CONTACT FORCES

Two boxes—one large and heavy, the other small and light—rest on a smooth, level floor. You push with a force $\vec{\mathbf{F}}$ on either the small box or the large box. Is the contact force between the two boxes **(a)** the same in either case, **(b)** larger when you push on the large box, or **(c)** larger when you push on the small box?



REASONING AND DISCUSSION

Since the same force pushes on the boxes, you might think the force of contact is the same in both cases. It is not. What we can conclude, however, is that the boxes have the same acceleration in either case—the same net force acts on the same total mass, so the same acceleration, a, results.

To find the contact force between the boxes, we focus our attention on each box individually, and note that *Newton's second law must be satisfied for each of the boxes, just as it is for the entire two-box system*. For example, when the external force is applied to the small box, the only force acting on the *large* box (mass m_1) is the contact force; hence, the contact force must have a magnitude equal to m_1a. In the second case, the only force acting on the *small* box (mass m_2) is the contact force, and so the magnitude of the contact force is m_2a. Since m_1 is greater than m_2, it follows that the force of contact is larger when you push on the small box, m_1a, than when you push on the large box, m_2a.

To summarize, the contact force is larger when *it* must push the larger box.

ANSWER

(c) The contact force is larger when you push on the small box.

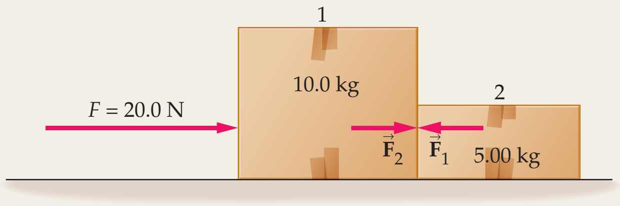
In the next Example, we calculate a numerical value for the contact force in a system similar to that described in Conceptual Checkpoint 5–2. We also show explicitly that Newton's third law is required for a full analysis of this system.

EXAMPLE 5–4 WHEN PUSH COMES TO SHOVE

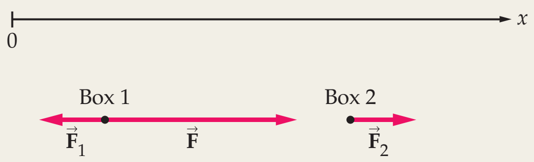
A box of mass $m_1 = 10.0$ kg rests on a smooth, horizontal floor next to a box of mass $m_2 = 5.00$ kg. If you push on box 1 with a horizontal force of magnitude $F = 20.0$ N, **(a)** what is the acceleration of the boxes? **(b)** What is the force of contact between the boxes?

PICTURE THE PROBLEM

We choose the x axis to be horizontal and pointing to the right. Thus, $\vec{\mathbf{F}} = (20.0\text{ N})\hat{\mathbf{x}}$. The contact forces are labeled as follows: $\vec{\mathbf{F}}_1$ is the contact force exerted on box 1; $\vec{\mathbf{F}}_2$ is the contact force exerted on box 2. By Newton's third law, the contact forces have the same magnitude, f, but point in opposite directions. With our coordinate system, we have $\vec{\mathbf{F}}_1 = -f\hat{\mathbf{x}}$ and $\vec{\mathbf{F}}_2 = f\hat{\mathbf{x}}$.



Physical picture



Free-body diagrams

INTERACTIVE FIGURE

STRATEGY

a. Since the two boxes are in contact, they have the same acceleration. We find this acceleration with Newton's second law; that is, we divide the net horizontal force by the total mass of the two boxes.

b. Now let's consider the system consisting solely of box 2. The mass in this case is 5.00 kg, and the only horizontal force acting on the system is $\vec{\mathbf{F}}_2$. Thus, we can find f, the magnitude of $\vec{\mathbf{F}}_2$, by requiring that box 2 have the acceleration found in part (a).

SOLUTION

Part (a)

1. Find the net horizontal force acting on the two boxes. Note that $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are equal in magnitude but opposite in direction. Hence, they sum to zero; $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$:

$$\sum_{\substack{\text{both}\\\text{boxes}}} F_x = F = 20.0\text{ N}$$

2. Divide the net force by the total mass, $m_1 + m_2$, to find the acceleration of the boxes:

$$a_x = \frac{\sum F_x}{m_1 + m_2} = \frac{20.0\text{ N}}{(10.0\text{ kg} + 5.00\text{ kg})} = \frac{20.0\text{ N}}{15.0\text{ kg}} = 1.33\text{ m/s}^2$$

Part (b)

3. Find the net horizontal force acting on box 2, and set it equal to the mass of box 2 times its acceleration:

$$\sum_{\text{box2}} F_x = F_{2,x} = f = m_2 a_x$$

4. Determine the magnitude of the contact force, f, by substituting numerical values for m_2 and a_x:

$$f = m_2 a_x = (5.00\text{ kg})(1.33\text{ m/s}^2) = 6.67\text{ N}$$

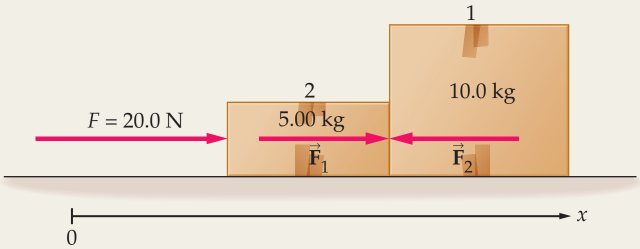
INSIGHT

Since the net horizontal force acting on box 1 is $F - f = 20.0\text{ N} - 6.67\text{ N} = 13.3\text{ N}$, it follows that its acceleration is $(13.3\text{ N})/(10.0\text{ kg}) = 1.33\text{ m/s}^2$. Thus, as expected, box 1 and box 2 have precisely the same acceleration.

If box 2 were not present, the 20.0-N force acting on box 1 would give it an acceleration of 2.00 m/s^2. As it is, the contact force between the boxes slows box 1 so that its acceleration is less than 2.00 m/s^2, and accelerates box 2 so that its acceleration is greater than zero. The precise value of the contact force is simply the value that gives both boxes the same acceleration.

PRACTICE PROBLEM

Suppose the relative positions of the boxes are reversed, so that F pushes on the small box, as shown here. Calculate the contact force for this case, and show that the force is greater than 6.67 N, as expected from Conceptual Checkpoint 5–2. [**Answer:** The contact force in this case is 13.3 N, double its previous value. This follows because the box being pushed has twice the mass of the box that was pushed originally.]



Some related homework problems: Problem 20, Problem 21

5–5 The Vector Nature of Forces: Forces in Two Dimensions

When we presented Newton's second law in Section 5–3, we said that an object's acceleration is equal to the net force acting on it divided by its mass. For example, if only a single force acts on an object, its acceleration is found to be in the same direction as the force. If more than one force acts on an object, experiments show that its acceleration is in the direction of the vector sum of the forces. Thus forces are indeed vectors, and they exhibit all the vector properties discussed in Chapter 3.

The mass of an object, on the other hand, is simply a positive number with no associated direction. It represents the amount of matter in an object.

As an example of the vector nature of forces, suppose two astronauts are using jet packs to push a 940-kg satellite toward the space shuttle, as shown in **Figure 5–9**. With the coordinate system indicated in the figure, astronaut 1 pushes in the positive x direction and astronaut 2 pushes in a direction 52° above the x axis.

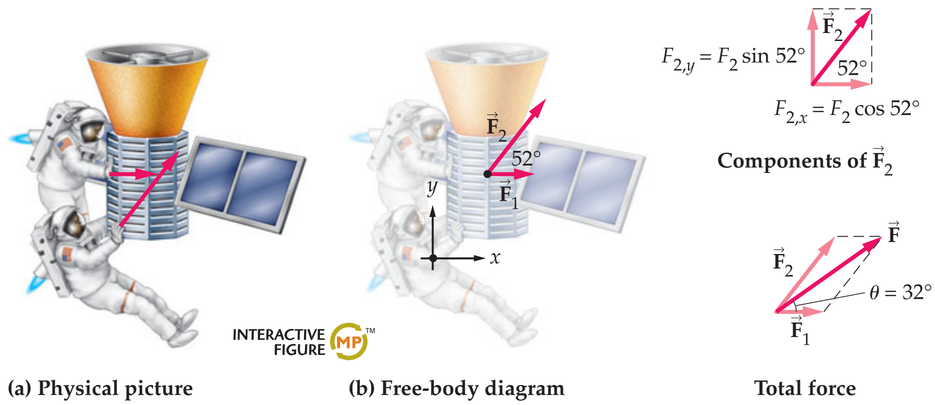


FIGURE 5–9 Two astronauts pushing a satellite with forces that differ in magnitude and direction
The acceleration of the satellite can be found by calculating a_x and a_y separately, then combining these components to find a and θ.

If astronaut 1 pushes with a force of magnitude $F_1 = 26$ N and astronaut 2 pushes with a force of magnitude $F_2 = 41$ N, what are the magnitude and direction of the satellite's acceleration?

The easiest way to solve a problem like this is to treat each coordinate direction independently of the other, just as we did many times when studying two-dimensional kinematics in Chapter 4. Thus, we first resolve each force into its x and y components. Referring to Figure 5–9, we see that for the x direction

$$F_{1,x} = F_1$$
$$F_{2,x} = F_2 \cos 52°$$

For the y direction

$$F_{1,y} = 0$$
$$F_{2,y} = F_2 \sin 52°$$

Next, we find the acceleration in the x direction by using the x component of Newton's second law:

$$\sum F_x = ma_x$$

Applied to this system, we have

$$\sum F_x = F_{1,x} + F_{2,x} = F_1 + F_2 \cos 52° = 26\text{ N} + (41\text{ N})\cos 52° = 51\text{ N}$$
$$= ma_x$$

Solving for the acceleration yields

$$a_x = \frac{\sum F_x}{m} = \frac{51\text{ N}}{940\text{ kg}} = 0.054\text{ m/s}^2$$

Similarly, in the y direction we start with

$$\sum F_y = ma_y$$

This gives

$$\sum F_y = F_{1,y} + F_{2,y} = 0 + F_2 \sin 52° = (41\text{ N})\sin 52° = 32\text{ N}$$
$$= ma_y$$

As a result, the y component of acceleration is:

$$a_y = \frac{\sum F_y}{m} = \frac{32\text{ N}}{940\text{ kg}} = 0.034\text{ m/s}^2$$

Thus, the satellite accelerates in both the x and the y directions. Its total acceleration has a magnitude of

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{(0.054\text{ m/s}^2)^2 + (0.034\text{ m/s}^2)^2} = 0.064\text{ m/s}^2$$

From Figure 5–9 we expect the total acceleration to be in a direction above the x axis but at an angle less than 52°. Straightforward calculation yields

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.034\text{ m/s}^2}{0.054\text{ m/s}^2}\right) = \tan^{-1}(0.63) = 32°$$

This is the same direction as the total force in Figure 5–9, as expected.

The following Example and Active Example give further practice with resolving force vectors and using Newton's second law in component form.

PROBLEM-SOLVING NOTE

Component-by-Component Application of Newton's Laws

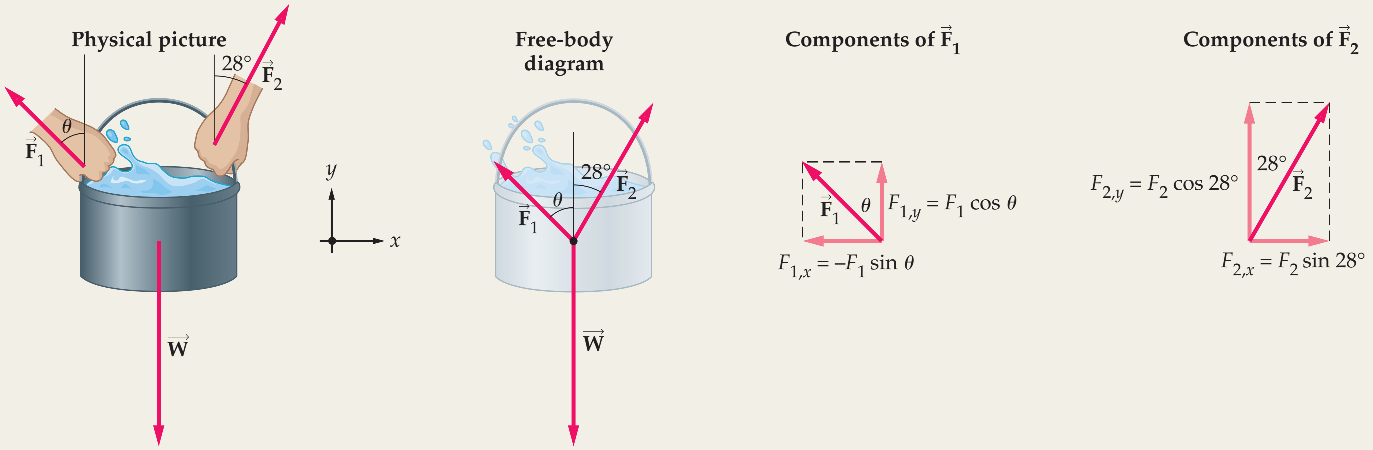
Newton's laws can be applied to each coordinate direction independently of the others. Therefore, when drawing a free-body diagram, be sure to include a coordinate system. Once the forces are resolved into their x and y components, the second law can be solved for each component separately. Working in a component-by-component fashion is the systematic way of using Newton's laws.

