FOURTH EDITION

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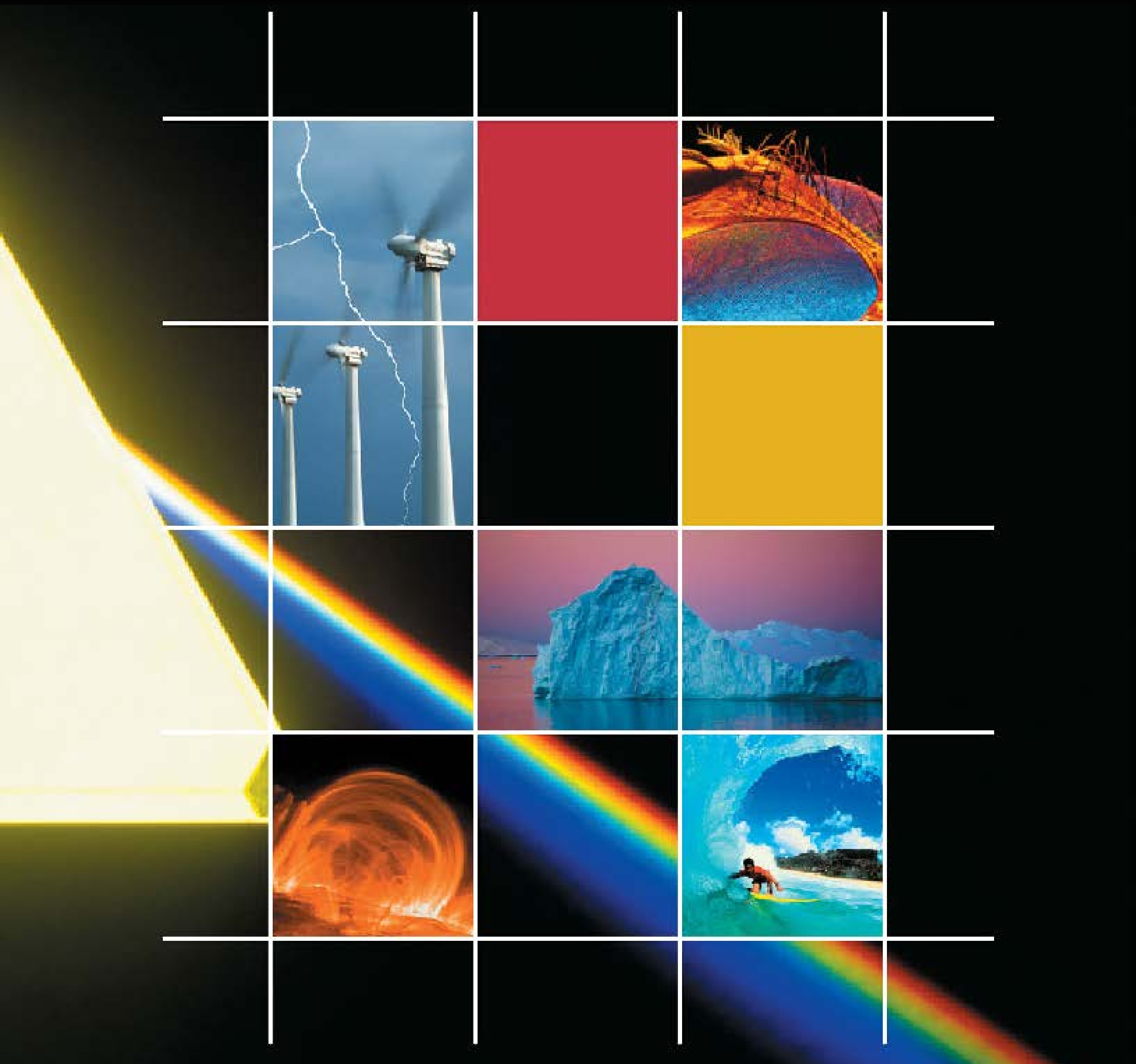
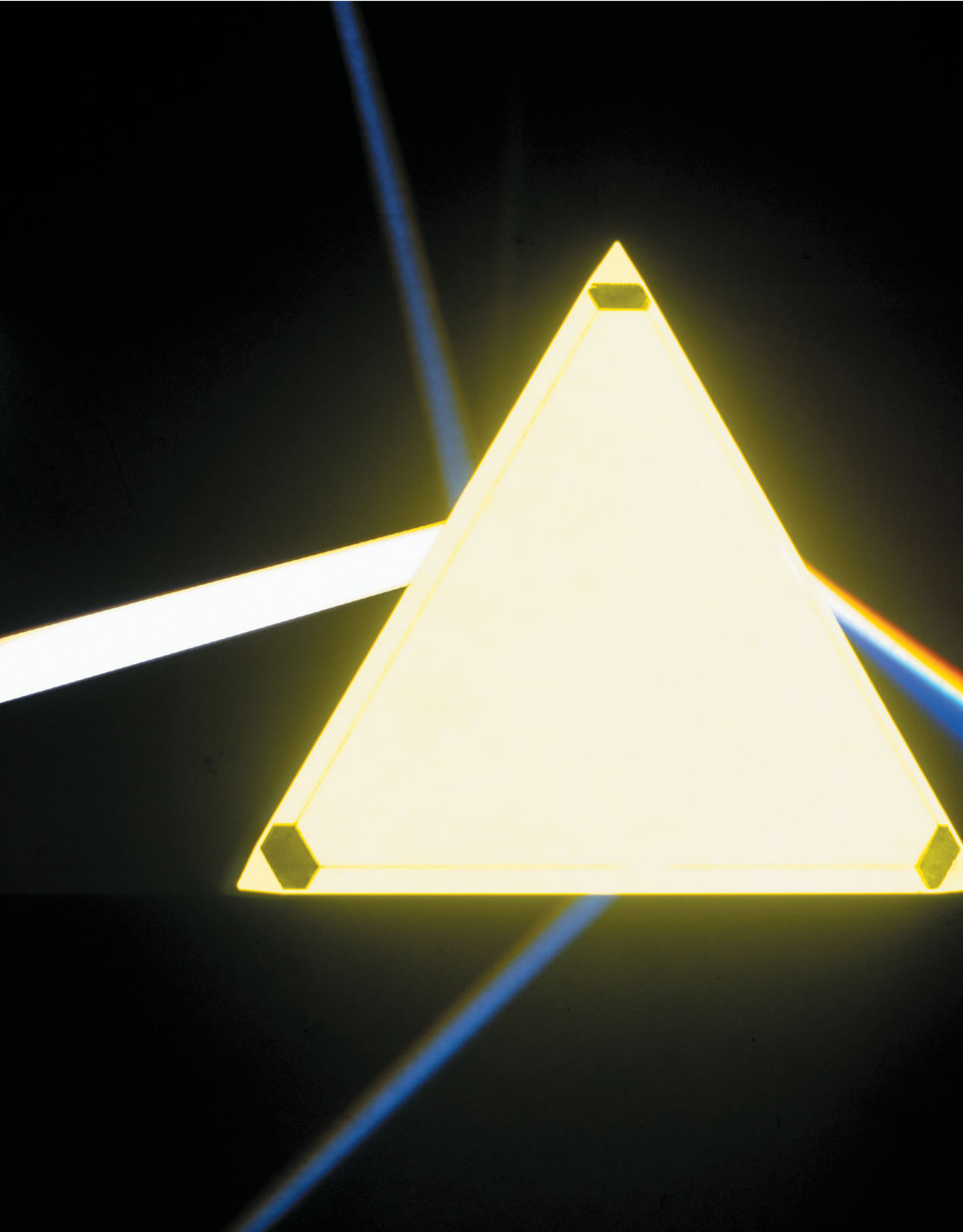


JAMES S. WALKER

PHYSICS

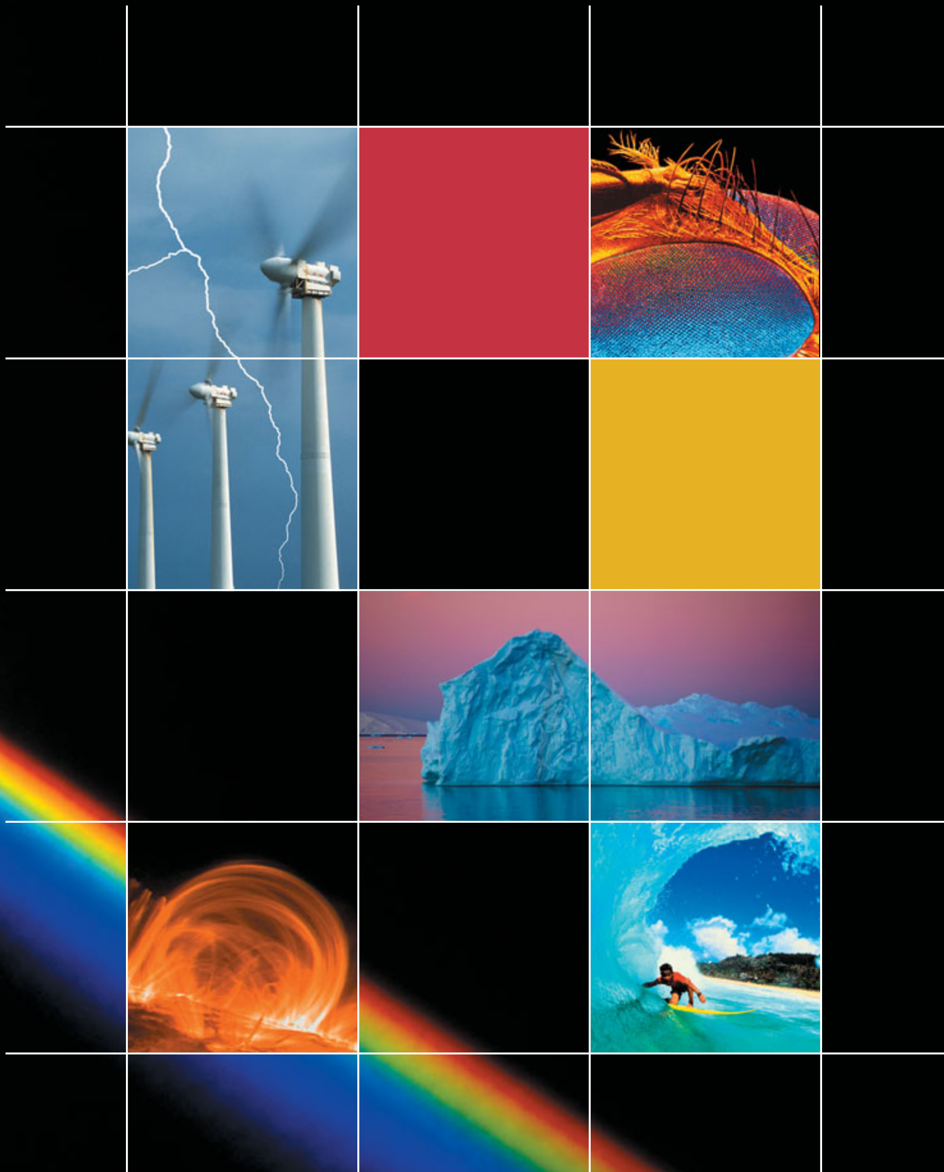


FOURTH EDITION

PHYSICS

JAMES S. WALKER

Western Washington University



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THIS BOOK IS DEDICATED TO MY PARENTS, IVAN AND JANET WALKER, AND TO MY WIFE, BETSY.

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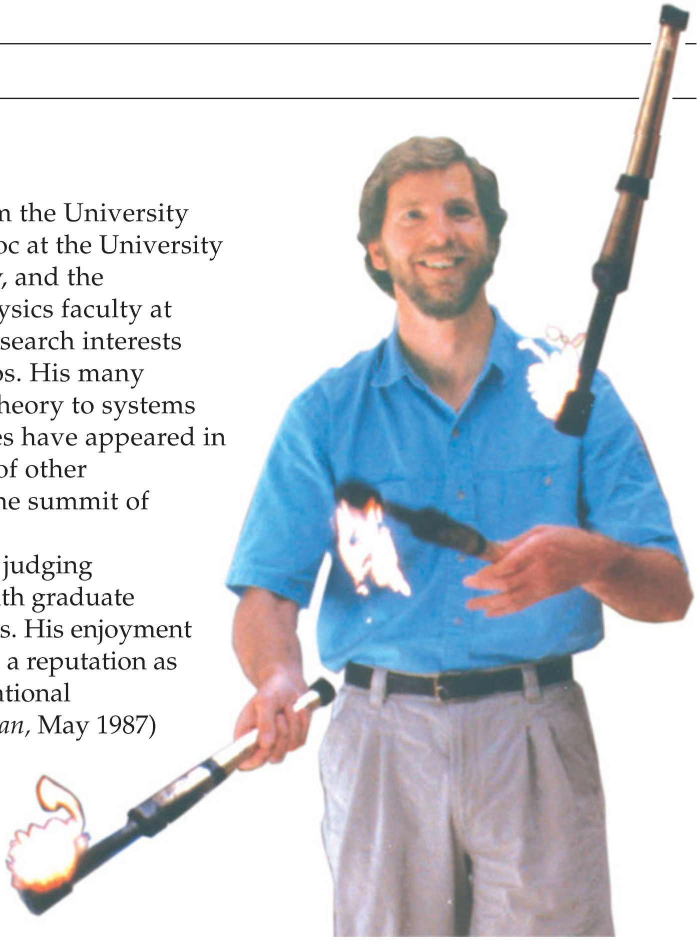
About the Author

JAMES S. WALKER



James Walker obtained his Ph.D. in theoretical physics from the University of Washington in 1978. He subsequently served as a post-doc at the University of Pennsylvania, the Massachusetts Institute of Technology, and the University of California at San Diego before joining the physics faculty at Washington State University in 1983. Professor Walker's research interests include statistical mechanics, critical phenomena, and chaos. His many publications on the application of renormalization-group theory to systems ranging from absorbed monolayers to binary-fluid mixtures have appeared in *Physical Review, Physical Review Letters, Physica,* and a host of other publications. He has also participated in observations on the summit of Mauna Kea, looking for evidence of extra-solar planets.

Jim Walker likes to work with students at all levels, from judging elementary school science fairs to writing research papers with graduate students, and has taught introductory physics for many years. His enjoyment of this course and his empathy for students have earned him a reputation as an innovative, enthusiastic, and effective teacher. Jim's educational publications include "Reappearing Phases" (*Scientific American,* May 1987) as well as articles in the *American Journal of Physics* and *The Physics Teacher.* In recognition of his contributions to the teaching of physics at Washington State University, Jim was named the Boeing Distinguished Professor of Science and Mathematics Education for 2001–2003. He currently teaches at Western Washington University.

When he is not writing, conducting research, teaching, or developing new classroom demonstrations and pedagogical materials, Jim enjoys amateur astronomy, eclipse chasing, bird and dragonfly watching, photography, juggling, unicycling, boogie boarding, and kayaking. Jim is also an avid jazz pianist and organist. He has served as ballpark organist for a number of Class A minor league baseball teams, including the Bellingham Mariners, an affiliate of the Seattle Mariners, and the Salem-Keizer Volcanoes, an affiliate of the San Francisco Giants. He can play "Take Me Out to the Ball Game" in his sleep.

About the Cover

The photographs on the cover of this book are a reminder of the wide "spectrum" of physics applications that are a part of our everyday lives.

Wind Turbines and Lightning Bolt: Wind turbines convert the mechanical energy of moving air into electrical energy to power our homes and cities. Electrical energy is also produced by nature, and occasionally unleashed in impressive bolts of lightning.

Scanning Electron Micrograph: Though electrons are usually thought of as "particles," they also have wave-like properties similar to light. The image of a fly's eye was taken with a beam of electrons.

Iceberg in the Errera Channel: A floating iceberg is a visual demonstration that ice has a lower density than liquid water.

Solar Coronal Loops: Magnetic storms often rage on the surface of the Sun. These glowing loops of ionized gas follow the curved lines of the magnetic field.

Surfer in the "Tube" on the North Shore of Oahu: The laws of physics determine the motion of the wave this surfer is riding.

As you study the material in this book, your understanding of physics will deepen, and your appreciation for the world around you will increase as you come to recognize the fundamental physical principles on which all of our lives are based.

Brief Contents

Foundations for Student Success

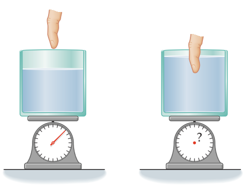
Walker's *Physics* has always been known for its integrated, coherent approach to teaching students the skills to solve problems successfully.

CONCEPTUAL CHECKPOINTS ➤ help students to master key ideas and relationships in a nonquantitative setting.

The end-of-chapter Conceptual Questions, Conceptual Exercises, and Predict/Explain problems further develop students' conceptual understanding.

CONCEPTUAL CHECKPOINT 15–3 HOW IS THE SCALE READING AFFECTED?

A flask of water rests on a scale. If you dip your finger into the water, without touching the flask, does the reading on the scale **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

Your finger experiences an upward buoyant force when it is dipped into the water. By Newton's third law, the water experiences an equal and opposite reaction force acting downward. This downward force is transmitted to the scale, which in turn gives a higher reading.

Another way to look at this result is to note that when you dip your finger into the water, its depth increases. This results in a greater pressure at the bottom of the flask, and hence a greater downward force on the flask. The scale reads this increased downward force.

ANSWER

(a) The reading on the scale increases.

EXERCISES ➤ present brief calculations which illustrate the application of important new relationships.

EXERCISE 7–1

One species of Darwin's finch, *Geospiza magnirostris*, can exert a force of 205 N with its beak as it cracks open a *Tribulus* seed case. If its beak moves through a distance of 0.40 cm during this operation, how much work does the finch do to get the seed?

SOLUTION

$W = Fd = (205\text{ N})(0.0040\text{ m}) = 0.82\text{ J}$

EXAMPLES ➤ model and explain how to solve a particular type of problem.

All Examples use a consistent strategy:

Picture the Problem
Strategy
Solution
Insight

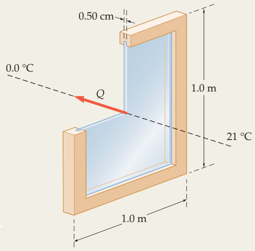
In response to user feedback, selected examples throughout the Fourth Edition are now more challenging.

EXAMPLE 16–6 WHAT A PANE!

One of the windows in a house has the shape of a square 1.0 m on a side. The glass in the window is 0.50 cm thick. **(a)** How much heat is lost through this window in one day if the temperature in the house is 21 °C and the temperature outside is 0.0 °C? **(b)** Suppose all the dimensions of the window—height, width, thickness—are doubled. If everything else remains the same, by what factor does the heat flow change?

PICTURE THE PROBLEM

The glass from the window is shown in our sketch, along with its relevant dimensions. Heat flows from the 21 °C side of the window to the 0.0 °C side.



STRATEGY

a. The heat flow is given by $Q = kA(\Delta T/L)t$ (Equation 16–16). Note that the area is $A = (1.0\text{ m})^2$ and that the length over which heat is conducted is, in this case, the thickness of the glass. Thus, $L = 0.0050$ m. The temperature difference is $\Delta T = 21\text{ C}° = 21$ K, and the thermal conductivity of glass (from Table 16–3) is $0.84\text{ W/(m}\cdot\text{K)}$. Also, recall from Section 7–4 that 1 W = 1 J/s.

b. Doubling all dimensions increases the thickness by a factor of 2 and increases the area by a factor of 4; that is, $L \rightarrow 2L$ and $A \rightarrow (2 \times \text{height}) \times (2 \times \text{width}) = 4A$. Use these results in $Q = kA(\Delta T/L)t$.

SOLUTION

Part (a)

1. Calculate the heat flow for a given time, t:

$$Q = kA\left(\frac{\Delta T}{L}\right)t$$

$$= [0.84\text{ W/(m}\cdot\text{K)}](1.0\text{ m})^2\left(\frac{21\text{ K}}{0.0050\text{ m}}\right)t = (3500\text{ W})t$$

2. Substitute the number of seconds in a day, 86,400 s, for the time t in the expression for Q:

$$Q = (3500\text{ W})t = (3500\text{ W})(86{,}400\text{ s}) = 3.0 \times 10^8\text{ J}$$

Part (b)

3. Replace L with $2L$ and A with $4A$ in Step 1. The result is a doubling of the heat flow, Q:

$$Q = kA\left(\frac{\Delta T}{L}\right)t \rightarrow k(4A)\left[\frac{\Delta T}{(2L)}\right]t \rightarrow 2\left[kA\left(\frac{\Delta T}{L}\right)t\right] = 2Q$$

INSIGHT

Q is a sizable amount of heat, roughly equivalent to the energy released in burning a gallon of gasoline. A considerable reduction in heat loss can be obtained by using a double-paned window, which has an insulating layer of air (actually argon or krypton) sandwiched between the two panes of glass. This is discussed in more detail later in this section, and is explored in Homework Problems 53 and 91.

PRACTICE PROBLEM

Suppose the window is replaced with a plate of solid silver. How thick must this plate be to have the same heat flow in a day as the glass? [**Answer:** The silver must have a thickness of $L = 2.5$ m.]

Some related homework problems: Problem 49, Problem 50

Unique two-column layout helps the students relate the strategy to the math.

Examples end with a related **Practice Problem.**

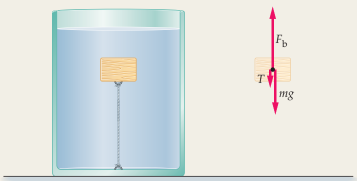
ACTIVE EXAMPLES ➤ provide a skeleton solution that the student must flesh out, helping to bridge the gap from the Examples to the end-of-chapter problems.

ACTIVE EXAMPLE 15–1 FIND THE TENSION IN THE STRING

A piece of wood with a density of 706 kg/m^3 is tied with a string to the bottom of a water-filled flask. The wood is completely immersed, and has a volume of 8.00×10^{-6} m^3. What is the tension in the string?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply Newton's second law to the wood: $F_b - T - mg = 0$
2. Solve for the tension, T: $T = F_b - mg$
3. Calculate the weight of the wood: $mg = 0.0554$ N
4. Calculate the buoyant force: $F_b = 0.0785$ N
5. Subtract to obtain the tension: $T = 0.0231$ N



INSIGHT

Since the wood floats in water, its buoyant force when completely immersed is greater than its weight.

YOUR TURN

What is the tension in the string if the piece of wood has a density of 822 kg/m^3?

*(Answers to **Your Turn** problems can be found in the back of the book.)*

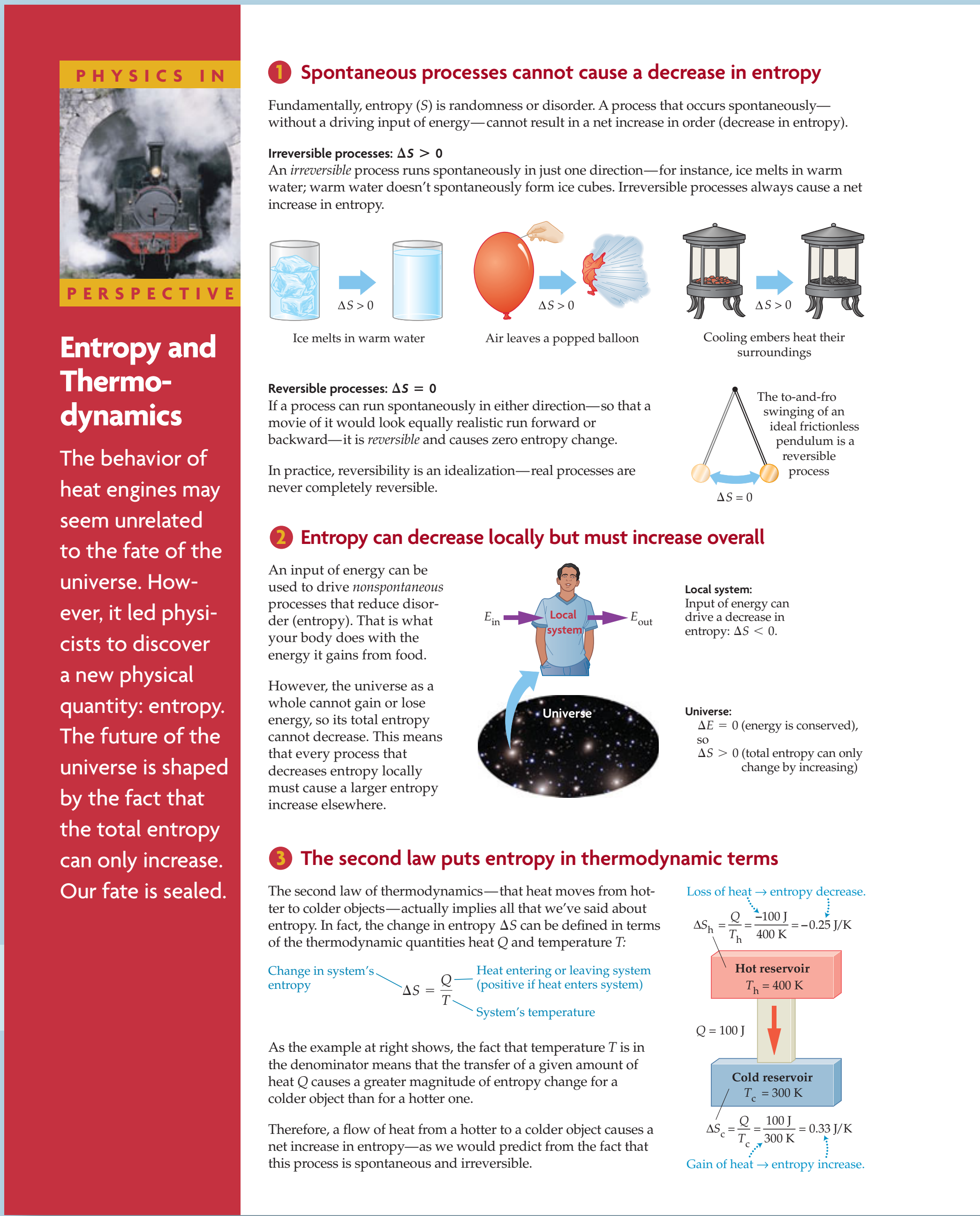
New to the Fourth Edition

PHYSICS IN PERSPECTIVE ➤
Located at key junctures in the book, **Physics in Perspective** two-page spreads focus on the core ideas developed in the preceding several chapters.

Looking back over several chapters, the spreads show unifying perspectives that the students are only now equipped to see.

For instance, the Physics in Perspective spread illustrated here, located after the final thermodynamics chapter, uses the second law to unify and explain ideas that initially had to be presented from a different perspective.

The **Big Picture** feature at the end of each chapter (not shown here) performs a similar function on a chapter level.



PHYSICS IN PERSPECTIVE

Entropy and Thermodynamics

The behavior of heat engines may seem unrelated to the fate of the universe. However, it led physicists to discover a new physical quantity: entropy. The future of the universe is shaped by the fact that the total entropy can only increase. Our fate is sealed.

1 Spontaneous processes cannot cause a decrease in entropy

Fundamentally, entropy (S) is randomness or disorder. A process that occurs spontaneously—without a driving input of energy—cannot result in a net increase in order (decrease in entropy).

Irreversible processes: $\Delta S > 0$
An *irreversible* process runs spontaneously in just one direction—for instance, ice melts in warm water; warm water doesn't spontaneously form ice cubes. Irreversible processes always cause a net increase in entropy.

$\Delta S > 0$ Ice melts in warm water

$\Delta S > 0$ Air leaves a popped balloon

$\Delta S > 0$ Cooling embers heat their surroundings

Reversible processes: $\Delta S = 0$
If a process can run spontaneously in either direction—so that a movie of it would look equally realistic run forward or backward—it is *reversible* and causes zero entropy change.

In practice, reversibility is an idealization—real processes are never completely reversible.

The to-and-fro swinging of an ideal frictionless pendulum is a reversible process

$\Delta S = 0$

2 Entropy can decrease locally but must increase overall

An input of energy can be used to drive *nonspontaneous* processes that reduce disorder (entropy). That is what your body does with the energy it gains from food.

E_{in} Local system E_{out}

Local system:
Input of energy can drive a decrease in entropy: $\Delta S < 0$.

However, the universe as a whole cannot gain or lose energy, so its total entropy cannot decrease. This means that every process that decreases entropy locally must cause a larger entropy increase elsewhere.

Universe

Universe:
$\Delta E = 0$ (energy is conserved), so
$\Delta S > 0$ (total entropy can only change by increasing)

3 The second law puts entropy in thermodynamic terms

The second law of thermodynamics—that heat moves from hotter to colder objects—actually implies all that we've said about entropy. In fact, the change in entropy ΔS can be defined in terms of the thermodynamic quantities heat Q and temperature T:

$$\Delta S = \frac{Q}{T}$$

Change in system's entropy
Heat entering or leaving system (positive if heat enters system)
System's temperature

As the example at right shows, the fact that temperature T is in the denominator means that the transfer of a given amount of heat Q causes a greater magnitude of entropy change for a colder object than for a hotter one.

Therefore, a flow of heat from a hotter to a colder object causes a net increase in entropy—as we would predict from the fact that this process is spontaneous and irreversible.

Loss of heat → entropy decrease.

$\Delta S_h = \frac{Q}{T_h} = \frac{-100\ \text{J}}{400\ \text{K}} = -0.25\ \text{J/K}$

Hot reservoir $T_h = 400$ K

$Q = 100$ J

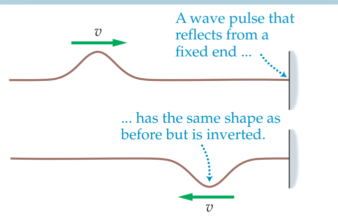
Cold reservoir $T_c = 300$ K

$\Delta S_c = \frac{Q}{T_c} = \frac{100\ \text{J}}{300\ \text{K}} = 0.33\ \text{J/K}$

Gain of heat → entropy increase.

Annotated equations help students to see the meaning in the math.

ANNOTATED FIGURES ➤
Blue explanatory annotations help students to read complex figures and to integrate verbal and visual knowledge.



▲ **FIGURE 14–7 A reflected wave pulse: fixed end**
A wave pulse on a string is inverted when it reflects from an end that is tied down.

PHYSICS DEMONSTRATION PHOTOS ➤
use high-speed time-lapse photography to illustrate phenomena that illuminate physical principles.

Building on strong pedagogic foundations, the Fourth Edition adds features that help students see beyond the mathematical details to the underlying ideas of physics.

PHYSICS IN PERSPECTIVE

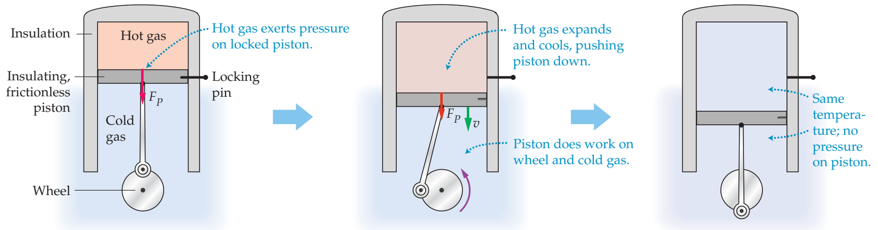
4 A temperature difference can be exploited to do work ...

The tendency of hotter and colder objects to come to the same temperature can be tapped to do work, as in this example:

Initial state: Gases at different temperatures are separated by a locked piston.

Piston unlocked: Pressure difference causes piston to move, doing work on wheel.

Final state: Gases at same temperature; no more work can be done.



The expansion shown above is a single process, not a cycle, so this piston-cylinder does not constitute a heat engine.

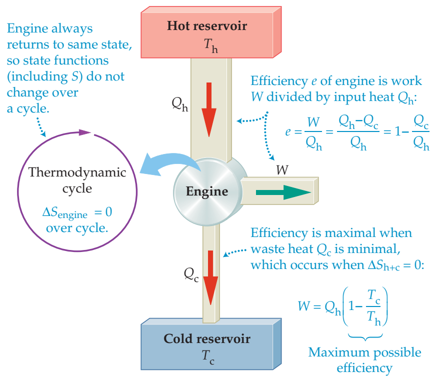
5 ... but entropy sets the limit of efficiency for a heat engine

A heat engine is a device that converts part of a heat flow into work. Entropy sets an absolute limit on the efficiency of this process.

To see why, we start with the fact that a heat engine operates on a thermodynamic cycle—it starts in a particular state, goes through a series of proc esses involving heat and work, and returns to its original state. (Think of the cyclic operation of a cylinder in a car engine.)

Because entropy S is a state function, the engine's entropy returns to it s original value at the end of each cycle—so over the course of a cycle, the entropy change ΔS_{engine} of a heat engine is zero. Therefore, the entropy of the engine's *environment*—specifically, of the hot and cold reservoir ($S_{\text{h+c}}$)—must increase or stay the same ($\Delta S_{\text{h+c}} \geq 0$).

The engine will have the highest efficiency $e = W/Q_{\text{h}}$ when $\Delta S_{\text{h+c}} = 0$, because higher values of $\Delta S_{\text{h+c}}$ entail more waste heat (Q_{c}) and thus yield less work W. To be more efficient than this, an engine would have to cause a net *decrease* in entropy, which is impossible. Actual engines all have $\Delta S_{\text{h+c}} \geq 0$.



6 Entropy spells the death of the universe

The night sky shows us a universe of stars and galaxies separated by cold, nearly empty space. Over time, the inexorable growth of entropy will erase these differences, leaving a universe that is uniform in temperature and density—unable ever again to create stars or give rise to life.

Nevertheless, the energy content of the universe will remain the same as at its birth.

The Physics in Perspective spreads blend words, equations, and pictures into an integrated and highly visual presentation.

Blue explanatory annotations guide the student through the diagrams.

NEW END-OF-CHAPTER PROBLEM TYPES

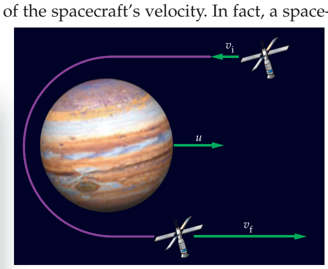
PASSAGE PROBLEMS ➤

offer a reading passage followed by a set of multiple-choice questions (the format used by most MCAT questions), testing students' ability to apply what they've learned to a real-world situation.

PASSAGE PROBLEMS

Navigating in Space: The Gravitational Slingshot

Many spacecraft navigate through space these days by using the "gravitational slingshot" effect, in which a close encounter with a planet results in a significant increase in magnitude and change in direction of the spacecraft's velocity. In fact, a space-



▲ **FIGURE 9–31** Problems 97, 98, 99, and 100

PREDICT/EXPLAIN PROBLEMS ➤

consist of two linked multiple choice questions — the first asking the student to *predict* the outcome of a situation and the second asking for its physical *explanation*.