EXAMPLE 5–5 JACK AND JILL

Jack and Jill lift upward on a 1.30-kg pail of water, with Jack exerting a force $\vec{\mathbf{F}}_1$ of magnitude 7.0 N and Jill exerting a force $\vec{\mathbf{F}}_2$ of magnitude 11 N. Jill's force is exerted at an angle of 28° with the vertical, as shown below. **(a)** At what angle θ with respect to the vertical should Jack exert his force if the pail is to accelerate straight upward? **(b)** Determine the acceleration of the pail of water, given that its weight, $\vec{\mathbf{W}}$, has a magnitude of 12.8 N. (The simple connection between an object's mass and weight is presented in the next section.)

PICTURE THE PROBLEM

Our physical picture and free-body diagram show the pail and the three forces acting on it, as well as the angles relative to the vertical. In the panels at the right, we show the x and y components of the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. Notice, in particular, that $F_{1,x} = -F_1 \sin\theta$ and $F_{1,y} = F_1 \cos\theta$. Similarly, $F_{2,x} = F_2 \sin 28°$ and $F_{2,y} = F_2 \cos 28°$.



STRATEGY

a. We want the acceleration to be purely vertical. This means that the x component of acceleration must be zero, $a_x = 0$. For a_x to be zero it is necessary that the sum of forces in the x direction be zero, $\Sigma F_x = 0$. Since the x component of $\vec{\mathbf{F}}_1$ depends on the angle θ, the equation $\Sigma F_x = 0$ can be used to find θ.

b. Once the appropriate angle is found, we can use it to find the y component of $\vec{\mathbf{F}}_1$. Add this result to the y component of $\vec{\mathbf{F}}_2$. We're not done yet, though—to find the total y component of the force, $\Sigma \vec{\mathbf{F}}_y$, we must also add the weight of the pail, which points in the negative y direction. Finally, divide the total force by the mass of the pail, $m = 1.30$ kg, to obtain its acceleration, $a_y = (\Sigma F_y)/m$.

SOLUTION

Part (a)

1. Begin by writing out the x component of each force. Note that $\vec{\mathbf{W}}$ has no x component and that the x component of $\vec{\mathbf{F}}_1$ points in the negative x direction:

$$F_{1,x} = -F_1 \sin\theta \qquad F_{2,x} = F_2 \sin 28° \qquad W_x = 0$$

2. Sum the x components of force and set equal to zero. Note that θ is the only unknown in this equation:

$$\sum F_x = -F_1 \sin\theta + F_2 \sin 28° + 0 = ma_x = 0 \text{ or}$$
$$F_1 \sin\theta = F_2 \sin 28°$$

3. Solve for $\sin\theta$ and then for θ:

$$\sin\theta = \frac{F_2 \sin 28°}{F_1} = \frac{(11\text{ N}) \sin 28°}{7.0\text{ N}} = 0.74$$
$$\theta = \sin^{-1}(0.74) = 48°$$

Part (b)

4. First, determine the y component of each force. Note that $\vec{\mathbf{W}}$ points in the negative y direction and that the y components of both $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are positive:

$$F_{1,y} = F_1 \cos\theta = (7.0\text{ N}) \cos 48° = 4.7\text{ N}$$
$$F_{2,y} = F_2 \cos 28° = (11\text{ N}) \cos 28° = 9.7\text{ N}$$
$$W_y = -W = -12.8\text{ N}$$

5. Sum the y components of force and divide by the mass m to obtain the acceleration of the pail of water:

$$\sum F_y = F_1 \cos\theta + F_2 \cos 28° - W$$
$$= 4.7\text{ N} + 9.7\text{ N} - 12.8\text{ N} = 1.6\text{ N}$$
$$a_y = \left(\sum F_y\right)/m = (1.6\text{ N})/(1.3\text{ kg}) = 1.2\text{ m/s}^2$$

INSIGHT

Note that only the y components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ contribute to the vertical acceleration of the pail. The x components of the applied forces influence only the horizontal motion—they have no effect at all on the vertical acceleration of the pail. In this case the horizontal components of the applied forces cancel, and hence the pail moves straight upward with an acceleration of 1.2 m/s².

Finally, in the next section we shall see that the weight W of an object of mass m is $W = mg$. In this case, $W = (1.3\text{ kg})(9.81\text{ m/s}^2) = 12.8$ N.

PRACTICE PROBLEM

At what angle must Jack exert his force for the pail to accelerate straight upward if **(a)** $\vec{\mathbf{F}}_2$ is at an angle of 19° with the vertical or **(b)** $\vec{\mathbf{F}}_2$ is at an angle of 35° with the vertical? [**Answer: (a)** 31°, **(b)** 64°]

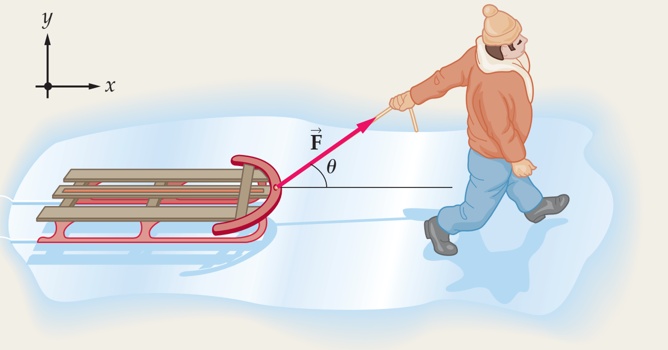
Some related homework problems: Problem 28, Problem 33

ACTIVE EXAMPLE 5–2 FIND THE SPEED OF THE SLED

A 4.60-kg sled is pulled across a smooth ice surface. The force acting on the sled is of magnitude 6.20 N and points in a direction 35.0° above the horizontal. If the sled starts at rest, how fast is it going after being pulled for 1.15 s?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the x component of $\vec{\mathbf{F}}$: $F_x = 5.08\text{ N}$
2. Apply Newton's second law to the x direction: $\sum F_x = F_x = ma_x$
3. Solve for the x component of acceleration: $a_x = 1.10\text{ m/s}^2$
4. Use $v_x = v_{0x} + a_x t$ to find the speed of the sled: $v_x = 1.27\text{ m/s}$



INSIGHT

Note that the y component of $\vec{\mathbf{F}}$ has no effect on the acceleration of the sled.

YOUR TURN

Suppose the angle of the force above the horizontal is decreased, and the sled is again pulled from rest for 1.15 s. **(a)** Is the final speed of the sled greater than, less than, or the same as before? Explain. **(b)** Find the final speed of the sled for the case $\theta = 25.0°$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

5–6 Weight

When you step onto a scale to weigh yourself, the scale gives a measurement of the pull of Earth's gravity. This is your weight, W. Similarly, the weight of any object on the Earth's surface is simply the gravitational force exerted on it by the Earth.

- The weight, W, of an object on the Earth's surface is the gravitational force exerted on it by the Earth.

As we know from everyday experience, the greater the mass of an object, the greater its weight. For example, if you put a brick on a scale and weigh it, you might get a reading of 9.0 N. If you put a second, identical brick on the scale—which doubles the mass—you will find a weight of $2(9.0\text{ N}) = 18\text{ N}$. Clearly, there must be a simple connection between weight, W, and mass, m.

To see exactly what this connection is, consider taking one of the bricks just mentioned and letting it drop in free fall. As indicated in **Figure 5–10**, the only force acting on the brick is its weight, W, which is downward. If we choose upward to be the positive direction, we have

$$\sum F_y = -W$$

In addition, we know from Chapter 2 that the brick moves downward with an acceleration of $g = 9.81\text{ m/s}^2$ regardless of its mass. Thus,

$$a_y = -g$$

Using these results in Newton's second law

$$\sum F_y = ma_y$$

we find

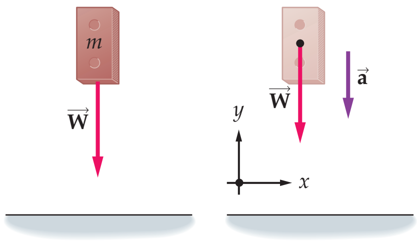
$$-W = -mg$$

Therefore, the weight of an object of mass m is $W = mg$:

Definition: Weight, W

$$W = mg \qquad 5\text{–}5$$

SI unit: newton, N



▲ **FIGURE 5–10 Weight and mass**

A brick of mass m has only one force acting on it in free fall—its weight, $\vec{\mathbf{W}}$. The resulting acceleration has a magnitude $a = g$; hence $W = mg$.

Note that there is a clear distinction between weight and mass. Weight is a gravitational force, measured in newtons; mass is a measure of the inertia of an object, and it is given in kilograms. For example, if you were to travel to the Moon, your mass would not change—you would have the same amount of matter in you, regardless of your location. On the other hand, the gravitational force on the Moon's surface is less than the gravitational force on the Earth's surface. As a result, you would weigh less on the Moon than on the Earth, even though your mass is the same.

To be specific, on Earth an 81.0-kg person has a weight given by

$$W_{\text{Earth}} = mg_{\text{Earth}} = (81.0 \text{ kg})(9.81 \text{ m/s}^2) = 795 \text{ N}$$

In contrast, the same person on the Moon, where the acceleration of gravity is 1.62 m/s^2, weighs only

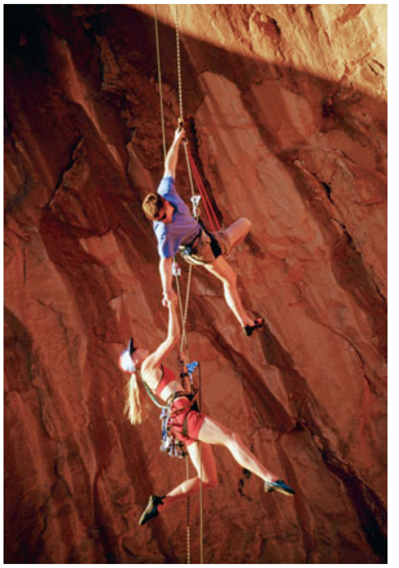
$$W_{\text{Moon}} = mg_{\text{Moon}} = (81.0 \text{ kg})(1.62 \text{ m/s}^2) = 131 \text{ N}$$

This is roughly one-sixth the weight on Earth. If, sometime in the future, there is a Lunar Olympics, the Moon's low gravity would be a boon for pole-vaulters, gymnasts, and others.

Finally, since weight is a force—which is a vector quantity—it has both a magnitude and a direction. Its magnitude, of course, is mg, and its direction is simply the direction of gravitational acceleration. Thus, if $\vec{\mathbf{g}}$ denotes a vector of magnitude g, pointing in the direction of free-fall acceleration, the weight of an object can be written in vector form as follows:

$$\vec{\mathbf{W}} = m\vec{\mathbf{g}}$$

We use the weight vector and its magnitude, mg, in the next Example.



▲ At the moment this picture was taken, the acceleration of both climbers was zero because the net force acting on them was zero. In particular, the upward forces exerted on the lower climber by the other climber and the ropes exactly cancel the downward force that gravity exerts on her.

EXAMPLE 5–6 WHERE'S THE FIRE?

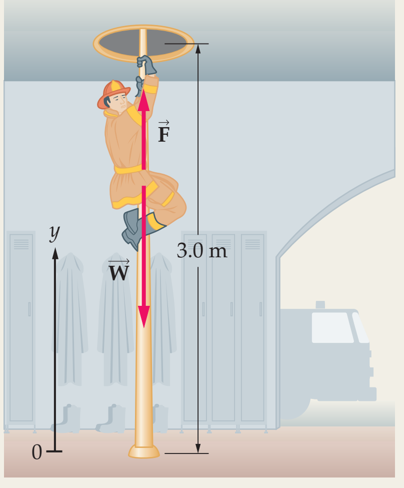
The fire alarm goes off, and a 97-kg fireman slides 3.0 m down a pole to the ground floor. Suppose the fireman starts from rest, slides with constant acceleration, and reaches the ground floor in 1.2 s. What was the upward force $\vec{\mathbf{F}}$ exerted by the pole on the fireman?

PICTURE THE PROBLEM

Our sketch shows the fireman sliding down the pole and the two forces acting on him: the upward force exerted by the pole, $\vec{\mathbf{F}}$, and the downward force of gravity, $\vec{\mathbf{W}}$. We choose the positive y direction to be upward, therefore $\vec{\mathbf{F}} = F\hat{\mathbf{y}}$ and $\vec{\mathbf{W}} = (-mg)\hat{\mathbf{y}}$. In addition, we choose $y = 0$ to be at ground level.

STRATEGY

The basic idea in approaching this problem is to apply Newton's second law to the y direction: $\sum F_y = ma_y$. The acceleration is not given directly, but we can find it using the kinematic equation $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$. Substituting the result for a_y into Newton's second law, along with $W_y = -W = -mg$, allows us to solve for the unknown force, $\vec{\mathbf{F}}$.



Physical picture

Free-body diagram

SOLUTION

1. Solve $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ for a_y, using the fact that $v_{0y} = 0$:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_yt^2 = y_0 + \tfrac{1}{2}a_yt^2$$

$$a_y = \frac{2(y - y_0)}{t^2}$$

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2. Substitute $y = 0$, $y_0 = 3.0$ m, and $t = 1.2$ s to find the acceleration: $a_y = \dfrac{2(0 - 3.0\text{ m})}{(1.2\text{ s})^2} = -4.2\text{ m/s}^2$
3. Sum the forces in the y direction: $\sum F_y = F - mg$
4. Set the sum of the forces equal to mass times acceleration: $F - mg = ma_y$
5. Solve for F, the y component of the force exerted by the pole. Use the result for F to write the force vector $\vec{\mathbf{F}}$: $F = mg + ma_y = m(g + a_y)$ $= (97\text{ kg})(9.81\text{ m/s}^2 - 4.2\text{ m/s}^2) = 540\text{ N}$ $\vec{\mathbf{F}} = (540\text{ N})\,\hat{\mathbf{y}}$

INSIGHT

How is it that the pole exerts a force on the fireman? Well, by wrapping his arms and legs around the pole as he slides, the fireman exerts a downward force on the pole. By Newton's third law, the pole exerts an upward force of equal magnitude on the fireman. These forces are due to friction, which we shall study in detail in Chapter 6.

PRACTICE PROBLEM

What is the fireman's acceleration if the force exerted on him by the pole is 650 N? **[Answer:** $a_y = -3.1\text{ m/s}^2$]

Some related homework problems: Problem 36, Problem 40

Apparent Weight

We have all had the experience of riding in an elevator and feeling either heavy or light, depending on its motion. For example, when an elevator moving downward comes to rest by accelerating upward, we feel heavier. On the other hand, we feel lighter when an elevator moving upward comes to rest by accelerating downward. In short, the motion of an elevator can give rise to an **apparent weight** that differs from our true weight. Why?

The reason is that our sensation of weight in this case is due to the force exerted on our feet by the floor of the elevator. If this force is greater than our weight, mg, we feel heavy; if it is less than mg, we feel light.

As an example, imagine you are in an elevator that is moving with an upward acceleration a, as indicated in **Figure 5–11**. Two forces act on you: (i) your weight, W, acting downward; and (ii) the upward normal force exerted on your feet by the floor of the elevator. Let's call the second force W_a, since it represents your apparent weight—that is, W_a is the force that pushes upward on your feet and gives you the sensation of your "weight" pushing down on the floor. We can find W_a by applying Newton's second law to the vertical direction.

To be specific, the sum of the forces acting on you is

$$\sum F_y = W_a - W$$

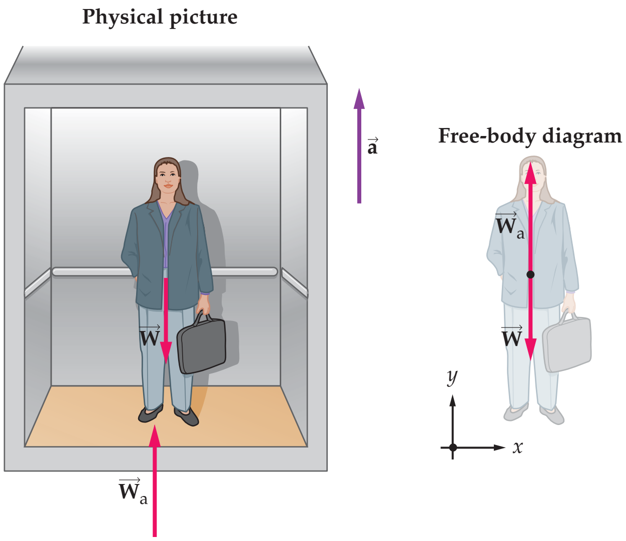


FIGURE 5–11 Apparent weight

A person rides in an elevator that is accelerating upward. Because the acceleration is upward, the net force must also be upward. As a result, the force exerted on the person by the floor of the elevator, $\vec{\mathbf{W}}_a$, must be greater than the person's weight, $\vec{\mathbf{W}}$. This means that the person feels heavier than normal.

By Newton's second law, this sum must equal ma_y. Since $a_y = a$, we find

$$W_a - W = ma$$

Solving for the apparent weight, W_a, yields

$$\begin{aligned} W_a &= W + ma \\ &= mg + ma = m(g + a) \end{aligned} \qquad 5\text{–}6$$

Note that W_a is greater than your weight, mg, and hence you feel heavier. In fact, your apparent weight is precisely what it would be if you were suddenly "transported" to a planet where the acceleration of gravity is $g + a$ instead of g.

On the other hand, if the elevator accelerates downward, so that $a_y = -a$, your apparent weight is found by simply replacing a with $-a$ in Equation 5–6:

$$\begin{aligned} W_a &= W - ma \\ &= mg - ma = m(g - a) \end{aligned} \qquad 5\text{–}7$$

In this case you feel lighter than usual.

We explore these results in the next Example, in which we consider weighing a fish on a scale. The reading on the scale is equal to the upward force it exerts on an object. Thus, the upward force exerted by the scale is the apparent weight, W_a.

EXAMPLE 5–7 HOW MUCH DOES THE SALMON WEIGH?

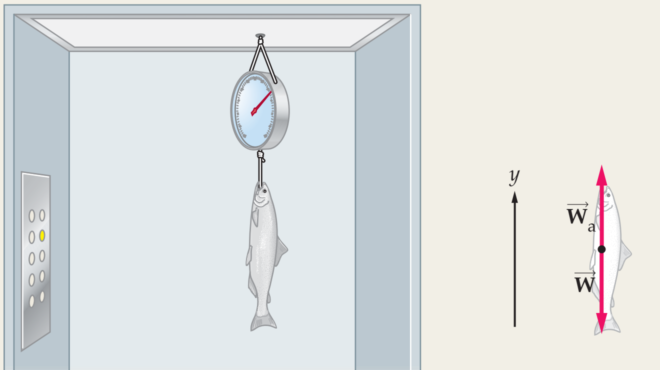
As part of an attempt to combine physics and biology in the same class, an instructor asks students to weigh a 5.0-kg salmon by hanging it from a fish scale attached to the ceiling of an elevator. What is the apparent weight of the salmon, $\vec{\mathbf{W}}_a$, if the elevator **(a)** is at rest, **(b)** moves with an upward acceleration of 2.5 m/s^2, or **(c)** moves with a downward acceleration of 3.2 m/s^2?

PICTURE THE PROBLEM

The free-body diagram for the salmon shows the weight of the salmon, $\vec{\mathbf{W}}$, and the force exerted by the scale, $\vec{\mathbf{W}}_a$. Note that upward is the positive direction. Therefore, the y component of $\vec{\mathbf{W}}$ is $-W = -mg$ and the y component of $\vec{\mathbf{W}}_a$ is W_a.

STRATEGY

We know the weight, $W = mg$, and the acceleration, a. To find the apparent weight, W_a, we use $\sum F_y = ma_y$. **(a)** Set $a_y = 0$. **(b)** Set $a_y = 2.5$ m/s^2. **(c)** Set $a_y = -3.2$ m/s^2.



Physical picture

Free-body diagram

SOLUTION

Part (a)

1. Sum the y component of the forces and set equal to mass times the y component of acceleration, with $a_y = 0$:

$$\sum F_y = W_a - W = ma_y = 0$$

2. Solve for W_a, then write the vector $\vec{\mathbf{W}}$:

$$W_a = W = mg = (5.0\text{ kg})(9.81\text{ m/s}^2) = 49\text{ N}$$

$$\vec{\mathbf{W}}_a = (49\text{ N})\hat{\mathbf{y}}$$

Part (b)

3. Again, sum the forces and set equal to mass times acceleration, this time with $a_y = a = 2.5$ m/s^2:

$$\sum F_y = W_a - W = ma_y = ma$$

4. Solve for W_a, then write the vector $\vec{\mathbf{W}}_a$:

$$\begin{aligned} W_a &= W + ma \\ &= mg + ma = 49\text{ N} + (5.0\text{ kg})(2.5\text{ m/s}^2) = 62\text{ N} \end{aligned}$$

$$\vec{\mathbf{W}}_a = (62\text{ N})\hat{\mathbf{y}}$$

Part (c)

5. Finally, sum the forces and set equal to mass times acceleration, with $a_y = -a = -3.2$ m/s^2:

$$\sum F_y = W_a - W = ma_y = -ma$$

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6. Solve for W_a, then write the vector $\vec{\mathbf{W}}_a$:

$$W_a = W - ma$$
$$= mg - ma = 49\text{ N} - (5.0\text{ kg})(3.2\text{ m/s}^2) = 33\text{ N}$$
$$\vec{\mathbf{W}}_a = (33\text{ N})\hat{\mathbf{y}}$$

INSIGHT

When the salmon is at rest, or moving with constant velocity, its acceleration is zero and the apparent weight is equal to the actual weight, mg. In part (b) the apparent weight is greater than the actual weight because the scale must exert an upward force capable not only of supporting the salmon, but of accelerating it upward as well. In part (c) the apparent weight is less than the actual weight. In this case the net force acting on the salmon is downward, and hence its acceleration is downward.

PRACTICE PROBLEM

What is the elevator's acceleration if the scale gives a reading of **(a)** 55 N or **(b)** 45 N? [**Answer:** **(a)** $a_y = 1.2\text{ m/s}^2$, **(b)** $a_y = -0.80\text{ m/s}^2$]

Some related homework problems: Problem 38, Problem 39

REAL-WORLD PHYSICS: BIO

Simulating weightlessness

▲ Astronaut candidates pose for a floating class picture during weightlessness training aboard the "vomit comet."

Let's return for a moment to Equation 5–7:

$$W_a = m(g - a)$$

This result indicates that a person feels lighter than normal when riding in an elevator with a downward acceleration a. In particular, if the elevator's downward acceleration is g—that is, if the elevator is in free fall—it follows that $W_a = m(g - g) = 0$. Thus, a person feels "weightless" (zero apparent weight) in a freely falling elevator!

NASA uses this effect when training astronauts. Trainees are sent aloft in a KC-135 airplane affectionately known as the "vomit comet" (since many trainees experience nausea along with the weightlessness). To generate an experience of weightlessness, the plane flies on a parabolic path—the same path followed by a projectile in free fall. Each round of weightlessness lasts about half a minute, after which the plane pulls up to regain altitude and start the cycle again. On a typical flight, trainees experience about 40 cycles of weightlessness. Many scenes in the movie *Apollo 13* were shot in 30-second takes aboard the vomit comet.

This idea of free-fall weightlessness applies to more than just the vomit comet. In fact, astronauts in orbit experience weightlessness for the same reason—they and their craft are actually in free fall. As we shall see in detail in Chapter 12 (Gravity), orbital motion is just a special case of free fall.

CONCEPTUAL CHECKPOINT 5–3 ELEVATOR RIDE

If you ride in an elevator moving upward with constant speed, is your apparent weight **(a)** the same as, **(b)** greater than, or **(c)** less than mg?

REASONING AND DISCUSSION

If the elevator is moving in a straight line with constant speed, its acceleration is zero. Now, if the acceleration is zero, the net force must also be zero. Hence, the upward force exerted by the floor of the elevator, W_a, must equal the downward force of gravity, mg. As a result, your apparent weight is equal to mg.

Note that this conclusion agrees with Equations 5–6 and 5–7, with $a = 0$.

ANSWER

(a) Your apparent weight is the same as mg.

5–7 Normal Forces

As you get ready for lunch, you take a can of soup from the cupboard and place it on the kitchen counter. The can is now at rest, which means that its acceleration is zero, so the net force acting on it is also zero. Thus, you know that the

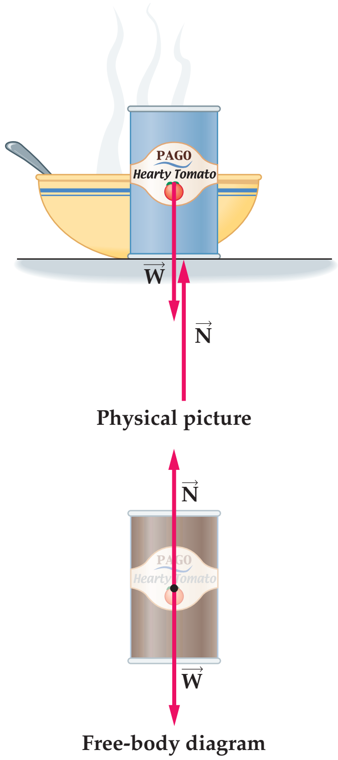


FIGURE 5–12 The normal force is perpendicular to a surface
A can of soup rests on a kitchen counter, which exerts a normal (perpendicular) force, $\vec{\mathbf{N}}$, to support it. In the special case shown here, the normal force is equal in magnitude to the weight, $W = mg$, and opposite in direction.

downward force of gravity is being opposed by an upward force exerted by the counter, as shown in **Figure 5–12**. As we have mentioned before, this force is referred to as the **normal force, $\vec{\mathbf{N}}$**. The reason the force is called normal is that it is *perpendicular to the surface*, and in mathematical terms, *normal* simply means perpendicular.

The origin of the normal force is the interaction between atoms in a solid that act to maintain its shape. When the can of soup is placed on the countertop, for example, it causes an imperceptibly small compression of the surface of the counter. This is similar to compressing a spring, and just like a spring, the countertop exerts a force to oppose the compression. Therefore, the greater the weight placed on the countertop, the greater the normal force it exerts to oppose being compressed.

In the example of the soup can and the countertop, the magnitude of the normal force is equal to the weight of the can. This is a special case, however. In general, the normal force may be greater than or less than the weight of an object.

To see how this can come about, consider pulling a 12.0-kg suitcase across a smooth floor by exerting a force, $\vec{\mathbf{F}}$, at an angle θ above the horizontal. The weight of the suitcase is $mg = (12.0\text{ kg})(9.81\text{ m/s}^2) = 118\text{ N}$. The normal force will have a magnitude less than this, however, because the force $\vec{\mathbf{F}}$ has an upward component that supports part of the suitcase's weight. To be specific, suppose that $\vec{\mathbf{F}}$ has a magnitude of 45.0 N and that $\theta = 20.0°$. What is the normal force exerted by the floor on the suitcase?