15\. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the acceleration experienced by the car greater than, less than, or equal to the acceleration experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. The truck exerts a larger force on the car, giving it the greater acceleration.

II. Both vehicles experience the same magnitude of force, therefore the lightweight car experiences the greater acceleration.

III. The greater force exerted on the truck gives it the greater acceleration.

Applications in the Text

Note: This list includes applied topics that receive significant discussion in the chapter text or a worked Example, as well as topics that are touched on in end-of-chapter Conceptual Questions, Conceptual Exercises, and Problems. Topics of particular relevance to the life sciences or medicine are marked **BIO**. Topics related to Passage Problems are marked **PP**.

ActivPhysics OnLine™ Activities

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1.1 Analyzing Motion Using Diagrams
1.2 Analyzing Motion Using Graphs
1.3 Predicting Motion from Graphs
1.4 Predicting Motion from Equations
1.5 Problem-Solving Strategies for Kinematics
1.6 Skier Races Downhill
1.7 Balloonist Drops Lemonade
1.8 Seat Belts Save Lives
1.9 Screeching to a Halt
1.10 Pole-Vaulter Lands
1.11 Car Starts, Then Stops
1.12 Solving Two-Vehicle Problems
1.13 Car Catches Truck
1.14 Avoiding a Rear-End Collision
2.1.1 Force Magnitudes
2.1.2 Skydiver
2.1.3 Tension Change
2.1.4 Sliding on an Incline
2.1.5 Car Race
2.2 Lifting a Crate
2.3 Lowering a Crate
2.4 Rocket Blasts Off
2.5 Truck Pulls Crate
2.6 Pushing a Crate Up a Wall
2.7 Skier Goes Down a Slope
2.8 Skier and Rope Tow
2.9 Pole-Vaulter Vaults
2.10 Truck Pulls Two Crates
2.11 Modified Atwood Machine
3.1 Solving Projectile Motion Problems
3.2 Two Balls Falling
3.3 Changing the x-Velocity
3.4 Projectile x- and y-Accelerations
3.5 Initial Velocity Components
3.6 Target Practice I
3.7 Target Practice II
4.1 Magnitude of Centripetal Acceleration
4.2 Circular Motion Problem Solving
4.3 Cart Goes Over Circular Path
4.4 Ball Swings on a String
4.5 Car Circles a Track
4.6 Satellites Orbit
5.1 Work Calculations
5.2 Upward-Moving Elevator Stops
5.3 Stopping a Downward-Moving Elevator
5.4 Inverse Bungee Jumper
5.5 Spring-Launched Bowler
5.6 Skier Speed
5.7 Modified Atwood Machine
6.1 Momentum and Energy Change
6.2 Collisions and Elasticity
6.3 Momentum Conservation and Collisions
6.4 Collision Problems
6.5 Car Collision: Two Dimensions
6.6 Saving an Astronaut
6.7 Explosion Problems
6.8 Skier and Cart
6.9 Pendulum Bashes Box
6.10 Pendulum Person-Projectile Bowling
7.1 Calculating Torques
7.2 A Tilted Beam: Torques and Equilibrium
7.3 Arm Levers
7.4 Two Painters on a Beam
7.5 Lecturing from a Beam
7.6 Rotational Inertia
7.7 Rotational Kinematics
7.8 Rotoride: Dynamics Approach
7.9 Falling Ladder
7.10 Woman and Flywheel Elevator: Dynamics Approach
7.11 Race Between a Block and a Disk
7.12 Woman and Flywheel Elevator: Energy Approach
7.13 Rotoride: Energy Approach
7.14 Ball Hits Bat
8.1 Characteristics of a Gas
8.2 Maxwell-Boltzmann Distribution: Conceptual Analysis
8.3 Maxwell-Boltzmann Distribution: Quantitative Analysis
8.4 State Variables and Ideal Gas Law
8.5 Work Done by a Gas
8.6 Heat, Internal Energy, and First Law of Thermodynamics
8.7 Heat Capacity
8.8 Isochoric Process
8.9 Isobaric Process
8.10 Isothermal Process
8.11 Adiabatic Process
8.12 Cyclic Process: Strategies
8.13 Cyclic Process: Problems
8.14 Carnot Cycle
9.1 Position Graphs and Equations
9.2 Describing Vibrational Motion
9.3 Vibrational Energy
9.4 Two Ways to Weigh Young Tarzan
9.5 Ape Drops Tarzan
9.6 Releasing a Vibrating Skier I
9.7 Releasing a Vibrating Skier II
9.8 One- and Two-Spring Vibrating Systems
9.9 Vibro-Ride
9.10 Pendulum Frequency
9.11 Risky Pendulum Walk
9.12 Physical Pendulum
10.1 Properties of Mechanical Waves
10.2 Speed of Waves on a String
10.3 Speed of Sound in a Gas
10.4 Standing Waves on Strings
10.5 Tuning a Stringed Instrument: Standing Waves
10.6 String Mass and Standing Waves
10.7 Beats and Beat Frequency
10.8 Doppler Effect: Conceptual Introduction
10.9 Doppler Effect: Problems
10.10 Complex Waves: Fourier Analysis
11.1 Electric Force: Coulomb's Law
11.2 Electric Force: Superposition Principle
11.3 Electric Force Superposition Principle (Quantitative)
11.4 Electric Field: Point Charge
11.5 Electric Field Due to a Dipole
11.6 Electric Field: Problems
11.7 Electric Flux
11.8 Gauss's Law
11.9 Motion of a Charge in an Electric Field: Introduction
11.10 Motion in an Electric Field: Problems
11.11 Electric Potential: Qualitative Introduction
11.12 Electric Potential, Field, and Force
11.13 Electrical Potential Energy and Potential
12.1 DC Series Circuits (Qualitative)
12.2 DC Parallel Circuits
12.3 DC Circuit Puzzles
12.4 Using Ammeters and Voltmeters
12.5 Using Kirchhoff's Laws
12.6 Capacitance
12.7 Series and Parallel Capacitors
12.8 *RC* Circuit Time Constants
13.1 Magnetic Field of a Wire
13.2 Magnetic Field of a Loop
13.3 Magnetic Field of a Solenoid
13.4 Magnetic Force on a Particle
13.5 Magnetic Force on a Wire
13.6 Magnetic Torque on a Loop
13.7 Mass Spectrometer
13.8 Velocity Selector
13.9 Electromagnetic Induction
13.10 Motional emf
14.1 The *RL* Circuit
14.2 The *RLC* Oscillator
14.3 The Driven Oscillator
15.1 Reflection and Refraction
15.2 Total Internal Reflection
15.3 Refraction Applications
15.4 Plane Mirrors
15.5 Spherical Mirrors: Ray Diagrams
15.6 Spherical Mirror: The Mirror Equation
15.7 Spherical Mirror: Linear Magnification
15.8 Spherical Mirror: Problems
15.9 Thin-Lens Ray Diagrams
15.10 Converging Lens Problems
15.11 Diverging Lens Problems
15.12 Two-Lens Optical Systems
16.1 Two-Source Interference: Introduction
16.2 Two-Source Interference: Qualitative Questions
16.3 Two-Source Interference: Problems
16.4 The Grating: Introduction and Qualitative Questions
16.5 The Grating: Problems
16.6 Single-Slit Diffraction
16.7 Circular Hole Diffraction
16.8 Resolving Power
16.9 Polarization
17.1 Relativity of Time
17.2 Relativity of Length
17.3 Photoelectric Effect
17.4 Compton Scattering
17.5 Electron Interference
17.6 Uncertainty Principle
17.7 Wave Packets
18.1 The Bohr Model
18.2 Spectroscopy
18.3 The Laser
19.1 Particle Scattering
19.2 Nuclear Binding Energy
19.3 Fusion
19.4 Radioactivity
19.5 Particle Physics
20.1 Potential Energy Diagrams
20.2 Particle in a Box
20.3 Potential Wells
20.4 Potential Barriers

Preface: To the Instructor

Teaching introductory algebra-based physics can be a most challenging—and rewarding—experience. Students enter the course with a wide range of backgrounds, interests, and skills and we, the instructors, strive not only to convey the basic concepts and fundamental laws of physics but also to give students an appreciation of its relevance and appeal.

I wrote this book to help with that task. It incorporates a number of unique and innovative pedagogical features that evolved from years of teaching experience. The materials have been tested extensively in the classroom and in focus groups, and refined based on comments from students and teachers who used the earlier editions of the text. The enthusiastic response I received from users of the first three editions was both flattering and motivating. The fourth edition has been improved in response to this feedback.

Learning Tools in the Text

A key goal of this text is to help students make the connection between a conceptual understanding of physics and the various skills necessary to solve quantitative problems. One of the chief means to that end is the replacement of traditional "textbook" Examples with an integrated system of learning tools: fully worked Examples with Solutions in Two-Column Format, Active Examples, Conceptual Checkpoints, and Exercises. Each of these tools is specialized to meet the needs of students at a particular point in the development of a chapter.

These needs are not always the same. Sometimes students require a detailed explanation of how to tackle a particular problem; at other times, they must be allowed to take an active role and work out the details for themselves. Sometimes it is important for them to perform calculations and concentrate on numerical precision; at other times it is more fruitful for them to explore a key idea in a conceptual context. And sometimes, all that is required is practice using a new equation or definition.

This text attempts to emulate the teaching style of successful instructors by providing the right tool at the right place and the right time.

Perspective Across Chapters

It's easy for students to miss the forest for the trees—to overlook the unifying concepts that are central to physics and that will make the details easier to learn and retain. To address this difficulty, the fourth edition adds two features. At key junctures in the text are six **Physics in Perspective** features, two-page spreads that take a highly visual look at core ideas whose significance students are now prepared to understand. For instance, after working through the energy chapters, do students *really* understand how conservation of energy relates to conservation of mechanical energy, and the role of work done by dissipative and nondissipative forces? And after working through the chapters on electricity and magnetism, do they have a clear view of how electric and magnetic forces relate to each other? These are two of the topics on which the Physics in Perspective pages focus. Each chapter now ends with a **Big Picture box** that links ideas covered in the chapter to related material from earlier and later chapters in the text.

NEW

NEW

WORKED EXAMPLES WITH SOLUTIONS IN TWO-COLUMN FORMAT

Examples model the most complete and detailed method of solving a particular type of problem. The Examples in this text are presented in a format that focuses on the basic strategies and thought processes involved in problem solving. This focus on the intimate relationship between conceptual insights and problem-solving techniques encourages students to view the ability to solve problems as a logical outgrowth of conceptual understanding rather than a kind of parlor trick.

Each Example has the same basic structure:

- **Picture the Problem** This first step discusses how the physical situation can be represented visually and what such a representation can tell us about how to analyze and solve the problem. At this step, always accompanied by a figure, we set up a coordinate system where appropriate, label important quantities, and indicate which values are known.
- **Strategy** The Strategy addresses the commonly asked question, "How do I get started?" by providing a clear overview of the problem and helping students to identify the relevant physical principles. It then guides the student in using known relationships to map a step-by-step path to the solution.
- **Solution in Two-Column Format** In the step-by-step Solution of the problem, each of the steps is presented with a prose statement in the left-hand column and the corresponding mathematical implementation in the right-hand column. Each step clearly translates the idea described in words into the appropriate equations.
- **Insight** Each Example wraps up with an Insight—a comment regarding the solution just obtained. Some Insights deal with possible alternative solution techniques, others with new ideas suggested by the results.
- **Practice Problem** Following the Insight is a Practice Problem, which gives the student a chance to practice the type of calculation just presented. The Practice Problems, always accompanied by their answers, provide students with a valuable check on their understanding of the material. Finally, each Example ends with a reference to some related end-of-chapter Problems to allow students to test their skills further.

ACTIVE EXAMPLES

Active Examples serve as a bridge between the fully worked Examples, in which every detail is fully discussed and every step is given, and the homework Problems, where no help is given at all. In an Active Example, students take an active role in solving the problem by thinking through the logic of the steps described on the left and checking their answers on the right. Students often find it useful to practice problem solving by covering one column of an Active Example with a sheet of paper and filling in the covered steps as they refer to the other column. Follow-up questions, called Your Turns, ask students to look at the problem in a slightly different way. Answers to Your Turns are provided at the end of the book.

CONCEPTUAL CHECKPOINTS

Conceptual Checkpoints help students sharpen their insight into key physical principles. A typical Conceptual Checkpoint presents a thought-provoking question that can be answered by logical reasoning based on physical concepts rather than by numerical calculations. The statement of the question is followed by a detailed discussion and analysis in the section titled Reasoning and Discussion, and the Answer is given at the end of the checkpoint for quick and easy reference.

EXERCISES

Exercises present brief calculations designed to illustrate the application of important new relationships, without the expenditure of time and space required by a fully worked Example. Exercises generally give students an opportunity to practice the use of a new equation, become familiar with the units of a new physical quantity, and get a feeling for typical magnitudes.

PROBLEM-SOLVING NOTES

Each chapter includes a number of Problem-Solving Notes in the margin. These practical hints are designed to highlight useful problem-solving methods while helping students avoid common pitfalls and misconceptions.

End-of-Chapter Learning Tools

The end-of-chapter material in this text also includes a number of innovations, along with refinements of more familiar elements.

- Each chapter concludes with a **Chapter Summary** presented in an easy-to-use outline style. Key concepts, equations, and important figures are organized by topic for convenient reference.
- A unique feature of this text is the **Problem-Solving Summary** at the end of the chapter. This summary addresses common sources of misconceptions in problem solving, and gives specific references to Examples and Active Examples illustrating the correct procedures.
- The homework for each chapter begins with a section of **Conceptual Questions**. Answers to the odd-numbered Questions can be found in the back of the book. Answers to even-numbered Conceptual Questions are available in the online Instructor Solutions Manual.
- **Conceptual Exercises (CE)** have been integrated into the homework section at the end of the chapter and consist of multiple-choice and ranking questions. These questions have been carefully selected and written for maximum effectiveness when used with classroom-response systems (clickers). Answers to the odd-numbered Exercises can be found in the back of the book. Answers to even-numbered Conceptual Exercises are available in the online Instructor Solutions Manual.
- **Predict/Explain problems** are new to this edition. These problems ask the student to predict what will happen in a given physical situation and then to choose an explanation for their prediction. NEW
- Also new to this edition, **Passage Problems** are similar to those found on MCAT exams, with associated multiple-choice questions. NEW
- **Interactive Problems** are based on the animations and simulations associated with the Interactive Figures and are found within MasteringPhysics.
- A popular feature within the homework section is the **Integrated Problems (IP).** These problems, labeled with the symbol IP, integrate a conceptual question with a numerical problem. Problems of this type, which stress the importance of reasoning from basic principles, show how conceptual insight and numerical calculation go hand in hand in physics.
- In addition, a section titled **General Problems** presents a variety of problems that use material from two or more sections within the chapter, or refer to material covered in earlier chapters.
- **Problems of special biological or medical relevance** are indicated with the symbol BIO.

Scope and Organization

TABLE OF CONTENTS

The presentation of physics in this text follows the standard practice for introductory courses, with only a few well-motivated refinements.

- First, note that Chapter 3 is devoted to **vectors and their application to physics.** My experience has been that students benefit greatly from a full discussion of vectors early in the course. Most students have seen vectors and trigonometric functions before, but rarely from the point of view of physics. Thus, including vectors in the text sends a message that this is important material, and it gives students an opportunity to brush up on their math skills.
- Note also that **additional time is given to some of the more fundamental aspects of physics,** such as Newton's laws and energy. Presenting such

material in two chapters gives the student a better opportunity to assimilate and master these crucial topics. Sections considered optional are marked with an asterisk.

REAL-WORLD PHYSICS

REAL-WORLD PHYSICS

REAL-WORLD PHYSICS: BIO

Since physics applies to everything in nature, it is only reasonable to point out applications of physics that students may encounter in the real world. Each chapter presents a number of discussions focusing on "Real-World Physics." Those of general interest are designated by a globe icon in the margin. Applications that pertain more specifically to biology and medicine are indicated by a green frog icon in the margin.

The Illustration Program

DRAWINGS

Many physics concepts are best conveyed by graphic means. Figures do far more than illustrate a physics text—often, they bear the main burden of the exposition. Accordingly, great attention has been paid to the figures in this book, with the primary emphasis always on the clarity of the analysis. Color has been used consistently throughout the text to reinforce concepts and make the diagrams easier for students to understand. NEW New to this edition, helpful **annotations in blue** are included on select figures to help guide students in "reading" graphs and other figures. This technique emulates what instructors do at the chalkboard when explaining figures.

PHOTOGRAPHS

One of the most fundamental ways in which we learn is by comparing and contrasting. Many **companion photos** are presented in groups of two or three that contrast opposing physical principles or illustrate a single concept in a variety of contexts. Grouping carefully chosen photographs in this way helps students to see the universality of physics. NEW In this edition, we have added new **demonstration photos** that use high-speed time-lapse photography to dramatically illustrate topics, such as standing waves, static versus kinetic friction, and the motion of center of mass, in a way that reveals physical principles in the world around us.

Resources

The fourth edition is supplemented by an ancillary package developed to address the needs of both students and instructors.

FOR THE INSTRUCTOR

Instructor Solutions Manual by Kenneth L. Menningen (University of Wisconsin–Stevens Point) is available online at the Instructor Resource Center:
www.pearsonhighered.com/educator
You will find detailed, worked solutions to every Problem and Conceptual Exercise in the text, all solved using the step-by-step problem-solving strategy of the in-chapter Examples (Picture the Problem, Strategy, two-column Solutions, and Insight). The solutions also contain answers to the even-numbered Conceptual Questions.

Instructor Resource Manual with Notes on ConcepTest Questions
Available at the Instructor Resource Center: www.pearsonhighered.com/educator, this online manual consists of two parts. The first part, prepared by Katherine Whatley and Judith Beck (both of University of North Carolina, Asheville), contains sample syllabi, lecture outlines, notes, demonstration suggestions, readings, and additional references and resources. The second part, prepared by Cornelius Bennhold and Gerald Feldman (both of George Washington University) contains an overview of the development and implementation of ConcepTests, as well as instructor notes for each ConcepTest found in the Instructor Resource Center and available on the Instructor Resource DVD.

Test Bank Available at the Instructor Resource Center: www.pearsonhighered.com/educator
Written by Delena Bell Gatch (Georgia Southern University), this online, cross-platform test bank contains approximately 3000 multiple-choice, short-answer, and true/false questions, many conceptual in nature. All are referenced to the corresponding text section and ranked by level of difficulty.

Instructor Resource DVD (ISBN 0-321-60193-9)
This cross-platform DVD provides virtually every electronic asset you'll need in and out of the classroom. The DVD is organized by chapter and includes all text illustrations and tables from *Physics,* Fourth Edition, in jpeg and PowerPoint formats. The IRDVD also contains the Interactive Figures, chapter-by-chapter lecture outlines in PowerPoint, ConcepTest "Clicker" Questions in PowerPoint, editable Word files of all numbered equations, the eleven "Physics You Can See" demonstration videos, and pdf files of the *Instructor Resource Manual with Notes on ConcepTest Questions.*

MasteringPhysics™ www.masteringphysics.com
This homework, tutorial, and assessment system is designed to assign, assess, and track each student's progress using a wide diversity of tutorials and extensively pre-tested problems. All the end-of-chapter problems from the text and the Interactive Figures are available in MasteringPhysics. MasteringPhysics provides instructors with a fast and effective way to assign uncompromising, wide-ranging online homework assignments of just the right difficulty and duration. The tutorials coach 90% of students to the correct answer with specific wrong-answer feedback. The powerful post-assignment diagnostics allow instructors to assess the progress of their class as a whole or to quickly identify individual student's areas of difficulty.

NEW

myeBook is available through MasteringPhysics either automatically when MasteringPhysics is packaged with new books, or available as a purchased upgrade online. Allowing students access to the text wherever they have access to the Internet, myeBook comprises the full text, including figures that can be enlarged for better viewing. Within myeBook, students are also able to pop up definitions and terms to help with vocabulary and the reading of the material. Students can also take notes in myeBook using the annotation feature at the top of each page.

NEW

ActivPhysics OnLine™ (accessed through the Self Study area within www.masteringphysics.com) provides a comprehensive library of more than 420 tried and tested *ActivPhysics* applets. In addition, it provides a suite of applet-based tutorials developed by education pioneers Alan Van Heuvelen and Paul D'Alessandris. The online exercises are designed to encourage students to confront misconceptions, reason qualitatively about physical processes, experiment quantitatively, and learn to think critically. They cover all topics from mechanics to electricity and magnetism and from optics to modern physics. The *ActivPhysics OnLine* companion workbooks help students work through complex concepts and understand them more clearly.

FOR THE STUDENT

Student Study Guide with Selected Solutions by David Reid (University of Chicago) Volume 1: ISBN 0-321-60200-5; Volume 2: ISBN 0-321-60199-8
The print study guide provides the following for each chapter:

Objectives; Warm-Up Questions from the Just-in-Time Teaching (JiTT) method by Gregor Novak and Andrew Gavrin (Indiana University–Purdue University, Indianapolis); Chapter Review with two-column Examples and integrated quizzes; Reference Tools & Resources (equation summaries, important tips, and tools); Puzzle Questions (also from Novak & Gavrin's JiTT method); Selected Solutions for several end-of-chapter questions and problems.

MasteringPhysics™ (www.masteringphysics.com)
This homework, tutorial, and assessment system is based on years of research into how students work physics problems and precisely where they need help. Studies show that students who use MasteringPhysics significantly increase their final scores compared to hand-written homework. MasteringPhysics achieves this improvement by providing students with instantaneous feedback specific to their wrong answers, simpler sub-problems upon request when they get stuck, and partial credit for their method(s) used. This individualized, 24/7 Socratic tutoring is recommended by nine out of ten students to their peers as the most effective and time-efficient way to study.

NEW

myeBook is available through MasteringPhysics either automatically when MasteringPhysics is packaged with new books, or available as a purchased upgrade online. Allowing students access to the text wherever they have access to the Internet, myeBook comprises the full text, including figures that can be enlarged for better viewing. Within myeBook, students are also able to pop up definitions and terms to help with vocabulary and the reading of the material. Students can also take notes in myeBook using the annotation feature at the top of each page.

NEW

ActivPhysics OnLine™ (accessed via www.masteringphysics.com) provides students with a suite of highly regarded applet-based self-study tutorials (see description on previous page). The following workbooks provide a range of tutorial problems designed to use the *ActivPhysics OnLine* simulations, helping students work through complex concepts and understand them more clearly:

- ***ActivPhysics OnLine Workbook*** Volume 1: Mechanics • Thermal Physics • Oscillations & Waves (ISBN 0-8053-9060-X)
- ***ActivPhysics OnLine Workbook*** Volume 2: Electricity & Magnetism • Optics • Modern Physics (ISBN 0-8053-9061-8)

Pearson Tutor Services (www.pearsontutorservices.com) Each student's subscription to MasteringPhysics also contains complimentary access to Pearson Tutor Services, powered by Smarthinking, Inc. By logging in with their MasteringPhysics ID and password, they will be connected to highly qualified e-structors™ who provide additional, interactive online tutoring on the major concepts of physics. Some restrictions apply; offer subject to change.

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Preface: To the Student

As a student preparing to take an algebra-based physics course, you are probably aware that physics applies to absolutely everything in the natural world, from raindrops and people to galaxies and atoms. Because physics is so wide-ranging and comprehensive, it can sometimes seem a bit overwhelming. This text, which reflects nearly two decades of classroom experience, is designed to help you deal with a large body of information and develop a working understanding of the basic concepts in physics. Now in its fourth edition, it incorporates many refinements that have come directly from interacting with students using the first three editions. As a result of these interactions, I am confident that as you develop a deeper understanding of physics, you will also enrich your experience of the world in which you live.

Now, I must admit that I like physics, and so I may be a bit biased in this respect. Still, the reason I teach and continue to study physics is that I enjoy the insight it gives into the physical world. I can't help but notice—and enjoy—aspects of physics all around me each and every day. As I always tell my students on the first day of class, I would like to share some of this enjoyment and delight in the natural world with you. It is for this reason that I undertook the task of writing this book.

To assist you in the process of studying physics, this text incorporates a number of learning aids, including Two-Column Examples, Active Examples, and Conceptual Checkpoints. These and other elements work together in a unified way to enhance your understanding of physics on both a conceptual and a quantitative level—they have been developed to give you the benefit of what we know about how students learn physics, and to incorporate strategies that have proven successful to students over the years. The pages that follow will introduce these elements to you, describe the purpose of each, and explain how they can help you.

As you progress through the text, you will encounter many interesting and intriguing applications of physics drawn from the world around you. Some of these, such as magnetically levitated trains or the satellite-based Global Positioning System that enables you to determine your position anywhere on Earth to within a few feet, are primarily technological in nature. Others focus on explaining familiar or not-so-familiar phenomena, such as why the Moon has no atmosphere, how sweating cools the body, or why flying saucer shaped clouds often hover over mountain peaks even when the sky is clear. Still others, such as countercurrent heat exchange in animals and humans or the use of sound waves to destroy kidney stones, are of particular relevance to students of biology and the other life sciences.

In many cases, you may find the applications to be a bit surprising. Did you know, for example, that you are shorter at the end of the day than when you first get up in the morning? (This is discussed in Chapter 5.) That an instrument called the ballistocardiograph can detect the presence of a person hiding in a truck, just by registering the minute recoil from the beating of the stowaway's heart? (This is discussed in Chapter 9.) That if you hum next to a spider's web at just the right pitch you can cause a resonance effect that sends the spider into a tizzy? (This is discussed in Chapter 13.) That powerful magnets can exploit the phenomenon of diamagnetism to levitate living creatures? (This is discussed in Chapter 22.)

Writing this textbook was a rewarding experience for me. I hope using it will prove equally rewarding to you, and that it will inspire an interest in and appreciation of physics that will last a lifetime.

James S. Walker

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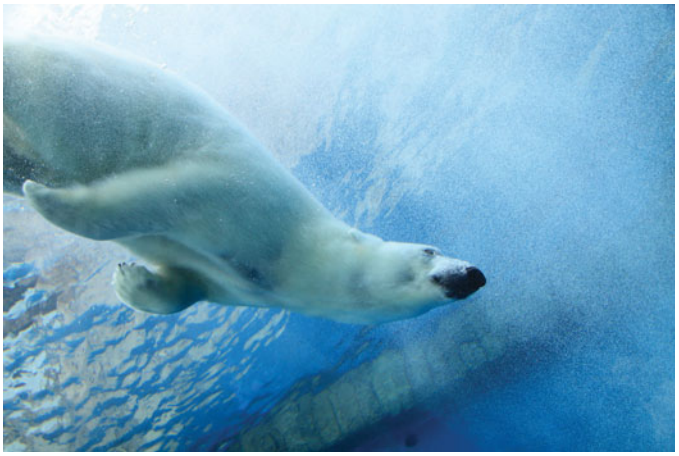
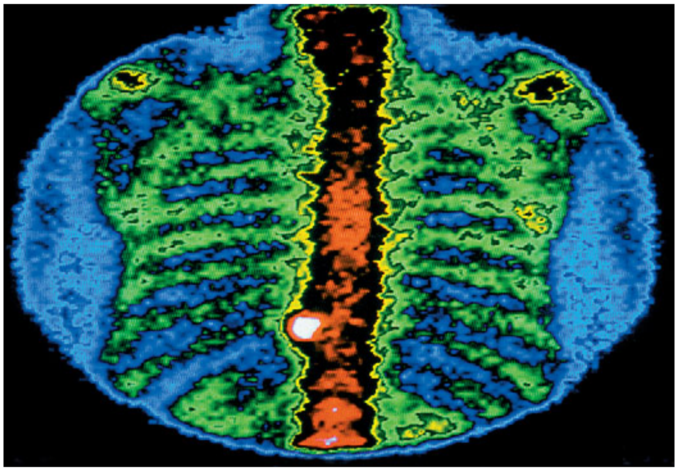
Detailed Contents



PART IV LIGHT AND OPTICS

PART V MODERN PHYSICS



1 Introduction to Physics

Physics is a quantitative science, based on careful measurements of quantities such as mass, length, and time. In the measurement shown here, a baby elephant is found to have a mass of approximately 425 kilograms, corresponding to a weight of about 935 pounds. Measurements of length and time indicate that the elephant's height is 1.25 meters, and its age is eleven months.

The goal of physics is to gain a deeper understanding of the world in which we live. For example, the laws of physics allow us to predict the behavior of everything from rockets sent to the Moon, to integrated chips in computers, to lasers used to perform eye surgery. In short, everything in nature—from atoms and subatomic particles to solar systems and galaxies—obeys the laws of physics.

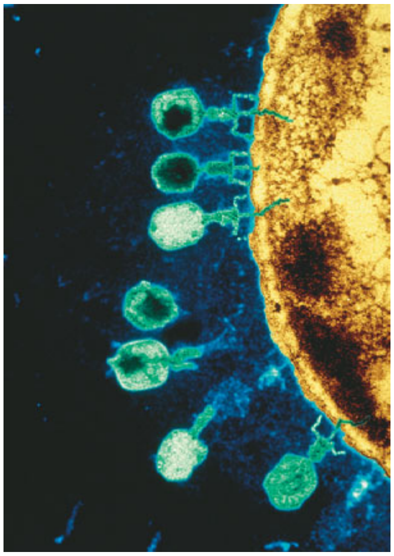
As we begin our study of physics, it is useful to consider a range of issues that underlies everything to follow. One of the most fundamental of these is the system of units we use when we measure such things as the mass of an object, its length, and the time between two events. Other equally important issues include methods for handling numerical calculations and basic conventions of mathematical notation. By the end of the chapter we will have developed a common "language" of physics that will be used throughout this book and probably in any science that you study.

1–1 Physics and the Laws of Nature

Physics is the study of the fundamental laws of nature, which, simply put, are the laws that underlie all physical phenomena in the universe. Remarkably, we have found that these laws can be expressed in terms of mathematical equations. As a result, it is possible to make precise, quantitative comparisons between the predictions of theory—derived from the mathematical form of the laws—and the observations of experiments. Physics, then, is a science rooted equally firmly in theory and experiment, and, as physicists make new observations, they constantly test and—if necessary—refine the present theories.

What makes physics particularly fascinating is the fact that it relates to everything in the universe. There is a great beauty in the vision that physics brings to our view of the universe; namely, that all the complexity and variety that we see in the world around us, and in the universe as a whole, are manifestations of a few fundamental laws and principles. That we can discover and apply these basic laws of nature is both astounding and exhilarating.

For those not familiar with the subject, physics may seem to be little more than a confusing mass of formulas. Sometimes, in fact, these formulas can be the trees that block the view of the forest. For a physicist, however, the many formulas of physics are simply different ways of expressing a few fundamental ideas. It is the forest—the basic laws and principles of physical phenomena in nature—that is the focus of this text.



▲ The size of these viruses, seen here attacking a bacterial cell, is about 10^{-7} m.

▲ The diameter of this typical galaxy is about 10^{21} m. (How many viruses would it take to span the galaxy?)

1–2 Units of Length, Mass, and Time

To make quantitative comparisons between the laws of physics and our experience of the natural world, certain basic physical quantities must be measured. The most common of these quantities are **length** (L), **mass** (M), and **time** (T). In fact, in the next several chapters these are the only quantities that arise. Later in the text, additional quantities, such as temperature and electric current, will be introduced as needed.

We begin by defining the units in which each of these quantities is measured. Once the units are defined, the values obtained in specific measurements can be expressed as multiples of them. For example, our unit of length is the **meter** (m). It follows, then, that a person who is 1.94 m tall has a height 1.94 times this unit of length. Similar comments apply to the unit of mass, the **kilogram,** and the unit of time, the **second.**

The detailed system of units used in this book was established in 1960 at the Eleventh General Conference of Weights and Measures in Paris, France, and goes by the name Système International d'Unités, or SI for short. Thus, when we refer to **SI units,** we mean units of meters (m), kilograms (kg), and seconds (s). Taking the first letter from each of these units leads to an alternate name that is often used—the **mks system.**

In the remainder of this section we define each of the SI units.

Length

Early units of length were often associated with the human body. For example, the Egyptians defined the cubit to be the distance from the elbow to the tip of the middle finger. Similarly, the foot was at one time defined to be the length of the royal foot of King Louis XIV. As colorful as these units may be, they are not particularly reproducible—at least not to great precision.

In 1793 the French Academy of Sciences, seeking a more objective and reproducible standard, decided to define a unit of length equal to one ten-millionth the distance from the North Pole to the equator. This new unit was named the metre (from the Greek *metron* for "measure"). The preferred spelling in the United States is *meter*. This definition was widely accepted, and in 1799 a "standard" meter was produced. It consisted of a platinum-iridium alloy rod with two marks on it one meter apart.

TABLE 1–1 Typical Distances

Distance from Earth to the nearest large galaxy (the Andromeda galaxy, M31)	2×10^{22} m
Diameter of our galaxy (the Milky Way)	8×10^{20} m
Distance from Earth to the nearest star (other than the Sun)	4×10^{16} m
One light-year	9.46×10^{15} m
Average radius of Pluto's orbit	6×10^{12} m
Distance from Earth to the Sun	1.5×10^{11} m
Radius of Earth	6.37×10^{6} m
Length of a football field	10^{2} m
Height of a person	2 m
Diameter of a CD	0.12 m
Diameter of the aorta	0.018 m
Diameter of a period in a sentence	5×10^{-4} m
Diameter of a red blood cell	8×10^{-6} m
Diameter of the hydrogen atom	10^{-10} m
Diameter of a proton	2×10^{-15} m

Since 1983 we have used an even more precise definition of the meter, based on the speed of light in a vacuum. In particular:

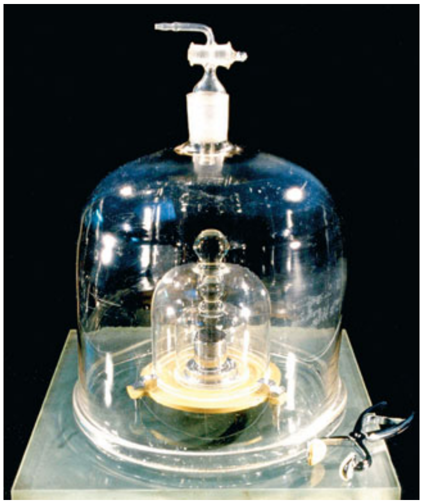
> One meter is defined to be the distance traveled by light in a vacuum in 1/299,792,458 of a second.

No matter how its definition is refined, however, a meter is still about 3.28 feet, which is roughly 10 percent longer than a yard. A list of typical lengths is given in Table 1–1.