The situation is illustrated in **Figure 5–13**, where we show the three forces acting on the suitcase: (i) the weight of the suitcase, $\vec{\mathbf{W}}$, (ii) the force $\vec{\mathbf{F}}$, and (iii) the normal force, $\vec{\mathbf{N}}$. We also indicate a typical coordinate system in the figure, with the x axis horizontal and the y axis vertical. Now, the key to solving a problem like this is to realize that since the suitcase does not move in the y direction, its y component of acceleration is zero; that is, $a_y = 0$. It follows, from Newton's second law, that the sum of the y components of force must also equal zero; that is, $\Sigma F_y = ma_y = 0$. Using this condition, we can solve for the one force that is unknown, $\vec{\mathbf{N}}$.

To find $\vec{\mathbf{N}}$, then, we start by writing out the y component of each force. For the weight we have $W_y = -mg = -118\text{ N}$; for the applied force, $\vec{\mathbf{F}}$, the y component is $F_y = F \sin 20.0° = (45.0\text{ N}) \sin 20.0° = 15.4\text{ N}$; finally, the y component of the normal force is $N_y = N$. Setting the sum of the y components of force equal to zero yields

$$\sum F_y = W_y + F_y + N_y = -mg + F \sin 20.0° + N = 0$$

Solving for N gives

$$N = mg - F \sin 20.0° = 118\text{ N} - 15.4\text{ N} = 103\text{ N}$$

In vector form,

$$\vec{\mathbf{N}} = N_y\hat{\mathbf{y}} = (103\text{ N})\hat{\mathbf{y}}$$

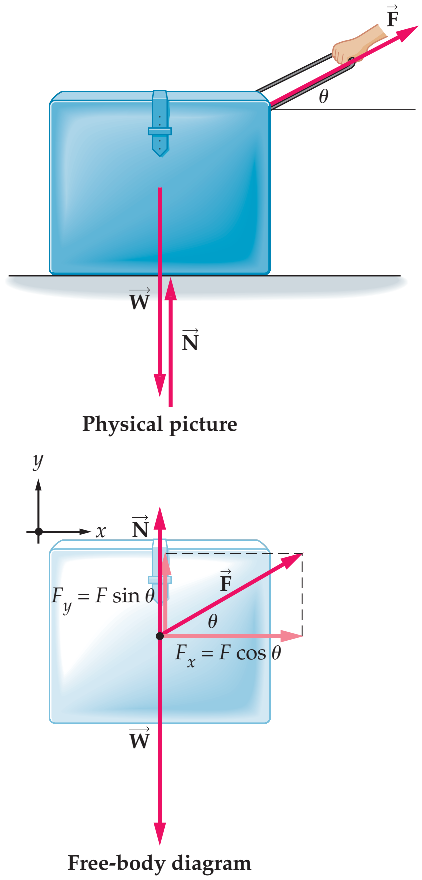


FIGURE 5–13 The normal force may differ from the weight
A suitcase is pulled across the floor by an applied force of magnitude F, directed at an angle θ above the horizontal. As a result of the upward component of $\vec{\mathbf{F}}$, the normal force $\vec{\mathbf{N}}$ will have a magnitude less than the weight of the suitcase.

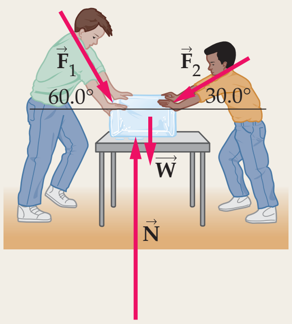
Thus, as mentioned, the normal force has a magnitude less than $mg = 118\text{ N}$ because the y component of $\vec{\mathbf{F}}$, $F_y = F \sin 20.0°$, supports part of the weight. In the following Example, however, the applied forces cause the normal force to be greater than the weight.

EXAMPLE 5–8 ICE BLOCK

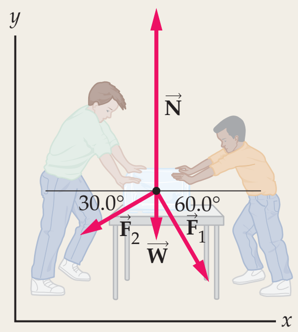
A 6.0-kg block of ice is acted on by two forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$, as shown in the diagram. If the magnitudes of the forces are $F_1 = 13\text{ N}$ and $F_2 = 11\text{ N}$, find **(a)** the acceleration of the ice and **(b)** the normal force exerted on it by the table.

PICTURE THE PROBLEM

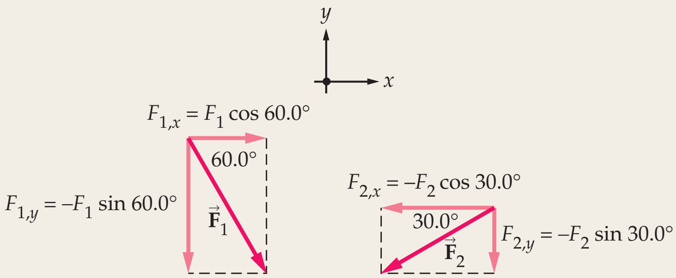
The sketch shows our choice of coordinate system, as well as all the forces acting on the block of ice. Note that $\vec{\mathbf{F}}_1$ has a positive x component and a negative y component; $\vec{\mathbf{F}}_2$ has negative x and y components. The weight and the normal force have only y components, therefore $W_x = 0$, $W_y = -W = -mg$, $N_x = 0$, and $N_y = N$.



Physical picture



Free-body diagram



Components of $\vec{\mathbf{F}}_1$ (enlarged)

Components of $\vec{\mathbf{F}}_2$ (enlarged)

STRATEGY

The basic idea in this problem is to apply Newton's second law to the x and y directions separately. **(a)** The block can accelerate only in the horizontal direction; thus we find the acceleration by solving $\Sigma F_x = ma_x$ for a_x. **(b)** There is no motion in the y direction, and therefore the acceleration in the y direction is zero. Hence, we can find the normal force $\vec{\mathbf{N}}$ by setting $\Sigma F_y = ma_y = 0$.

SOLUTION

Part (a)

1. Write out the x component of each force:

$$F_{1,x} = F_1 \cos 60.0° = (13\text{ N}) \cos 60.0° = 6.5\text{ N}$$
$$F_{2,x} = -F_2 \cos 30.0° = -(11\text{ N}) \cos 30.0° = -9.5\text{ N}$$
$$N_x = 0 \qquad W_x = 0$$

2. Sum the x components of force:

$$\sum F_x = F_{1,x} + F_{2,x} + N_x + W_x$$
$$= 6.5\text{ N} - 9.5\text{ N} + 0 + 0 = -3.0\text{ N}$$

3. Divide by the mass to obtain the acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{-3.0\text{ N}}{6.0\text{ kg}} = -0.50\text{ m/s}^2$$
$$\vec{\mathbf{a}} = (-0.50\text{ m/s}^2)\hat{\mathbf{x}}$$

Part (b)

4. Write out the y component of each force:

The only force we don't know is the normal. We represent its magnitude by N:

$$F_{1,y} = -F_1 \sin 60° = -(13\text{ N}) \sin 60.0° = -11\text{ N}$$
$$F_{2,y} = -F_2 \sin 30° = -(11\text{ N}) \sin 30.0° = -5.5\text{ N}$$
$$N_y = N \qquad W_y = -W = -mg$$

5. Sum the y components of force:

$$\sum F_y = F_{1,y} + F_{2,y} + N_y + W_y$$
$$= -11\text{ N} - 5.5\text{ N} + N - mg$$

6. Set this sum equal to 0 since the acceleration in the y direction is zero, and solve for N:

$$-11\text{ N} - 5.5\text{ N} + N - mg = 0$$
$$N = 11\text{ N} + 5.5\text{ N} + mg$$
$$= 11\text{ N} + 5.5\text{ N} + (6.0\text{ kg})(9.81\text{ m/s}^2) = 75\text{ N}$$

7. Finally, we write the normal force in vector form:

$$\vec{\mathbf{N}} = (75\text{ N})\hat{\mathbf{y}}$$

INSIGHT

The block accelerates to the left, even though the force acting to the right, $\vec{\mathbf{F}}_1$, has a greater magnitude than the force acting to the left, $\vec{\mathbf{F}}_2$. This is because $\vec{\mathbf{F}}_2$ has the greater x component. Also, note that the normal force is greater in magnitude than the weight, $mg = 59$ N.

In general, the normal force exerted by a surface is just as large as is necessary to prevent motion of an object into the surface. If the required force is larger than the material can provide, the surface will break.

PRACTICE PROBLEM

At what angle must $\vec{\mathbf{F}}_2$ be applied if the block of ice is to have zero acceleration? [**Answer:** $a_x = 0$ implies $F_1 \cos 60.0° = F_2 \cos\theta$. Thus, $\theta = 54°$.]

Some related homework problems: Problem 44, Problem 50

To this point, we have considered surfaces that are horizontal, in which case the normal force is vertical. When a surface is inclined, the normal force is still at right angles to the surface, even though it is no longer vertical. This is illustrated in **Figure 5–14**. (If friction is present, a surface may also exert a force that is parallel to its surface. This will be considered in detail in Chapter 6.)

When choosing a coordinate system for an inclined surface, it is generally best to have the x and y axes of the system parallel and perpendicular to the surface, respectively, as in **Figure 5–15**. One can imagine the coordinate system to be "bolted down" to the surface, so that when the surface is tilted the coordinate system tilts along with it.

With this choice of coordinate system, there is no motion in the y direction, even on the inclined surface, and the normal force points in the positive y direction. Thus, the condition that determines the normal force is still $\Sigma F_y = ma_y = 0$, as before. In addition, if the object slides on the surface, its motion is purely in the x direction.

Finally, if the surface is inclined by an angle θ, note that the weight—which is still vertically downward—is at the same angle θ with respect to the negative

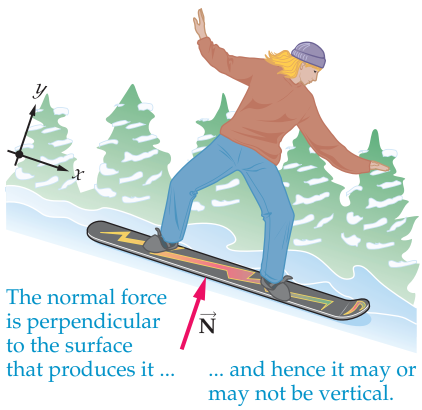


FIGURE 5–14 An object on an inclined surface

The normal force $\vec{\mathbf{N}}$ is always at right angles to the surface; hence, it is not always in the vertical direction.

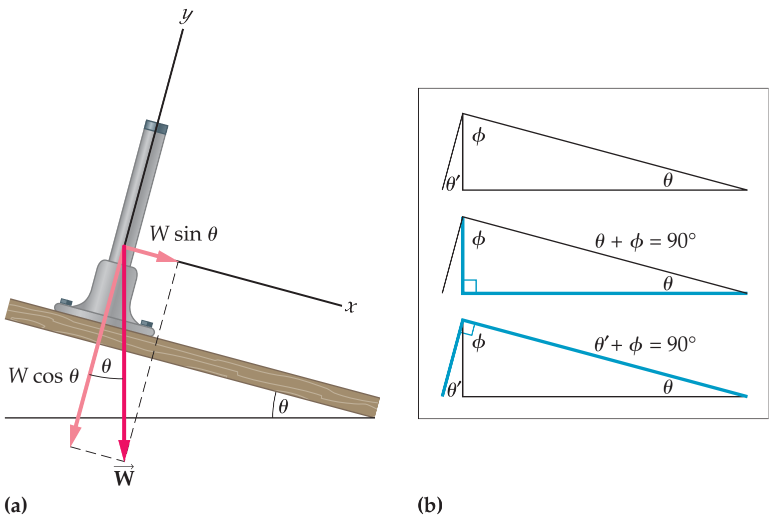


FIGURE 5–15 Components of the weight on an inclined surface

Whenever a surface is tilted by an angle θ, the weight $\vec{\mathbf{W}}$ makes the same angle θ with respect to the negative y axis. This is proven in part (b), where we show that $\theta + \phi = 90°$, and that $\theta' + \phi = 90°$. From these results it follows that $\theta' = \theta$. The component of the weight perpendicular to the surface is $W_y = -W\cos\theta$; the component parallel to the surface is $W_x = W\sin\theta$.

y axis, as shown in Figure 5–15. As a result, the x and y components of the weight are

$$W_x = W \sin\theta = mg \sin\theta \qquad \text{5–8}$$

and

$$W_y = -W \cos\theta = -mg \cos\theta \qquad \text{5–9}$$

Let's quickly check some special cases of these results. First, if $\theta = 0$ the surface is horizontal, and we find $W_x = 0$, $W_y = -mg$, as expected. Second, if $\theta = 90°$ the surface is vertical; therefore, the weight is parallel to the surface, pointing in the positive x direction. In this case, $W_x = mg$ and $W_y = 0$.

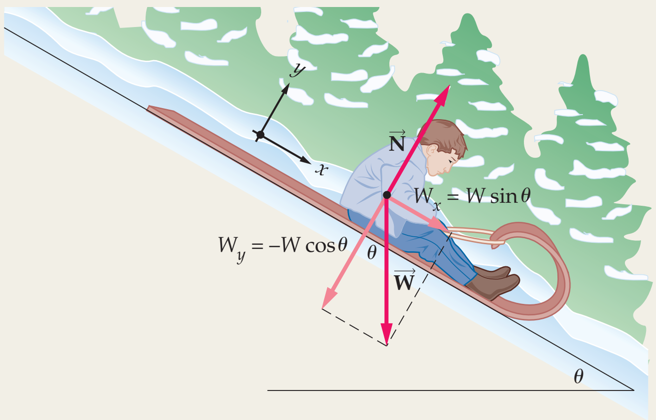
The next Example shows how to use the weight components to find the acceleration of an object on an inclined surface.

EXAMPLE 5–9 TOBOGGAN TO THE BOTTOM

A child of mass m rides on a toboggan down a slick, ice-covered hill inclined at an angle θ with respect to the horizontal. **(a)** What is the acceleration of the child? **(b)** What is the normal force exerted on the child by the toboggan?

PICTURE THE PROBLEM

We choose the x axis to be parallel to the slope, with the positive direction pointing downhill. Similarly, we choose the y axis to be perpendicular to the slope, pointing up and to the right. With these choices, the x component of $\vec{\mathbf{W}}$ is positive, $W_x = W \sin\theta$, and its y component is negative, $W_y = -W\cos\theta$. Finally, the x component of the normal force is zero, $N_x = 0$, and its y component is positive, $N_y = N$.



STRATEGY

Note that only two forces act on the child: (i) the weight, $\vec{\mathbf{W}}$, and (ii) the normal force, $\vec{\mathbf{N}}$. **(a)** We find the child's acceleration by solving $\sum F_x = ma_x$ for a_x. **(b)** Because there is no motion in the y direction, the y component of acceleration is zero. Therefore, we can find the normal force by setting $\sum F_y = ma_y = 0$.

SOLUTION

Part (a)

1. Write out the x components of the forces acting on the child: $N_x = 0 \qquad W_x = W\sin\theta = mg\sin\theta$
2. Sum the x components of the forces and set equal to ma_x: $\sum F_x = N_x + W_x = mg\sin\theta = ma_x$
3. Divide by the mass m to find the acceleration in the x direction: $a_x = \dfrac{\sum F_x}{m} = \dfrac{mg\sin\theta}{m} = g\sin\theta$

Part (b)

4. Write out the y components of the forces acting on the child: $N_y = N \qquad W_y = -W\cos\theta = -mg\cos\theta$
5. Sum the y components of the forces and set the sum equal to zero, since $a_y = 0$: $\sum F_y = N_y + W_y = N - mg\cos\theta = ma_y = 0$
6. Solve for the magnitude of the normal force, N: $N - mg\cos\theta = 0$ or $N = mg\cos\theta$
7. Write the normal force in vector form: $\vec{\mathbf{N}} = (mg\cos\theta)\hat{\mathbf{y}}$

INSIGHT

Note that for θ between 0 and 90° the acceleration of the child is *less* than the acceleration of gravity. This is because only a *component* of the weight is causing the acceleration.

Let's check some special cases of our general result, $a_x = g\sin\theta$. First, let $\theta = 0$. In this case, we find zero acceleration; $a_x = g\sin 0 = 0$. This makes sense because with $\theta = 0$ the hill is actually level, and we don't expect an acceleration. Second, let $\theta = 90°$. In this case, the hill is vertical, and the toboggan should drop straight down in free fall. This also agrees with our general result; $a_x = g\sin 90° = g$.

PRACTICE PROBLEM

What is the child's acceleration if its mass is doubled to $2m$? [**Answer:** The acceleration is still $a_x = g \sin\theta$. As in free fall, the acceleration produced by gravity is independent of mass.]

Some related homework problems: Problem 45, Problem 49

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The fact that a constant force produces a constant acceleration gives special significance to the discussion of constant acceleration in Chapters 2 and 4.

All forces are vectors, and therefore the ability to use and manipulate vectors confidently is essential to a full and complete understanding of forces. Again, we see the importance of the vector material presented in Chapter 3.

As with two-dimensional kinematics in Chapter 4, where motion in the x and y directions were seen to be independent, the x and y components of force are independent as well. In particular, acceleration in the x direction depends only on the x component of force, and acceleration in the y direction depends only on the y component of force.

LOOKING AHEAD

Forces are a central theme throughout physics. In particular, we shall see in Chapters 7 and 8 that a force acting on an object over a distance changes its energy.

Another important application of forces is in the study of collisions. Central to this topic is the concept of momentum, a physical quantity that is changed when a force acts on an object over a period of time.

In this chapter we introduced the force law for gravity near the Earth's surface, $F = mg$. The more general law of gravity, valid at any location, is introduced in Chapter 12. Similarly, the force laws for electricity and magnetism are presented in Chapters 19 and 22, respectively.

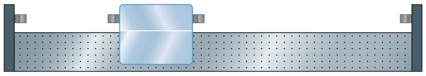
CHAPTER SUMMARY

5–1 FORCE AND MASS

Force
A push or a pull.

Mass
A measure of the difficulty in accelerating an object. Equivalently, a measure of the quantity of matter in an object.

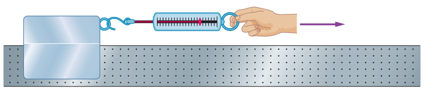
5–2 NEWTON'S FIRST LAW OF MOTION



First Law (Law of Inertia)
If the net force on an object is zero, its velocity is constant.

Inertial Frame of Reference
Frame of reference in which the first law holds. All inertial frames of reference move with constant velocity relative to one another.

5–3 NEWTON'S SECOND LAW OF MOTION



Second Law
An object of mass m has an acceleration $\vec{\mathbf{a}}$ given by the net force $\Sigma\vec{\mathbf{F}}$ divided by m. That is

$$\vec{\mathbf{a}} = \sum \vec{\mathbf{F}}/m \qquad \text{5–1}$$

Component Form

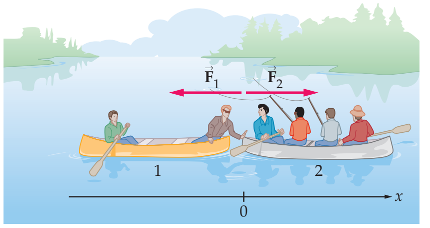
$$a_x = \sum F_x/m \qquad a_y = \sum F_y/m \qquad a_z = \sum F_z/m \qquad \text{5–2}$$

SI Unit: Newton (N)

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2 \qquad \text{5–3}$$

Free-Body Diagram
A sketch showing all external forces acting on an object.

5–4 NEWTON'S THIRD LAW OF MOTION

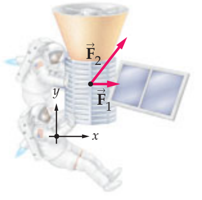


Physical picture

Third Law
For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.

Contact Forces
Action-reaction pair of forces produced by physical contact of two objects.

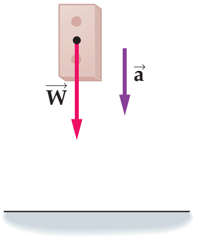
5–5 THE VECTOR NATURE OF FORCES: FORCES IN TWO DIMENSIONS



Forces are vectors.

Newton's second law can be applied to each component of force separately and independently.

5–6 WEIGHT



Gravitational force exerted *by the Earth on an object*.

On the surface of the Earth the weight, W, of an object of mass m has the magnitude

$$W = mg \tag{5–5}$$

Apparent Weight
Force felt from contact with the floor or a scale in an accelerating system. For example, the sensation of feeling heavier or lighter in an accelerating elevator.

5–7 NORMAL FORCES

Force exerted by a surface that is *perpendicular* to the surface.

The normal force is equal to the weight of an object only in special cases. In general, the normal force is greater than or less than the object's weight.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Find the acceleration of an object.	Solve Newton's second law for each component of the acceleration; that is, $a_x = \Sigma F_x/m$ and $a_y = \Sigma F_y/m$.	Examples 5–1, 5–3, 5–4, 5–5, 5–8, 5–9 Active Examples 5–1, 5–2
Solve problems involving action-reaction forces.	Apply Newton's third law, being careful to note that the action-reaction forces act on different objects.	Examples 5–3, 5–4
Find the normal force exerted on an object.	Since there is no acceleration in the normal direction, set the sum of the normal components of force equal to zero.	Examples 5–8, 5–9

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Driving down the road, you hit the brakes suddenly. As a result, your body moves toward the front of the car. Explain, using Newton's laws.
2. You've probably seen pictures of someone pulling a tablecloth out from under glasses, plates, and silverware set out for a formal dinner. Perhaps you've even tried it yourself. Using Newton's laws of motion, explain how this stunt works.
3. As you read this, you are most likely sitting quietly in a chair. Can you conclude, therefore, that you are at rest? Explain.

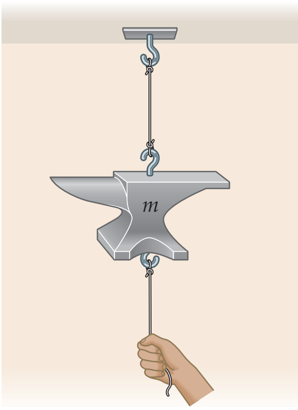
4. When a dog gets wet, it shakes its body from head to tail to shed the water. Explain, in terms of Newton's first law, why this works.

A dog uses the principle of inertia to shake water from its coat. (Conceptual Question 4)

5. A young girl slides down a rope. As she slides faster and faster she tightens her grip, increasing the force exerted on her by the rope. What happens when this force is equal in magnitude to her weight? Explain.

6. A drag-racing car accelerates forward because of the force exerted on it by the road. Why, then, does it need an engine? Explain.

7. A block of mass m hangs from a string attached to a ceiling, as shown in **Figure 5–16**. An identical string hangs down from the bottom of the block. Which string breaks if **(a)** the lower string is pulled with a slowly increasing force or **(b)** the lower string is jerked rapidly downward? Explain.



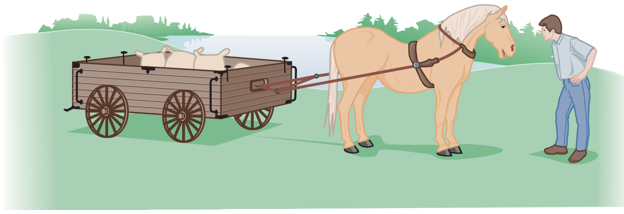
▲ **FIGURE 5–16** Conceptual Question 7

8. An astronaut on a space walk discovers that his jet pack no longer works, leaving him stranded 50 m from the spacecraft. If the jet pack is removable, explain how the astronaut can still use it to return to the ship.

9. Two untethered astronauts on a space walk decide to take a break and play catch with a baseball. Describe what happens as the game of catch progresses.

10. What are the action-reaction forces when a baseball bat hits a fast ball? What is the effect of each force?

11. In **Figure 5–17** Wilbur asks Mr. Ed, the talking horse, to pull a cart. Mr. Ed replies that he would like to, but the laws of nature just won't allow it. According to Newton's third law, he says, if he pulls on the wagon it pulls back on him with an equal force. Clearly, then, the net force is zero and the wagon will stay put. How should Wilbur answer the clever horse?



▲ **FIGURE 5–17** Conceptual Question 11

12. A whole brick has more mass than half a brick, thus the whole brick is harder to accelerate. Why doesn't a whole brick fall more slowly than half a brick? Explain.

13. The force exerted by gravity on a whole brick is greater than the force exerted by gravity on half a brick. Why, then, doesn't a whole brick fall faster than half a brick? Explain.

14. Is it possible for an object at rest to have only a single force acting on it? If your answer is yes, provide an example. If your answer is no, explain why not.

15. Is it possible for an object to be in motion and yet have zero net force acting on it? Explain.

16. A bird cage, with a parrot inside, hangs from a scale. The parrot decides to hop to a higher perch. What can you say about the reading on the scale **(a)** when the parrot jumps, **(b)** when the parrot is in the air, and **(c)** when the parrot lands on the second perch? Assume that the scale responds rapidly so that it gives an accurate reading at all times.

17. Suppose you jump from the cliffs of Acapulco and perform a perfect swan dive. As you fall, you exert an upward force on the Earth equal in magnitude to the downward force the Earth exerts on you. Why, then, does it seem that you are the one doing all the accelerating? Since the forces are the same, why aren't the accelerations?

18. A friend tells you that since his car is at rest, there are no forces acting on it. How would you reply?

19. Since all objects are "weightless" in orbit, how is it possible for an orbiting astronaut to tell if one object has more mass than another object? Explain.

20. To clean a rug, you can hang it from a clothesline and beat it with a tennis racket. Use Newton's laws to explain why beating the rug should have a cleansing effect.

21. If you step off a high board and drop to the water below, you plunge into the water without injury. On the other hand, if you were to drop the same distance onto solid ground, you might break a leg. Use Newton's laws to explain the difference.

22. A moving object is acted on by a net force. Give an example of a situation in which the object moves **(a)** in the same direction as the net force, **(b)** at right angles to the net force, or **(c)** in the opposite direction of the net force.

23. Is it possible for an object to be moving in one direction while the net force acting on it is in another direction? If your answer is yes, provide an example. If your answer is no, explain why not.

24. Since a bucket of water is "weightless" in space, would it hurt to kick the bucket? Explain.

25. In the movie *The Rocketeer*, a teenager discovers a jet-powered backpack in an old barn. The backpack allows him to fly at incredible speeds. In one scene, however, he uses the backpack to rapidly accelerate an old pickup truck that is being chased by "bad guys." He does this by bracing his arms against the cab of the pickup and firing the backpack, giving the truck the acceleration of a drag racer. Is the physics of this scene "Good," "Bad," or "Ugly?" Explain.

26. List three common objects that have a weight of approximately 1 N.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 5–3 NEWTON'S SECOND LAW OF MOTION

1. • **CE** An object of mass m is initially at rest. After a force of magnitude F acts on it for a time T, the object has a speed v. Suppose the mass of the object is doubled, and the magnitude of the force acting on it is quadrupled. In terms of T, how long does it take for the object to accelerate from rest to a speed v now?