Mass

In SI units, mass is measured in kilograms. Unlike the meter, the kilogram is not based on any natural physical quantity. By convention, the kilogram has been defined as follows:

> The kilogram, by definition, is the mass of a particular platinum-iridium alloy cylinder at the International Bureau of Weights and Standards in Sèvres, France.



▲ The standard kilogram, a cylinder of platinum and iridium 0.039 m in height and diameter, is kept under carefully controlled conditions in Sèvres, France. Exact replicas are maintained in other laboratories around the world.

To put the kilogram in everyday terms, a quart of milk has a mass slightly less than 1 kilogram. Additional masses, in kilograms, are given in Table 1–2.

Note that we do not define the kilogram to be the *weight* of the platinum-iridium cylinder. In fact, weight and mass are quite different quantities, even though they are often confused in everyday language. Mass is an intrinsic, unchanging property of an object. Weight, in contrast, is a measure of the gravitational force acting on an object, which can vary depending on the object's location. For example, if you are fortunate enough to travel to Mars someday, you will find that your weight is less than on Earth, though your mass is unchanged. The force of gravity will be discussed in detail in Chapter 12.

TABLE 1–2 Typical Masses

Galaxy (Milky Way)	4×10^{41} kg
Sun	2×10^{30} kg
Earth	5.97×10^{24} kg
Space shuttle	2×10^{6} kg
Elephant	5400 kg
Automobile	1200 kg
Human	70 kg
Baseball	0.15 kg
Honeybee	1.5×10^{-4} kg
Red blood cell	10^{-13} kg
Bacterium	10^{-15} kg
Hydrogen atom	1.67×10^{-27} kg
Electron	9.11×10^{-31} kg

Time

Nature has provided us with a fairly accurate timepiece in the revolving Earth. In fact, prior to 1956 the mean solar day was defined to consist of 24 hours, with 60 minutes per hour, and 60 seconds per minute, for a total of $(24)(60)(60) = 84{,}400$ seconds. Even the rotation of the Earth is not completely regular, however.

Today, the most accurate timekeepers known are "atomic clocks," which are based on characteristic frequencies of radiation emitted by certain atoms. These

▲ This atomic clock, which keeps time on the basis of radiation from cesium atoms, is accurate to about three millionths of a second per year. (How long would it take for it to gain or lose an hour?)

TABLE 1–3 Typical Times

Age of the universe	5×10^{17} s
Age of the Earth	1.3×10^{17} s
Existence of human species	6×10^{13} s
Human lifetime	2×10^{9} s
One year	3×10^{7} s
One day	8.6×10^{4} s
Time between heartbeats	0.8 s
Human reaction time	0.1 s
One cycle of a high-pitched sound wave	5×10^{-5} s
One cycle of an AM radio wave	10^{-6} s
One cycle of a visible light wave	2×10^{-15} s

"MY GOODNESS, IT'S 12:15:0936420175! TIME FOR LUNCH."

clocks have typical accuracies of about 1 second in 300,000 years. The atomic clock used for defining the second operates with cesium-133 atoms. In particular, the second is defined as follows:

> One second is defined to be the time it takes for radiation from a cesium-133 atom to complete 9,192,631,770 cycles of oscillation.

A range of characteristic time intervals is given in Table 1–3.

The nation's time and frequency standard is determined by a *cesium fountain atomic clock* developed at the National Institute of Standards and Technology (NIST) in Boulder, Colorado. The fountain atomic clock, designated NIST-F1, produces a "fountain" of cesium atoms that are projected upward in a vacuum to a height of about a meter. It takes roughly a second for the atoms to rise and fall through this height (as we shall see in the next chapter), and during this relatively long period of time the frequency of their oscillation can be measured with great precision. In fact, the NIST-F1 will gain or lose no more than one second in every 20 million years of operation.

Atomic clocks are almost commonplace these days. For example, the satellites that participate in the Global Positioning System (GPS) actually carry atomic clocks with them in orbit. This allows them to make the precision time measurements that are needed for an equally precise determination of position and speed. Similarly, the "atomic clocks" that are advertised for use in the home, while not atomic in their operation, nonetheless get their time from radio signals sent out from the atomic clocks at NIST in Boulder. You can access the official U.S. time on your computer by going to http://time.gov on the Web.

Other Systems of Units and Standard Prefixes

Although SI units are used throughout most of this book and are used almost exclusively in scientific research and in industry, we will occasionally refer to other systems that you may encounter from time to time.

For example, a system of units similar to the mks system, though comprised of smaller units, is the **cgs system,** which stands for centimeter (cm), gram (g), and second (s). In addition, the British engineering system is often encountered in everyday usage in the United States. Its basic units are the slug for mass, the foot (ft) for length, and the second (s) for time.

Finally, multiples of the basic units are common no matter which system is used. Standard prefixes are used to designate common multiples in powers of ten. For example, the prefix *kilo* means one thousand, or, equivalently, 10^3. Thus, 1 kilogram is 10^3 grams, and 1 kilometer is 10^3 meters. Similarly, *milli* is the prefix for one thousandth, or 10^{-3}. Thus, a millimeter is 10^{-3} meter, and so on. The most common prefixes are listed in Table 1–4.

EXERCISE 1–1

a. A minivan sells for 33,200 dollars. Express the price of the minivan in kilodollars and megadollars.

b. A typical *E. coli* bacterium is about 5 micrometers (or microns) in length. Give this length in millimeters and kilometers.

SOLUTION

a. 33.2 kilodollars, 0.0332 megadollars

b. 0.005 mm, 0.000000005 km

1–3 Dimensional Analysis

In physics, when we speak of the **dimension** of a physical quantity, we refer to the *type* of quantity in question, regardless of the units used in the measurement. For example, a distance measured in cubits and another distance measured in light-years both have the same dimension—length. The same is true of compound units such as velocity, which has the dimensions of length per unit time (length/time). A velocity measured in miles per hour has the same dimensions—length/time—as one measured in inches per century.

Now, any valid formula in physics must be **dimensionally consistent;** that is, each term in the equation must have the same dimensions. It simply doesn't make sense to add a distance to a time, for example, any more than it makes sense to add apples and oranges. They are different things.

To check the dimensional consistency of an equation, it is convenient to introduce a special notation for the dimension of a quantity. We will use square brackets, [ ], for this purpose. Thus, if x represents a distance, which has dimensions of length [L], we write this as $x = [\mathrm{L}]$. Similarly, a velocity, v, has dimensions of length per time [T]; thus we write $v = [\mathrm{L}]/[\mathrm{T}]$ to indicate its dimensions. Acceleration, a, which is the change in velocity per time, has the dimensions $a = ([\mathrm{L}]/[\mathrm{T}])/[\mathrm{T}] = [\mathrm{L}]/[\mathrm{T}^2]$. The dimensions of some common physical quantities are summarized in Table 1–5.

Let's use this notation to check the dimensional consistency of a simple equation. Consider the following formula:

$$x = x_0 + vt$$

In this equation, x and x_0 represent distances, v is a velocity, and t is time. Writing out the dimensions of each term, we have

$$[\mathrm{L}] = [\mathrm{L}] + \frac{[\mathrm{L}]}{[\mathrm{T}]}[\mathrm{T}]$$

It might seem at first that the last term has different dimensions than the other two. However, dimensions obey the same rules of algebra as other quantities. Thus the dimensions of time cancel in the last term:

$$[\mathrm{L}] = [\mathrm{L}] + \frac{[\mathrm{L}]}{[\cancel{\mathrm{T}}]}[\cancel{\mathrm{T}}] = [\mathrm{L}] + [\mathrm{L}]$$

As a result, we see that each term in this formula has the same dimensions. This type of calculation with dimensions is referred to as **dimensional analysis.**

EXERCISE 1–2

Show that $x = x_0 + v_0t + \frac{1}{2}at^2$ is dimensionally consistent. The quantities x and x_0 are distances, v_0 is a velocity, and a is an acceleration.

SOLUTION

Using the dimensions given in Table 1–5, we have

$$[\mathrm{L}] = [\mathrm{L}] + \frac{[\mathrm{L}]}{[\cancel{\mathrm{T}}]}[\cancel{\mathrm{T}}] + \frac{[\mathrm{L}]}{[\cancel{\mathrm{T}}]^2}[\cancel{\mathrm{T}}^2] = [\mathrm{L}] + [\mathrm{L}] + [\mathrm{L}]$$

Note that $\frac{1}{2}$ is ignored in this analysis because it has no dimensions.

Later in this text you will derive your own formulas from time to time. As you do so, it is helpful to check dimensional consistency at each step of the derivation. If at any time the dimensions don't agree, you will know that a mistake has been made, and you can go back and look for it. If the dimensions check, however, it's not a guarantee the formula is correct—after all, dimensionless factors, like 1/2 or 2, don't show up in a dimensional check.

1–4 Significant Figures

When a mass, a length, or a time is measured in a scientific experiment, the result is known only to within a certain accuracy. The inaccuracy or uncertainty can be caused by a number of factors, ranging from limitations of the measuring device itself to limitations associated with the senses and the skill of the person performing the experiment. In any case, the fact that observed values of experimental

TABLE 1–4 Common Prefixes

Power	Prefix	Abbreviation
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

TABLE 1–5 Dimensions of Some Common Physical Quantities

Quantity	Dimension
Distance	[L]
Area	$[\mathrm{L}^2]$
Volume	$[\mathrm{L}^3]$
Velocity	$[\mathrm{L}]/[\mathrm{T}]$
Acceleration	$[\mathrm{L}]/[\mathrm{T}^2]$
Energy	$[\mathrm{M}][\mathrm{L}^2]/[\mathrm{T}^2]$

quantities have inherent uncertainties should always be kept in mind when performing calculations with those values.

▲ Every measurement has some degree of uncertainty associated with it. How precise would you expect this measurement to be?

Suppose, for example, that you want to determine the walking speed of your pet tortoise. To do so, you measure the time, t, it takes for the tortoise to walk a distance, d, and then you calculate the quotient, d/t. When you measure the distance with a ruler, which has one tick mark per millimeter, you find that $d = 21.2$ cm, with the precise value of the digit in the second decimal place uncertain. Defining the number of **significant figures** in a physical quantity to be equal to the number of digits in it that are known with certainty, we say that d is known to *three* significant figures.

Similarly, you measure the time with an old pocket watch, and as best you can determine it, $t = 8.5$ s, with the second decimal place uncertain. Note that t is known to only *two* significant figures. If we were to make this measurement with a digital watch, with a readout giving the time to 1/100 of a second, the accuracy of the result would still be limited by the finite reaction time of the experimenter. The reaction time would have to be predetermined in a separate experiment. (See Problem 77 in Chapter 2 for a simple way to determine your reaction time.)

Returning to the problem at hand, we would now like to calculate the speed of the tortoise. Using the above values for d and t and a calculator with eight digits in its display, we find $(21.2 \text{ cm})/(8.5 \text{ s}) = 2.4941176 \text{ cm/s}$. Clearly, such an accurate value for the speed is unjustified, considering the limitations of our measurements. After all, we can't expect to measure quantities to two and three significant figures and from them obtain results with eight significant figures. In general, the number of significant figures that result when we multiply or divide physical quantities is given by the following rule of thumb:

> The number of significant figures after multiplication or division is equal to the number of significant figures in the *least* accurately known quantity.

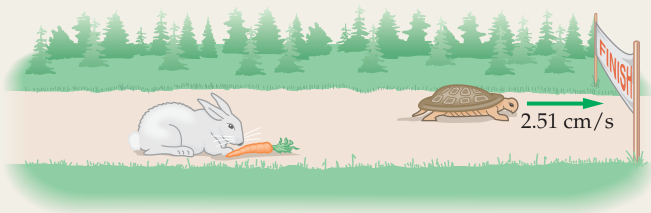
In our speed calculation, for example, we know the distance to three significant figures, but the time to only two significant figures. As a result, the speed should be given with just two significant figures, $d/t = (21.2 \text{ cm})/(8.5 \text{ s}) = 2.5 \text{ cm/s}$. Note that we didn't just keep the first two digits in 2.4941176 cm/s and drop the rest. Instead, we "rounded up"; that is, because the first digit to be dropped (9 in this case) is greater than or equal to 5, we increase the previous digit (4 in this case) by 1. Thus, 2.5 cm/s is our best estimate for the tortoise's speed.

EXAMPLE 1–1 IT'S THE TORTOISE BY A HARE

A tortoise races a rabbit by walking with a constant speed of 2.51 cm/s for 12.23 s. How much distance does the tortoise cover?

PICTURE THE PROBLEM

The race between the rabbit and the tortoise is shown in our sketch. The rabbit pauses to eat a carrot while the tortoise walks with a constant speed.



STRATEGY

The distance covered by the tortoise is the speed of the tortoise multiplied by the time during which it walks.

SOLUTION

1. Multiply the speed by the time to find the distance d:

$$d = (\text{speed})(\text{time}) = (2.51 \text{ cm/s})(12.23 \text{ s}) = 30.7 \text{ cm}$$

INSIGHT

Notice that if we simply multiply 2.51 cm/s by 12.23 s, we obtain 30.6973 cm. We don't give all of these digits in our answer, however. In particular, because the quantity that is known with the least accuracy (the speed) has only three significant figures, we give a result with three significant figures. Note, in addition, that the third digit in our answer has been rounded up from 6 to 7.

PRACTICE PROBLEM

How long does it take for the tortoise to walk 17 cm? [**Answer:** $t = (17\text{ cm})/(2.51\text{ cm/s}) = 6.8\text{ s}$]

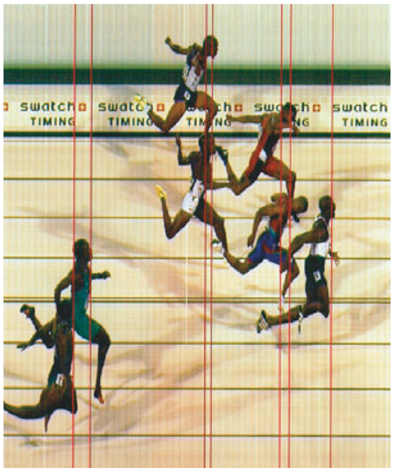
Some related homework problems: Problem 14, Problem 18

Note that the distance of 17 cm in the Practice Problem has only two significant figures because we don't know the digits to the right of the decimal place. If the distance were given as 17.0 cm, on the other hand, it would have three significant figures.

When physical quantities are added or subtracted, we use a slightly different rule of thumb. In this case, the rule involves the number of decimal places in each of the terms:

> The number of decimal places after addition or subtraction is equal to the smallest number of decimal places in any of the individual terms.

Thus, if you make a time measurement of 16.74 s, and then a subsequent time measurement of 5.1 s, the total time of the two measurements should be given as 21.8 s, rather than 21.84 s.



▲ The finish of the 100-meter race at the 1996 Atlanta Olympics. This official timing photo shows Donovan Bailey setting a new world record of 9.84 s. (If the timing had been accurate to only tenths of a second—as would probably have been the case before electronic devices came into use—how many runners would have shared the winning time? How many would have shared the second-place and third-place times?)

EXERCISE 1–3

You and a friend pick some raspberries. Your flat weighs 12.7 lb, and your friend's weighs 7.25 lb. What is the combined weight of the raspberries?

SOLUTION

Just adding the two numbers gives 19.95 lb. According to our rule of thumb, however, the final result must have only a single decimal place (corresponding to the term with the smallest number of decimal places). Rounding off to one place, then, gives 20.0 lb as the acceptable result.

Scientific Notation

The number of significant figures in a given quantity may be ambiguous due to the presence of zeros at the beginning or end of the number. For example, if a distance is stated to be 2500 m, the two zeros could be significant figures, or they could be zeros that simply show where the decimal point is located. If the two zeros are significant figures, the uncertainty in the distance is roughly a meter; if they are not significant figures, however, the uncertainty is about 100 m.

To remove this type of ambiguity, we can write the distance in **scientific notation**—that is, as a number of order unity times an appropriate power of ten. Thus, in this example, we would express the distance as 2.5×10^3 m if there are only two significant figures, or as 2.500×10^3 m to indicate four significant figures. Likewise, a time given as 0.000036 s has only two significant figures—the preceding zeros only serve to fix the decimal point. If this quantity were known to three significant figures, we would write it as 3.60×10^{-5} s to remove any ambiguity. See Appendix A for a more detailed discussion of scientific notation.

EXERCISE 1–4

How many significant figures are there in **(a)** 21.00, **(b)** 21, **(c)** 2.1×10^{-2}, **(d)** 2.10×10^{-3}?

SOLUTION

(a) 4, **(b)** 2, **(c)** 2, **(d)** 3

Round-Off Error

Finally, even if you perform all your calculations to the same number of significant figures as in the text, you may occasionally obtain an answer that differs in its last digit from that given in the book. In most cases this is not an issue as far as understanding the physics is concerned—usually it is due to **round-off error.**

Round-off error occurs when numerical results are rounded off at different times during a calculation. To see how this works, let's consider a simple example. Suppose you are shopping for knickknacks, and you buy one item for \$2.21, plus 8 percent sales tax. The total price is \$2.3868, or, rounded off to the nearest penny, \$2.39. Later, you buy another item for \$1.35. With tax this becomes \$1.458 or, again to the nearest penny, \$1.46. The total expenditure for these two items is \$2.39 + \$1.46 = \$3.85.

Now, let's do the rounding off in a different way. Suppose you buy both items at the same time for a total before-tax price of \$2.21 + \$1.35 = \$3.56. Adding in the 8% tax gives \$3.8448, which rounds off to \$3.84, one penny different from the previous amount. This same type of discrepancy can occur in physics problems. In general, it's a good idea to keep one extra digit throughout your calculations whenever possible, rounding off only the final result. But while this practice can help to reduce the likelihood of round-off error, there is no way to avoid it in every situation.

1–5 Converting Units

It is often convenient to convert from one set of units to another. For example, suppose you would like to convert 316 ft to its equivalent in meters. Looking at the conversion factors on the inside front cover of the text, we see that

$$1\ \text{m} = 3.281\ \text{ft} \qquad 1\text{–}1$$

Equivalently,

$$\frac{1\ \text{m}}{3.281\ \text{ft}} = 1 \qquad 1\text{–}2$$

Now, to make the conversion, we simply multiply 316 ft by this expression, which is equivalent to multiplying by 1:

$$(316\ \cancel{\text{ft}})\left(\frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}}\right) = 96.3\ \text{m}$$

Note that the conversion factor is written in this particular way, as 1 m divided by 3.281 ft, so that the units of feet cancel out, leaving the final result in the desired units of meters.

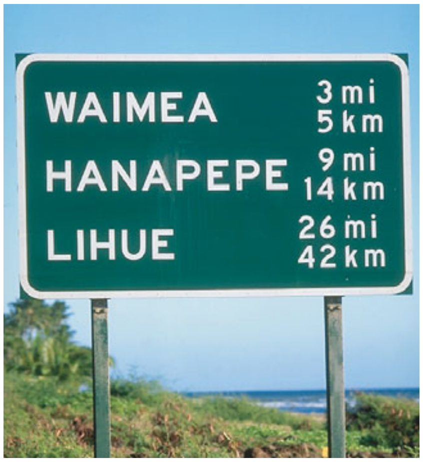
Of course, we can just as easily convert from meters to feet if we use the reciprocal of this conversion factor—which is also equal to 1:

$$1 = \frac{3.281\ \text{ft}}{1\ \text{m}}$$

For example, a distance of 26.4 m is converted to feet by canceling out the units of meters, as follows:

$$(26.4\ \cancel{\text{m}})\left(\frac{3.281\ \text{ft}}{1\ \cancel{\text{m}}}\right) = 86.6\ \text{ft}$$

Thus, we see that converting units is as easy as multiplying by 1—because that's really what you're doing.



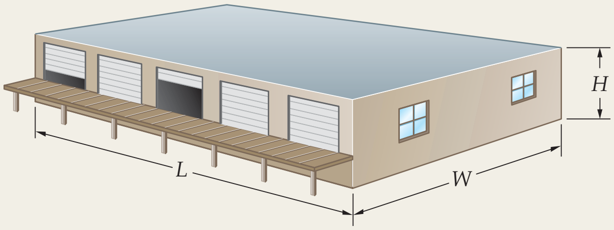
▲ From this sign, you can calculate factors for converting miles to kilometers and vice versa. (Why do you think the conversion factors seem to vary for different destinations?)

EXAMPLE 1–2 A HIGH-VOLUME WAREHOUSE

A warehouse is 20.0 yards long, 10.0 yards wide, and 15.0 ft high. What is its volume in SI units?

PICTURE THE PROBLEM

In our sketch we picture the warehouse, and indicate the relevant lengths for each of its dimensions.



STRATEGY

We begin by converting the length, width, and height of the warehouse to meters. Once this is done, the volume in SI units is simply the product of the three dimensions.

SOLUTION

1. Convert the length of the warehouse to meters: $L = (20.0 \text{ yard})\left(\frac{3 \text{ ft}}{1 \text{ yard}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 18.3 \text{ m}$
2. Convert the width to meters: $W = (10.0 \text{ yard})\left(\frac{3 \text{ ft}}{1 \text{ yard}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 9.14 \text{ m}$
3. Convert the height to meters: $H = (15.0 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 4.57 \text{ m}$
4. Calculate the volume of the warehouse: $V = L \times W \times H = (18.3 \text{ m})(9.14 \text{ m})(4.57 \text{ m}) = 764 \text{ m}^3$

INSIGHT

We would say, then, that the warehouse has a volume of 764 cubic meters—the same as 764 cubical boxes that are 1 m on a side.

PRACTICE PROBLEM

What is the volume of the warehouse if its length is one-hundredth of a mile, and the other dimensions are unchanged? [**Answer:** $V = 672 \text{ m}^3$]

Some related homework problems: Problem 20, Problem 21

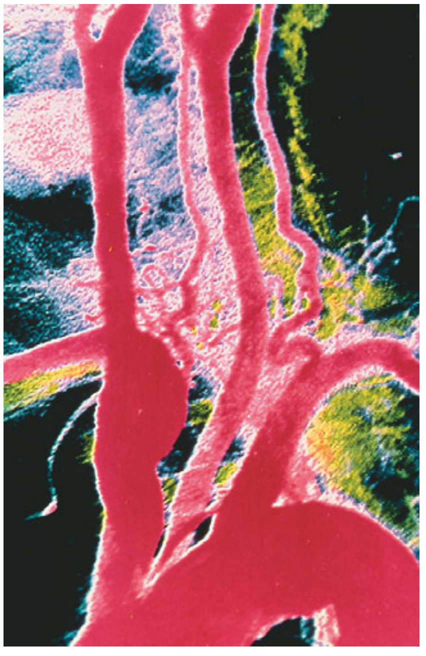
Finally, the same procedure can be applied to conversions involving any number of units. For instance, if you walk at 3.00 mi/h, how fast is that in m/s? In this case we need the following additional conversion factors:

$$1 \text{ mi} = 5280 \text{ ft} \qquad 1 \text{ h} = 3600 \text{ s}$$

With these factors at hand, we carry out the conversion as follows:

$$(3.00 \cancel{\text{mi}}/\cancel{\text{h}})\left(\frac{5280 \cancel{\text{ft}}}{1 \cancel{\text{mi}}}\right)\left(\frac{1 \text{ m}}{3.281 \cancel{\text{ft}}}\right)\left(\frac{1 \cancel{\text{h}}}{3600 \text{ s}}\right) = 1.34 \text{ m/s}$$

Note that in each conversion factor the numerator is equal to the denominator. In addition, each conversion factor is written in such a way that the unwanted units cancel, leaving just meters per second in our final result.



▲ Major blood vessels branch from the aorta (bottom), the artery that receives blood directly from the heart.

ACTIVE EXAMPLE 1–1 FIND THE SPEED OF BLOOD

Blood in the human aorta can attain speeds of 35.0 cm/s. How fast is this in **(a)** ft/s and **(b)** mi/h?

SOLUTION

(Test your understanding by performing the calculations indicated in each step.)

Part (a)

1. Convert centimeters to meters and then to feet: 1.15 ft/s

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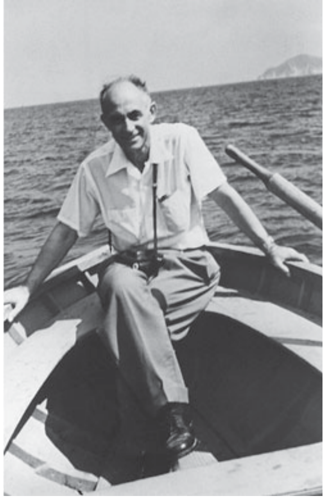


▲ Enrico Fermi (1901–1954) was renowned for his ability to pose and solve interesting order-of-magnitude problems. A winner of the 1938 Nobel Prize in physics, Fermi would ask his classes to obtain order-of-magnitude estimates for questions such as "How many piano tuners are there in Chicago?" or "How much is a tire worn down during one revolution?" Estimation questions like these are known to physicists today as "Fermi Problems."

CONTINUED FROM PREVIOUS PAGE

Part (b)

2. First, convert centimeters to miles: 2.17×10^{-4} mi/s
3. Next, convert seconds to hours: 0.783 mi/h

INSIGHT

Of course, the conversions in part (b) can be carried out in a single calculation if desired.

YOUR TURN

Find the speed of blood in units of km/h. *(Answers to* **Your Turn** *problems are given in the back of the book.)*

1–6 Order-of-Magnitude Calculations

An **order-of-magnitude** calculation is a rough "ballpark" estimate designed to be accurate to within a factor of about 10. One purpose of such a calculation is to give a quick idea of what order of magnitude should be expected from a complete, detailed calculation. If an order-of-magnitude calculation indicates that a distance should be on the order of 10^4 m, for example, and your calculator gives an answer on the order of 10^7 m, then there is an error somewhere that needs to be resolved.

For example, suppose you would like to estimate the speed of a cliff diver on entering the water. First, the cliff may be 20 or 30 feet high; thus in SI units we would say that the order of magnitude of the cliff's height is 10 m—certainly not 1 m or 10^2 m. Next, the diver hits the water something like a second later—certainly not 0.1 s later nor 10 s later. Thus, a reasonable order-of-magnitude estimate of the diver's speed is $10 \text{ m}/1 \text{ s} = 10 \text{ m/s}$, or roughly 20 mi/h. If you do a detailed calculation and your answer is on the order of 10^4 m/s, you probably entered one of your numbers incorrectly.

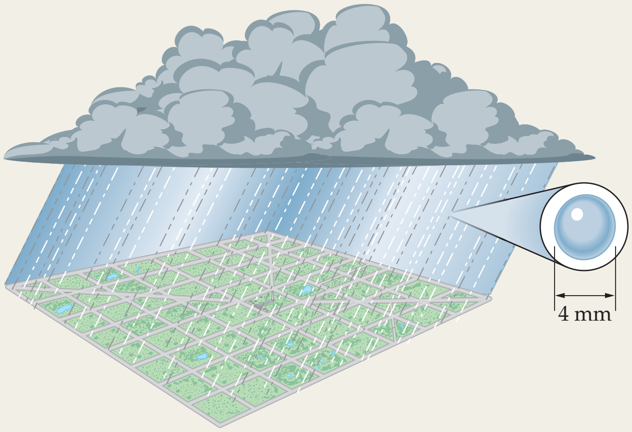
Another reason for doing an order-of-magnitude calculation is to get a feeling for what size numbers we are talking about in situations where a precise count is not possible. This is illustrated in the following Example.

EXAMPLE 1–3 ESTIMATION: HOW MANY RAINDROPS IN A STORM

A thunderstorm drops half an inch (~0.01 m) of rain on Washington D.C., which covers an area of about 70 square miles (~10^8 m^2). Estimate the number of raindrops that fell during the storm.

PICTURE THE PROBLEM

Our sketch shows an area $A = 10^8 \text{ m}^2$ covered to a depth $d = 0.01$ m by rainwater from the storm. Each drop of rain is approximated by a small sphere with a diameter of 4 mm.



STRATEGY

To find the number of raindrops, we first calculate the volume of water required to cover 10^8 m^2 to a depth of 0.01 m. Next, we calculate the volume of an individual drop of rain, recalling that the volume of a sphere of radius r is $4\pi r^3/3$. We estimate the diameter of a raindrop to be about 4 mm. Finally, dividing the volume of a drop into the volume of water that fell during the storm gives the number of drops.

SOLUTION

1. Calculate the order of magnitude of the volume of water, V_{water}, that fell during the storm:

$$V_{water} = Ad = (10^8\ \mathrm{m}^2)(0.01\ \mathrm{m}) \approx 10^6\ \mathrm{m}^3$$

2. Calculate the order of magnitude of the volume of a drop of rain, V_{drop}. Note that if the diameter of a drop is 4 mm, its radius is $r = 2\ \mathrm{mm} = 0.002\ \mathrm{m}$:

$$V_{drop} = \frac{4}{3}\pi r^3 \approx \frac{4}{3}\pi(0.002\ \mathrm{m})^3 \approx 10^{-8}\ \mathrm{m}^3$$

3. Divide V_{drop} into V_{water} to find the order of magnitude of the number of drops that fell during the storm:

$$\textit{number of raindrops} \approx \frac{V_{water}}{V_{drop}} \approx \frac{10^6\ \mathrm{m}^3}{10^{-8}\ \mathrm{m}^3} = 10^{14}$$

INSIGHT

Thus the number of raindrops in this one small storm is roughly 100,000 times greater than the current population of the Earth.

PRACTICE PROBLEM

If a storm pelts Washington D.C. with 10^{15} raindrops, how many inches of rain fall on the city? [**Answer:** About 5 inches]

Some related homework problems: Problem 36, Problem 38

Appendix B provides a number of interesting "typical values" for length, mass, speed, acceleration, and many other quantities. You may find these to be of use in making your own order-of-magnitude estimates.

1–7 Scalars and Vectors

Physical quantities are sometimes defined solely in terms of a number and the corresponding unit, like the volume of a room or the temperature of the air it contains. Other quantities require both a numerical value *and* a direction. For example, suppose a car is traveling at a rate of 25 m/s in a direction that is due north. Both pieces of information—the rate of travel (25 m/s) and the direction (north)—are required to fully specify the motion of the car. The rate of travel is given the name **speed;** the rate of travel combined with the direction is referred to as the **velocity.**

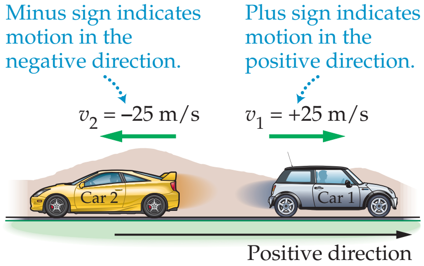
In general, quantities that are specified by a numerical value only are referred to as **scalars;** quantities that require both a numerical value and a direction are called **vectors:**

- A scalar is a numerical value, expressed in terms of appropriate units. An example would be the temperature of a room or the speed of a car.
- A vector is a mathematical quantity with both a numerical value and a direction. An example would be the velocity of a car.

All the physical quantities discussed in this text are either vectors or scalars. The properties of numbers (scalars) are well known, but the properties of vectors are sometimes less well known—though no less important. For this reason, you will find that Chapter 3 is devoted entirely to a discussion of vectors in two and three dimensions and, more specifically, to how they are used in physics.

The rather straightforward special case of vectors in one dimension is discussed in Chapter 2. There, we see that the direction of a velocity vector, for example, can only be to the left or to the right, up or down, and so on. That is, only two choices are available for the direction of a vector in one dimension. This is illustrated in **Figure 1–1**, where we see two cars, each traveling with a speed of 25 m/s. We also see that the cars are traveling in opposite directions, with car 1 moving to the right and car 2 moving to the left. We indicate the direction of travel with a plus sign for motion to the right, and a negative sign for motion to the left. Thus, the velocity of car 1 is written $v_1 = +25\ \mathrm{m/s}$, and the velocity of car 2 is $v_2 = -25\ \mathrm{m/s}$. The speed of each car is the absolute value, or **magnitude,** of the velocity; that is, speed $= |v_1| = |v_2| = 25\ \mathrm{m/s}$.

Whenever we deal with one-dimensional vectors, we shall indicate their direction with the appropriate sign. Many examples are found in Chapter 2 and, again, in later chapters where the simplicity of one dimension can again be applied.



▲ **FIGURE 1–1 Velocity vectors in one dimension**

The two cars shown in this figure have equal speeds of 25 m/s, but are traveling in opposite directions. To indicate the direction of travel, we first choose a positive direction (to the right in this case), and then give appropriate signs to the velocity of each car. For example, car 1 moves to the right, and hence its velocity is positive, $v_1 = +25\ \mathrm{m/s}$; the velocity of car 2 is negative, $v_2 = -25\ \mathrm{m/s}$, because it moves to the left.

1–8 Problem Solving in Physics

Physics is a lot like swimming—you have to learn by doing. You could read a book on swimming and memorize every word in it, but when you jump into a pool the first time you are going to have problems. Similarly, you could read this book carefully, memorizing every formula in it, but when you finish, you still haven't learned physics. To learn physics, you have to go beyond passive reading; you have to interact with physics and experience it by doing problems.

In this section we present a general overview of problem solving in physics. The suggestions given below, which apply to problems in all areas of physics, should help to develop a systematic approach.

We should emphasize at the outset that there is no recipe for solving problems in physics—it is a creative activity. In fact, the opportunity to be creative is one of the attractions of physics. The following suggestions, then, are not intended as a rigid set of steps that must be followed like the steps in a computer program. Rather, they provide a general guideline that experienced problem solvers find to be effective.

- **Read the problem carefully** Before you can solve a problem, you need to know exactly what information it gives and what it asks you to determine. Some information is given explicitly, as when a problem states that a person has a mass of 70 kg. Other information is implicit; for example, saying that a ball is dropped from rest means that its initial speed is zero. Clearly, a *careful* reading is the essential first step in problem solving.
- **Sketch the system** This may seem like a step you can skip—but don't. A sketch helps you to acquire a physical feeling for the system. It also provides an opportunity to label those quantities that are known and those that are to be determined. All Examples in this text begin with a sketch of the system, accompanied by a brief description in a section labeled "Picture the Problem."
- **Visualize the physical process** Try to visualize what is happening in the system as if you were watching it in a movie. Your sketch should help. This step ties in closely with the next step.
- **Strategize** This may be the most difficult, but at the same time the most creative, part of the problem-solving process. From your sketch and visualization, try to identify the physical processes at work in the system. Ask yourself what concepts or principles are involved in this situation. Then, develop a strategy—a game plan—for solving the problem. All Examples in this book have a "Strategy" spelled out before the solution begins.
- **Identify appropriate equations** Once a strategy has been developed, find the specific equations that are needed to carry it out.
- **Solve the equations** Use basic algebra to solve the equations identified in the previous step. Work with symbols such as x or y for the most part, substituting numerical values near the end of the calculations. Working with symbols will make it easier to go back over a problem to locate and identify mistakes, if there are any, and to explore limits and special cases.
- **Check your answer** Once you have an answer, check to see if it makes sense: (i) Does it have the correct dimensions? (ii) Is the numerical value reasonable?
- **Explore limits/special cases** Getting the correct answer is nice, but it's not all there is to physics. You can learn a great deal about physics and about the connection between physics and mathematics by checking various limits of your answer. For example, if you have two masses in your system, m_1 and m_2, what happens in the special case that $m_1 = 0$ or $m_1 = m_2$? Check to see whether your answer and your physical intuition agree.