2. • On a planet far, far away, an astronaut picks up a rock. The rock has a mass of 5.00 kg, and on this particular planet its weight is 40.0 N. If the astronaut exerts an upward force of 46.2 N on the rock, what is its acceleration?

3. • In a grocery store, you push a 12.3-kg shopping cart with a force of 10.1 N. If the cart starts at rest, how far does it move in 2.50 s?

4. • You are pulling your little sister on her sled across an icy (frictionless) surface. When you exert a constant horizontal force of 120 N, the sled has an acceleration of 2.5 m/s^2. If the sled has a mass of 7.4 kg, what is the mass of your little sister?

5. • A 0.53-kg billiard ball initially at rest is given a speed of 12 m/s during a time interval of 4.0 ms. What average force acted on the ball during this time?

6. • A 92-kg water skier floating in a lake is pulled from rest to a speed of 12 m/s in a distance of 25 m. What is the net force exerted on the skier, assuming his acceleration is constant?

7. •• **CE Predict/Explain** You drop two balls of equal diameter from the same height at the same time. Ball 1 is made of metal and has a greater mass than ball 2, which is made of wood. The upward force due to air resistance is the same for both balls. **(a)** Is the drop time of ball 1 greater than, less than, or equal to the drop time of ball 2? **(b)** Choose the *best explanation* from among the following:

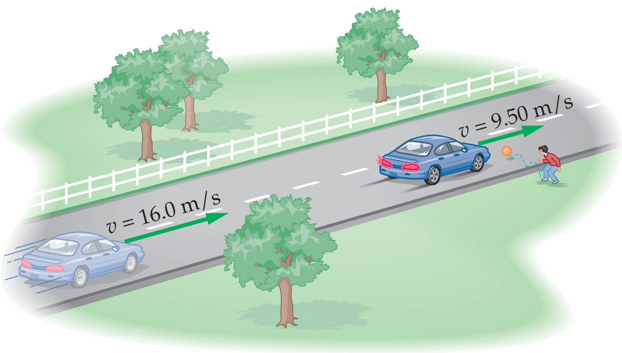
I. The acceleration of gravity is the same for all objects, regardless of mass.
II. The more massive ball is harder to accelerate.
III. Air resistance has less effect on the more massive ball.

8. •• **IP** A 42.0-kg parachutist is moving straight downward with a speed of 3.85 m/s. **(a)** If the parachutist comes to rest with constant acceleration over a distance of 0.750 m, what force does the ground exert on her? **(b)** If the parachutist comes to rest over a shorter distance, is the force exerted by the ground greater than, less than, or the same as in part (a)? Explain.

9. •• **IP** In baseball, a pitcher can accelerate a 0.15-kg ball from rest to 98 mi/h in a distance of 1.7 m. **(a)** What is the average force exerted on the ball during the pitch? **(b)** If the mass of the ball is increased, is the force required of the pitcher increased, decreased, or unchanged? Explain.

10. •• A major-league catcher gloves a 92-mi/h pitch and brings it to rest in 0.15 m. If the force exerted by the catcher is 803 N, what is the mass of the ball?

11. •• Driving home from school one day, you spot a ball rolling out into the street (**Figure 5–18**). You brake for 1.20 s, slowing your 950-kg car from 16.0 m/s to 9.50 m/s. **(a)** What was the average force exerted on your car during braking? **(b)** How far did you travel while braking?



▲ **FIGURE 5–18** Problem 11

12. •• **Stopping a 747** A 747 jetliner lands and begins to slow to a stop as it moves along the runway. If its mass is 3.50×10^5 kg, its speed is 27.0 m/s, and the net braking force is 4.30×10^5 N, **(a)** what is its speed 7.50 s later? **(b)** How far has it traveled in this time?

13. •• **IP** A drag racer crosses the finish line doing 202 mi/h and promptly deploys her drag chute (the small parachute used for braking). **(a)** What force must the drag chute exert on the 891-kg car to slow it to 45.0 mi/h in a distance of 185 m? **(b)** Describe the strategy you used to solve part (a).

SECTION 5–4 NEWTON'S THIRD LAW OF MOTION

14. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the magnitude of the force experienced by the car greater than, less than, or equal to the magnitude of the force experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. Action-reaction forces always have equal magnitude.
II. The truck has more mass, and hence the force exerted on it is greater.
III. The massive truck exerts a greater force on the lightweight car.

15. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the acceleration experienced by the car greater than, less than, or equal to the acceleration experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. The truck exerts a larger force on the car, giving it the greater acceleration.

II. Both vehicles experience the same magnitude of force, therefore the lightweight car experiences the greater acceleration.

III. The greater force exerted on the truck gives it the greater acceleration.

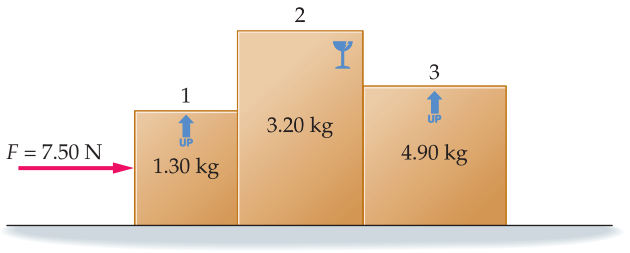
16. • You hold a brick at rest in your hand. **(a)** How many forces act on the brick? **(b)** Identify these forces. **(c)** Are these forces equal in magnitude and opposite in direction? **(d)** Are these forces an action-reaction pair? Explain.

17. • Referring to Problem 16, you are now accelerating the brick upward. **(a)** How many forces act on the brick in this case? **(b)** Identify these forces. **(c)** Are these forces equal in magnitude and opposite in direction? **(d)** Are these forces an action-reaction pair? Explain.

18. •• On vacation, your 1400-kg car pulls a 560-kg trailer away from a stoplight with an acceleration of 1.85 m/s^2. **(a)** What is the net force exerted on the trailer? **(b)** What force does the trailer exert on the car? **(c)** What is the net force acting on the car?

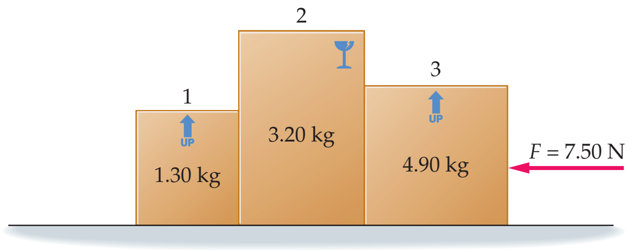
19. •• **IP** A 71-kg parent and a 19-kg child meet at the center of an ice rink. They place their hands together and push. **(a)** Is the force experienced by the child more than, less than, or the same as the force experienced by the parent? **(b)** Is the acceleration of the child more than, less than, or the same as the acceleration of the parent? Explain. **(c)** If the acceleration of the child is 2.6 m/s^2 in magnitude, what is the magnitude of the parent's acceleration?

20. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in **Figure 5–19**. Find the magnitude of the contact force **(a)** between boxes 1 and 2, and **(b)** between boxes 2 and 3.



▲ **FIGURE 5–19** Problem 20

21. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in **Figure 5–20**. Find the magnitude of the contact force **(a)** between boxes 1 and 2, and **(b)** between boxes 2 and 3.



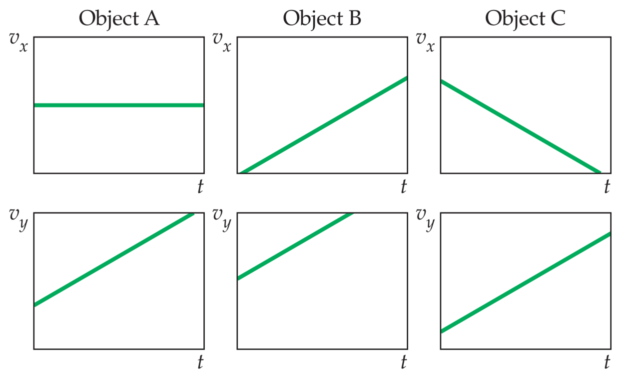
▲ **FIGURE 5–20** Problem 21

22. •• **IP** Two boxes sit side-by-side on a smooth horizontal surface. The lighter box has a mass of 5.2 kg; the heavier box has a mass of 7.4 kg. **(a)** Find the contact force between these boxes when a horizontal force of 5.0 N is applied to the light box. **(b)** If the 5.0-N force is applied to the heavy box instead, is the contact force between the boxes the same as, greater than, or less than the contact force in part (a)? Explain. **(c)** Verify your answer to part (b) by calculating the contact force in this case.

SECTION 5–5 THE VECTOR NATURE OF FORCES

23. • **CE** A skateboarder on a ramp is accelerated by a nonzero net force. For each of the following statements, state whether it is always true, never true, or sometimes true. **(a)** The skateboarder is moving in the direction of the net force. **(b)** The acceleration of the skateboarder is at right angles to the net force. **(c)** The acceleration of the skateboarder is in the same direction as the net force. **(d)** The skateboarder is instantaneously at rest.

24. • **CE** Three objects, A, B, and C, have x and y components of velocity that vary with time as shown in **Figure 5–21**. What is the direction of the net force acting on **(a)** object A, **(b)** object B, and **(c)** object C, as measured from the positive x axis? (All of the nonzero slopes have the same magnitude.)

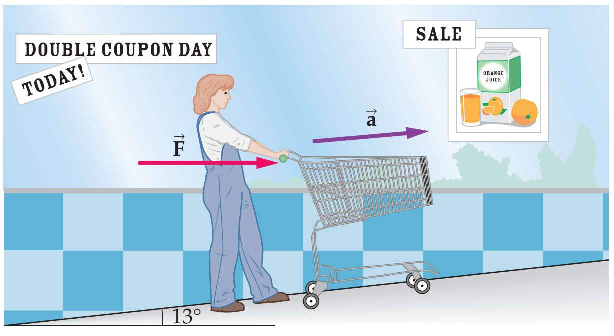


▲ **FIGURE 5–21** Problem 24

25. • A farm tractor tows a 3700-kg trailer up an 18° incline with a steady speed of 3.2 m/s. What force does the tractor exert on the trailer? (Ignore friction.)

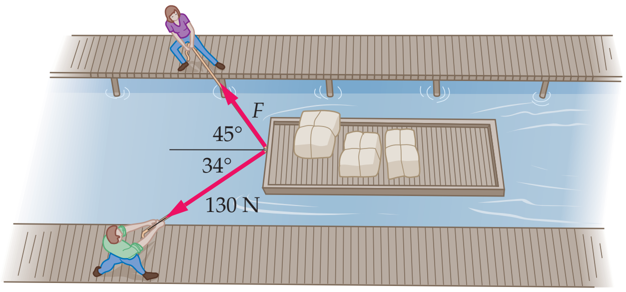
26. • A surfer "hangs ten," and accelerates down the sloping face of a wave. If the surfer's acceleration is 3.25 m/s^2 and friction can be ignored, what is the angle at which the face of the wave is inclined above the horizontal?

27. • A shopper pushes a 7.5-kg shopping cart up a 13° incline, as shown in **Figure 5–22**. Find the magnitude of the horizontal force, $\vec{F}$, needed to give the cart an acceleration of 1.41 m/s^2.



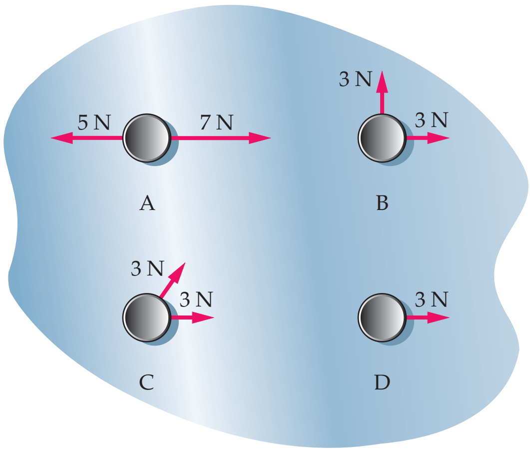
▲ **FIGURE 5–22** Problem 27

28. • Two crewmen pull a raft through a lock, as shown in **Figure 5–23**. One crewman pulls with a force of 130 N at an angle of 34° relative to the forward direction of the raft. The second crewman, on the opposite side of the lock, pulls at an angle of 45°. With what force should the second crewman pull so that the net force of the two crewmen is in the forward direction?



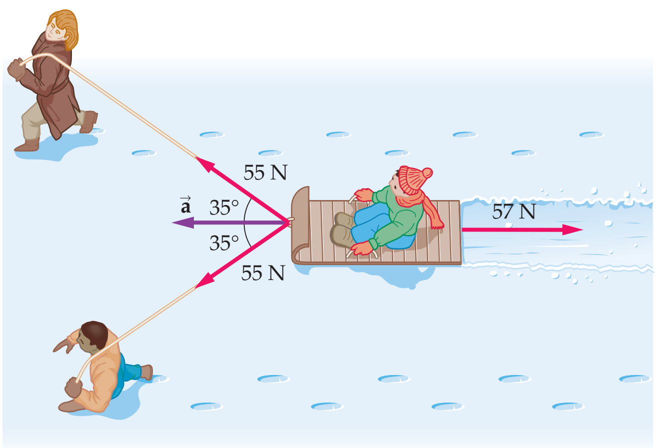
▲ **FIGURE 5–23** Problem 28

29. •• **CE** A hockey puck is acted on by one or more forces, as shown in **Figure 5–24**. Rank the four cases, A, B, C, and D, in order of the magnitude of the puck's acceleration, starting with the smallest. Indicate ties where appropriate.



▲ **FIGURE 5–24** Problem 29

30. •• To give a 19-kg child a ride, two teenagers pull on a 3.7-kg sled with ropes, as indicated in **Figure 5–25**. Both teenagers pull with a force of 55 N at an angle of 35° relative to the forward direction, which is the direction of motion. In addition, the snow exerts a retarding force on the sled that points opposite to the direction of motion, and has a magnitude of 57 N. Find the acceleration of the sled and child.



▲ **FIGURE 5–25** Problem 30

31. •• **IP** Before practicing his routine on the rings, a 67-kg gymnast stands motionless, with one hand grasping each ring and his feet touching the ground. Both arms slope upward at an angle of 24° above the horizontal. **(a)** If the force exerted by the rings on each arm has a magnitude of 290 N, and is directed along the length of the arm, what is the magnitude of the force exerted by the floor on his feet? **(b)** If the angle his arms make with the horizontal is greater that 24°, and everything else remains the same, is the force exerted by the floor on his feet greater than, less than, or the same as the value found in part (a)? Explain.

32. •• **IP** A 65-kg skier speeds down a trail, as shown in **Figure 5–26**. The surface is smooth and inclined at an angle of 22° with the horizontal. **(a)** Find the direction and magnitude of the net force acting on the skier. **(b)** Does the net force exerted on the skier increase, decrease, or stay the same as the slope becomes steeper? Explain.

▲ **FIGURE 5–26** Problems 32 and 45

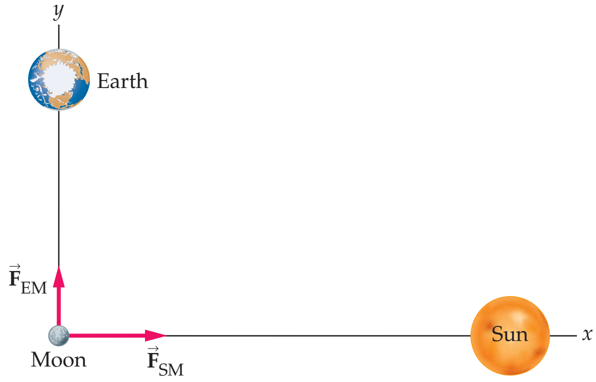
33. •• An object acted on by three forces moves with constant velocity. One force acting on the object is in the positive x direction and has a magnitude of 6.5 N; a second force has a magnitude of 4.4 N and points in the negative y direction. Find the direction and magnitude of the third force acting on the object.

34. •• A train is traveling up a 3.73° incline at a speed of 3.25 m/s when the last car breaks free and begins to coast without friction. **(a)** How long does it take for the last car to come to rest momentarily? **(b)** How far did the last car travel before momentarily coming to rest?

35. •• **The Force Exerted on the Moon** **Figure 5–27** shows the Earth, Moon, and Sun (not to scale) in their relative positions at the time when the Moon is in its third-quarter phase. Though few people realize it, the force exerted on the Moon by the Sun is actually greater than the force exerted on the Moon by the Earth. In fact, the force exerted on the Moon by the Sun has a magnitude of $F_{SM} = 4.34 \times 10^{20}$ N, whereas the force exerted by the Earth has a magnitude of only $F_{EM} = 1.98 \times 10^{20}$ N. These forces are indicated to scale in Figure 5–27. Find **(a)** the direction and **(b)** the magnitude of the net force acting on the Moon. **(c)** Given that the mass of the Moon is $M_M = 7.35 \times 10^{22}$ kg, find the magnitude of its acceleration at the time of the third-quarter phase.

SECTION 5–6 WEIGHT

36. • You pull upward on a stuffed suitcase with a force of 105 N, and it accelerates upward at 0.705 m/s^2. What are **(a)** the mass and **(b)** the weight of the suitcase?



▲ **FIGURE 5–27** Problem 35

37. • **BIO** **Brain Growth** A newborn baby's brain grows rapidly. In fact, it has been found to increase in mass by about 1.6 mg per minute. **(a)** How much does the brain's weight increase in one day? **(b)** How long does it take for the brain's weight to increase by 0.15 N?

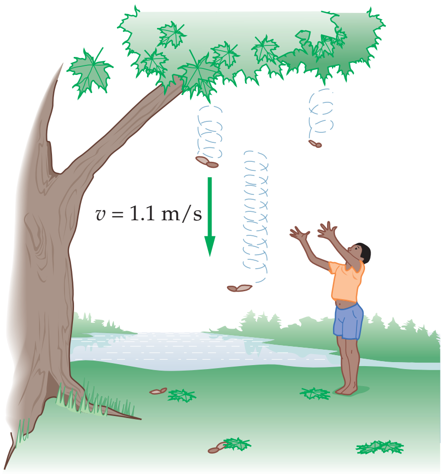
38. • Suppose a rocket launches with an acceleration of 30.5 m/s^2. What is the apparent weight of an 92-kg astronaut aboard this rocket?

39. • At the bow of a ship on a stormy sea, a crewman conducts an experiment by standing on a bathroom scale. In calm waters, the scale reads 182 lb. During the storm, the crewman finds a maximum reading of 225 lb and a minimum reading of 138 lb. Find **(a)** the maximum upward acceleration and **(b)** the maximum downward acceleration experienced by the crewman.

40. •• **IP** As part of a physics experiment, you stand on a bathroom scale in an elevator. Though your normal weight is 610 N, the scale at the moment reads 730 N. **(a)** Is the acceleration of the elevator upward, downward, or zero? Explain. **(b)** Calculate the magnitude of the elevator's acceleration. **(c)** What, if anything, can you say about the velocity of the elevator? Explain.

41. •• When you weigh yourself on good old *terra firma* (solid ground), your weight is 142 lb. In an elevator your apparent weight is 121 lb. What are the direction and magnitude of the elevator's acceleration?

42. •• **IP** **BIO** **Flight of the Samara** A 1.21-g samara—the winged fruit of a maple tree—falls toward the ground with a constant speed of 1.1 m/s (**Figure 5–28**). **(a)** What is the force of air resistance exerted on the samara? **(b)** If the constant speed of descent is greater than 1.1 m/s, is the force of air resistance greater than, less than, or the same as in part (a)? Explain.



▲ **FIGURE 5–28** Problem 42

43. ••• When you lift a bowling ball with a force of 82 N, the ball accelerates upward with an acceleration a. If you lift with a force of 92 N, the ball's acceleration is $2a$. Find **(a)** the weight of the bowling ball, and **(b)** the acceleration a.

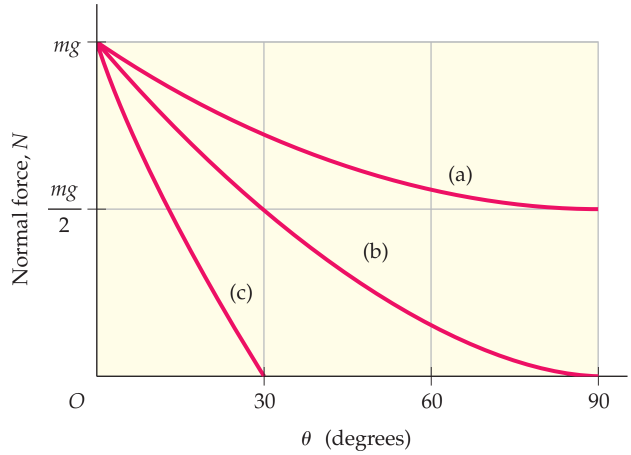
SECTION 5–7 NORMAL FORCES

44. • A 23-kg suitcase is being pulled with constant speed by a handle that is at an angle of 25° above the horizontal. If the normal force exerted on the suitcase is 180 N, what is the force F applied to the handle?

45. • **(a)** Draw a free-body diagram for the skier in Problem 32. **(b)** Determine the normal force acting on the skier.

46. • A 9.3-kg child sits in a 3.7-kg high chair. **(a)** Draw a free-body diagram for the child, and find the normal force exerted by the chair on the child. **(b)** Draw a free-body diagram for the chair, and find the normal force exerted by the floor on the chair.

47. •• **Figure 5–29** shows the normal force as a function of the angle θ for the suitcase shown in Figure 5–13. Determine the magnitude of the force $\vec{F}$ for each of the three curves shown in Figure 5–29. Give your answer in terms of the weight of the suitcase, mg.



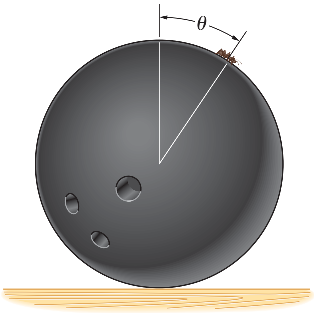
▲ **FIGURE 5–29** Problem 47

48. •• A 5.0-kg bag of potatoes sits on the bottom of a stationary shopping cart. **(a)** Sketch a free-body diagram for the bag of potatoes. **(b)** Now suppose the cart moves with a constant velocity. How does this affect your free-body diagram? Explain.

49. •• **IP** **(a)** Find the normal force exerted on a 2.9-kg book resting on a surface inclined at 36° above the horizontal. **(b)** If the angle of the incline is reduced, do you expect the normal force to increase, decrease, or stay the same? Explain.

50. •• **IP** A gardener mows a lawn with an old-fashioned push mower. The handle of the mower makes an angle of 35° with the surface of the lawn. **(a)** If a 219-N force is applied along the handle of the 19-kg mower, what is the normal force exerted by the lawn on the mower? **(b)** If the angle between the surface of the lawn and the handle of the mower is increased, does the normal force exerted by the lawn increase, decrease, or stay the same? Explain.

51. ••• An ant walks slowly away from the top of a bowling ball, as shown in **Figure 5–30**. If the ant starts to slip when the normal force on its feet drops below one-half its weight, at what angle θ does slipping begin?



▲ **FIGURE 5–30** Problem 51

GENERAL PROBLEMS

52. • **CE Predict/Explain** Riding in an elevator moving upward with constant speed, you begin a game of darts. **(a)** Do you have to aim your darts higher than, lower than, or the same as when you play darts on solid ground? **(b)** Choose the *best explanation* from among the following:
    I. The elevator rises during the time it takes for the dart to travel to the dartboard.
    II. The elevator moves with constant velocity. Therefore, Newton's laws apply within the elevator in the same way as on the ground.
    III. You have to aim lower to compensate for the upward speed of the elevator.

53. • **CE Predict/Explain** Riding in an elevator moving with a constant upward acceleration, you begin a game of darts. **(a)** Do you have to aim your darts higher than, lower than, or the same as when you play darts on solid ground? **(b)** Choose the *best explanation* from among the following:
    I. The elevator accelerates upward, giving its passengers a greater "effective" acceleration of gravity.
    II. You have to aim lower to compensate for the upward acceleration of the elevator.
    III. Since the elevator moves with a constant acceleration, Newton's laws apply within the elevator the same as on the ground.

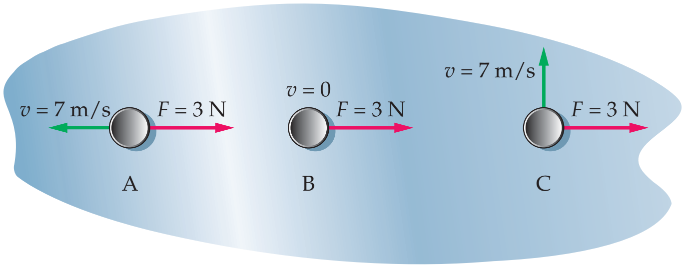
54. • **CE** Give the direction of the net force acting on each of the following objects. If the net force is zero, state "zero." **(a)** A car accelerating northward from a stoplight. **(b)** A car traveling southward and slowing down. **(c)** A car traveling westward with constant speed. **(d)** A skydiver parachuting downward with constant speed. **(e)** A baseball during its flight from pitcher to catcher (ignoring air resistance).

55. • **CE Predict/Explain** You jump out of an airplane and open your parachute after an extended period of free fall. **(a)** To decelerate your fall, must the force exerted on you by the parachute be greater than, less than, or equal to your weight? **(b)** Choose the *best explanation* from among the following:
    I. Parachutes can only exert forces that are less than the weight of the skydiver.
    II. The parachute exerts a force exactly equal to the skydiver's weight.
    III. To decelerate after free fall, the net force acting on a skydiver must be upward.

56. • In a tennis serve, a 0.070-kg ball can be accelerated from rest to 36 m/s over a distance of 0.75 m. Find the magnitude of the average force exerted by the racket on the ball during the serve.

57. • A 51.5-kg swimmer with an initial speed of 1.25 m/s decides to coast until she comes to rest. If she slows with constant acceleration and stops after coasting 2.20 m, what was the force exerted on her by the water?

58. •• **CE** Each of the three identical hockey pucks shown in **Figure 5–31** is acted on by a 3-N force. Puck A moves with a speed of 7 m/s in a direction opposite to the force; puck B is instantaneously at rest; puck C moves with a speed of 7 m/s at right angles to the force. Rank the three pucks in order of the magnitude of their acceleration, starting with the smallest. Indicate ties with an equal sign.



▲ **FIGURE 5–31** Problem 58

59. •• **IP The VASIMR Rocket** NASA plans to use a new type of rocket, a Variable Specific Impulse Magnetoplasma Rocket (VASIMR), on future missions. A VASIMR can produce 1200 N of thrust (force) when in operation. If a VASIMR has a mass of 2.2×10^5 kg, **(a)** what acceleration will it experience? Assume that the only force acting on the rocket is its own thrust, and that the mass of the rocket is constant. **(b)** Over what distance must the rocket accelerate from rest to achieve a speed of 9500 m/s? **(c)** When the rocket has covered one-quarter the acceleration distance found in part (b), is its average speed 1/2, 1/3, or 1/4 its average speed during the final three-quarters of the acceleration distance? Explain.