The **Examples** in this text are designed to deepen your understanding of physics and at the same time develop your problem-solving skills. They all have the same basic structure: Problem Statement; Picture the Problem; Strategy; Solution, presenting the flow of ideas and the mathematics side-by-side in a two-column format; Insight; and a Practice Problem related to the one just solved. As you work through the Examples in the chapters to come, notice how the basic problem-solving guidelines outlined above are implemented in a consistent way.

In addition to the Examples, this text contains a new and innovative type of worked-out problem called the **Active Example,** the first one of which appears on page 9. The purpose of Active Examples is to encourage active participation in the solution of a problem and, in so doing, to act as a "bridge" between Examples—where each and every detail is worked out—and homework problems—where you are completely on your own. An analogy would be to think of Examples as like a tricycle, with no balancing required; homework problems as like a bicycle, where balancing is initially difficult to master; and Active Examples as like a bicycle with training wheels that give just enough help to prevent a fall. When you work through an Active Example, keep in mind that the work you are doing as you progress step-by-step through the problem is just the kind of work you'll be doing later in your homework assignments.

Finally, it is tempting to look for shortcuts when doing a problem—to look for a formula that seems to fit and some numbers to plug into it. It may seem harder to think ahead, to be systematic as you solve the problem, and then to think back over what you have done at the end of the problem. The extra effort is worth it, however, because by doing these things you will develop powerful problem-solving skills that can be applied to unexpected problems you may encounter on exams—and in life in general.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The three physical dimensions introduced in this chapter—mass, length, time—are the only ones we'll use until Chapter 19, when we introduce electric charge. Other quantities found in the next several chapters, like force, momentum, and energy, are combinations of these three basic dimensions. In this chapter we discussed the idea of a vector in one spatial dimension and showed how the direction of the vector can be indicated by its sign. These concepts are developed in more detail in Chapter 2.	Dimensional analysis is used frequently in the coming chapters to verify that each term in an equation has the correct dimensions. See, for example, the discussion following Equation 2–7, where we show that each term has the dimensions of velocity. We also use dimensional analysis to help derive some results, such as the speed of waves on a string in Section 14–2. Vectors are extended to two and three spatial dimensions in Chapter 3. After that, they are a standard tool throughout mechanics, and they appear again in electricity and magnetism.

CHAPTER SUMMARY

1–1 PHYSICS AND THE LAWS OF NATURE

Physics is based on a small number of fundamental laws and principles.

1–2 UNITS OF LENGTH, MASS, AND TIME

Length

One meter is defined as the distance traveled by light in a vacuum in 1/299,792,458 second.

Mass
One kilogram is the mass of a metal cylinder kept at the International Bureau of Weights and Standards.

Time
One second is the time required for a particular type of radiation from cesium-133 to undergo 9,192,631,770 oscillations.

1–3 DIMENSIONAL ANALYSIS

Dimension
The dimension of a quantity is the type of quantity it is, for example, length [L], mass [M], or time [T].

Dimensional Consistency
An equation is dimensionally consistent if each term in it has the same dimensions. All valid physical equations are dimensionally consistent.

Dimensional Analysis
A calculation based on the dimensional consistency of an equation.

1–4 SIGNIFICANT FIGURES

Significant Figures
The number of digits reliably known, excluding digits that simply indicate the decimal place. For example, 3.45 and 0.0000345 both have three significant figures.

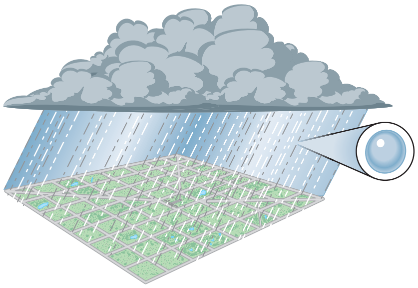
Round-off Error
Discrepancies caused by rounding off numbers in intermediate results.

1–5 CONVERTING UNITS

Multiply by the ratio of two units to convert from one to another. As an example, to convert 3.5 m to feet, you multiply by the factor (1 ft/0.3048 m).

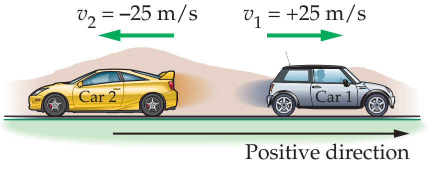
1–6 ORDER-OF-MAGNITUDE CALCULATIONS

A ballpark estimate designed to be accurate to within the nearest power of ten.



1–7 SCALARS AND VECTORS

A physical quantity that can be represented by a numerical value only is called a scalar. Quantities that require a direction in addition to the numerical value are called vectors.



1–8 PROBLEM SOLVING IN PHYSICS

A good general approach to problem solving is as follows: read; sketch; visualize; strategize; identify equations; solve; check; explore limits.

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Can dimensional analysis determine whether the area of a circle is πr^2 or $2\pi r^2$? Explain.

2. If a distance d has units of meters, and a time T has units of seconds, does the quantity $T + d$ make sense physically? What about the quantity d/T? Explain in both cases.

3. Is it possible for two quantities to **(a)** have the same units but different dimensions or **(b)** have the same dimensions but different units? Explain.
4. Give an order-of-magnitude estimate for the time in seconds of the following: **(a)** a year; **(b)** a baseball game; **(c)** a heartbeat; **(d)** the age of the Earth; **(e)** the age of a person.
5. Give an order-of-magnitude estimate for the length in meters of the following: **(a)** a person; **(b)** a fly; **(c)** a car; **(d)** a 747 airplane; **(e)** an interstate freeway stretching coast-to-coast.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 1–2 UNITS OF LENGTH, MASS, AND TIME

1. • **Spiderman** The movie *Spiderman* brought in \$114,000,000 in its opening weekend. Express this amount in **(a)** gigadollars and **(b)** teradollars.

2. • **BIO The Thickness of Hair** A human hair has a thickness of about 70 μm. What is this in **(a)** meters and **(b)** kilometers?

3. • The speed of light in a vacuum is approximately 0.3 Gm/s. Express the speed of light in meters per second.

4. • **A Fast Computer** IBM has a computer it calls the Blue Gene/L that can do 136.8 teracalculations per second. How many calculations can it do in a microsecond?

SECTION 1–3 DIMENSIONAL ANALYSIS

5. • **CE** Which of the following equations are dimensionally consistent? **(a)** $x = vt$, **(b)** $x = \frac{1}{2}at^2$, **(c)** $t = (2x/a)^{1/2}$.

6. • **CE** Which of the following quantities have the dimensions of a distance? **(a)** vt, **(b)** $\frac{1}{2}at^2$, **(c)** $2at$, **(d)** v^2/a.

7. • **CE** Which of the following quantities have the dimensions of a speed? **(a)** $\frac{1}{2}at^2$, **(b)** at, **(c)** $(2x/a)^{1/2}$, **(d)** $(2ax)^{1/2}$.

8. • Velocity is related to acceleration and distance by the following expression: $v^2 = 2ax^p$. Find the power p that makes this equation dimensionally consistent.

9. • Acceleration is related to distance and time by the following expression: $a = 2xt^p$. Find the power p that makes this equation dimensionally consistent.

10. • Show that the equation $v = v_0 + at$ is dimensionally consistent. Note that v and v_0 are velocities and that a is an acceleration.

11. •• Newton's second law (to be discussed in Chapter 5) states that acceleration is proportional to the force acting on an object and is inversely proportional to the object's mass. What are the dimensions of force?

12. •• The time T required for one complete oscillation of a mass m on a spring of force constant k is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Find the dimensions k must have for this equation to be dimensionally correct.

SECTION 1–4 SIGNIFICANT FIGURES

13. • The first several digits of π are known to be $\pi = 3.14159265358979\ldots$. What is π to **(a)** three significant figures, **(b)** five significant figures, and **(c)** seven significant figures?

14. • The speed of light to five significant figures is 2.9979×10^8 m/s. What is the speed of light to three significant figures?

15. • A parking lot is 144.3 m long and 47.66 m wide. What is the perimeter of the lot?

16. • On a fishing trip you catch a 2.35-lb bass, a 12.1-lb rock cod, and a 12.13-lb salmon. What is the total weight of your catch?

17. •• How many significant figures are there in **(a)** 0.000054 and **(b)** 3.001×10^5?

18. •• What is the area of a circle of radius **(a)** 14.37 m and **(b)** 3.8 m?

SECTION 1–5 CONVERTING UNITS

19. • **BIO Mantis Shrimp** Peacock mantis shrimps (*Odontodactylus scyllarus*) feed largely on snails. They shatter the shells of their prey by delivering a sharp blow with their front legs, which have been observed to reach peak speeds of 23 m/s. What is this speed in **(a)** feet per second and **(b)** miles per hour?

20. • **(a)** The largest building in the world by volume is the Boeing 747 plant in Everett, Washington. It measures approximately 631 m long, 707 yards wide, and 110 ft high. What is its volume in cubic feet? **(b)** Convert your result from part (a) to cubic meters.

21. • The Ark of the Covenant is described as a chest of acacia wood 2.5 cubits in length and 1.5 cubits in width and height. Given that a cubit is equivalent to 17.7 in., find the volume of the ark in cubic feet.

22. • How long does it take for radiation from a cesium-133 atom to complete 1.5 million cycles?

23. • **Angel Falls** Water going over Angel Falls, in Venezuela, the world's highest waterfall, drops through a distance of 3212 ft. What is this distance in km?

24. • An electronic advertising sign repeats a message every 7 seconds, day and night, for a week. How many times did the message appear on the sign?

25. • **BIO Blue Whales** The blue whale (*Balaenoptera musculus*) is thought to be the largest animal ever to inhabit the Earth. The longest recorded blue whale had a length of 108 ft. What is this length in meters?

26. • **The Star of Africa** The Star of Africa, a diamond in the royal scepter of the British crown jewels, has a mass of 530.2 carats, where 1 carat = 0.20 g. Given that 1 kg has an approximate weight of 2.21 lb, what is the weight of this diamond in pounds?

27. • **IP** Many highways have a speed limit of 55 mi/h. **(a)** Is this speed greater than, less than, or equal to 55 km/h? Explain. **(b)** Find the speed limit in km/h that corresponds to 55 mi/h.

28. • What is the speed in miles per hour of a beam of light traveling at 3.00×10^8 m/s?

29. • **BIO Woodpecker Impact** When red-headed woodpeckers (*Melanerpes erythrocephalus*) strike the trunk of a tree, they can experience an acceleration ten times greater than the acceleration of gravity, or about 98.1 m/s^2. What is this acceleration in ft/s^2?

30. •• **A Jiffy** The American physical chemist Gilbert Newton Lewis (1875–1946) proposed a unit of time called the "jiffy." According to Lewis, 1 jiffy = the time it takes light to travel one centimeter. **(a)** If you perform a task in a jiffy, how long has it taken in seconds? **(b)** How many jiffys are in one minute? (Use the fact that the speed of light is approximately 2.9979×10^8 m/s.)

31. •• **The Mutchkin and the Noggin** **(a)** A mutchkin is a Scottish unit of liquid measure equal to 0.42 L. How many mutchkins are required to fill a container that measures one foot on a side? **(b)** A noggin is a volume equal to 0.28 mutchkin. What is the conversion factor between noggins and gallons?

32. •• Suppose 1.0 cubic meter of oil is spilled into the ocean. Find the area of the resulting slick, assuming that it is one molecule thick, and that each molecule occupies a cube 0.50 μm on a side.

33. •• **IP** **(a)** A standard sheet of paper measures 8 1/2 by 11 inches. Find the area of one such sheet of paper in m^2. **(b)** A second sheet of paper is half as long and half as wide as the one described in part (a). By what factor is its area less than the area found in part (a)?

34. •• **BIO Squid Nerve Impulses** Nerve impulses in giant axons of the squid can travel with a speed of 20.0 m/s. How fast is this in **(a)** ft/s and **(b)** mi/h?

35. •• The acceleration of gravity is approximately 9.81 m/s^2 (depending on your location). What is the acceleration of gravity in feet per second squared?

SECTION 1–6 ORDER-OF-MAGNITUDE CALCULATIONS

36. • Give a ballpark estimate of the number of seats in a typical major league ballpark.

Shea Stadium, in New York. How many fans can it hold? (Problem 36)

37. • Milk is often sold by the gallon in plastic containers. **(a)** Estimate the number of gallons of milk that are purchased in the United States each year. **(b)** What approximate weight of plastic does this represent?

38. •• New York is roughly 3000 miles from Seattle. When it is 10:00 A.M. in Seattle, it is 1:00 P.M. in New York. Using this information, estimate **(a)** the rotational speed of the surface of Earth, **(b)** the circumference of Earth, and **(c)** the radius of Earth.

39. •• You've just won the $12 million cash lottery, and you go to pick up the prize. What is the approximate weight of the cash if you request payment in **(a)** quarters or **(b)** dollar bills?

GENERAL PROBLEMS

40. • **CE** Which of the following equations are dimensionally consistent? **(a)** $v = at$, **(b)** $v = \frac{1}{2}at^2$, **(c)** $t = a/v$, **(d)** $v^2 = 2ax$.

41. • **CE** Which of the following quantities have the dimensions of an acceleration? **(a)** xt^2, **(b)** v^2/x, **(c)** x/t^2, **(d)** v/t.

42. • **BIO Photosynthesis** The light that plants absorb to perform photosynthesis has a wavelength that peaks near 675 nm. Express this distance in **(a)** millimeters and **(b)** inches.

43. • **Glacial Speed** On June 9, 1983, the lower part of the Variegated Glacier in Alaska was observed to be moving at a rate of 210 feet per day. What is this speed in meters per second?

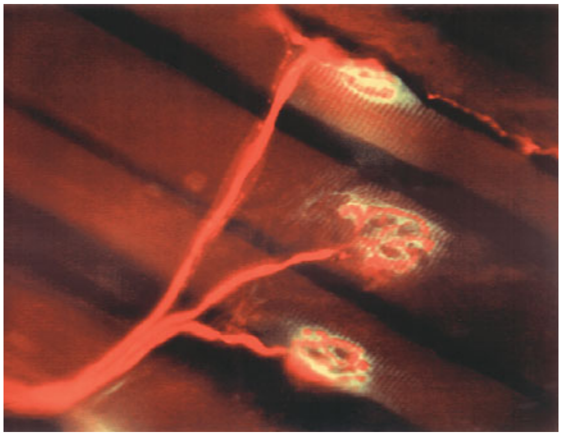
Alaska's Variegated Glacier (Problem 43)

44. •• **BIO Mosquito Courtship** Male mosquitoes in the mood for mating find female mosquitoes of their own species by listening for the characteristic "buzzing" frequency of the female's wing beats. This frequency is about 605 wing beats per second. **(a)** How many wing beats occur in one minute? **(b)** How many cycles of oscillation does the radiation from a cesium-133 atom complete during one mosquito wing beat?

45. •• **Ten and Ten** When Coast Guard pararescue jumpers leap from a helicopter to save a person in the water, they like to jump when the helicopter is flying "ten and ten," which means it is 10 feet above the water and moving forward with a speed of 10 knots. What is "ten and ten" in SI units? (A knot is one nautical mile per hour, where a nautical mile is 1.852 km.)

46. •• **IP** A Porsche sports car can accelerate at 14 m/s^2. **(a)** Is this acceleration greater than, less than, or equal to 14 ft/s^2? Explain. **(b)** Determine the acceleration of a Porsche in ft/s^2. **(c)** Determine its acceleration in km/h^2.

47. •• **BIO** **Human Nerve Fibers** Type A nerve fibers in humans can conduct nerve impulses at speeds up to 140 m/s. **(a)** How fast are the nerve impulses in miles per hour? **(b)** How far (in meters) can the impulses travel in 5.0 ms?



The impulses in these nerve axons, which carry commands to the skeletal muscle fibers in the background, travel at speeds of up to 140 m/s. (Problem 47)

48. •• **BIO** **Brain Growth** The mass of a newborn baby's brain has been found to increase by about 1.6 mg per minute. **(a)** How much does the brain's mass increase in one day? **(b)** How long does it take for the brain's mass to increase by 0.0075 kg?

49. •• **The Huygens Probe** NASA's Cassini mission to Saturn released a probe on December 25, 2004, that landed on the Saturnian moon Titan on January 14, 2005. The probe, which was named Huygens, was released with a gentle relative speed of 31 cm/s. As Huygens moved away from the main spacecraft, it rotated at a rate of seven revolutions per minute. **(a)** How many revolutions had Huygens completed when it was 150 yards from the mother ship? **(b)** How far did Huygens move away from the mother ship during each revolution? Give your answer in feet.

50. ••• Acceleration is related to velocity and time by the following expression: $a = v^p t^q$. Find the powers p and q that make this equation dimensionally consistent.

51. ••• The period T of a simple pendulum is the amount of time required for it to undergo one complete oscillation. If the length of the pendulum is L and the acceleration of gravity is g, then T is given by

$$T = 2\pi L^p g^q$$

Find the powers p and q required for dimensional consistency.

52. ••• Driving along a crowded freeway, you notice that it takes a time t to go from one mile marker to the next. When you increase your speed by 7.9 mi/h, the time to go one mile decreases by 13 s. What was your original speed?

PASSAGE PROBLEMS

BIO Using a Cricket as a Thermometer

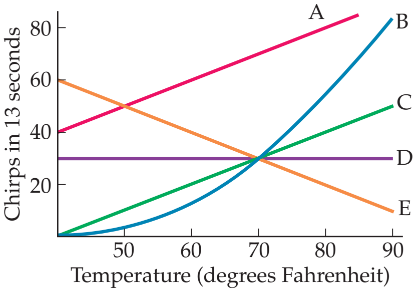
All chemical reactions, whether organic or inorganic, proceed at a rate that depends on temperature—the higher the temperature, the higher the rate of reaction. This can be understood in terms of molecules moving with increased energy as the temperature is increased, and colliding with other molecules more frequently. In the case of organic reactions, the result is that metabolic processes speed up with increasing temperature.

An increased or decreased metabolic rate can manifest itself in a number of ways. For example, a cricket trying to attract a mate chirps at a rate that depends on its overall rate of metabolism. As a result, the chirping rate of crickets depends directly on temperature. In fact, some people even use a pet cricket as a thermometer.

The cricket that is most accurate as a thermometer is the snowy tree cricket (*Oecanthus fultoni* Walker). Its rate of chirping is described by the following formula:

$$\begin{aligned} N &= \text{number of chirps per 13.0 seconds} \\ &= T - 40.0 \end{aligned}$$

In this expression, T is the temperature in degrees Fahrenheit.



Chirping rate versus temperature

▲ **FIGURE 1–2** Problem 53

53. • Which plot in **Figure 1–2** represents the chirping rate of the snowy tree cricket?

A B C D E

54. • If the temperature is 43 degrees Fahrenheit, how long does it take for the cricket to chirp 12 times?

A. 12 s **B.** 24 s **C.** 43 s **D.** 52 s

55. •• Your pet cricket chirps 112 times in one minute (60.0 s). What is the temperature in degrees Fahrenheit?

A. 41.9 **B.** 47.0 **C.** 64.3 **D.** 74.7

56. •• Suppose a snowy cricket is chirping when the temperature is 65.0 degrees Fahrenheit. How many oscillations does the radiation from a cesium-133 atom complete between successive chirps?

A. 7.98×10^7 **B.** 3.68×10^8

C. 4.78×10^9 **D.** 9.58×10^9

2 One-Dimensional Kinematics

These sprinters, crossing the finish line of the 100-m dash, illustrate one-dimensional motion. At the moment shown in the photograph the runners are moving with constant velocity at a speed of approximately 10 m/s.

We begin our study of physics with **mechanics,** the area of physics perhaps most apparent to us in our everyday lives. Every time you raise an arm, stand up or sit down, throw a ball, or open a door, your actions are governed by the laws of mechanics. Basically, mechanics is the study of how objects move, how they respond to external forces, and how other factors, such as size, mass, and mass distribution, affect their motion. This is a lot to cover, and we certainly won't try to tackle it all in one chapter.

Furthermore, in this chapter we treat all physical objects as *point particles;* that is, we consider all the mass of the object to be concentrated at a single point. This is a common practice in physics. For example, if you are interested in calculating the time it takes the Earth to complete a revolution about the Sun, it is reasonable to consider the Earth and the Sun as simple particles. In later chapters, we extend our studies to increasingly realistic situations, involving motion in more than one dimension and physical objects with shape and size.

2–1 Position, Distance, and Displacement

The first step in describing the motion of a particle is to set up a **coordinate system** that defines its position. An example of a coordinate system in one dimension is shown in **Figure 2–1**. This is simply an x axis, with an origin (where $x = 0$) and an arrow indicating the positive direction—the direction in which x increases. In setting up a coordinate system, we are free to choose the origin and the positive direction as we like, but once we make a choice we must be consistent with it throughout any calculations that follow.

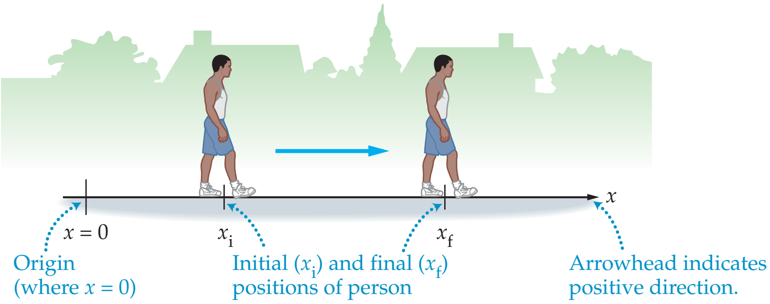


FIGURE 2–1 A one-dimensional coordinate system
You are free to choose the origin and positive direction as you like, but once your choice is made, stick with it.

The particle in Figure 2–1 is a person who has moved to the right from an initial position, x_i, to a final position, x_f. Because the positive direction is to the right, it follows that x_f is greater than x_i; that is, $x_f > x_i$.

Now that we've seen how to set up a coordinate system, let's use one to investigate the situation sketched in **Figure 2–2**. Suppose that you leave your house, drive to the grocery store, and then return home. The **distance** you've covered in your trip is 8.6 mi. In general, distance is defined as follows:

Definition: Distance

distance = total length of travel

SI unit: meter, m

Using SI units, the distance in this case is

$$8.6\text{ mi} = (8.6\text{ mi})\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 1.4 \times 10^4\text{ m}$$

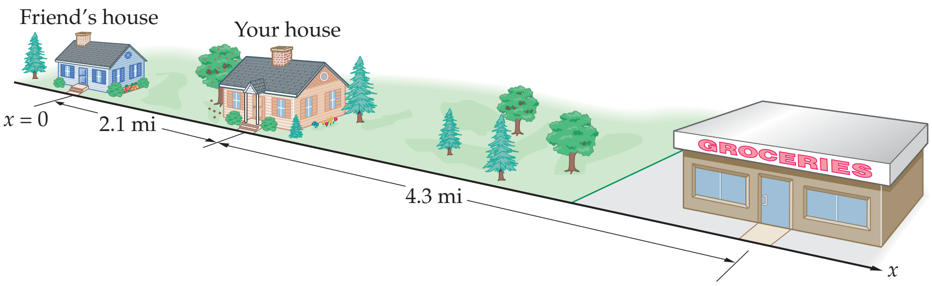


FIGURE 2–2 One-dimensional coordinates
The locations of your house, your friend's house, and the grocery store in terms of a one-dimensional coordinate system.

In a car, the distance traveled is indicated by the odometer. Note that distance is always positive and, because it has no direction associated with it, it is a scalar, as discussed in Chapter 1.

Another useful way to characterize a particle's motion is in terms of the **displacement,** Δx, which is simply the change in position.

Definition: Displacement, Δx

displacement = change in position = final position − initial position

$$\text{displacement} = \Delta x = x_f - x_i \qquad \text{2–1}$$

SI unit: meter, m

Notice that we use the delta notation, Δx, as a convenient shorthand for the quantity $x_f - x_i$. (See Appendix A for a complete discussion of delta notation.) Also, note that Δx can be positive (if the final position is to the right of the initial position, $x_f > x_i$), negative (if the final position is to the left of the initial position, $x_f < x_i$), or zero (if the final and initial positions are the same, $x_f = x_i$). In fact, the displacement is a one-dimensional vector, as defined in Chapter 1, and its direction (right or left) is given by its sign (positive or negative, respectively).

The SI units of displacement are meters—the same as for distance—but displacement and distance are really quite different. For example, in the round trip from your house to the grocery store and back the distance traveled is 8.6 mi, whereas the displacement is zero because $x_f = 2.1 \text{ mi} = x_i$. Suppose, instead, that you go from your house to the grocery store and then to your friend's house. On this trip the distance is 10.7 mi, but the displacement is

$$\Delta x = x_f - x_i = (0) - (2.1 \text{ mi}) = -2.1 \text{ mi}$$

As mentioned in the previous paragraph, the minus sign means your displacement is in the negative direction, that is, to the left.

ACTIVE EXAMPLE 2–1 FIND THE DISTANCE AND DISPLACEMENT

Calculate **(a)** the distance and **(b)** the displacement for a trip from your friend's house to the grocery store and then to your house.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Add the distances for the various parts of the total trip:	$2.1 \text{ mi} + 4.3 \text{ mi} + 4.3 \text{ mi} = 10.7 \text{ mi}$

Part (b)

2. Determine the initial position for the trip, using Figure 2–2:	$x_i = 0$
3. Determine the final position for the trip, using Figure 2–2:	$x_f = 2.1 \text{ mi}$
4. Subtract x_i from x_f to find the displacement:	$\Delta x = 2.1 \text{ mi}$

YOUR TURN

Suppose we choose the origin in Figure 2–2 to be at your house, rather than at your friend's house. In this case, find **(a)** the distance and **(b)** the displacement for the trip from your friend's house to the grocery store and then to your house.
(Answers to **Your Turn** *problems are given in the back of the book.)*

2–2 Average Speed and Velocity

The next step in describing motion is to consider how rapidly an object moves. For example, how long does it take for a Randy Johnson fastball to reach home plate? How far does an orbiting space shuttle travel in one hour? How fast do your eyelids move when you blink? These are examples of some of the most basic questions regarding motion, and in this section we learn how to answer them.

The simplest way to characterize the rate of motion is with the **average speed:**

$$\text{average speed} = \frac{\text{distance}}{\text{elapsed time}} \qquad 2\text{–}2$$

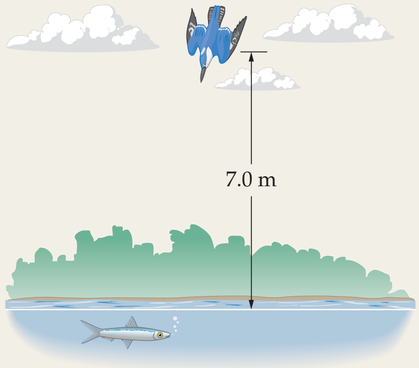
The dimensions of average speed are distance per time or, in SI units, meters per second, m/s. Both distance and elapsed time are positive; thus average speed is always positive.

EXAMPLE 2–1 THE KINGFISHER TAKES A PLUNGE

A kingfisher is a bird that catches fish by plunging into water from a height of several meters. If a kingfisher dives from a height of 7.0 m with an average speed of 4.00 m/s, how long does it take for it to reach the water?

PICTURE THE PROBLEM

As shown in the sketch, the kingfisher moves in a straight line through a vertical distance of 7.0 m. The average speed of the bird is 4.00 m/s.



STRATEGY

By rearranging Equation 2–2 we can solve for the elapsed time.

SOLUTION

1. Rearrange Equation 2–2 to solve for elapsed time: $$\text{elapsed time} = \frac{\text{distance}}{\text{average speed}}$$
2. Substitute numerical values to find the time: $$\text{elapsed time} = \frac{7.0\ \cancel{\text{m}}}{4.00\ \cancel{\text{m}}/\text{s}} = \frac{7.0}{4.00}\,\text{s} = 1.8\ \text{s}$$

INSIGHT

Note that Equation 2–2 is not just a formula for calculating the average speed. It relates speed, time, and distance. Given any two of these quantities, Equation 2–2 can be used to find the third.

PRACTICE PROBLEM

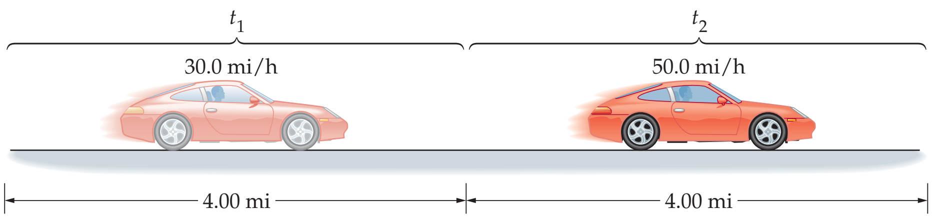
A kingfisher dives with an average speed of 4.6 m/s for 1.4 s. What was the height of the dive? [**Answer:** distance = (average speed) (elapsed time) = (4.6 m/s) (1.4 s) = 6.4 m]

Some related homework problems: Problem 13, Problem 15

Next, we calculate the average speed for a trip consisting of two parts of equal length, each traveled with a different speed.

CONCEPTUAL CHECKPOINT 2–1 AVERAGE SPEED

You drive 4.00 mi at 30.0 mi/h and then another 4.00 mi at 50.0 mi/h. Is your average speed for the 8.00-mi trip **(a)** greater than 40.0 mi/h, **(b)** equal to 40.0 mi/h, or **(c)** less than 40.0 mi/h?



REASONING AND DISCUSSION

At first glance it might seem that the average speed is definitely 40.0 mi/h. On further reflection, however, it is clear that it takes more time to travel 4.00 mi at 30.0 mi/h than it does to travel 4.00 mi at 50.0 mi/h. Therefore, you will be traveling at the lower speed for a greater period of time, and hence your average speed will be *less* than 40.0 mi/h—that is, closer to 30.0 mi/h than to 50.0 mi/h.

ANSWER

(c) The average speed is less than 40.0 mi/h.

To confirm the conclusion of the Conceptual Checkpoint, we simply apply the definition of average speed to find its value for this trip. We already know that the distance traveled is 8.00 mi; what we need now is the elapsed time. On the first 4.00 mi the time is

$$t_1 = \frac{4.00\text{ mi}}{30.0\text{ mi/h}} = (4.00/30.0)\text{ h}$$

The time required to cover the second 4.00 mi is

$$t_2 = \frac{4.00\text{ mi}}{50.0\text{ mi/h}} = (4.00/50.0)\text{ h}$$

Therefore, the elapsed time for the entire trip is

$$t_1 + t_2 = (4.00/30.0)\text{ h} + (4.00/50.0)\text{ h} = 0.213\text{ h}$$

This gives the following average speed:

$$\text{average speed} = \frac{8.00\text{ mi}}{0.213\text{ h}} = 37.6\text{ mi/h} < 40.0\text{ mi/h}$$

Note that a "guess" will never give a detailed result like 37.6 mi/h; a systematic, step-by-step calculation is required.

In many situations, there is a quantity that is even more useful than the average speed. It is the **average velocity,** v_{av}, and it is defined as displacement per time:

Definition: Average velocity, v_{av}

$$\text{average velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad \text{2–3}$$

SI unit: meter per second, m/s

Not only does the average velocity tell us, on average, how fast something is moving, it also tells us the *direction* the object is moving. For example, if an object moves in the positive direction, then $x_f > x_i$, and $v_{av} > 0$. On the other hand, if an object moves in the negative direction, it follows that $x_f < x_i$, and $v_{av} < 0$. As with displacement, the average velocity is a one-dimensional vector, and its direction is given by its sign. Average velocity gives more information than average speed; hence it is used more frequently in physics.

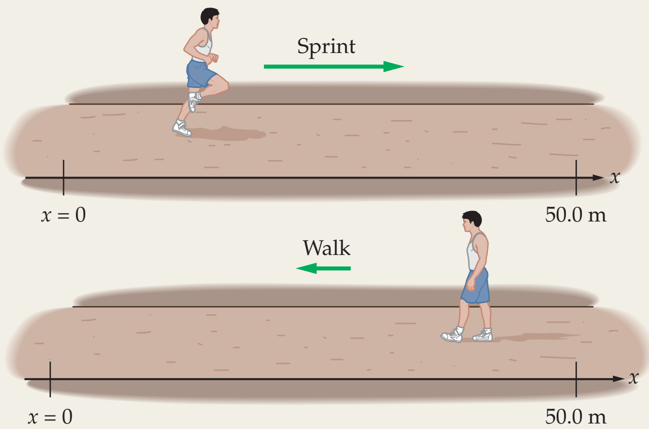
In the next Example, pay close attention to the positive and negative signs.

EXAMPLE 2–2 SPRINT TRAINING

An athlete sprints 50.0 m in 8.00 s, stops, and then walks slowly back to the starting line in 40.0 s. If the "sprint direction" is taken to be positive, what are **(a)** the average sprint velocity, **(b)** the average walking velocity, and **(c)** the average velocity for the complete round trip?

PICTURE THE PROBLEM

In our sketch we set up a coordinate system with the sprint going in the positive x direction, as described in the problem. For convenience, we choose the origin to be at the starting line. The finish line, then, is at $x = 50.0$ m.



STRATEGY

In each part of the problem we are asked for the average velocity and we are given information for times and distances. All that is needed, then, is to determine $\Delta x = x_f - x_i$ and $\Delta t = t_f - t_i$ in each case and apply Equation 2–3.

SOLUTION

Part (a)

1. Apply Equation 2–3 to the sprint, with $x_f = 50.0$ m, $x_i = 0$, $t_f = 8.00$ s, and $t_i = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{50.0 \text{ m} - 0}{8.00 \text{ s} - 0} = \frac{50.0}{8.00} \text{ m/s} = 6.25 \text{ m/s}$$

Part (b)

2. Apply Equation 2–3 to the walk. In this case, $x_f = 0$, $x_i = 50.0$ m, $t_f = 48.0$ s, and $t_i = 8.00$ s:

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 50.0 \text{ m}}{48.0 \text{ s} - 8.00 \text{ s}} = -\frac{50.0}{40.0} \text{ m/s} = -1.25 \text{ m/s}$$

Part (c)

3. For the round trip, $x_f = x_i = 0$; thus $\Delta x = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{0}{48.0 \text{ s}} = 0$$

INSIGHT

Note that the sign of the velocities in parts (a) and (b) indicates the direction of motion; positive for motion to the right, negative for motion to the left. Also, notice that the average *speed* for the entire 100.0-m trip $(100.0 \text{ m}/48.0 \text{ s} = 2.08 \text{ m/s})$ is nonzero, even though the average velocity vanishes.

PRACTICE PROBLEM

If the average velocity during the walk is -1.50 m/s, how long does it take the athlete to walk back to the starting line? [**Answer:** $\Delta t = \Delta x/v_{av} = (-50.0 \text{ m})/(-1.50 \text{ m/s}) = 33.3 \text{ s}$]

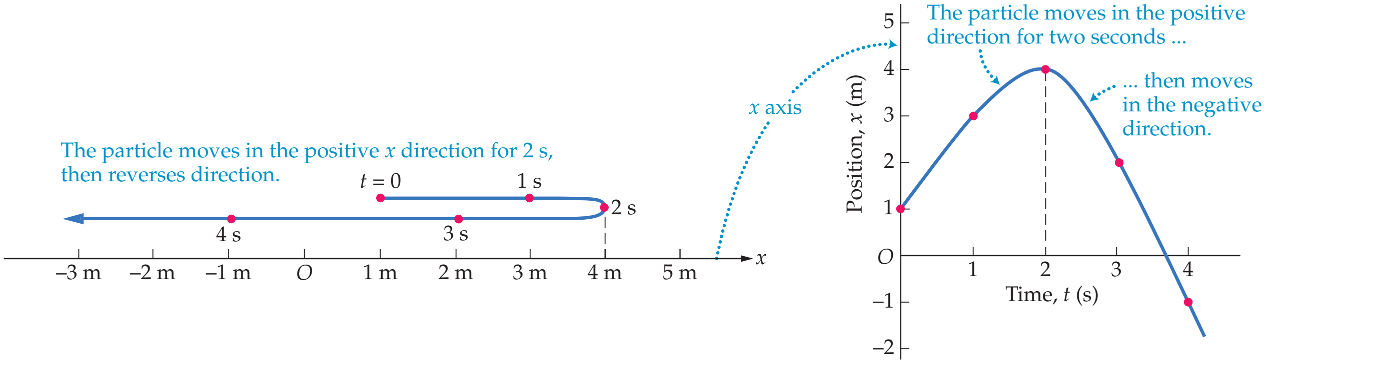
Some related homework problems: Problem 9, Problem 17, Problem 18

Graphical Interpretation of Average Velocity

It is often useful to "visualize" a particle's motion by sketching its position as a function of time. For example, consider a particle moving back and forth along the x axis, as shown in **Figure 2–3 (a)**. In this plot, we have indicated the position of a particle at a variety of times.

This way of keeping track of a particle's position and the corresponding time is a bit messy, though, so let's replot the same information with a different type of graph. In **Figure 2–3 (b)** we again plot the motion shown in Figure 2–3 (a), but this time with the vertical axis representing the position, x, and the horizontal axis representing time, t. An ***x*-versus-*t* graph** like this makes it considerably easier to visualize a particle's motion.

An x-versus-t plot also leads to a particularly useful interpretation of average velocity. To see how, suppose you would like to know the average velocity of the particle in Figures 2–3 (a) and 2–3 (b) from $t = 0$ to $t = 3$ s. From our definition of average velocity in Equation 2–3, we know that $v_{av} = \Delta x/\Delta t = (2 \text{ m} - 1 \text{ m})/(3 \text{ s} - 0) = +0.3$ m/s. To relate this to the

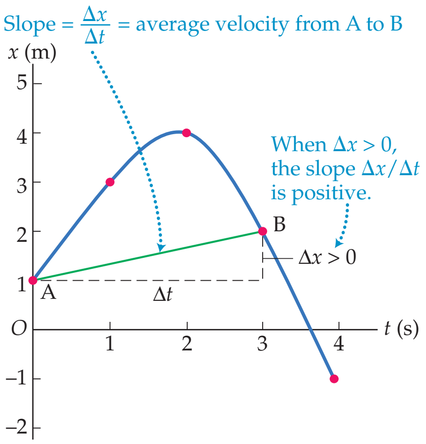


(a) The particle's path shown on a coordinate axis

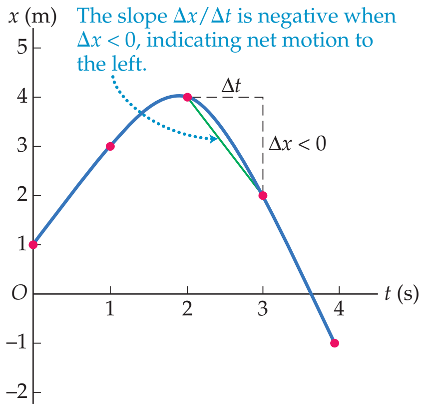
(b) The same path as a graph of position x versus time t

FIGURE 2–3 Two ways to visualize one-dimensional motion

Although the path in (a) is shown as a "U" for clarity, the particle actually moves straight back and forth along the x axis.



(a) Average velocity between $t = 0$ and $t = 3$ s



(b) Average velocity between $t = 2$ s and $t = 3$ s

▲ **FIGURE 2–4 Average velocity on an *x*-versus-*t* graph**

The slope of a straight line between any two points on an x-versus-t graph equals the average velocity between those points. Positive slopes indicate net motion to the right; negative slopes indicate net motion to the left.

▲ A speedometer indicates the instantaneous speed of a car. Note that the speedometer gives no information about the *direction* of motion. Thus, the speedometer is truly a "speed meter," not a velocity meter.

x-versus-t plot, draw a straight line connecting the position at $t = 0$ (call this point A) and the position at $t = 3$ s (point B). The result is shown in **Figure 2–4 (a)**.

The slope of the straight line from A to B is equal to the rise over the run, which in this case is $\Delta x/\Delta t$. But $\Delta x/\Delta t$ is the average velocity. Thus we see that:

- The slope of a line connecting two points on an x-versus-t plot is equal to the average velocity during that time interval.

As an additional example, let's calculate the average velocity between times $t = 2$ s and $t = 3$ s in Figure 2–3 (b). A line connecting the corresponding points is shown in **Figure 2–4 (b)**.

The first thing we notice about this line is that it has a negative slope; thus $v_{av} < 0$ and the particle is moving to the left. We also note that it is inclined more steeply than the line in Figure 2–4 (a), hence the magnitude of its slope is greater. In fact, if we calculate the slope of this line we find that $v_{av} = -2$ m/s for this time interval.

Thus, connecting points on an x-versus-t plot gives an immediate "feeling" for the average velocity over a given time interval. This type of graphical analysis will be particularly useful in the next section.

2–3 Instantaneous Velocity

Though average velocity is a useful way to characterize motion, it can miss a lot. For example, suppose you travel by car on a long, straight highway, covering 92 mi in 2.0 hours. Your average velocity is 46 mi/h. Even so, there may have been only a few times during the trip when you were actually driving at 46 mi/h. You may have sped along at 65 mi/h during most of the time, except when you stopped to have a bite to eat at a roadside diner, during which time your average velocity was zero.

To have a more accurate representation of your trip, you should average your velocity over shorter periods of time. If you calculate your average velocity every 15 minutes, you have a better picture of what the trip was like. An even better, more realistic picture of the trip is obtained if you calculate the average velocity every minute or every second. Ideally, when dealing with the motion of any particle, it is desirable to know the velocity of the particle at each instant of time.

This idea of a velocity corresponding to an instant of time is just what is meant by the **instantaneous velocity.** Mathematically, we define the instantaneous velocity as follows:

Definition: Instantaneous Velocity, *v*

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \text{2–4}$$

SI unit: meter per second, m/s

In this expression the notation $\lim_{\Delta t \to 0}$ means "evaluate the average velocity, $\Delta x/\Delta t$, over shorter and shorter time intervals, approaching zero in the limit." Note that the instantaneous velocity can be positive, negative, or zero, just like the average velocity—and just like the average velocity, the instantaneous velocity is a one-dimensional vector. The magnitude of the instantaneous velocity is called the **instantaneous speed.** In a car, the speedometer gives a reading of the vehicle's instantaneous speed.

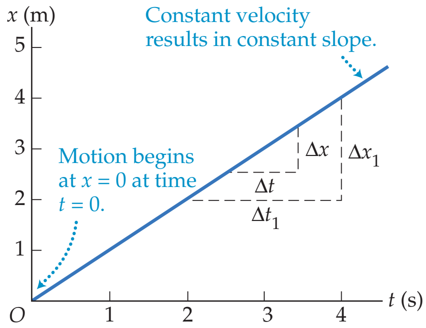
As Δt becomes smaller, Δx becomes smaller as well, but the ratio $\Delta x/\Delta t$ approaches a constant value. To see how this works, consider first the simple case of a particle moving with a constant velocity of $+1$ m/s. If the particle starts at $x = 0$ at $t = 0$, then its position at $t = 1$ s is $x = 1$ m, its position at $t = 2$ s is $x = 2$ m, and so on. Plotting this motion in an x-versus-t plot gives a straight line, as shown in **Figure 2–5**.

Now, suppose we want to find the instantaneous velocity at $t = 3$ s. To do so, we calculate the average velocity over small intervals of time centered at 3 s, and let the time intervals become arbitrarily small, as shown in the Figure. Since x-versus-t is a straight line, it is clear that $\Delta x/\Delta t = \Delta x_1/\Delta t_1$, no matter how small the time interval Δt. As Δt becomes smaller, so does Δx, but the ratio $\Delta x/\Delta t$ is simply the slope of the line, 1 m/s. Thus, the instantaneous velocity at $t = 3$ s is 1 m/s.

Of course, in this example the instantaneous velocity is 1 m/s for any instant of time, not just $t = 3$ s. Therefore:

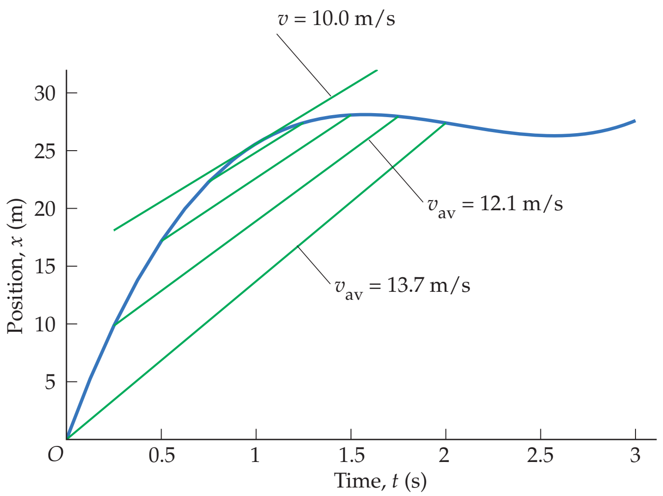
- When velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.

In general, a particle's velocity varies with time, and the x-versus-t plot is not a straight line. An example is shown in **Figure 2–6**, with the corresponding numerical values of x and t given in Table 2–1.



▲ FIGURE 2–5 Constant velocity corresponds to constant slope on an *x*-versus-*t* graph

The slope $\Delta x_1/\Delta t_1$ is equal to $(4\text{ m} - 2\text{ m})/(4\text{ s} - 2\text{ s}) = (2\text{ m})/(2\text{ s}) = 1$ m/s. Because x-versus-t is a straight line, the slope $\Delta x/\Delta t$ is also equal to 1 m/s for any value of Δt.



▲ FIGURE 2–6 Instantaneous velocity

An x-versus-t plot for motion with variable velocity. The instantaneous velocity at $t = 1$ s is equal to the slope of the tangent line at that time. The average velocity for a small time interval centered on $t = 1$ s approaches the instantaneous velocity at $t = 1$ s as the time interval goes to zero.

In this case, what is the instantaneous velocity at, say, $t = 1.00$ s? As a first approximation, let's calculate the average velocity for the time interval from $t_i = 0$ to $t_f = 2.00$ s. Note that this time interval is centered at $t = 1.00$ s. From Table 2–1 we see that $x_i = 0$ and $x_f = 27.4$ m, thus $v_{av} = 13.7$ m/s. The corresponding straight line connecting these two points is the lowest straight line in Figure 2–6.

The next three lines, in upward progression, refer to time intervals from 0.250 s to 1.75 s, 0.500 s to 1.50 s, and 0.750 s to 1.25 s, respectively. The corresponding average velocities, given in Table 2–2, are 12.1 m/s, 10.9 m/s, and 10.2 m/s. Table 2–2 also gives results for even smaller time intervals. In particular, for the interval from 0.900 s to 1.10 s the average velocity is 10.0 m/s. Smaller intervals also give 10.0 m/s. Thus, we can conclude that the instantaneous velocity at $t = 1.00$ s is $v = 10.0$ m/s.

The uppermost straight line in Figure 2–6 is the tangent line to the x-versus-t curve at the time $t = 1.00$ s; that is, it is the line that touches the curve at just a single point. Its slope is 10.0 m/s. Clearly, the average-velocity lines have slopes that

TABLE 2–1
***x*-versus-*t* Values for Figure 2–6**

t (s)	x (m)
0	0
0.25	9.85
0.50	17.2
0.75	22.3
1.00	25.6
1.25	27.4
1.50	28.1
1.75	28.0
2.00	27.4

TABLE 2–2 Calculating the Instantaneous Velocity at $t = 1$ s

t_i (s)	t_f (s)	Δt (s)	x_i (m)	x_f (m)	Δx (m)	$v_{av} = \Delta x/\Delta t$ (m/s)
0	2.00	2.00	0	27.4	27.4	13.7
0.250	1.75	1.50	9.85	28.0	18.2	12.1
0.500	1.50	1.00	17.2	28.1	10.9	10.9
0.750	1.25	0.50	22.3	27.4	5.10	10.2
0.900	1.10	0.20	24.5	26.5	2.00	10.0
0.950	1.05	0.10	25.1	26.1	1.00	10.0

approach the slope of the tangent line as the time intervals become smaller. This is an example of the following general result:

- The instantaneous velocity at a given time is equal to the slope of the tangent line at that point on an x-versus-t graph.

Thus, a visual inspection of an x-versus-t graph gives information not only about the location of a particle, but also about its velocity.

CONCEPTUAL CHECKPOINT 2–2 INSTANTANEOUS VELOCITY

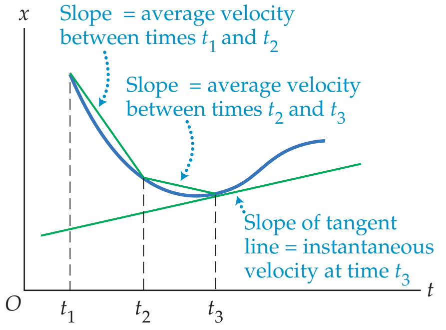
Referring to Figure 2–6, is the instantaneous velocity at $t = 0.500$ s **(a)** greater than, **(b)** less than, or **(c)** the same as the instantaneous velocity at $t = 1.00$ s?

REASONING AND DISCUSSION

From the x-versus-t graph in Figure 2–6 it is clear that the slope of a tangent line drawn at $t = 0.500$ s is greater than the slope of the tangent line at $t = 1.00$ s. It follows that the particle's velocity at 0.500 s is greater than its velocity at 1.00 s.

ANSWER

(a) The instantaneous velocity is greater at $t = 0.500$ s.



▲ **FIGURE 2–7 Graphical interpretation of average and instantaneous velocity**
Average velocities correspond to the slope of straight-line segments connecting different points on an x-versus-t graph. Instantaneous velocities are given by the slope of the tangent line at a given time.

In the remainder of the book, when we say velocity it is to be understood that we mean *instantaneous* velocity. If we want to refer to the average velocity, we will specifically say average velocity.

Graphical Interpretation of Average and Instantaneous Velocity

Let's summarize the graphical interpretations of average and instantaneous velocity on an x-versus-t graph:

- Average velocity is the slope of the straight line connecting two points corresponding to a given time interval.
- Instantaneous velocity is the slope of the tangent line at a given instant of time.

These relations are illustrated in **Figure 2–7**.

▲ The space shuttle *Discovery* accelerates upward on the initial phase of its journey into orbit. During this time the astronauts on board the shuttle experience an approximately linear acceleration that may be as great as 20 m/s^2.

2–4 Acceleration

Just as velocity is the rate of change of *displacement* with time, **acceleration** is the rate of change of *velocity* with time. Thus, an object accelerates whenever its velocity *changes,* no matter what the change—it accelerates when its velocity increases, it accelerates when its velocity decreases. Of all the concepts discussed in this chapter, perhaps none is more central to physics than acceleration. Galileo, for example, showed that falling bodies move with constant acceleration. Newton showed that acceleration and force are directly related, as we shall see in Chapter 5. Thus, it is particularly important to have a clear, complete understanding of acceleration before leaving this chapter.

We begin, then, with the definition of **average acceleration:**

Definition: Average Acceleration, a_{av}

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad \text{2–5}$$

SI unit: meter per second per second, m/s^2

Note that the dimensions of average acceleration are the dimensions of velocity per time, or (meters per second) per second:

$$\frac{\text{meters per second}}{\text{second}} = \frac{m/s}{s} = \frac{m}{s^2}$$

This is generally expressed as meters per second squared. For example, the acceleration of gravity on the Earth's surface is approximately 9.81 m/s^2, which means that the velocity of a falling object changes by 9.81 meters per second (m/s) every second (s). In addition, we see that the average acceleration can be positive, negative, or zero. In fact, it is a one-dimensional vector, just like displacement, average velocity, and instantaneous velocity. Typical magnitudes of acceleration are given in Table 2–3.

TABLE 2–3 Typical Accelerations (m/s^2)

Ultracentrifuge	3×10^6
Bullet fired from a rifle	4.4×10^5
Batted baseball	3×10^4
Click beetle righting itself	400
Acceleration required to deploy airbags	60
Bungee jump	30
High jump	15
Acceleration of gravity on Earth	9.81
Emergency stop in a car	8
Airplane during takeoff	5
An elevator	3
Acceleration of gravity on the Moon	1.62

EXERCISE 2–1

a. Saab advertises a car that goes from 0 to 60.0 mi/h in 6.2 s. What is the average acceleration of this car?

b. An airplane has an average acceleration of 5.6 m/s^2 during takeoff. How long does it take for the plane to reach a speed of 150 mi/h?

SOLUTION

a. average acceleration $= a_{av} = (60.0 \text{ mi/h})/(6.2 \text{ s})$

$$= (26.8 \text{ m/s})/(6.2 \text{ s}) = 4.3 \text{ m/s}^2$$

b. $\Delta t = \Delta v/a_{av} = (150 \text{ mi/h})/(5.6 \text{ m/s}^2) = (67.0 \text{ m/s})/(5.6 \text{ m/s}^2) = 12 \text{ s}$

Next, just as we considered the limit of smaller and smaller time intervals to find an instantaneous velocity, we can do the same to define an **instantaneous acceleration:**

Definition: Instantaneous Acceleration, *a*

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{2–6}$$

SI unit: meter per second per second, m/s^2

As you might expect, the instantaneous acceleration is a one-dimensional vector, just like the average acceleration, and its direction is given by its sign. For simplicity, when we say acceleration in the future we are referring to the instantaneous acceleration.

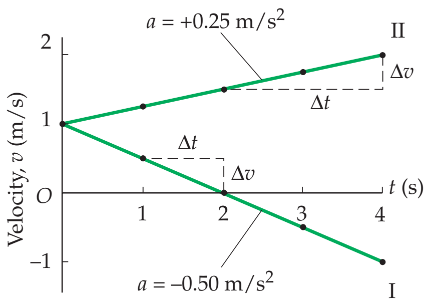
One final note before we go on to some examples. If the acceleration is constant, it has the same value at all times. Therefore:

- When acceleration is constant, the instantaneous and average accelerations are the same.

We shall make use of this fact when we return to the special case of constant acceleration in the next section.

Graphical Interpretation of Acceleration

To see how acceleration can be interpreted graphically, suppose that a particle has a constant acceleration of -0.50 m/s^2. This means that the velocity of the particle *decreases* by 0.50 m/s each second. Thus, if its velocity is 1.0 m/s at $t = 0$, then at $t = 1$ s its velocity is 0.50 m/s, at $t = 2$ s its velocity is 0, at $t = 3$ s its velocity is -0.50 m/s, and so on. This is illustrated by curve I in **Figure 2–8**, where we see that a plot of v-versus-t results in a straight line with a negative slope. Curve II in Figure 2–8 has a positive slope, corresponding to a constant acceleration of $+0.25 \text{ m/s}^2$. Thus, in terms of a v-versus-t plot, a constant acceleration results in a straight line with a slope equal to the acceleration.



▲ **FIGURE 2–8 *v*-versus-*t* plots for motion with constant acceleration**
Curve I represents the movement of a particle with constant acceleration $a = -0.50 \text{ m/s}^2$. Curve II represents the motion of a particle with constant acceleration $a = +0.25 \text{ m/s}^2$.

CONCEPTUAL CHECKPOINT 2–3 SPEED AS A FUNCTION OF TIME

The speed of a particle with the v-versus-t graph shown by curve II in Figure 2–8 increases steadily with time. Consider, instead, a particle whose v-versus-t graph is given by curve I in Figure 2–8. As a function of time, does the speed of this particle **(a)** increase, **(b)** decrease, or **(c)** decrease and then increase?

REASONING AND DISCUSSION

Recall that speed is the *magnitude* of velocity. In curve I of Figure 2–8 the speed starts out at 1.0 m/s, then *decreases* to 0 at $t = 2$ s. After $t = 2$ s the speed *increases* again. For example, at $t = 3$ s the speed is 0.50 m/s, and at $t = 4$ s the speed is 1 m/s.

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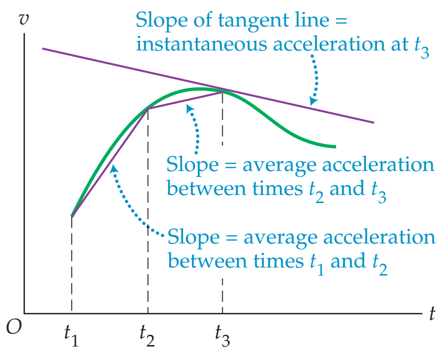


▲ **FIGURE 2–9 Graphical interpretation of average and instantaneous acceleration**

Average accelerations correspond to the slope of straight-line segments connecting different points on a v-versus-t graph. Instantaneous accelerations are given by the slope of the tangent line at a given time.

CONTINUED FROM PREVIOUS PAGE

Did you realize that the particle represented by curve I in Figure 2–8 changes direction at $t = 2\,\mathrm{s}$? It certainly does. Before $t = 2\,\mathrm{s}$ the particle moves in the positive direction; after $t = 2\,\mathrm{s}$ it moves in the negative direction. At precisely $t = 2\,\mathrm{s}$ the particle is momentarily at rest. However, regardless of whether the particle is moving in the positive direction, moving in the negative direction, or instantaneously at rest, it still has the same constant acceleration. Acceleration has to do only with the way the velocity is *changing* at a given moment.

ANSWER

(c) The speed decreases and then increases.

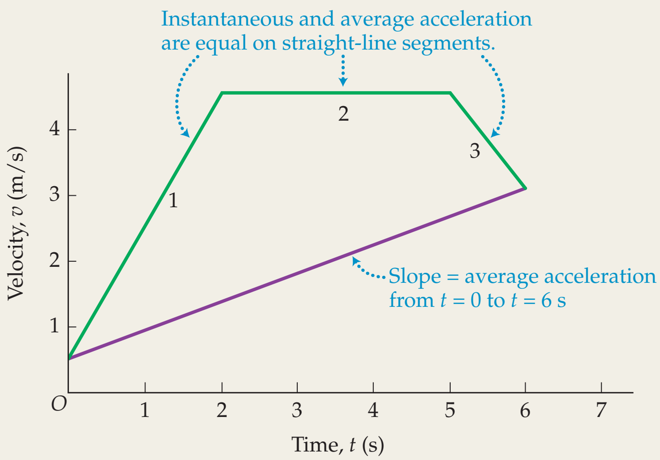
The graphical interpretations for velocity presented in Figure 2–7 apply equally well to acceleration, with just one small change: Instead of an x-versus-t graph, we use a v-versus-t graph, as in **Figure 2–9**. Thus, the average acceleration in a v-versus-t plot is the slope of a straight line connecting points corresponding to two different times. Similarly, the instantaneous acceleration is the slope of the tangent line at a particular time.

EXAMPLE 2–3 AN ACCELERATING TRAIN

A train moving in a straight line with an initial velocity of 0.50 m/s accelerates at $2.0\,\mathrm{m/s^2}$ for 2.0 seconds, coasts with zero acceleration for 3.0 seconds, and then accelerates at $-1.5\,\mathrm{m/s^2}$ for 1.0 second. **(a)** What is the final velocity of the train? **(b)** What is the average acceleration of the train?

PICTURE THE PROBLEM

We begin by sketching a v-versus-t plot for the train. The basic idea is that each interval of constant acceleration is represented by a straight line of the appropriate slope. Therefore, we draw a straight line with the slope $2.0\,\mathrm{m/s^2}$ from $t = 0$ to $t = 2.0\,\mathrm{s}$, a line with zero slope from $t = 2.0\,\mathrm{s}$ to $t = 5.0\,\mathrm{s}$, and a line with the slope $-1.5\,\mathrm{m/s^2}$ from $t = 5.0\,\mathrm{s}$ to $t = 6.0\,\mathrm{s}$. The line connecting the initial and final points determines the average acceleration.



STRATEGY

During each period of constant acceleration the change in velocity is $\Delta v = a_{av}\Delta t = a\Delta t$.

a. Adding the individual changes in velocity gives the total change, $\Delta v = v_f - v_i$. Since v_i is known, this expression can be solved for the final velocity, v_f.

b. The average acceleration can be calculated using Equation 2–5, $a_{av} = \Delta v/\Delta t$. Note that Δv has been obtained in part (a), and that the total time interval is $\Delta t = 6.0\,\mathrm{s}$, as is clear from the graph.

SOLUTION

Part (a)

1. Find the change in velocity during each of the three periods of constant acceleration:

$$\Delta v_1 = a_1\Delta t_1 = (2.0\,\mathrm{m/s^2})(2.0\,\mathrm{s}) = 4.0\,\mathrm{m/s}$$
$$\Delta v_2 = a_2\Delta t_2 = (0)(3.0\,\mathrm{s}) = 0$$
$$\Delta v_3 = a_3\Delta t_3 = (-1.5\,\mathrm{m/s^2})(1.0\,\mathrm{s}) = -1.5\,\mathrm{m/s}$$

2. Sum the change in velocity for each period to obtain the total Δv:

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3 = 4.0\,\mathrm{m/s} + 0 - 1.5\,\mathrm{m/s} = 2.5\,\mathrm{m/s}$$

3. Use Δv to find v_f, recalling that $v_i = 0.50\,\mathrm{m/s}$:

$$\Delta v = v_f - v_i$$
$$v_f = \Delta v + v_i = 2.5\,\mathrm{m/s} + 0.50\,\mathrm{m/s} = 3.0\,\mathrm{m/s}$$

Part (b)

4. The average acceleration is $\Delta v/\Delta t$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{2.5\ \mathrm{m/s}}{6.0\ \mathrm{s}} = 0.42\ \mathrm{m/s^2}$$

INSIGHT

Note that the average acceleration for these six seconds is not simply the average of the individual accelerations, $2.0\ \mathrm{m/s^2}$, $0\ \mathrm{m/s^2}$, and $-1.5\ \mathrm{m/s^2}$. The reason is that different amounts of time are spent with each acceleration. In addition, the average acceleration can be found graphically, as indicated in the v-versus-t sketch on the previous page. Specifically, the graph shows that Δv is 2.5 m/s for the time interval from $t = 0$ to $t = 6.0$ s.

PRACTICE PROBLEM

What is the average acceleration of the train between $t = 2.0$ s and $t = 6.0$ s?
[**Answer:** $a_{av} = \Delta v/\Delta t = (3.0\ \mathrm{m/s} - 4.5\ \mathrm{m/s})/(6.0\ \mathrm{s} - 2.0\ \mathrm{s}) = -0.38\ \mathrm{m/s^2}$]

Some related homework problems: Problem 36, Problem 38

In one dimension, nonzero velocities and accelerations are either positive or negative, depending on whether they point in the positive or negative direction of the coordinate system chosen. Thus, the velocity and acceleration of an object may have the same or opposite signs. (Of course, in two or three dimensions the relationship between velocity and acceleration can be much more varied, as we shall see in the next several chapters.) This leads to the following two possibilities:

- When the velocity and acceleration of an object have the same sign, the speed of the object increases. In this case, the velocity and acceleration point in the same direction.
- When the velocity and acceleration of an object have opposite signs, the speed of the object decreases. In this case, the velocity and acceleration point in opposite directions.

These two possibilities are illustrated in **Figure 2–10**. Notice that when a particle's speed increases, it means either that its velocity becomes more positive, as in **Figure 2–10 (a)**, or more negative, as in **Figure 2–10 (d)**. In either case, it is the magnitude of the velocity—the speed—that increases.

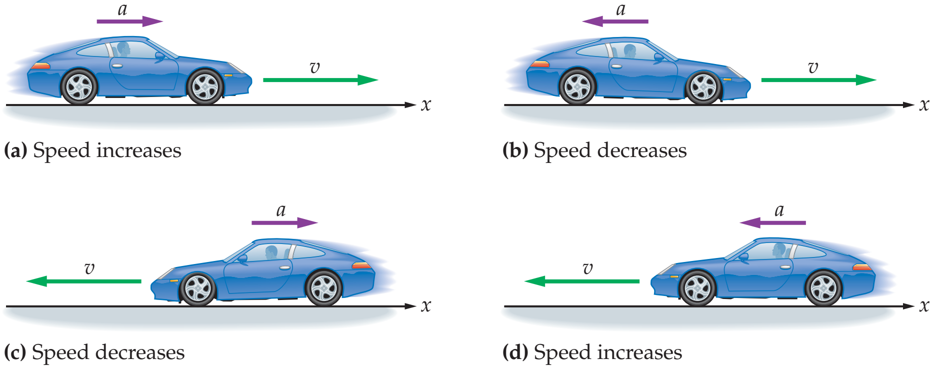
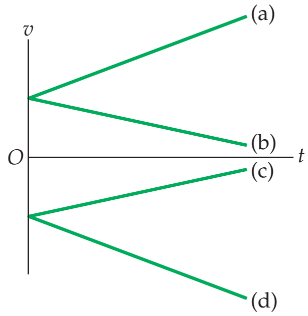


FIGURE 2–10 Cars accelerating or decelerating
A car's speed increases when its velocity and acceleration point in the same direction, as in cases **(a)** and **(d).** When the velocity and acceleration point in opposite directions, as in cases **(b)** and **(c),** the car's speed decreases.

The winner of this race was traveling at a speed of 313.91 mi/h at the end of the quarter-mile course. Since the winning time was just 4.607 s, the *average* acceleration during this race was approximately three times the acceleration of gravity (Section 2–7).



▲ **FIGURE 2–11** ***v*-versus-*t* plots with constant acceleration**

Four plots of v versus t corresponding to the four situations shown in Figure 2–10. Note that the speed increases in cases **(a)** and **(d),** but decreases in cases **(b)** and **(c).**

When a particle's speed decreases, it is often said to be *decelerating*. A common misconception is that deceleration implies a negative acceleration. This is not true. Deceleration can be caused by a positive or a negative acceleration, depending on the direction of the initial velocity. For example, the car in **Figure 2–10 (b)** has a positive velocity and a negative acceleration, while the car in **Figure 2–10 (c)** has a negative velocity and a positive acceleration. In both cases, the speed of the car decreases. Again, all that is required for deceleration in one dimension is that the velocity and acceleration have *opposite signs*; that is, they must point in *opposite directions*, as in parts (b) and (c) of Figure 2–10.

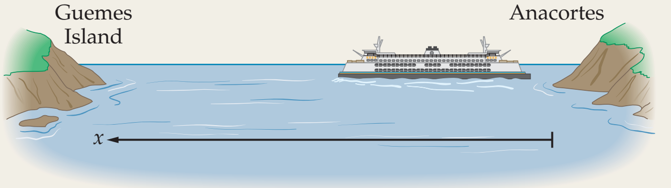
Velocity-versus-time plots for the four situations shown in Figure 2–10 are presented in **Figure 2–11**. In each of the four plots in Figure 2–11 we assume constant acceleration. Be sure to understand clearly the connection between the v-versus-t plots in Figure 2–11 and the corresponding physical motions indicated in Figure 2–10.

EXAMPLE 2–4 THE FERRY DOCKS

A ferry makes a short run between two docks; one in Anacortes, Washington, the other on Guemes Island. As the ferry approaches Guemes Island (traveling in the positive x direction), its speed is 7.4 m/s. **(a)** If the ferry slows to a stop in 12.3 s, what is its average acceleration? **(b)** As the ferry returns to the Anacortes dock, its speed is 7.3 m/s. If it comes to rest in 13.1 s, what is its average acceleration?

PICTURE THE PROBLEM

Our sketch shows the locations of the two docks and the positive direction indicated in the problem. Note that the distance between docks is not given, nor is it needed.



STRATEGY

We are given the initial and final velocities (the ferry comes to a stop in each case, so its final speed is zero) and the relevant times. Therefore, we can find the average acceleration using $a_{av} = \Delta v/\Delta t$, being careful to get the signs right.

SOLUTION

Part (a)

1. Calculate the average acceleration, noting that $v_i = 7.4$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 7.4\text{ m/s}}{12.3\text{ s}} = -0.60\text{ m/s}^2$$

Part (b)

2. In this case, $v_i = -7.3$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - (-7.3\text{ m/s})}{13.1\text{ s}} = 0.56\text{ m/s}^2$$

INSIGHT

In each case, the acceleration of the ferry is opposite in sign to its velocity; therefore the ferry decelerates.

PRACTICE PROBLEM

When the ferry leaves Guemes Island, its speed increases from 0 to 5.8 m/s in 9.25 s. What is its average acceleration? [**Answer:** $a_{av} = -0.63\text{ m/s}^2$]

Some related homework problems: Problem 34, Problem 35

2–5 Motion with Constant Acceleration

In this section, we derive equations describing the motion of particles moving with **constant acceleration.** These "equations of motion" can be used to describe a wide range of everyday phenomena. For example, in an idealized world with no air resistance, falling bodies have constant acceleration.