60. •• An object of mass $m = 5.95$ kg has an acceleration $\vec{\mathbf{a}} = (1.17\ \mathrm{m/s^2})\hat{\mathbf{x}} + (-0.664\ \mathrm{m/s^2})\hat{\mathbf{y}}$. Three forces act on this object: $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, and $\vec{\mathbf{F}}_3$. Given that $\vec{\mathbf{F}}_1 = (3.22\ \mathrm{N})\hat{\mathbf{x}}$ and $\vec{\mathbf{F}}_2 = (-1.55\ \mathrm{N})\hat{\mathbf{x}} + (2.05\ \mathrm{N})\hat{\mathbf{y}}$, find $\vec{\mathbf{F}}_3$.

61. •• At the local grocery store, you push a 14.5-kg shopping cart. You stop for a moment to add a bag of dog food to your cart. With a force of 12.0 N, you now accelerate the cart from rest through a distance of 2.29 m in 3.00 s. What was the mass of the dog food?

62. •• **IP BIO The Force of Running** Biomechanical research has shown that when a 67-kg person is running, the force exerted on each foot as it strikes the ground can be as great as 2300 N. **(a)** What is the ratio of the force exerted on the foot by the ground to the person's body weight? **(b)** If the only forces acting on the person are (i) the force exerted by the ground and (ii) the person's weight, what are the magnitude and direction of the person's acceleration? **(c)** If the acceleration found in part (b) acts for 10.0 ms, what is the resulting change in the vertical component of the person's velocity?

63. •• **IP BIO Grasshopper Liftoff** To become airborne, a 2.0-g grasshopper requires a takeoff speed of 2.7 m/s. It acquires this speed by extending its hind legs through a distance of 3.7 cm. **(a)** What is the average acceleration of the grasshopper during takeoff? **(b)** Find the magnitude of the average net force exerted

on the grasshopper by its hind legs during takeoff. **(c)** If the mass of the grasshopper increases, does the takeoff acceleration increase, decrease, or stay the same? **(d)** If the mass of the grasshopper increases, does the required takeoff force increase, decrease, or stay the same? Explain.

64. •• **Takeoff from an Aircraft Carrier** On an aircraft carrier, a jet can be catapulted from 0 to 155 mi/h in 2.00 s. If the average force exerted by the catapult is 9.35×10^5 N, what is the mass of the jet?

A jet takes off from the flight deck of an aircraft carrier. (Problem 64)

65. •• **IP** An archer shoots a 0.024-kg arrow at a target with a speed of 54 m/s. When it hits the target, it penetrates to a depth of 0.083 m. **(a)** What was the average force exerted by the target on the arrow? **(b)** If the mass of the arrow is doubled, and the force exerted by the target on the arrow remains the same, by what multiplicative factor does the penetration depth change? Explain.

66. •• An apple of mass $m = 0.13$ kg falls out of a tree from a height $h = 3.2$ m. **(a)** What is the magnitude of the force of gravity, mg, acting on the apple? **(b)** What is the apple's speed, v, just before it lands? **(c)** Show that the force of gravity times the height, mgh, is equal to $\frac{1}{2}mv^2$. (We shall investigate the significance of this result in Chapter 8.) Be sure to show that the dimensions are in agreement as well as the numerical values.

67. •• An apple of mass $m = 0.22$ kg falls from a tree and hits the ground with a speed of $v = 14$ m/s. **(a)** What is the magnitude of the force of gravity, mg, acting on the apple? **(b)** What is the time, t, required for the apple to reach the ground? **(c)** Show that the force of gravity times the time, mgt, is equal to mv. (We shall investigate the significance of this result in Chapter 9.) Be sure to show that the dimensions are in agreement as well as the numerical values.

68. •• **BIO The Fall of *T. rex*** Paleontologists estimate that if a *Tyrannosaurus rex* were to trip and fall, it would have experienced a force of approximately 260,000 N acting on its torso when it hit the ground. Assuming the torso has a mass of 3800 kg, **(a)** find the magnitude of the torso's upward acceleration as it comes to rest. (For comparison, humans lose consciousness with an acceleration of about $7g$.) **(b)** Assuming the torso is in free fall for a distance of 1.46 m as it falls to the ground, how much time is required for the torso to come to rest once it contacts the ground?

69. •• **Deep Space I** The NASA spacecraft *Deep Space I* was shut down on December 18, 2001, following a three-year journey to the asteroid Braille and the comet Borrelly. This spacecraft used a solar-powered ion engine to produce 0.064 ounces of thrust (force) by stripping electrons from neon atoms and accelerating the resulting ions to 70,000 mi/h. The thrust was only as much as the weight of a couple sheets of paper, but the engine operated continuously for 16,000 hours. As a result, the speed of the spacecraft increased by 7900 mi/h. What was the mass of *Deep Space I*? (Assume that the mass of the neon gas is negligible.)

70. •• Your groceries are in a bag with paper handles. The handles will tear off if a force greater than 51.5 N is applied to them. What is the greatest mass of groceries that can be lifted safely with this bag, given that the bag is raised **(a)** with constant speed, or **(b)** with an acceleration of 1.25 m/s^2?

71. •• **IP** While waiting at the airport for your flight to leave, you observe some of the jets as they take off. With your watch you find that it takes about 35 seconds for a plane to go from rest to takeoff speed. In addition, you estimate that the distance required is about 1.5 km. **(a)** If the mass of a jet is 1.70×10^5 kg, what force is needed for takeoff? **(b)** Describe the strategy you used to solve part (a).

72. •• **BIO Gecko Feet** Researchers have found that a gecko's foot is covered with hundreds of thousands of small hairs (*setae*) that allow it to walk up walls and even across ceilings. A single foot pad, which has an area of 1.0 cm^2, can attach to a wall or ceiling with a force of 11 N. **(a)** How many 250-g geckos could be suspended from the ceiling by a single foot pad? **(b)** Estimate the force per square centimeter that your body exerts on the soles of your shoes, and compare with the 11 N/cm^2 of the sticky gecko foot.

A Tokay gecko (*Gekko gecko*) shows off its famous feet. (Problem 72)

73. •• Two boxes are at rest on a smooth, horizontal surface. The boxes are in contact with one another. If box 1 is pushed with a force of magnitude $F = 12.00$ N, the contact force between the boxes is 8.50 N; if, instead, box 2 is pushed with the force F, the contact force is 12.00 N − 8.50 N = 3.50 N. In either case, the boxes move together with an acceleration of 1.70 m/s^2. What is the mass of **(a)** box 1 and **(b)** box 2?

74. ••• **IP** Responding to an alarm, a 102-kg fireman slides down a pole to the ground floor, 3.3 m below. The fireman starts at rest and lands with a speed of 4.2 m/s. **(a)** Find the average force exerted on the fireman by the pole. **(b)** If the landing speed is half that in part (a), is the average force exerted on the fireman by the pole doubled? Explain. **(c)** Find the average force exerted on the fireman by the pole when the landing speed is 2.1 m/s.

75. ••• For a birthday gift, you and some friends take a hot-air balloon ride. One friend is late, so the balloon floats a couple of feet off the ground as you wait. Before this person arrives, the combined weight of the basket and people is 1220 kg, and the balloon is neutrally buoyant. When the late arrival climbs up into the basket, the balloon begins to accelerate downward at $0.56\ \mathrm{m/s^2}$. What was the mass of the last person to climb aboard?

76. ••• A baseball of mass m and initial speed v strikes a catcher's mitt. If the mitt moves a distance Δx as it brings the ball to rest, what is the average force it exerts on the ball?

77. ••• When two people push in the same direction on an object of mass m they cause an acceleration of magnitude a_1. When the same people push in opposite directions, the acceleration of the object has a magnitude a_2. Determine the magnitude of the force exerted by each of the two people in terms of m, a_1, and a_2.

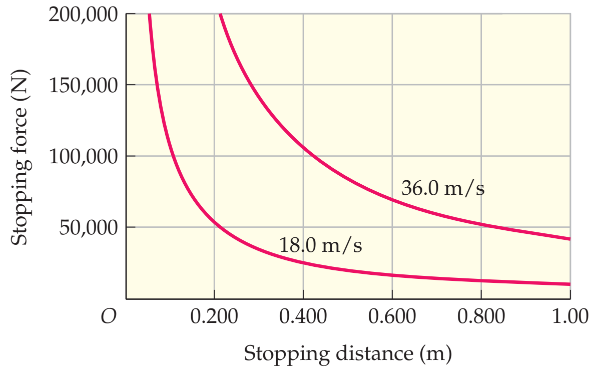
78. ••• An air-track cart of mass $m_1 = 0.14$ kg is moving with a speed $v_0 = 1.3$ m/s to the right when it collides with a cart of mass $m_2 = 0.25$ kg that is at rest. Each cart has a wad of putty on its bumper, and hence they stick together as a result of their collision. Suppose the average contact force between the carts is $F = 1.5$ N during the collision. **(a)** What is the acceleration of cart 1? Give direction and magnitude. **(b)** What is the acceleration of cart 2? Give direction and magnitude. **(c)** How long does it take for both carts to have the same speed? (Once the carts have the same speed the collision is over and the contact force vanishes.) **(d)** What is the final speed of the carts, v_f? **(e)** Show that m_1v_0 is equal to $(m_1 + m_2)v_f$. (We shall investigate the significance of this result in Chapter 9.)

PASSAGE PROBLEMS

BIO Increasing Safety in a Collision

Safety experts say that an automobile accident is really a succession of three separate collisions. These can be described as follows: (1) the automobile collides with an obstacle and comes to rest; (2) people within the car continue to move forward until they collide with the interior of the car, or are brought to rest by a restraint system like a seatbelt or an air bag; (3) organs within the occupants' bodies continue to move forward until they collide with the body wall and are brought to rest. Not much can be done about the third collision, but the effects of the first two can be mitigated by increasing the distance over which the car and its occupants are brought to rest.

For example, the severity of the first collision is reduced by building collapsible "crumple zones" into the body of a car, and by placing compressible collision barriers near dangerous obstacles like bridge supports. The second collision is addressed primarily through the use of seatbelts and air bags. These devices reduce the force that acts on an occupant to survivable levels by increasing the distance over which he or she comes to rest. This is illustrated in Figure 5–32, where we see the force exerted on a 65.0-kg driver who slows from an initial speed of 18.0 m/s (lower curve) or 36.0 m/s (upper curve) to rest in a distance ranging from 5.00 cm to 1.00 m.



▲ **FIGURE 5–32** Problems 79, 80, 81, and 82

79. • The combination of "crumple zones" and air bags/seatbelts might increase the distance over which a person stops in a collision to as great as 1.00 m. What is the magnitude of the force exerted on a 65.0-kg driver who decelerates from 18.0 m/s to 0.00 m/s over a distance of 1.00 m?

A. 162 N **B.** 585 N

C. 1.05×10^4 N **D.** 2.11×10^4 N

80. • A driver who does not wear a seatbelt continues to move forward with a speed of 18.0 m/s (due to inertia) until something solid like the steering wheel is encountered. The driver now comes to rest in a much shorter distance—perhaps only a few centimeters. Find the magnitude of the net force acting on a 65.0-kg driver who is decelerated from 18.0 m/s to rest in 5.00 cm.

A. 3240 N **B.** 1.17×10^4 N

C. 2.11×10^5 N **D.** 4.21×10^5 N

81. • Suppose the initial speed of the driver is doubled to 36.0 m/s. If the driver still has a mass of 65.0 kg, and comes to rest in 1.00 m, what is the magnitude of the force exerted on the driver during this collision?

A. 648 N **B.** 1170 N

C. 2.11×10^4 N **D.** 4.21×10^4 N

82. • If both the speed and stopping distance of a driver are doubled, by what factor does the force exerted on the driver change?

A. 0.5 **B.** 1

C. 2 **D.** 4

INTERACTIVE PROBLEMS

83. •• **IP Referring to Example 5–4** Suppose that we would like the contact force between the boxes to have a magnitude of 5.00 N, and that the only thing in the system we are allowed to change is the mass of box 2—the mass of box 1 is 10.0 kg and the applied force is 20.0 N. **(a)** Should the mass of box 2 be increased or decreased? Explain. **(b)** Find the mass of box 2 that results in a contact force of magnitude 5.00 N. **(c)** What is the acceleration of the boxes in this case?

84. •• **Referring to Example 5–4** Suppose the force of 20.0 N pushes on two boxes of unknown mass. We know, however, that the acceleration of the boxes is $1.20\ \mathrm{m/s^2}$ and the contact force has a magnitude of 4.45 N. Find the mass of **(a)** box 1 and **(b)** box 2.

85. •• **IP Referring to Figure 5–9** Suppose the magnitude of $\vec{\mathbf{F}}_2$ is increased from 41 N to 55 N, and that everything else in the system remains the same. **(a)** Do you expect the direction of the satellite's acceleration to be greater than, less than, or equal to 32°? Explain. Find **(b)** the direction and **(c)** the magnitude of the satellite's acceleration in this case.

86. •• **IP Referring to Figure 5–9** Suppose we would like the acceleration of the satellite to be at an angle of 25°, and that the only quantity we can change in the system is the magnitude of $\vec{\mathbf{F}}_1$. **(a)** Should the magnitude of $\vec{\mathbf{F}}_1$ be increased or decreased? Explain. **(b)** What is the magnitude of the satellite's acceleration in this case?