As mentioned in the previous section, if a particle has constant acceleration—that is, the same acceleration at every instant of time—then its instantaneous acceleration, a, is equal to its average acceleration, a_{av}. Recalling the definition of average acceleration, Equation 2–5, we have

$$a_{av} = \frac{v_f - v_i}{t_f - t_i} = a$$

where the initial and final times may be chosen arbitrarily. For example, let $t_i = 0$ for the initial time, and let $v_i = v_0$ denote the velocity at time zero. For the final time and velocity we drop the subscripts to simplify notation; thus we let $t_f = t$ and $v_f = v$. With these identifications we have

$$a_{av} = \frac{v - v_0}{t - 0} = a$$

Therefore,

$$v - v_0 = a(t - 0) = at$$

or

Constant-Acceleration Equation of Motion: Velocity as a Function of Time

$$v = v_0 + at \qquad 2\text{–}7$$

Note that Equation 2–7 describes a straight line on a v-versus-t plot. The line crosses the velocity axis at the value v_0 and has a slope a, in agreement with the graphical interpretations discussed in the previous section. For example, in curve I of Figure 2–8, the equation of motion is $v = v_0 + at = (1\ \mathrm{m/s}) + (-0.5\ \mathrm{m/s^2})t$. Also, note that $(-0.5\ \mathrm{m/s^2})t$ has the units $(\mathrm{m/s^2})\ (\mathrm{s}) = \mathrm{m/s}$; thus each term in Equation 2–7 has the same dimensions (as it must to be a valid physical equation).

EXERCISE 2–2

A ball is thrown straight upward with an initial velocity of $+8.2$ m/s. If the acceleration of the ball is $-9.81\ \mathrm{m/s^2}$, what is its velocity after

a. 0.50 s, and **b.** 1.0 s?

SOLUTION

a. Substituting $t = 0.50$ s in Equation 2–7 yields

$$v = 8.2\ \mathrm{m/s} + (-9.81\ \mathrm{m/s^2})(0.50\ \mathrm{s}) = 3.3\ \mathrm{m/s}$$

b. Similarly, using $t = 1.0$ s in Equation 2–7 gives

$$v = 8.2\ \mathrm{m/s} + (-9.81\ \mathrm{m/s^2})(1.0\ \mathrm{s}) = -1.6\ \mathrm{m/s}$$

Next, how far does a particle move in a given time if its acceleration is constant? To answer this question, recall the definition of average velocity:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Using the same identifications given previously for initial and final times, and letting $x_i = x_0$ and $x_f = x$, we have

$$v_{av} = \frac{x - x_0}{t - 0}$$

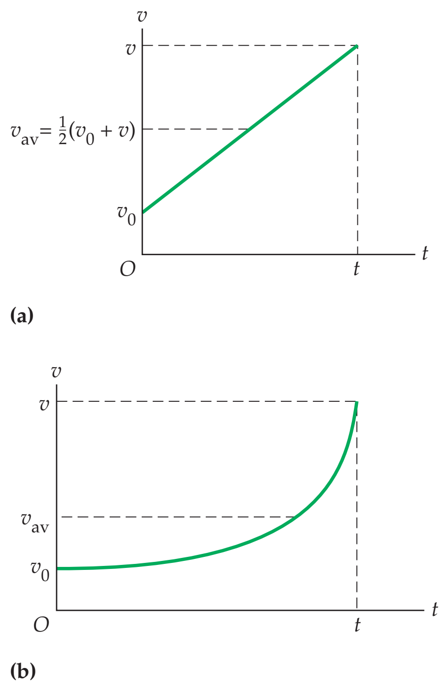
Thus,

$$x - x_0 = v_{av}(t - 0) = v_{av}t$$

or

$$x = x_0 + v_{av}t \qquad 2\text{–}8$$

Now, Equation 2–8 is fine as it is. In fact, it applies whether the acceleration is constant or not. A more useful expression, for the case of constant acceleration, is obtained by writing v_{av} in terms of the initial and final velocities. This can be done by referring to **Figure 2–12 (a)**. Here the velocity changes linearly (since a is



▲ **FIGURE 2–12 The average velocity**
(a) When acceleration is constant, the velocity varies linearly with time. As a result, the average velocity, v_{av}, is simply the average of the initial velocity, v_0, and the final velocity, v. **(b)** The velocity curve for nonconstant acceleration is nonlinear. In this case, the average velocity is no longer midway between the initial and final velocities.

constant) from v_0 at $t = 0$ to v at some later time t. The average velocity during this period of time is simply the average of the initial and final velocities; that is, the sum of the two velocities divided by two:

Constant-Acceleration Equation of Motion: Average Velocity

$$v_{av} = \tfrac{1}{2}(v_0 + v) \qquad \text{2–9}$$

The average velocity is indicated in the figure. Note that if the acceleration is not constant, as in **Figure 2–12 (b)**, this simple averaging of initial and final velocities is no longer valid.

Substituting the expression for v_{av} from Equation 2–9 into Equation 2–8 yields

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad \text{2–10}$$

This equation, like Equation 2–7, is valid *only* for constant acceleration. The utility of Equations 2–7 and 2–10 is illustrated in the next Example.

PROBLEM-SOLVING NOTE

"Coordinate" the Problem

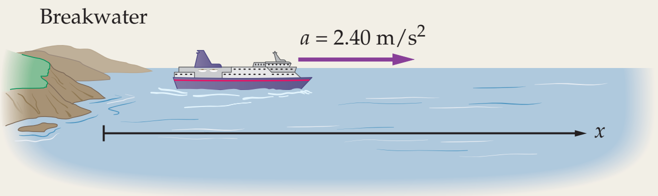
The first step in solving a physics problem is to produce a simple sketch of the system. Your sketch should include a coordinate system, along with an origin and a positive direction. Next, you should identify quantities that are given in the problem, such as initial position, initial velocity, acceleration, and so on. These preliminaries will help in producing a mathematical representation of the problem.

EXAMPLE 2–5 FULL SPEED AHEAD

A boat moves slowly inside a marina (so as not to leave a wake) with a constant speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. **(a)** How fast is the boat moving after accelerating for 5.00 s? **(b)** How far has the boat traveled in this time?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the breakwater, and the positive x direction to be the direction of motion. With this choice the initial position is $x_0 = 0$, and the initial velocity is $v_0 = 1.50$ m/s.



STRATEGY

The acceleration is constant, so we can use Equations 2–7 to 2–10. In part (a) we want to relate velocity to time, so we use Equation 2–7, $v = v_0 + at$. In part (b) our knowledge of the initial and final velocities allows us to relate position to time using Equation 2–10, $x = x_0 + \frac{1}{2}(v_0 + v)t$.

SOLUTION

Part (a)

1. Use Equation 2–7 with $v_0 = 1.50$ m/s and $a = 2.40$ m/s^2:

$$v = v_0 + at = 1.50\ \text{m/s} + (2.40\ \text{m/s}^2)(5.00\ \text{s})$$
$$= 1.50\ \text{m/s} + 12.0\ \text{m/s} = 13.5\ \text{m/s}$$

Part (b)

2. Apply Equation 2–10, using the result for v obtained in part (a):

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$
$$= 0 + \tfrac{1}{2}(1.50\ \text{m/s} + 13.5\ \text{m/s})(5.00\ \text{s})$$
$$= (7.50\ \text{m/s})(5.00\ \text{s}) = 37.5\ \text{m}$$

INSIGHT

Since the boat has a constant acceleration between $t = 0$ and $t = 5.00$ s, its velocity-versus-time curve is linear during this time interval. As a result, the average velocity for these 5.00 seconds is the average of the initial and final velocities, $v_{av} = \frac{1}{2}(1.50\ \text{m/s} + 13.5\ \text{m/s}) = 7.50$ m/s. Multiplying the average velocity by the time, 5.00 s, gives the distance traveled—which is exactly what Equation 2–10 does in Step 2.

PRACTICE PROBLEM

At what time is the boat's speed equal to 10.0 m/s? [**Answer:** $t = 3.54$ s]

Some related homework problems: Problem 47, Problem 48

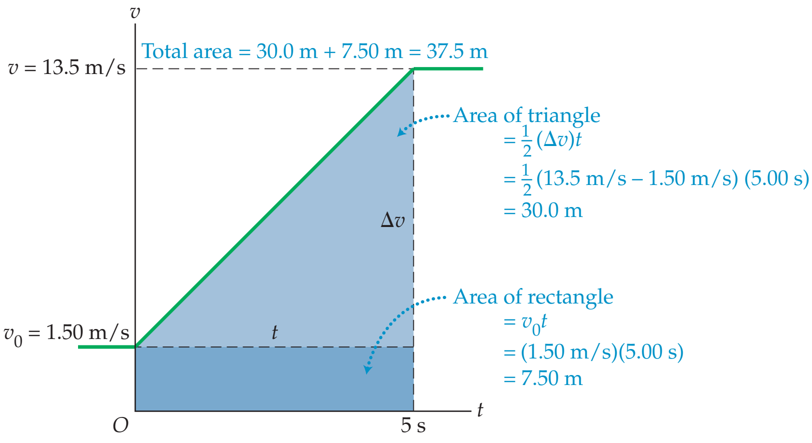


FIGURE 2–13 Velocity versus time for the boat in Example 2–5
The distance traveled by the boat between $t = 0$ and $t = 5.00$ s is equal to the corresponding area under the velocity curve.

The velocity of the boat in Example 2–5 is plotted as a function of time in **Figure 2–13**, with the acceleration starting at time $t = 0$ and ending at $t = 5.00$ s. We will now show that the *distance* traveled by the boat from $t = 0$ to $t = 5.00$ s *is equal to the corresponding area under the velocity-versus-time curve*. This is a general result, valid for any velocity curve and any time interval:

- The distance traveled by an object from a time t_1 to a time t_2 is equal to the area under the velocity curve between those two times.

In this case, the area is the sum of the areas of a rectangle and a triangle. The rectangle has a base of 5.00 s and a height of 1.50 m/s, which gives an area of $(5.00\text{ s})(1.50\text{ m/s}) = 7.50$ m. Similarly, the triangle has a base of 5.00 s and a height of $(13.5\text{ m/s} - 1.50\text{ m/s}) = 12.0$ m/s, for an area of $\frac{1}{2}(5.00\text{ s})(12.0\text{ m/s}) = 30.0$ m. Clearly, the total area is 37.5 m, just as found in Example 2–5.

Staying with Example 2–5 for a moment, let's repeat the calculation of part (b), only this time for the general case. First, we use the final velocity from part (a), calculated with $v = v_0 + at$, in the expression for the average velocity, $v_{av} = \frac{1}{2}(v_0 + v)$. Symbolically, this gives the following:

$$\tfrac{1}{2}(v_0 + v) = \tfrac{1}{2}[v_0 + (v_0 + at)] = v_0 + \tfrac{1}{2}at \qquad \textit{(constant acceleration)}$$

Next, we substitute this result into Equation 2–10, which yields

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = x_0 + \left(v_0 + \tfrac{1}{2}at\right)t$$

Multiplying through by t gives the following result:

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \text{2–11}$$

Here we have an expression for position versus time that is explicitly in terms of the acceleration, a.

Note that each term in Equation 2–11 has the same dimensions, as they must. For example, the velocity term, $v_0 t$, has the units $(\text{m/s})(\text{s}) = \text{m}$. Similarly, the acceleration term, $\frac{1}{2}at^2$, has the units $(\text{m/s}^2)(\text{s}^2) = \text{m}$.

EXERCISE 2–3

Repeat part (b) of Example 2–5 using Equation 2–11.

SOLUTION

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + (1.50\text{ m/s})(5.00\text{ s}) + \tfrac{1}{2}(2.40\text{ m/s}^2)(5.00\text{ s})^2 = 37.5\text{ m}$$

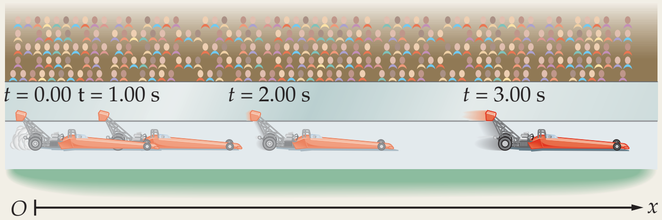
The next Example gives further insight into the physical meaning of Equation 2–11.

EXAMPLE 2–6 PUT THE PEDAL TO THE METAL

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in **(a)** 1.00 s, **(b)** 2.00 s, **(c)** 3.00 s?

PICTURE THE PROBLEM

We set up a coordinate system in which the drag racer starts at the origin and accelerates in the positive x direction. With this choice, it follows that $x_0 = 0$ and $a = +7.40 \text{ m/s}^2$. Also, since the racer starts from rest, its initial velocity is zero, $v_0 = 0$. Incidentally, the positions of the racer in the sketch have been drawn to scale.



STRATEGY

Since this problem gives the acceleration, which is constant, and asks for a relationship between position and time, we use Equation 2–11.

SOLUTION

Part (a)

1. Evaluate Equation 2–11 with $a = 7.40 \text{ m/s}^2$ and $t = 1.00$ s:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2 = \tfrac{1}{2}at^2$$
$$x = \tfrac{1}{2}(7.40 \text{ m/s}^2)(1.00 \text{ s})^2 = 3.70 \text{ m}$$

Part (b)

2. From the calculation in part (a), Equation 2–11 reduces to $x = \frac{1}{2}at^2$ in this situation. Evaluate $x = \frac{1}{2}at^2$ at $t = 2.00$ s:

$$x = \tfrac{1}{2}at^2$$
$$= \tfrac{1}{2}(7.40 \text{ m/s}^2)(2.00 \text{ s})^2 = 14.8 \text{ m} = 4(3.70 \text{ m})$$

Part (c)

3. Repeat with $t = 3.00$ s:

$$x = \tfrac{1}{2}at^2$$
$$= \tfrac{1}{2}(7.40 \text{ m/s}^2)(3.00 \text{ s})^2 = 33.3 \text{ m} = 9(3.70 \text{ m})$$

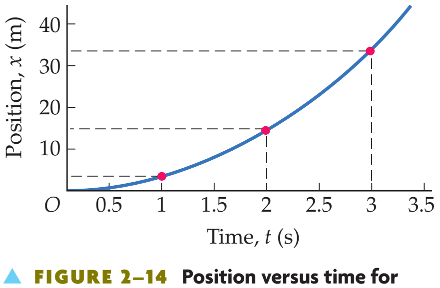
INSIGHT

This Example illustrates one of the key features of accelerated motion—position does not change uniformly with time when an object accelerates. In this case, the distance traveled in the first two seconds is 4 times the distance traveled in the first second, and the distance traveled in the first three seconds is 9 times the distance traveled in the first second. This kind of behavior is a direct result of the fact that x depends on t^2 when the acceleration is nonzero.

PRACTICE PROBLEM

In one second the racer travels 3.70 m. How long does it take for the racer to travel $2(3.70 \text{ m}) = 7.40$ m? [**Answer:** $t = \sqrt{2}$ s $= 1.41$ s]

Some related homework problems: Problem 49, Problem 64



▲ **FIGURE 2–14 Position versus time for Example 2–6**

The upward-curving, parabolic shape of this x-versus-t plot indicates a positive, constant acceleration. The dots on the curve show the position of the drag racer in Example 2–6 at the times 1.00 s, 2.00 s, and 3.00 s.

Figure 2–14 shows a graph of x-versus-t for Example 2–6. Notice the parabolic shape of the x-versus-t curve, which is due to the $\frac{1}{2}at^2$ term, and is characteristic of constant acceleration. In particular, if acceleration is positive ($a > 0$), then a plot of x-versus-t curves upward; if acceleration is negative ($a < 0$), a plot of x-versus-t curves downward. The greater the magnitude of a, the greater the curvature. In contrast, if a particle moves with constant velocity ($a = 0$) the t^2 dependence vanishes, and the x-versus-t plot is a straight line.

Our final equation of motion with constant acceleration relates velocity to position. We start by solving for the time, t, in Equation 2–7:

$$v = v_0 + at \quad \text{or} \quad t = \frac{v - v_0}{a}$$

Next, we substitute this result into Equation 2–10, thus eliminating t:

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = x_0 + \tfrac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right)$$

Noting that $(v_0 + v)(v - v_0) = v_0 v - v_0^2 + v^2 - v v_0 = v^2 - v_0^2$, we have

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

Finally, a straightforward rearrangement of terms yields

Constant-Acceleration Equation of Motion: Velocity in Terms of Displacement

$$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x \qquad 2\text{–}12$$

This equation allows us to relate the velocity at one position to the velocity at another position, without knowing how much time is involved. The next Example shows how Equation 2–12 can be used.

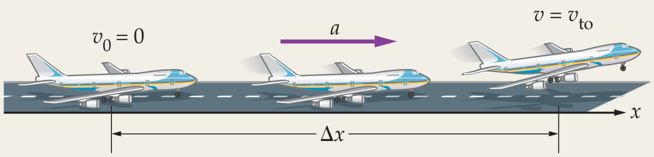
EXAMPLE 2–7 TAKEOFF DISTANCE FOR AN AIRLINER

REAL-WORLD PHYSICS

Jets at JFK International Airport accelerate from rest at one end of a runway, and must attain takeoff speed before reaching the other end of the runway. **(a)** Plane A has acceleration a and takeoff speed v_{to}. What is the minimum length of runway, Δx_{A}, required for this plane? Give a symbolic answer. **(b)** Plane B has the same acceleration as plane A, but requires twice the takeoff speed. Find Δx_{B} and compare with Δx_{A}. **(c)** Find the minimum runway length for plane A if $a = 2.20\ \text{m/s}^2$ and $v_{\text{to}} = 95.0\ \text{m/s}$. (These values are typical for a 747 jetliner.)

PICTURE THE PROBLEM

In our sketch, we choose the positive x direction to be the direction of motion. With this choice, it follows that the acceleration of the plane is positive, $a = +2.20\ \text{m/s}^2$. Similarly, the takeoff velocity is positive as well, $v_{\text{to}} = +95.0\ \text{m/s}$.



STRATEGY

From the sketch it is clear that we want to express Δx, the distance the plane travels in attaining takeoff speed, in terms of the acceleration, a, and the takeoff speed, v_{to}. Equation 2–12, which relates distance to velocity, allows us to do this.

SOLUTION

Part (a)

1. Solve Equation 2–12 for Δx. To find Δx_{A}, set $v_0 = 0$ and $v = v_{\text{to}}$:

$$\Delta x = \frac{v^2 - v_0^2}{2a} \qquad \Delta x_{\text{A}} = \frac{v_{\text{to}}^2}{2a}$$

Part (b)

2. To find Δx_{B}, simply change v_{to} to $2v_{\text{to}}$ in part (a):

$$\Delta x_{\text{B}} = \frac{(2v_{\text{to}})^2}{2a} = \frac{4v_{\text{to}}^2}{2a} = 4\Delta x_{\text{A}}$$

Part (c)

3. Substitute numerical values into the result found in part (a):

$$\Delta x_{\text{A}} = \frac{v_{\text{to}}^2}{2a} = \frac{(95.0\ \text{m/s})^2}{2(2.20\ \text{m/s}^2)} = 2050\ \text{m}$$

INSIGHT

For purposes of comparison, the shortest runway at JFK International Airport is 04R/22L, which has a length of 2560 m.

This Example illustrates the fact that there are many advantages to obtaining symbolic results before substituting numerical values. In this case, we find that the takeoff distance is proportional to v^2; hence, we conclude immediately that doubling v results in a fourfold increase of Δx.

PRACTICE PROBLEM

Find the minimum acceleration needed for a takeoff speed of $v_{\text{to}} = (95.0\ \text{m/s})/2 = 47.5\ \text{m/s}$ on a runway of length $\Delta x = (2050\ \text{m})/4 = 513\ \text{m}$. [**Answer:** $a = v_{\text{to}}^2/2\Delta x = 2.20\ \text{m/s}^2$]

Some related homework problems: Problem 55, Problem 57

Finally, all of our constant-acceleration equations of motion are collected for easy reference in Table 2–4.

TABLE 2–4 Constant-Acceleration Equations of Motion

Variables Related	Equation	Number
velocity, time, acceleration	$v = v_0 + at$	2–7
initial, final, and average velocity	$v_{\text{av}} = \frac{1}{2}(v_0 + v)$	2–9
position, time, velocity	$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10
position, time, acceleration	$x = x_0 + v_0t + \frac{1}{2}at^2$	2–11
velocity, position, acceleration	$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x$	2–12

2–6 Applications of the Equations of Motion

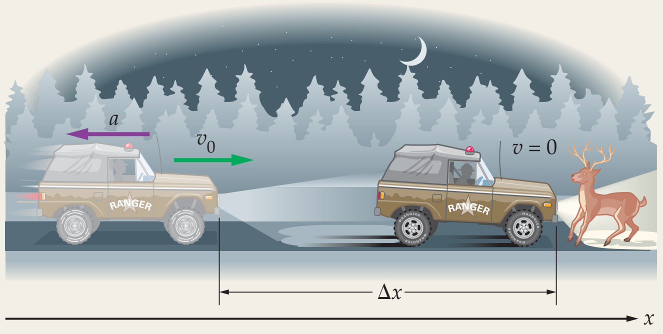
We devote this section to a variety of examples further illustrating the use of the constant-acceleration equations of motion. In our first Example, we consider the distance and time needed to brake a vehicle to a complete stop.

EXAMPLE 2–8 HIT THE BRAKES!

A park ranger driving on a back country road suddenly sees a deer "frozen" in the headlights. The ranger, who is driving at 11.4 m/s, immediately applies the brakes and slows with an acceleration of $3.80\ \mathrm{m/s^2}$. **(a)** If the deer is 20.0 m from the ranger's vehicle when the brakes are applied, how close does the ranger come to hitting the deer? **(b)** How much time is needed for the ranger's vehicle to stop?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion. With this choice it follows that $v_0 = +11.4\ \mathrm{m/s}$. In addition, the fact that the ranger's vehicle is slowing down means its acceleration points in the *opposite* direction to that of the velocity [see Figure 2–10 (b) and (c)]. Therefore, the vehicle's acceleration is $a = -3.80\ \mathrm{m/s^2}$. Finally, when the vehicle comes to rest its velocity is zero, $v = 0$.



STRATEGY

The acceleration is constant, so we can use the equations listed in Table 2–4. In part (a) we want to find a distance when we know the velocity and acceleration, so we use a rearranged version of Equation 2–12. In part (b) we want to find a time when we know the velocity and acceleration, so we use a rearranged version of Equation 2–7.

SOLUTION

Part (a)

1. Solve Equation 2–12 for Δx:

$$\Delta x = \frac{v^2 - v_0^2}{2a}$$

2. Set $v = 0$, and substitute numerical values:

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{(11.4\ \mathrm{m/s})^2}{2(-3.80\ \mathrm{m/s^2})} = 17.1\ \mathrm{m}$$

3. Subtract Δx from 20.0 m to find the distance between the stopped vehicle and the deer:

$$20.0\ \mathrm{m} - 17.1\ \mathrm{m} = 2.9\ \mathrm{m}$$

Part (b)

4. Set $v = 0$ in Equation 2–7 and solve for t:

$$v = v_0 + at = 0$$

$$t = -\frac{v_0}{a} = -\frac{11.4\ \mathrm{m/s}}{(-3.80\ \mathrm{m/s^2})} = 3.00\ \mathrm{s}$$

INSIGHT

Note the difference in the way t and Δx depend on the initial speed. If the initial speed is doubled, for example, the time needed to stop also doubles, but the distance needed to stop increases by a factor of four. This is one reason why speed on the highway has such a great influence on safety.

PRACTICE PROBLEM

Show that using $t = 3.00\ \mathrm{s}$ in Equation 2–11 results in the same distance needed to stop.
[**Answer:** $x = x_0 + v_0t + \frac{1}{2}at^2 = 0 + (11.4\ \mathrm{m/s})(3.00\ \mathrm{s}) + \frac{1}{2}(-3.80\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = 17.1\ \mathrm{m}$, as expected.]

Some related homework problems: Problem 57, Problem 58

In Example 2–8, we calculated the distance necessary for a vehicle to come to a complete stop. But how does v vary with distance as the vehicle slows down? The next Conceptual Checkpoint deals with this topic.

CONCEPTUAL CHECKPOINT 2–4 STOPPING DISTANCE

REAL-WORLD PHYSICS

The stopping distance of a car

The ranger in Example 2–8 brakes for 17.1 m. After braking for only half that distance, $\frac{1}{2}(17.1\text{ m}) = 8.55$ m, is the ranger's speed **(a)** equal to $\frac{1}{2}v_0$, **(b)** greater than $\frac{1}{2}v_0$, or **(c)** less than $\frac{1}{2}v_0$?

REASONING AND DISCUSSION

As pointed out in the Insight for Example 2–8, the fact that the stopping distance, Δx, depends on $v_0{}^2$ means that this distance increases by a factor of four when the speed is doubled. For example, the stopping distance with an initial speed of v_0 is four times the stopping distance when the initial speed is $v_0/2$.

To apply this observation to the ranger, suppose that the stopping distance with an initial speed of v_0 is Δx. It follows that the stopping distance for an initial speed of $v_0/2$ is $\Delta x/4$. This means that as the ranger slows from v_0 to 0, it takes a distance $\Delta x/4$ to slow from $v_0/2$ to 0, and the remaining distance, $3\Delta x/4$, to slow from v_0 to $v_0/2$. Thus, at the halfway point the ranger has not yet slowed to half of the initial velocity—the speed at this point is greater than $v_0/2$.

ANSWER

(b) The ranger's speed is greater than $\frac{1}{2}v_0$.

Clearly, v does not decrease uniformly with distance. A plot showing v as a function of x for Example 2–8 is shown in **Figure 2–15**. As we can see from the graph, v changes more in the second half of the braking distance than in the first half.

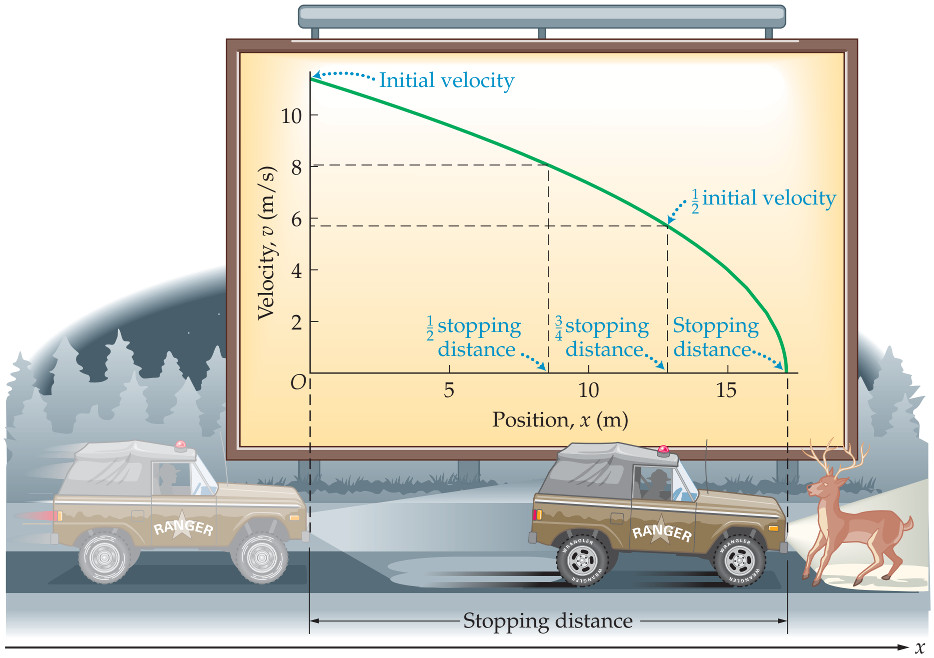


FIGURE 2–15 Velocity as a function of position for the ranger in Example 2–8

The ranger's vehicle in Example 2–8 comes to rest with constant acceleration, which means that its velocity decreases uniformly with time. The velocity *does not* decrease uniformly with distance, however. In particular, note how rapidly the velocity decreases in the final one-quarter of the stopping distance.

We close this section with a familiar, everyday example: a police car accelerating to overtake a speeder. This is the first time that we use two equations of motion for two different objects to solve a problem—but it won't be the last. Problems of this type are often more interesting than problems involving only a single object, and they relate to many types of situations in everyday life.

PROBLEM-SOLVING NOTE

Strategize

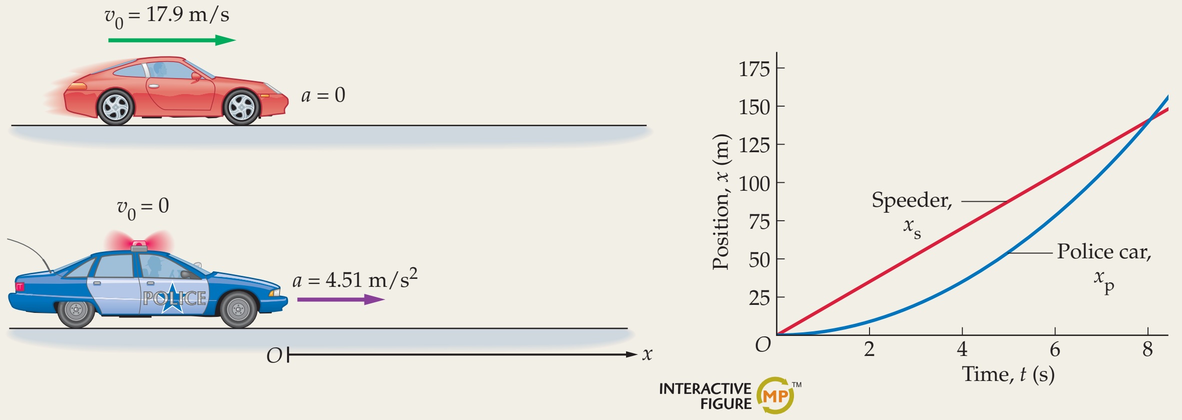
Before attempting to solve a problem, it is a good idea to have some sort of plan, or "strategy," for how to proceed. It may be as simple as saying, "The problem asks me to relate velocity and time, therefore I will use Equation 2–7." In other cases the strategy is a bit more involved. Producing effective strategies is one of the most challenging—and creative—aspects of problem solving.

EXAMPLE 2–9 CATCHING A SPEEDER

A speeder doing 40.0 mi/h (about 17.9 m/s) in a 25 mi/h zone approaches a parked police car. The instant the speeder passes the police car, the police begin their pursuit. If the speeder maintains a constant velocity, and the police car accelerates with a constant acceleration of 4.51 m/s^2, **(a)** how long does it take for the police car to catch the speeder, **(b)** how far have the two cars traveled in this time, and **(c)** what is the velocity of the police car when it catches the speeder?

PICTURE THE PROBLEM

Our sketch shows the two cars at the moment the speeder passes the resting police car. At this instant, which we take to be $t = 0$, both the speeder and the police car are at the origin, $x = 0$. In addition, we choose the positive x direction to be the direction of motion; therefore, the speeder's initial velocity is given by $v_s = +17.9\ \text{m/s}$, and the police car's initial velocity is zero. The speeder's acceleration is zero, but the police car has an acceleration given by $a = +4.51\ \text{m/s}^2$. Finally, our plot shows the linear x-versus-t plot for the speeder, and the parabolic x-versus-t plot for the police car.



STRATEGY

To solve this problem, first write down a position-versus-time equation for the police car, x_p, and a separate equation for the speeder, x_s. Next, we find the time it takes the police car to catch the speeder by setting $x_p = x_s$ and solving the resulting equation for t. Once the catch time is determined, it is straightforward to calculate the distance traveled and the velocity of the police car.

SOLUTION

Part (a)

1. Write equations of motion for the two vehicles. For the police car, $v_0 = 0$ and $a = 4.51\ \text{m/s}^2$. For the speeder, $v_0 = 17.9\ \text{m/s} = v_s$ and $a = 0$:

$$x_p = \tfrac{1}{2}at^2$$
$$x_s = v_s t$$

2. Set $x_p = x_s$ and solve for the time:

$$\tfrac{1}{2}at^2 = v_s t \text{ or } (\tfrac{1}{2}at - v_s)t = 0$$
$$\text{two solutions: } t = 0 \quad \text{or} \quad t = \frac{2v_s}{a}$$

3. Clearly, $t = 0$ corresponds to the initial conditions, because both vehicles started at $x = 0$ at that time. The time of interest is obtained by substituting numerical values into the other solution:

$$t = \frac{2v_s}{a} = \frac{2(17.9\ \text{m/s})}{4.51\ \text{m/s}^2} = 7.94\ \text{s}$$

Part (b)

4. Substitute $t = 7.94$ s into the equations of motion for x_p and x_s. Note that $x_p = x_s$, as expected:

$$x_p = \tfrac{1}{2}at^2 = \tfrac{1}{2}(4.51\ \text{m/s}^2)(7.94\ \text{s})^2 = 142$$
$$x_s = v_s t = (17.9\ \text{m/s})(7.94\ \text{s}) = 142\ \text{m}$$

Part (c)

5. To find the velocity of the police car use Equation 2–7, which relates velocity to time:

$$v_p = v_0 + at = 0 + (4.51\ \text{m/s}^2)(7.94\ \text{s}) = 35.8\ \text{m/s}$$

INSIGHT

When the police car catches up with the speeder, its velocity is 35.8 m/s, which is exactly twice the velocity of the speeder. A coincidence? Not at all. When the police car catches the speeder, both have traveled the same distance (142 m) in the same time (7.94 s), therefore they have the same average velocity. Of course, the average velocity of the speeder is simply v_s. The average velocity of the police car is $\frac{1}{2}(v_0 + v)$, since its acceleration is constant, and thus $\frac{1}{2}(v_0 + v) = v_s$. Since $v_0 = 0$ for the police car, it follows that $v = 2v_s$. Notice that this result is independent of the acceleration of the police car, as we show in the following Practice Problem.

PRACTICE PROBLEM

Repeat this Example for the case where the acceleration of the police car is $a = 3.25\ \text{m/s}^2$. [**Answer:** **(a)** $t = 11.0$ s, **(b)** $x_p = x_s = 197$ m, **(c)** $v_p = 35.8$ m/s]

Some related homework problems: Problem 54, Problem 65

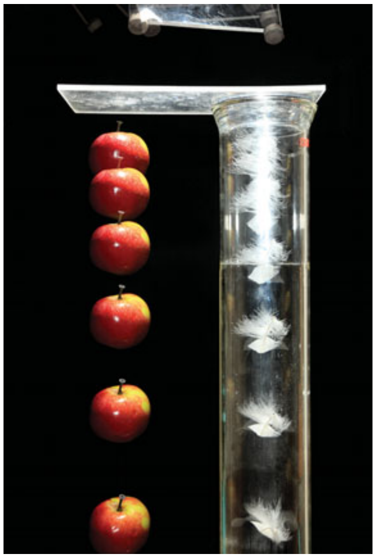
2–7 Freely Falling Objects

The most famous example of motion with constant acceleration is **free fall**—the motion of an object falling freely under the influence of gravity. It was Galileo (1564–1642) who first showed, with his own experiments, that falling bodies move with constant acceleration. His conclusions were based on experiments done by rolling balls down inclines of various steepness. By using an incline, Galileo was able to reduce the acceleration of the balls, thus producing motion slow enough to be timed with the rather crude instruments available.

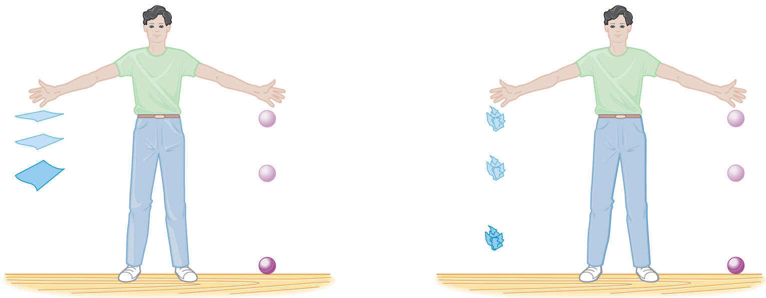
Galileo also pointed out that objects of different weight fall with the *same* constant acceleration—provided air resistance is small enough to be ignored. Whether he dropped objects from the Leaning Tower of Pisa to demonstrate this fact, as legend has it, will probably never be known for certain, but we do know that he performed extensive experiments to support his claim.

Today it is easy to verify Galileo's assertion by dropping objects in a vacuum chamber, where the effects of air resistance are essentially removed. In a standard classroom demonstration, a feather and a coin are dropped in a vacuum, and both fall at the same rate. In 1971, a novel version of this experiment was carried out on the Moon by astronaut David Scott. In the near-perfect vacuum on the Moon's surface he dropped a feather and a hammer and showed a worldwide television audience that they fell to the ground in the same time.

To illustrate the effect of air resistance in everyday terms, consider dropping a sheet of paper and a rubber ball (**Figure 2–16**). The paper drifts slowly to the ground, taking much longer to fall than the ball. Now, wad the sheet of paper into a tight ball and repeat the experiment. This time the ball of paper and the rubber ball reach the ground in nearly the same time. What was different in the two experiments? Clearly, when the sheet of paper was wadded into a ball, the effect of air resistance on it was greatly reduced, so that both objects fell almost as they would in a vacuum.



▲ In the absence of air resistance, all bodies fall with the same acceleration, regardless of their mass.



(a) Dropping a sheet of paper and a rubber ball

(b) Dropping a wadded-up sheet of paper and a rubber ball

◀ **FIGURE 2–16 Free fall and air resistance**

Before considering a few examples, let's first discuss exactly what is meant by "free fall." To begin, the word *free* in free fall means free from any effects other than gravity. For example, in free fall we assume that an object's motion is not influenced by any form of friction or air resistance.

- Free fall is the motion of an object subject *only* to the influence of gravity.

Though free fall is an idealization—which does not apply to many real-world situations—it is still a useful approximation in many other cases. In the following examples we assume that the motion may be considered as free fall.

Next, it should be realized that the word *fall* in free fall does not mean the object is necessarily moving downward. By free fall, we mean *any* motion under the influence of gravity alone. If you drop a ball, it is in free fall. If you throw a ball upward or downward, it is in free fall as soon as it leaves your hand.

- An object is in free fall as soon as it is released, whether it is dropped from rest, thrown downward, or thrown upward.

Finally, the acceleration produced by gravity on the Earth's surface (sometimes called the gravitational strength) is denoted with the symbol g. As a shorthand

▲ Whether she is on the way up, at the peak of her flight, or on the way down, this girl is in free fall, accelerating downward with the acceleration of gravity. Only when she is in contact with the blanket does her acceleration change.

TABLE 2–5 Values of g at Different Locations on Earth $(\mathrm{m/s^2})$

Location	Latitude	g
North Pole	90° N	9.832
Oslo, Norway	60° N	9.819
Hong Kong	30° N	9.793
Quito, Ecuador	0°	9.780

name, we will frequently refer to g simply as "the acceleration due to gravity." In fact, as we shall see in Chapter 12, the value of g varies according to one's location on the surface of the Earth, as well as one's altitude above it. Table 2–5 shows how g varies with latitude.

In all the calculations that follow in this book, we shall use $g = 9.81\ \mathrm{m/s^2}$ for the acceleration due to gravity. Note, in particular, that g always stands for $+9.81\ \mathrm{m/s^2}$, never $-9.81\ \mathrm{m/s^2}$. For example, if we choose a coordinate system with the positive direction upward, the acceleration in free fall is $a = -g$. If the positive direction is downward, then free-fall acceleration is $a = g$.

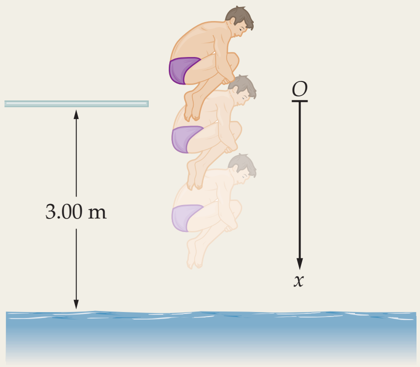
With these comments, we are ready to explore a variety of free-fall examples.

EXAMPLE 2–10 DO THE CANNONBALL!

A person steps off the end of a 3.00-m-high diving board and drops to the water below. **(a)** How long does it take for the person to reach the water? **(b)** What is the person's speed on entering the water?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the height of the diving board, and we let the positive direction be downward. With these choices, $x_0 = 0$, $a = g$, and the water is at $x = 3.00$ m. Of course, $v_0 = 0$ since the person simply steps off the board.



STRATEGY

We can neglect air resistance in this case and model the motion as free fall. This means we can assume a constant acceleration equal to g and use the equations of motion in Table 2–4. For part (a) we want to find the time of fall when we know the distance and acceleration, so we use Equation 2–11. For part (b) we can relate velocity to time by using Equation 2–7, or we can relate velocity to position by using Equation 2–12. We will implement both approaches and show that the results are the same.

SOLUTION

Part (a)

1. Write Equation 2–11 with $x_0 = 0$, $v_0 = 0$, and $a = g$:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}gt^2 = \tfrac{1}{2}gt^2$$

2. Solve for the time, t, and set $x = 3.00$ m:

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(3.00\ \mathrm{m})}{9.81\ \mathrm{m/s^2}}} = 0.782\ \mathrm{s}$$

Part (b)

3. Use the time found in part (a) in Equation 2–7:

$$v = v_0 + gt = 0 + (9.81\ \mathrm{m/s^2})(0.782\ \mathrm{s}) = 7.67\ \mathrm{m/s}$$

4. We can also find the velocity without knowing the time by using Equation 2–12 with $\Delta x = 3.00$ m:

$$v^2 = v_0{}^2 + 2a\Delta x = 0 + 2g\Delta x$$

$$v = \sqrt{2g\Delta x} = \sqrt{2(9.81\ \mathrm{m/s^2})(3.00\ \mathrm{m})} = 7.67\ \mathrm{m/s}$$

INSIGHT

Let's put these results in more common, everyday units. If you step off a diving board 9.84 ft (3.00 m) above the water, you enter the water with a speed of 17.2 mi/h (7.67 m/s).

PRACTICE PROBLEM

What is your speed on entering the water if you step off a 10.0-m diving tower? [**Answer:** $v = \sqrt{2(9.81\ \mathrm{m/s^2})(10.0\ \mathrm{m})} = 14.0\ \mathrm{m/s} = 31.4\ \mathrm{mi/h}$]

Some related homework problems: Problem 71, Problem 83

The special case of free fall from rest occurs so frequently, and in so many different contexts, that it deserves special attention. If we take x_0 to be zero, and positive to be downward, then position as a function of time is $x = x_0 + v_0 t + \frac{1}{2}gt^2 = 0 + 0 + \frac{1}{2}gt^2$, or

$$x = \tfrac{1}{2}gt^2 \qquad 2\text{–}13$$

Similarly, velocity as a function of time is

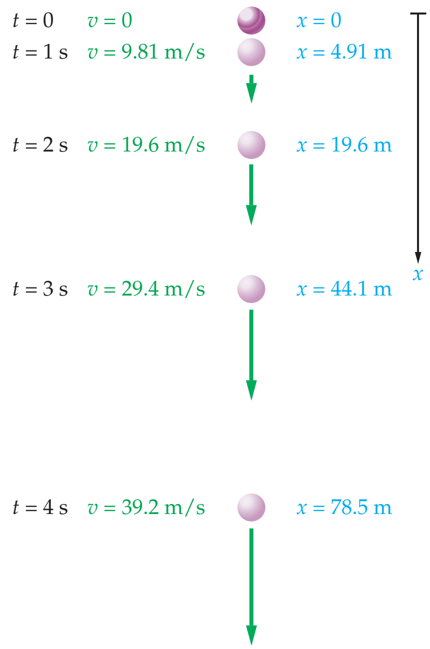
$$v = gt \qquad 2\text{–}14$$

and velocity as a function of position is

$$v = \sqrt{2gx} \tag{2–15}$$

The behavior of these functions is illustrated in **Figure 2–17**. Note that position increases with time squared, whereas velocity increases linearly with time.

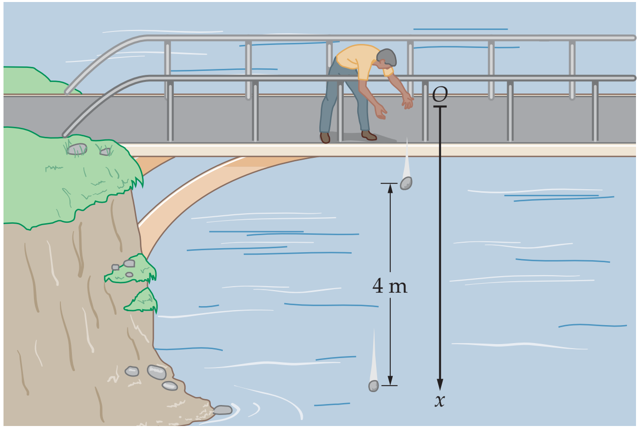
Next we consider two objects dropped from rest, one after the other, and discuss how their separation varies with time.



▲ **FIGURE 2–17 Free fall from rest**
Position and velocity are shown as functions of time. It is apparent that velocity depends linearly on t, whereas position depends on t^2.

CONCEPTUAL CHECKPOINT 2–5
FREE-FALL SEPARATION

You drop a rock from a bridge to the river below. When the rock has fallen 4 m, you drop a second rock. As the rocks continue their free fall, does their separation **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION
It might seem that since both rocks are in free fall, their separation remains the same. This is not so. The rock that has a head start always has a greater velocity than the later one; thus it covers a greater distance in any interval of time. As a result, the separation between the rocks increases.

ANSWER
(a) The separation between the rocks increases.

PROBLEM-SOLVING NOTE
Check Your Solution

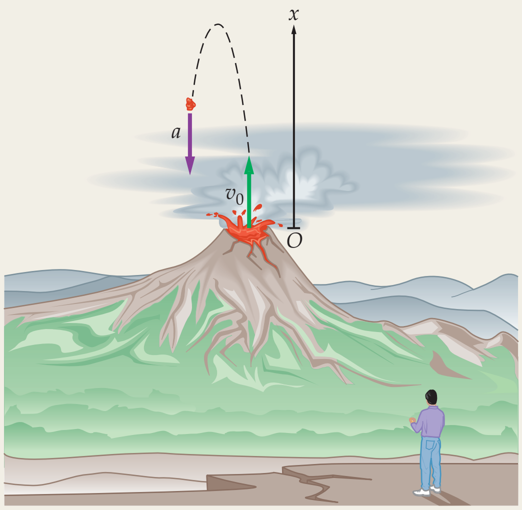
Once you have a solution to a problem, check to see whether it makes sense. First, make sure the units are correct; m/s for speed, m/s^2 for acceleration, and so on. Second, check the numerical value of your answer. If you are solving for the speed of a diver dropping from a 3.0-m diving board and you get an unreasonable value like 200 m/s ($\approx$450 mi/h), chances are good that you've made a mistake.

An erupting volcano shooting out fountains of lava is an impressive sight. In the next Example we show how a simple timing experiment can determine the initial velocity of the erupting lava.

EXAMPLE 2–11 BOMBS AWAY: CALCULATING THE SPEED OF A LAVA BOMB

REAL-WORLD PHYSICS

A volcano shoots out blobs of molten lava, called lava bombs, from its summit. A geologist observing the eruption uses a stopwatch to time the flight of a particular lava bomb that is projected straight upward. If the time for it to rise and fall back to its launch height is 4.75 s, and its acceleration is 9.81 m/s^2 downward, what is its initial speed?



PICTURE THE PROBLEM
Our sketch shows a coordinate system with upward as the positive x direction. For clarity, we offset the upward and downward trajectories slightly. In addition, we choose $t = 0$ to be the time at which the lava bomb is launched. With these choices it follows that $x_0 = 0$ and the acceleration is $a = -g = -9.81\ m/s^2$. The initial speed to be determined is v_0.

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STRATEGY

Once again, we can neglect air resistance and model the motion of the lava bomb as free fall—this time with an initial upward velocity. We know that the lava bomb starts at $x = 0$ at the time $t = 0$ and returns to $x = 0$ at the time $t = 4.75$ s. This means that we know the bomb's position, time, and acceleration $(a = -g)$, from which we would like to determine the initial velocity. A reasonable approach is to use Equation 2–11 and solve it for the one unknown it contains, v_0.

SOLUTION

1. Write out $x = x_0 + v_0t + \frac{1}{2}at^2$ with $x_0 = 0$ and $a = -g$. Factor out a time, t, from the two remaining terms:

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 = v_0t - \tfrac{1}{2}gt^2 = \left(v_0 - \tfrac{1}{2}gt\right)t$$

2. Set x equal to zero, since this is the position of the lava bomb at $t = 0$ and $t = 4.75$ s:

$x = \left(v_0 - \frac{1}{2}gt\right)t = 0$ two solutions:

(i) $t = 0$

(ii) $v_0 - \frac{1}{2}gt = 0$

3. The first solution is simply the initial condition; that is, $x = 0$ at $t = 0$. Solve the second solution for the initial speed:

$$v_0 - \tfrac{1}{2}gt = 0 \quad \text{or} \quad v_0 = \tfrac{1}{2}gt$$

4. Substitute numerical values for g and the time the lava bomb lands:

$$v_0 = \tfrac{1}{2}gt = \tfrac{1}{2}(9.81\ \text{m/s}^2)(4.75\ \text{s}) = 23.3\ \text{m/s}$$

INSIGHT

A geologist can determine a lava bomb's initial speed by simply observing its flight time. Knowing the lava bomb's initial speed can help geologists determine how severe a volcanic eruption will be, and how dangerous it might be to people in the surrounding area.

PRACTICE PROBLEM

A second lava bomb is projected straight upward with an initial speed of 25 m/s. How long is it in the air? [**Answer:** $t = 5.1$ s]

Some related homework problems: Problem 73, Problem 86

▲ In the absence of air resistance, these lava bombs from the Kilauea volcano on the big island of Hawaii would strike the water with the same speed they had when they were blasted into the air.

What is the speed of a lava bomb when it returns to Earth; that is, when it returns to the same level from which it was launched? Physical intuition might suggest that, in the absence of air resistance, it should be the same as the initial speed. To show that this hypothesis is indeed correct, write out Equation 2–7 for this case:

$$v = v_0 - gt$$

Substituting numerical values, we find

$$v = v_0 - gt = 23.3\ \text{m/s} - (9.81\ \text{m/s}^2)(4.75\ \text{s}) = -23.3\ \text{m/s}$$

Thus, the velocity of the lava when it lands is just the negative of the velocity it had when launched upward. Or put another way, when the lava lands, it has the same speed as when it was launched; it's just traveling in the opposite direction.

It is instructive to verify this result symbolically. Recall from Example 2–11 that $v_0 = \frac{1}{2}gt$, where t is the time the bomb lands. Substituting this result into Equation 2–7 we find

$$v = \tfrac{1}{2}gt - gt = -\tfrac{1}{2}gt = -v_0$$

The advantage of the symbolic solution lies in showing that the result is not a fluke—no matter what the initial velocity, no matter what the acceleration, the bomb lands with the velocity $-v_0$.

These results hint at a symmetry relating the motion on the way up to the motion on the way down. To make this symmetry more apparent, we first solve for the time when the lava bomb lands. Using the result $v_0 = \frac{1}{2}gt$ from Example 2–11, we find

$$t = \frac{2v_0}{g} \qquad \textit{(time of landing)}$$

Next, we find the time when the velocity of the lava is zero, which is at its highest point. Setting $v = 0$ in Equation 2–7, we have $v = v_0 - gt = 0$, or

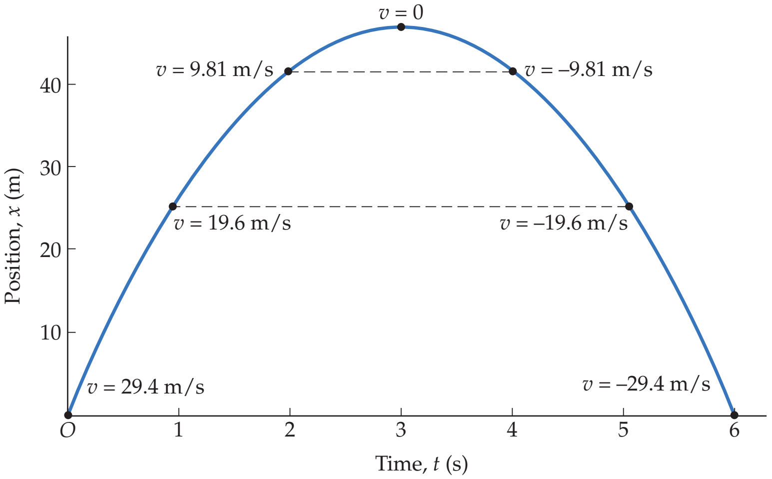
$$t = \frac{v_0}{g} \qquad \textit{(time when } v = 0\textit{)}$$

Note that this is exactly half the time required for the lava to make the round trip. Thus, the velocity of the lava is zero and the height of the lava is greatest exactly halfway between launch and landing.

This symmetry is illustrated in **Figure 2–18**. In this case we consider a lava bomb that is in the air for 6.00 s, moving without air resistance. Note that at $t = 3.00$ s the lava is at its highest point and its velocity is zero. At times equally spaced before and after $t = 3.00$ s, the lava is at the same height, has the same speed, but is moving in opposite directions. As a result of this symmetry, a movie of the lava bomb's flight would look the same whether run forward or in reverse.

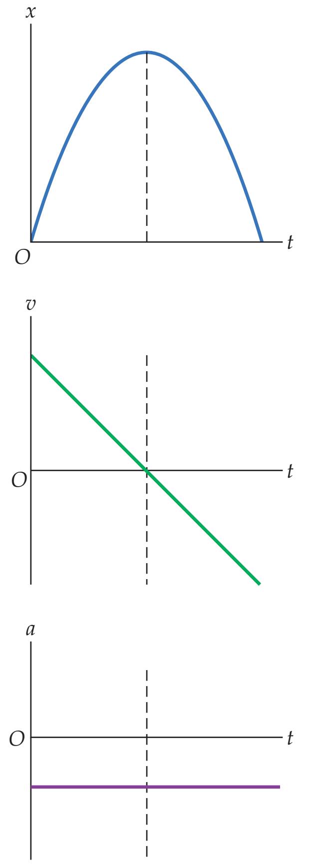
Figure 2–19 shows the time dependence of position, velocity, and acceleration for an object in free fall without air resistance after being thrown upward. As soon as the object is released, it begins to accelerate downward—as indicated by the negative slope of the velocity-versus-time plot—though it isn't necessarily moving downward. For example, if you throw a ball *upward* it begins to accelerate *downward* the moment it leaves your hand. It continues moving upward, however, until its speed diminishes to zero. Since gravity is causing the downward acceleration, and gravity doesn't turn off just because the ball's velocity goes through zero, the ball continues to accelerate downward even when it is momentarily at rest.

Similarly, in the next Example we consider a sandbag that falls from an ascending hot-air balloon. This means that before the bag is in free fall it was moving upward—just like a ball thrown upward. And just like the ball, the sandbag continues moving upward for a brief time before momentarily stopping and then moving downward.



▲ **FIGURE 2–18 Position and velocity of a lava bomb**

This lava bomb is in the air for 6 seconds. Note the symmetry about the midpoint of the bomb's flight.



▲ **FIGURE 2–19 Position, velocity, and acceleration of a lava bomb as functions of time**

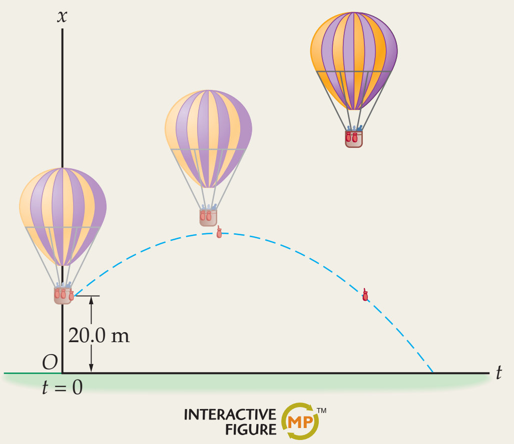
The fact that x versus t is curved indicates an acceleration; the downward curvature shows that the acceleration is negative. This is also clear from v versus t, which has a negative slope. The constant slope of the straight line in the v-versus-t plot indicates a constant acceleration, as shown in the a-versus-t plot.

EXAMPLE 2–12 LOOK OUT BELOW! A SANDBAG IN FREE FALL

A hot-air balloon is rising straight upward with a constant speed of 6.5 m/s. When the basket of the balloon is 20.0 m above the ground, a bag of sand tied to the basket comes loose. **(a)** How long is the bag of sand in the air before it hits the ground? **(b)** What is the greatest height of the bag of sand during its fall to the ground?

PICTURE THE PROBLEM

We choose the origin to be at ground level and positive to be upward. This means that, for the bag, we have $x_0 = 20.0$ m, $v_0 = 6.5$ m/s, and $a = -g$. Our sketch also shows snapshots of the balloon and bag of sand at three different times, starting at $t = 0$ when the bag comes loose. Note that the bag is moving upward with the balloon at the time it comes loose. It therefore continues to move upward for a short time after it separates from the basket, exactly as if it had been thrown upward.



STRATEGY

The effects of air resistance on the sandbag can be ignored. As a result, we can use the equations in Table 2–4 with a constant acceleration $a = -g$.

In part (a) we want to relate position and time—knowing the initial position and initial velocity—so we use Equation 2–11. To find the time the bag hits the ground, we set $x = 0$ and solve for t.

For part (b) we have no expression that gives the maximum height of a particle—so we will have to come up with something on our own. We can start with the fact that $v = 0$ at the greatest height, since it is there the bag momentarily stops as it changes direction. Therefore, we can find the time t when $v = 0$ by using Equation 2–7, and then substitute t into Equation 2–11 to find x_{max}.

SOLUTION

Part (a)

1. Apply Equation 2–11 to the bag of sand, where x_0 and v_0 have the values given. Set $x = 0$:

$$x = x_0 + v_0 t - \tfrac{1}{2}gt^2 = 0$$

2. Note that we have a quadratic equation for t in the form $v_f = v_i$ where $A = -\frac{1}{2}g$, $B = v_0$, and $C = x_0$. Solve this equation for t. The positive solution, 2.78 s, applies to this problem: (Quadratic equations and their solutions are discussed in Appendix A. In general, one can expect two solutions to a quadratic equation.)

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(-\frac{1}{2}g\right)(x_0)}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-(6.5\ \text{m/s}) \pm \sqrt{(6.5\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2)(20.0\ \text{m})}}{(-9.81\ \text{m/s}^2)}$$

$$= \frac{-(6.5\ \text{m/s}) \pm 20.8\ \text{m/s}}{(-9.81\ \text{m/s}^2)} = 2.78\ \text{s}, -1.46\ \text{s}$$

Part (b)

3. Apply Equation 2–7 to the bag of sand, then find the time when the velocity equals zero:

$$v = v_0 + at = v_0 - gt$$

$$v_0 - gt = 0 \quad \text{or} \quad t = \frac{v_0}{g} = \frac{6.5\ \text{m/s}}{9.81\ \text{m/s}^2} = 0.66\ \text{s}$$

4. Use $t = 0.66$ s in Equation 2–11 to find the maximum height:

$$x_{max} = 20.0\ \text{m} + (6.5\ \text{m/s})\ (0.66\ \text{s}) - \tfrac{1}{2}(9.81\ \text{m/s}^2)\ (0.66\ \text{s})^2$$
$$= 22\ \text{m}$$

INSIGHT

The positive solution to the quadratic equation is certainly the one that applies here, but the negative solution is not completely without meaning. What physical meaning might it have? Well, if the balloon had been *descending* with a speed of 6.5 m/s, instead of rising, then the time for the bag to reach the ground would have been 1.46 s. Try it! Let $v_0 = -6.5$ m/s and repeat the calculation given in part (a).

PRACTICE PROBLEM

What is the velocity of the bag of sand just before it hits the ground? [**Answer:** $v = v_0 - gt = (6.5\ \text{m/s}) - (9.81\ \text{m/s}^2)(2.78\ \text{s}) = -20.8\ \text{m/s}$; the minus sign indicates the bag is moving downward.]

Some related homework problems: Problem 90, Problem 107

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In this chapter we have made extensive use of the sign conventions for one-dimensional vectors—positive for one direction, negative for the opposite direction—as introduced in Chapter 1. See, for example, the positive and negative velocities in Figure 2–18.

We have been careful to check the dimensional consistency of our equations in this chapter. For example, the discussion following Equation 2–11 shows that all the terms in that equation have the dimensions of length.

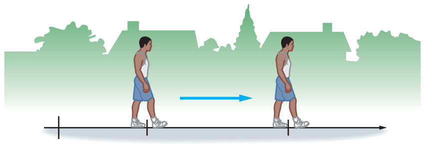
LOOKING AHEAD

The distinctions developed in this chapter between velocity and acceleration will play a key role in our understanding of Newton's laws of motion in Chapters 5 and 6, and everywhere else that Newton's laws are used throughout the text.

The equations developed for motion with constant acceleration in this chapter (Equations 2–7, 2–10, 2–11, and 2–12) will be used again with slightly different symbols when we study rotational motion in Chapter 10. See, in particular, Equations 10–8, 10–9, 10–10, and 10–11.

CHAPTER SUMMARY

2–1 POSITION, DISTANCE, AND DISPLACEMENT



Distance

Total length of travel, from beginning to end. The distance is always positive.

Displacement

Displacement, Δx, is the change in position:

$$\Delta x = x_f - x_i \qquad 2\text{–}1$$

When calculating displacement, it is important to remember that it is always *final* position minus *initial* position—never the other way. Displacement can be positive, negative, or zero.

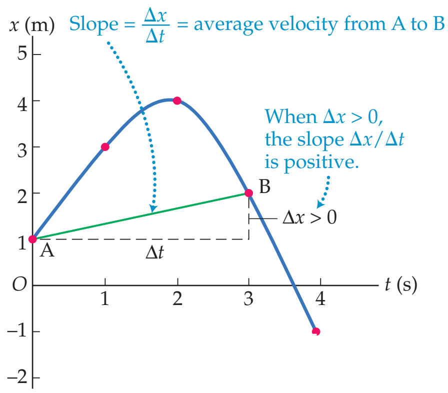
Positive and Negative Displacement

The *sign* of the displacement indicates the *direction* of motion. For example, suppose we choose the positive direction to be to the right. Then $\Delta x > 0$ means motion to the right, and $\Delta x < 0$ means motion to the left.

Units

The SI unit of distance and displacement is the meter, m.

2–2 AVERAGE SPEED AND VELOCITY



Average Speed

Average speed is *distance* divided by elapsed time:

$$\text{average speed} = \text{distance/time} \qquad 2\text{–}2$$

Average speed is never negative.

Average Velocity

Average velocity, v_{av}, is *displacement* divided by time:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad 2\text{–}3$$

Average velocity is positive if motion is in the positive direction, and negative if motion is in the negative direction.

Graphical Interpretation of Velocity

In an x-versus-t plot, the average velocity is the slope of a line connecting two points.

Units

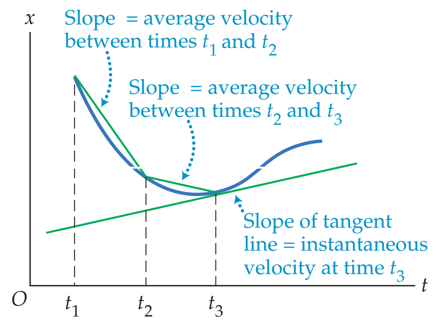
The SI unit of speed and velocity is meters per second, m/s.

2–3 INSTANTANEOUS VELOCITY

The velocity at an instant of time is the limit of the average velocity over shorter and shorter time intervals:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \text{2–4}$$

Instantaneous velocity can be positive, negative, or zero, with the sign indicating the direction of motion.



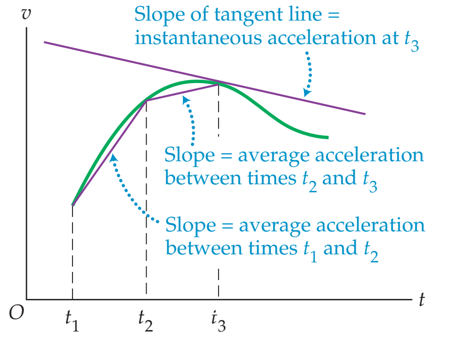
Constant Velocity
When velocity is constant, the instantaneous velocity is equal to the average velocity.

Graphical Interpretation
In an x-versus-t plot, the instantaneous velocity at a given time is equal to the slope of the tangent line at that time.

2–4 ACCELERATION

Average Acceleration
Average acceleration is the change in velocity divided by the change in time:

$$a_{\mathrm{av}} = \frac{\Delta v}{\Delta t} = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} \qquad \text{2–5}$$



Average acceleration is positive if $v_{\mathrm{f}} > v_{\mathrm{i}}$, is negative if $v_{\mathrm{f}} < v_{\mathrm{i}}$, and is zero if $v_{\mathrm{f}} = v_{\mathrm{i}}$.

Instantaneous Acceleration
Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad \text{2–6}$$

Instantaneous acceleration can be positive, negative, or zero, depending on whether the velocity is becoming more positive, more negative, or is staying the same. Knowing the sign of the acceleration *does not* tell you whether an object is speeding up or slowing down, and it *does not* give the direction of motion.

Constant Acceleration
When acceleration is constant, the instantaneous acceleration is equal to the average acceleration.

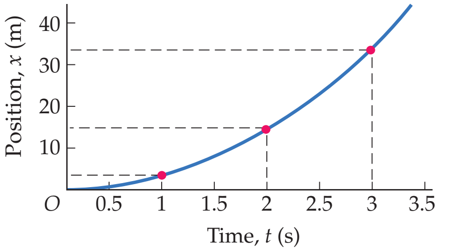
Deceleration
An object whose speed is decreasing is said to be decelerating. Deceleration occurs whenever the velocity and acceleration have opposite signs.

Graphical Interpretation
In a v-versus-t plot, the instantaneous acceleration is equal to the slope of the tangent line at a given time.

Units
The SI unit of acceleration is meters per second per second, or $\mathrm{m/s^2}$.

2–5 MOTION WITH CONSTANT ACCELERATION

Several different "equations of motion" describe particles moving with constant acceleration. Each equation relates a different set of variables:



Velocity as a Function of Time

$$v = v_0 + at \qquad \text{2–7}$$

Initial, Final, and Average Velocity

$$v_{\mathrm{av}} = \tfrac{1}{2}(v_0 + v) \qquad \text{2–9}$$

Position as a Function of Time and Velocity

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad \text{2–10}$$

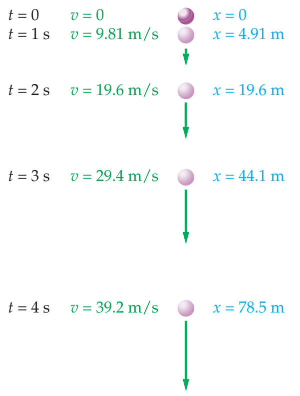
Position as a Function of Time and Acceleration

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \text{2–11}$$

Velocity as a Function of Position

$$v^2 = v_0{}^2 + 2a(x - x_0) = v_0{}^2 + 2a\Delta x \qquad \text{2–12}$$

2–7 FREELY FALLING OBJECTS



Objects in free fall move under the influence of gravity alone. An object is in free fall as soon as it is released, whether it is thrown upward, thrown downward, or released from rest.

Acceleration Due to Gravity
The acceleration due to gravity on the Earth's surface varies slightly from place to place. In this book we shall define the acceleration of gravity to have the following magnitude:

$$g = 9.81 \text{ m/s}^2$$

Note that g is always a positive quantity. If we choose the positive direction of our coordinate system to be downward (in the direction of the acceleration of gravity), it follows that the acceleration of an object in free fall is $a = +g$. On the other hand, if we choose our positive direction to be upward, the acceleration of a freely falling object is in the negative direction; hence $a = -g$.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Relate velocity to time.	In motion with uniform acceleration a, the velocity changes with time as $v = v_0 + at$ (Equation 2–7).	Examples 2–5, 2–8, 2–9, 2–10, 2–11, 2–12
Relate velocity to position.	If an object with an initial velocity v_0 accelerates with a uniform acceleration a for a distance Δx, the final velocity, v, is given by $v^2 = v_0{}^2 + 2a\Delta x$ (Equation 2–12).	Examples 2–7, 2–8, 2–10
Relate position to time.	The position of an object moving with constant acceleration a varies with time as follows: $x = x_0 + \frac{1}{2}(v_0 + v)t$ (Equation 2–10) or equivalently $x = x_0 + v_0t + \frac{1}{2}at^2$ (Equation 2–11).	Examples 2–5, 2–6, 2–9, 2–10, 2–11, 2–12

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)
(The effects of air resistance are to be ignored in this chapter.)

1. You and your dog go for a walk to a nearby park. On the way, your dog takes many short side trips to chase squirrels, examine fire hydrants, and so on. When you arrive at the park, do you and your dog have the same displacement? Have you traveled the same distance? Explain.
2. Does an odometer in a car measure distance or displacement? Explain.
3. Can you drive your car in such a way that the distance it covers is **(a)** greater than, **(b)** equal to, or **(c)** less than the magnitude of its displacement? In each case, give an example if your answer is yes, explain why not if your answer is no.
4. An astronaut orbits Earth in the space shuttle. In one complete orbit, is the magnitude of the displacement the same as the distance traveled? Explain.
5. After a tennis match the players dash to the net to congratulate one another. If they both run with a speed of 3 m/s, are their velocities equal? Explain.
6. Does a speedometer measure speed or velocity? Explain.
7. Is it possible for a car to circle a race track with constant velocity? Can it do so with constant speed? Explain.
8. Friends tell you that on a recent trip their average velocity was +20 m/s. Is it possible that their instantaneous velocity was negative at any time during the trip? Explain.
9. For what kind of motion are the instantaneous and average velocities equal?
10. If the position of an object is zero, does its speed have to be zero? Explain.
11. Assume that the brakes in your car create a constant deceleration, regardless of how fast you are going. If you double your driving speed, how does this affect **(a)** the time required to come to a stop, and **(b)** the distance needed to stop?
12. The velocity of an object is zero at a given instant of time. **(a)** Is it possible for the object's acceleration to be zero at this time? Explain. **(b)** Is it possible for the object's acceleration to be nonzero at this time? Explain.
13. If the velocity of an object is nonzero, can its acceleration be zero? Give an example if your answer is yes, explain why not if your answer is no.
14. Is it possible for an object to have zero average velocity over a given interval of time, yet still be accelerating during the interval? Give an example if your answer is yes, explain why not if your answer is no.
15. A batter hits a pop fly straight up. **(a)** Is the acceleration of the ball on the way up different from its acceleration on the way down? **(b)** Is the acceleration of the ball at the top of its flight different from its acceleration just before it lands?

16. A person on a trampoline bounces straight upward with an initial speed of 4.5 m/s. What is the person's speed when she returns to her initial height?

17. After winning a baseball game, one player drops a glove, while another tosses a glove into the air. How do the accelerations of the two gloves compare?

18. A volcano shoots a lava bomb straight upward. Does the displacement of the lava bomb depend on **(a)** your choice of origin for your coordinate system, or **(b)** your choice of a positive direction? Explain in each case.

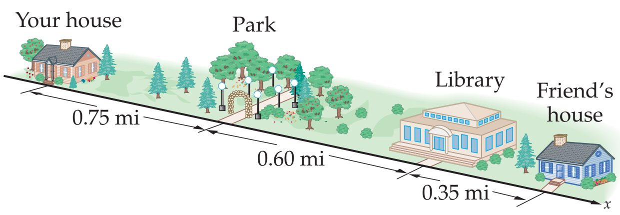
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(The effects of air resistance are to be ignored in this chapter.)

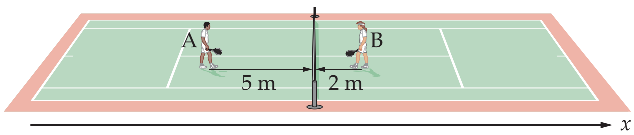
SECTION 2–1 POSITION, DISTANCE, AND DISPLACEMENT

1. • Referring to **Figure 2–20**, you walk from your home to the library, then to the park. **(a)** What is the distance traveled? **(b)** What is your displacement?



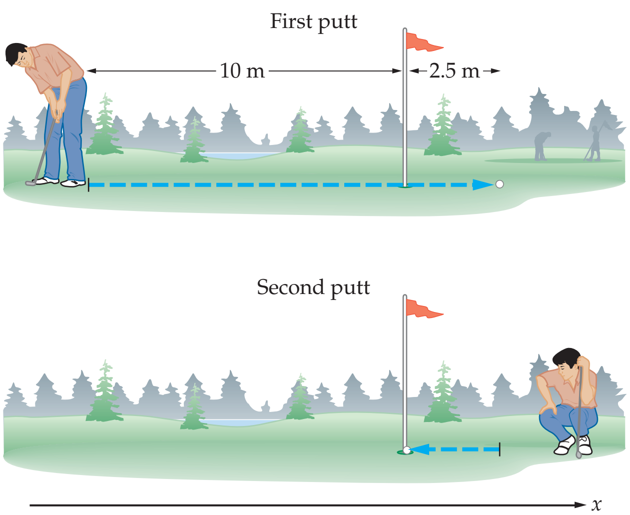
▲ **FIGURE 2–20** Problems 1 and 4

2. • The two tennis players shown in **Figure 2–21** walk to the net to congratulate one another. **(a)** Find the distance traveled and the displacement of player A. **(b)** Repeat for player B.



▲ **FIGURE 2–21** Problem 2

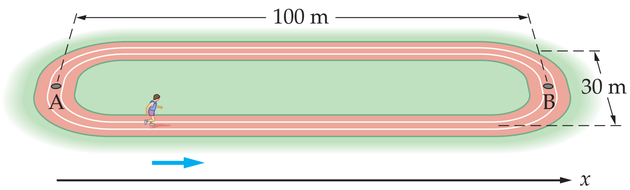
3. • The golfer in **Figure 2–22** sinks the ball in two putts, as shown. What are **(a)** the distance traveled by the ball, and **(b)** the displacement of the ball?



▲ **FIGURE 2–22** Problem 3

4. • In Figure 2–20, you walk from the park to your friend's house, then back to your house. What is your **(a)** distance traveled, and **(b)** displacement?

5. • A jogger runs on the track shown in **Figure 2–23**. Neglecting the curvature of the corners, **(a)** what is the distance traveled and the displacement in running from point A to point B? **(b)** Find the distance and displacement for a complete circuit of the track.



▲ **FIGURE 2–23** Problem 5

6. •• **IP** A child rides a pony on a circular track whose radius is 4.5 m. **(a)** Find the distance traveled and the displacement after the child has gone halfway around the track. **(b)** Does the distance traveled increase, decrease, or stay the same when the child completes one circuit of the track? Explain. **(c)** Does the displacement increase, decrease, or stay the same when the child completes one circuit of the track? Explain. **(d)** Find the distance and displacement after a complete circuit of the track.

SECTION 2–2 AVERAGE SPEED AND VELOCITY

7. • **CE Predict/Explain** You drive your car in a straight line at 15 m/s for 10 kilometers, then at 25 m/s for another 10 kilometers. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:
   - **I.** More time is spent at 15 m/s than at 25 m/s.
   - **II.** The average of 15 m/s and 25 m/s is 20 m/s.
   - **III.** Less time is spent at 15 m/s than at 25 m/s.

8. • **CE Predict/Explain** You drive your car in a straight line at 15 m/s for 10 minutes, then at 25 m/s for another 10 minutes. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:
   - **I.** More time is required to drive at 15 m/s than at 25 m/s.
   - **II.** Less distance is covered at 25 m/s than at 15 m/s.
   - **III.** Equal time is spent at 15 m/s and 25 m/s.

9. • Joseph DeLoach of the United States set an Olympic record in 1988 for the 200-meter dash with a time of 19.75 seconds. What was his average speed? Give your answer in meters per second and miles per hour.

10. • In 1992 Zhuang Yong of China set a women's Olympic record in the 100-meter freestyle swim with a time of 54.64 seconds. What was her average speed in m/s and mi/h?

11. • **BIO** Kangaroos have been clocked at speeds of 65 km/h. **(a)** How far can a kangaroo hop in 3.2 minutes at this speed? **(b)** How long will it take a kangaroo to hop 0.25 km at this speed?

12. • **Rubber Ducks** A severe storm on January 10, 1992, caused a cargo ship near the Aleutian Islands to spill 29,000 rubber ducks and other bath toys into the ocean. Ten months later hundreds of rubber ducks began to appear along the shoreline near Sitka, Alaska, roughly 1600 miles away. What was the approximate average speed of the ocean current that carried the ducks to shore in **(a)** m/s and **(b)** mi/h? (Rubber ducks from the same spill began to appear on the coast of Maine in July 2003.)

13. • Radio waves travel at the speed of light, approximately 186,000 miles per second. How long does it take for a radio message to travel from Earth to the Moon and back? (See the inside back cover for the necessary data.)

14. • It was a dark and stormy night, when suddenly you saw a flash of lightning. Three-and-a-half seconds later you heard the thunder. Given that the speed of sound in air is about 340 m/s, how far away was the lightning bolt?

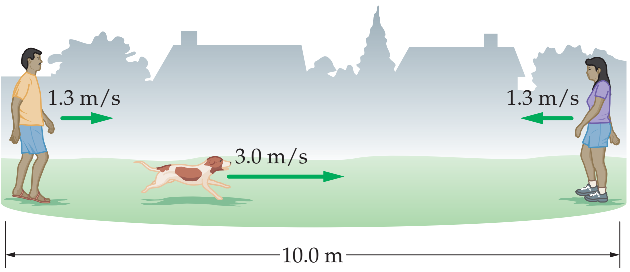
15. • **BIO Nerve Impulses** The human nervous system can propagate nerve impulses at about 10^2 m/s. Estimate the time it takes for a nerve impulse generated when your finger touches a hot object to travel to your brain.

16. • Estimate how fast your hair grows in miles per hour.

17. •• A finch rides on the back of a Galapagos tortoise, which walks at the stately pace of 0.060 m/s. After 1.2 minutes the finch tires of the tortoise's slow pace, and takes flight in the same direction for another 1.2 minutes at 12 m/s. What was the average speed of the finch for this 2.4-minute interval?

18. •• You jog at 9.5 km/h for 8.0 km, then you jump into a car and drive an additional 16 km. With what average speed must you drive your car if your average speed for the entire 24 km is to be 22 km/h?

19. •• A dog runs back and forth between its two owners, who are walking toward one another (**Figure 2–24**). The dog starts running when the owners are 10.0 m apart. If the dog runs with a speed of 3.0 m/s, and the owners each walk with a speed of 1.3 m/s, how far has the dog traveled when the owners meet?



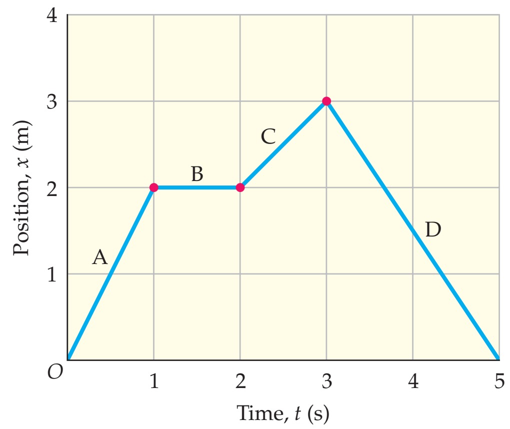
▲ **FIGURE 2–24** Problem 19

20. •• **IP** You drive in a straight line at 20.0 m/s for 10.0 minutes, then at 30.0 m/s for another 10.0 minutes. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

21. •• In heavy rush-hour traffic you drive in a straight line at 12 m/s for 1.5 minutes, then you have to stop for 3.5 minutes, and finally you drive at 15 m/s for another 2.5 minutes. **(a)** Plot a position-versus-time graph for this motion. Your plot should extend from $t = 0$ to $t = 7.5$ minutes. **(b)** Use your plot from part (a) to calculate the average velocity between $t = 0$ and $t = 7.5$ minutes.

22. •• **IP** You drive in a straight line at 20.0 m/s for 10.0 miles, then at 30.0 m/s for another 10.0 miles. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

23. •• **IP** An expectant father paces back and forth, producing the position-versus-time graph shown in **Figure 2–25**. Without performing a calculation, indicate whether the father's velocity is positive, negative, or zero on each of the following segments of the graph: **(a)** A, **(b)** B, **(c)** C, and **(d)** D. Calculate the numerical value of the father's velocity for the segments **(e)** A, **(f)** B, **(g)** C, and **(h)** D, and show that your results verify your answers to parts (a)–(d).

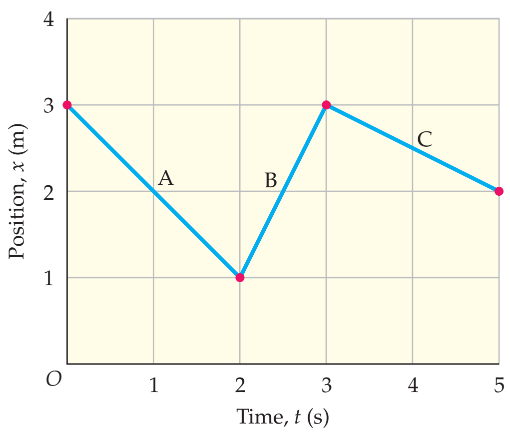


▲ **FIGURE 2–25** Problem 23

24. •• The position of a particle as a function of time is given by $x = (-5\text{ m/s})t + (3\text{ m/s}^2)t^2$. **(a)** Plot x versus t for $t = 0$ to $t = 2$ s. **(b)** Find the average velocity of the particle from $t = 0$ to $t = 1$ s. **(c)** Find the average speed from $t = 0$ to $t = 1$ s.

25. •• The position of a particle as a function of time is given by $x = (6\text{ m/s})t + (-2\text{ m/s}^2)t^2$. **(a)** Plot x versus t for $t = 0$ to $t = 2$ s. **(b)** Find the average velocity of the particle from $t = 0$ to $t = 1$ s. **(c)** Find the average speed from $t = 0$ to $t = 1$ s.

26. •• **IP** A tennis player moves back and forth along the baseline while waiting for her opponent to serve, producing the position-versus-time graph shown in **Figure 2–26.** **(a)** Without performing a calculation, indicate on which of the segments of the graph, A, B, or C, the player has the greatest speed. Calculate the player's speed for **(b)** segment A, **(c)** segment B, and **(d)** segment C, and show that your results verify your answers to part (a).

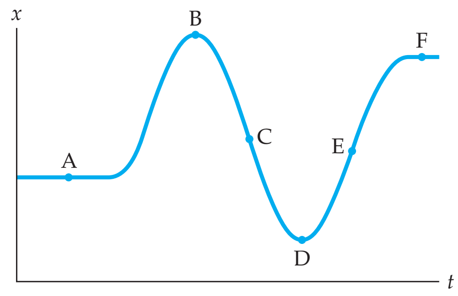


▲ **FIGURE 2–26** Problem 26

27. ••• On your wedding day you leave for the church 30.0 minutes before the ceremony is to begin, which should be plenty of time since the church is only 10.0 miles away. On the way, however, you have to make an unanticipated stop for construction work on the road. As a result, your average speed for the first 15 minutes is only 5.0 mi/h. What average speed do you need for the rest of the trip to get you to the church on time?

SECTION 2–3 INSTANTANEOUS VELOCITY

28. • **CE** The position-versus-time plot of a boat positioning itself next to a dock is shown in **Figure 2–27**. Rank the six points indicated in the plot in order of increasing value of the velocity v, starting with the most negative. Indicate a tie with an equal sign.



▲ **FIGURE 2–27** Problem 28

29. •• The position of a particle as a function of time is given by $x = (2.0\ \mathrm{m/s})t + (-3.0\ \mathrm{m/s^3})t^3$. **(a)** Plot x versus t for time from $t = 0$ to $t = 1.0$ s. **(b)** Find the average velocity of the particle from $t = 0.35$ s to $t = 0.45$ s. **(c)** Find the average velocity from $t = 0.39$ s to $t = 0.41$ s. **(d)** Do you expect the instantaneous velocity at $t = 0.40$ s to be closer to 0.54 m/s, 0.56 m/s, or 0.58 m/s? Explain.

30. •• The position of a particle as a function of time is given by $x = (-2.00\ \mathrm{m/s})t + (3.00\ \mathrm{m/s^3})t^3$. **(a)** Plot x versus t for time from $t = 0$ to $t = 1.00$ s. **(b)** Find the average velocity of the particle from $t = 0.150$ s to $t = 0.250$ s. **(c)** Find the average velocity from $t = 0.190$ s to $t = 0.210$ s. **(d)** Do you expect the instantaneous velocity at $t = 0.200$ s to be closer to -1.62 m/s, or -1.66 m/s? Explain.

SECTION 2–4 ACCELERATION

31. • **CE Predict/Explain** Two bows shoot identical arrows with the same launch speed. To accomplish this, the string in bow 1 must be pulled back farther when shooting its arrow than the string in bow 2. **(a)** Is the acceleration of the arrow shot by bow 1 greater than, less than, or equal to the acceleration of the arrow shot by bow 2? **(b)** Choose the *best explanation* from among the following:
   I. The arrow in bow 2 accelerates for a greater time.
   II. Both arrows start from rest.
   III. The arrow in bow 1 accelerates for a greater time.

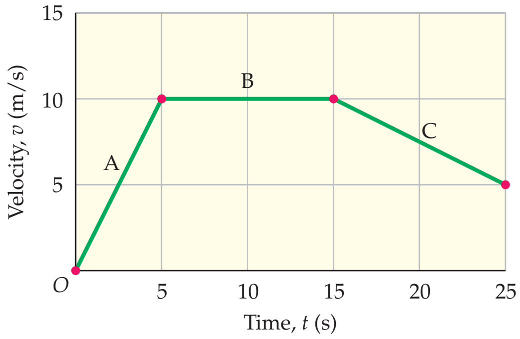
32. • A 747 airliner reaches its takeoff speed of 173 mi/h in 35.2 s. What is the magnitude of its average acceleration?

33. • At the starting gun, a runner accelerates at 1.9 $\mathrm{m/s^2}$ for 5.2 s. The runner's acceleration is zero for the rest of the race. What is the speed of the runner **(a)** at $t = 2.0$ s, and **(b)** at the end of the race?

34. • A jet makes a landing traveling due east with a speed of 115 m/s. If the jet comes to rest in 13.0 s, what are the magnitude and direction of its average acceleration?

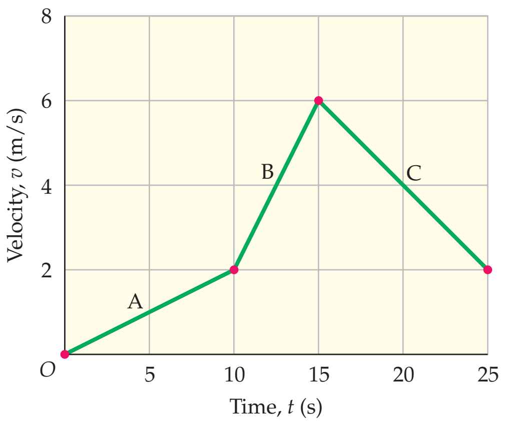
35. • A car is traveling due north at 18.1 m/s. Find the velocity of the car after 7.50 s if its acceleration is **(a)** 1.30 $\mathrm{m/s^2}$ due north, or **(b)** 1.15 $\mathrm{m/s^2}$ due south.

36. •• A motorcycle moves according to the velocity-versus-time graph shown in **Figure 2–28**. Find the average acceleration of the motorcycle during each of the following segments of the motion: **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 2–28** Problem 36

37. •• A person on horseback moves according to the velocity-versus-time graph shown in **Figure 2–29.** Find the displacement of the person for each of the following segments of the motion: **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 2–29** Problem 37

38. •• Running with an initial velocity of +11 m/s, a horse has an average acceleration of $-1.81\ \mathrm{m/s^2}$. How long does it take for the horse to decrease its velocity to +6.5 m/s?

39. •• **IP** Assume that the brakes in your car create a constant deceleration of 4.2 $\mathrm{m/s^2}$ regardless of how fast you are driving. If you double your driving speed from 16 m/s to 32 m/s, **(a)** does the time required to come to a stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping times for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

40. •• **IP** In the previous problem, **(a)** does the distance needed to stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping distances for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

41. •• As a train accelerates away from a station, it reaches a speed of 4.7 m/s in 5.0 s. If the train's acceleration remains constant, what is its speed after an additional 6.0 s has elapsed?

42. •• A particle has an acceleration of +6.24 $\mathrm{m/s^2}$ for 0.300 s. At the end of this time the particle's velocity is +9.31 m/s. What was the particle's initial velocity?

SECTION 2–5 MOTION WITH CONSTANT ACCELERATION

43. • Landing with a speed of 81.9 m/s, and traveling due south, a jet comes to rest in 949 m. Assuming the jet slows with constant acceleration, find the magnitude and direction of its acceleration.

44. • When you see a traffic light turn red, you apply the brakes until you come to a stop. If your initial speed was 12 m/s, and

you were heading due west, what was your average velocity during braking? Assume constant deceleration.

45. **CE** •• A ball is released at the point $x = 2$ m on an inclined plane with a nonzero initial velocity. After being released, the ball moves with constant acceleration. The acceleration and initial velocity of the ball are described by one of the following four cases: case 1, $a > 0$, $v_0 > 0$; case 2, $a > 0$, $v_0 < 0$; case 3, $a < 0$, $v_0 > 0$; case 4, $a < 0$, $v_0 < 0$. **(a)** In which of these cases will the ball definitely pass $x = 0$ at some later time? **(b)** In which of these cases is more information needed to determine whether the ball will cross $x = 0$? **(c)** In which of these cases will the ball come to rest momentarily at some time during its motion?

46. •• Suppose the car in Problem 44 comes to rest in 35 m. How much time does this take?

47. •• Starting from rest, a boat increases its speed to 4.12 m/s with constant acceleration. **(a)** What is the boat's average speed? **(b)** If it takes the boat 4.77 s to reach this speed, how far has it traveled?

48. •• **IP BIO** A cheetah can accelerate from rest to 25.0 m/s in 6.22 s. Assuming constant acceleration, **(a)** how far has the cheetah run in this time? **(b)** After sprinting for just 3.11 s, is the cheetah's speed 12.5 m/s, more than 12.5 m/s, or less than 12.5 m/s? Explain. **(c)** What is the cheetah's average speed for the first 3.11 s of its sprint? For the second 3.11 s of its sprint? **(d)** Calculate the distance covered by the cheetah in the first 3.11 s and the second 3.11 s.

SECTION 2–6 APPLICATIONS OF THE EQUATIONS OF MOTION

49. • A child slides down a hill on a toboggan with an acceleration of 1.8 m/s^2. If she starts at rest, how far has she traveled in **(a)** 1.0 s, **(b)** 2.0 s, and **(c)** 3.0 s?

50. • **The Detonator** On a ride called the Detonator at Worlds of Fun in Kansas City, passengers accelerate straight downward from rest to 45 mi/h in 2.2 seconds. What is the average acceleration of the passengers on this ride?

The Detonator (Problem 50)

51. • **Air Bags** Air bags are designed to deploy in 10 ms. Estimate the acceleration of the front surface of the bag as it expands. Express your answer in terms of the acceleration of gravity g.

52. • **Jules Verne** In his novel *From the Earth to the Moon* (1866), Jules Verne describes a spaceship that is blasted out of a cannon, called the *Columbiad*, with a speed of 12,000 yards/s. The *Columbiad* is 900 ft long, but part of it is packed with powder, so the spaceship accelerates over a distance of only 700 ft. Estimate the acceleration experienced by the occupants of the spaceship during launch. Give your answer in m/s^2. (Verne realized that the "travelers would . . . encounter a violent recoil," but he probably didn't know that people generally lose consciousness if they experience accelerations greater than about $7g \sim 70$ m/s^2.)

53. •• **BIO** **Bacterial Motion** Approximately 0.1% of the bacteria in an adult human's intestines are *Escherichia coli*. These bacteria have been observed to move with speeds up to 15 μm/s and maximum accelerations of 166 μm/s^2. Suppose an *E. coli* bacterium in your intestines starts at rest and accelerates at 156 μm/s^2. How much **(a)** time and **(b)** distance are required for the bacterium to reach a speed of 12 μm/s?

54. •• Two cars drive on a straight highway. At time $t = 0$, car 1 passes mile marker 0 traveling due east with a speed of 20.0 m/s. At the same time, car 2 is 1.0 km east of mile marker 0 traveling at 30.0 m/s due west. Car 1 is speeding up with an acceleration of magnitude 2.5 m/s^2, and car 2 is slowing down with an acceleration of magnitude 3.2 m/s^2. **(a)** Write x-versus-t equations of motion for both cars, taking east as the positive direction. **(b)** At what time do the cars pass next to one another?

55. •• **A Meteorite Strikes** On October 9, 1992, a 27-pound meteorite struck a car in Peekskill, NY, leaving a dent 22 cm deep in the trunk. If the meteorite struck the car with a speed of 130 m/s, what was the magnitude of its deceleration, assuming it to be constant?

56. •• A rocket blasts off and moves straight upward from the launch pad with constant acceleration. After 3.0 s the rocket is at a height of 77 m. **(a)** What are the magnitude and direction of the rocket's acceleration? **(b)** What is its speed at this time?

57. •• **IP** You are driving through town at 12.0 m/s when suddenly a ball rolls out in front of you. You apply the brakes and begin decelerating at 3.5 m/s^2. **(a)** How far do you travel before stopping? **(b)** When you have traveled only half the distance in part (a), is your speed 6.0 m/s, greater than 6.0 m/s, or less than 6.0 m/s? Support your answer with a calculation.

58. •• **IP** You are driving through town at 16 m/s when suddenly a car backs out of a driveway in front of you. You apply the brakes and begin decelerating at 3.2 m/s^2. **(a)** How much time does it take to stop? **(b)** After braking half the time found in part (a), is your speed 8.0 m/s, greater than 8.0 m/s, or less than 8.0 m/s? Support your answer with a calculation. **(c)** If the car backing out was initially 55 m in front of you, what is the maximum reaction time you can have before hitting the brakes and still avoid hitting the car?

59. •• **IP BIO** **A Tongue's Acceleration** When a chameleon captures an insect, its tongue can extend 16 cm in 0.10 s. **(a)** Find the magnitude of the tongue's acceleration, assuming it to be constant. **(b)** In the first 0.050 s, does the tongue extend 8.0 cm, more than 8.0 cm, or less than 8.0 cm? Support your conclusion with a calculation.


It's not polite to reach! (Problem 59)

60. •• **IP** Coasting due west on your bicycle at 8.4 m/s, you encounter a sandy patch of road 7.2 m across. When you leave the sandy patch your speed has been reduced by 2.0 m/s to 6.4 m/s. **(a)** Assuming the sand causes a constant acceleration, what was the bicycle's acceleration in the sandy patch? Give both magnitude and direction. **(b)** How long did it take to cross the sandy patch? **(c)** Suppose you enter the sandy patch with a speed of only 5.4 m/s. Is your final speed in this case 3.4 m/s, more than 3.4 m/s, or less than 3.4 m/s? Explain.

61. •• **BIO Surviving a Large Deceleration** On July 13, 1977, while on a test drive at Britain's Silverstone racetrack, the throttle on David Purley's car stuck wide open. The resulting crash subjected Purley to the greatest "g-force" ever survived by a human—he decelerated from 173 km/h to zero in a distance of only about 0.66 m. Calculate the magnitude of the acceleration experienced by Purley (assuming it to be constant), and express your answer in units of the acceleration of gravity, $g = 9.81\ \mathrm{m/s^2}$.

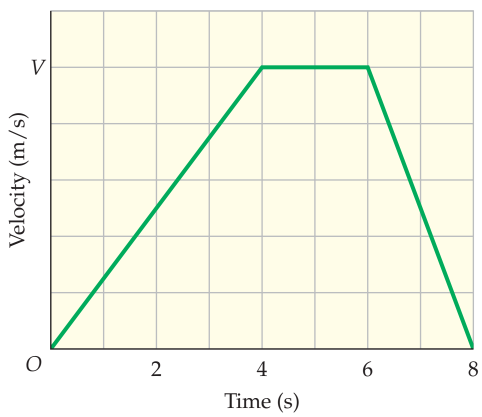
62. •• **IP** A boat is cruising in a straight line at a constant speed of 2.6 m/s when it is shifted into neutral. After coasting 12 m the engine is engaged again, and the boat resumes cruising at the reduced constant speed of 1.6 m/s. Assuming constant acceleration while coasting, **(a)** how long did it take for the boat to coast the 12 m? **(b)** What was the boat's acceleration while it was coasting? **(c)** When the boat had coasted for 6.0 m, was its speed 2.1 m/s, more than 2.1 m/s, or less than 2.1 m/s? Explain.

63. •• A model rocket rises with constant acceleration to a height of 3.2 m, at which point its speed is 26.0 m/s. **(a)** How much time does it take for the rocket to reach this height? **(b)** What was the magnitude of the rocket's acceleration? **(c)** Find the height and speed of the rocket 0.10 s after launch.

64. •• The infamous chicken is dashing toward home plate with a speed of 5.8 m/s when he decides to hit the dirt. The chicken slides for 1.1 s, just reaching the plate as he stops (safe, of course). **(a)** What are the magnitude and direction of the chicken's acceleration? **(b)** How far did the chicken slide?

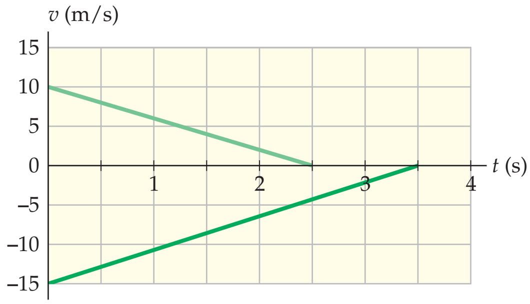
65. •• A bicyclist is finishing his repair of a flat tire when a friend rides by with a constant speed of 3.5 m/s. Two seconds later the bicyclist hops on his bike and accelerates at 2.4 m/s^2 until he catches his friend. **(a)** How much time does it take until he catches his friend? **(b)** How far has he traveled in this time? **(c)** What is his speed when he catches up?

66. •• A car in stop-and-go traffic starts at rest, moves forward 13 m in 8.0 s, then comes to rest again. The velocity-versus-time plot for this car is given in **Figure 2–30.** What distance does the car cover in **(a)** the first 4.0 seconds of its motion and **(b)** the last 2.0 seconds of its motion? **(c)** What is the constant speed V that characterizes the middle portion of its motion?



▲ **FIGURE 2–30** Problem 66

67. ••• A car and a truck are heading directly toward one another on a straight and narrow street, but they avoid a head-on collision by simultaneously applying their brakes at $t = 0$. The resulting velocity-versus-time graphs are shown in **Figure 2–31.** What is the separation between the car and the truck when they have come to rest, given that at $t = 0$ the car is at $x = 15$ m and the truck is at $x = -35$ m? (Note that this information determines which line in the graph corresponds to which vehicle.)



▲ **FIGURE 2–31** Problem 67

68. ••• In a physics lab, students measure the time it takes a small cart to slide a distance of 1.00 m on a smooth track inclined at an angle θ above the horizontal. Their results are given in the following table.

θ	**10.0°**	**20.0°**	**30.0°**
time, s	1.08	0.770	0.640

(a) Find the magnitude of the cart's acceleration for each angle. **(b)** Show that your results for part (a) are in close agreement with the formula, $a = g \sin \theta$. (We will derive this formula in Chapter 5.)

SECTION 2–7 FREELY FALLING OBJECTS

(Problems 71 and 72)

69. • **CE** At the edge of a roof you throw ball 1 upward with an initial speed v_0; a moment later you throw ball 2 downward with the same initial speed. The balls land at the same time. Which of the following statements is true for the instant just before the balls hit the ground? **A.** The speed of ball 1 is greater than the speed of ball 2; **B.** The speed of ball 1 is equal to the speed of ball 2; **C.** The speed of ball 1 is less than the speed of ball 2.

70. • Legend has it that Isaac Newton was hit on the head by a falling apple, thus triggering his thoughts on gravity. Assuming the story to be true, estimate the speed of the apple when it struck Newton.

71. • The cartoon shows a car in free fall. Is the statement made in the cartoon accurate? Justify your answer.

72. • Referring to the cartoon in Problem 71, how long would it take for the car to go from 0 to 30 mi/h?

73. • **Jordan's Jump** Michael Jordan's vertical leap is reported to be 48 inches. What is his takeoff speed? Give your answer in meters per second.

74. • **BIO** Gulls are often observed dropping clams and other shellfish from a height to the rocks below, as a means of opening the shells. If a seagull drops a shell from rest at a height of 14 m, how fast is the shell moving when it hits the rocks?

75. • A volcano launches a lava bomb straight upward with an initial speed of 28 m/s. Taking upward to be the positive direction, find the speed and direction of motion of the lava bomb **(a)** 2.0 seconds and **(b)** 3.0 seconds after it is launched.

76. • **An Extraterrestrial Volcano** The first active volcano observed outside the Earth was discovered in 1979 on Io, one of the moons of Jupiter. The volcano was observed to be ejecting material to a height of about 2.00×10^5 m. Given that the acceleration of gravity on Io is 1.80 m/s^2, find the initial velocity of the ejected material.

77. • **BIO Measure Your Reaction Time** Here's something you *can* try at home—an experiment to measure your reaction time. Have a friend hold a ruler by one end, letting the other end hang down vertically. At the lower end, hold your thumb and index finger on either side of the ruler, ready to grip it. Have your friend release the ruler without warning. Catch it as quickly as you can. If you catch the ruler 5.2 cm from the lower end, what is your reaction time?

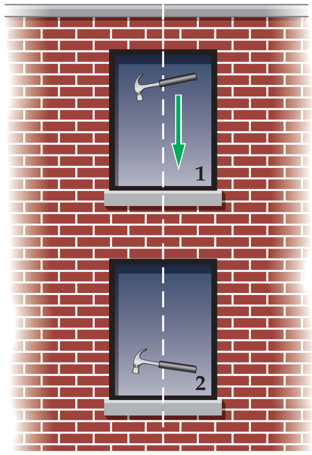
How fast are your reactions? (Problem 77)

78. •• **CE Predict/Explain** A carpenter on the roof of a building accidentally drops her hammer. As the hammer falls it passes two windows of equal height, as shown in **Figure 2–32.** **(a)** Is the *increase* in speed of the hammer as it drops past window 1 greater than, less than, or equal to the *increase* in speed as it drops past window 2? **(b)** Choose the *best explanation* from among the following:

I. The greater speed at window 2 results in a greater increase in speed.

II. Constant acceleration means the hammer speeds up the same amount for each window.

III. The hammer spends more time dropping past window 1.



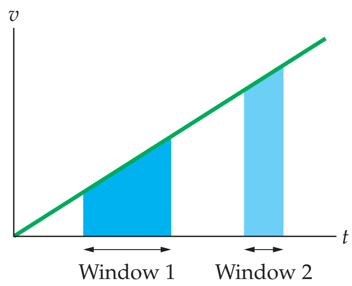
▲ **FIGURE 2–32** Problem 78

79. •• **CE Predict/Explain** **Figure 2–33** shows a v-versus-t plot for the hammer dropped by the carpenter in Problem 78. Notice that the times when the hammer passes the two windows are indicated by shaded areas. **(a)** Is the area of the shaded region corresponding to window 1 greater than, less than, or equal to the area of the shaded region corresponding to window 2? **(b)** Choose the *best explanation* from among the following:

I. The shaded area for window 2 is higher than the shaded area for window 1.

II. The windows are equally tall.

III. The shaded area for window 1 is wider than the shaded area for window 2.



▲ **FIGURE 2–33** Problem 79

80. •• **CE** A ball is thrown straight upward with an initial speed v_0. When it reaches the top of its flight at height h, a second ball is thrown straight upward with the same initial speed. Do the balls cross paths at height $\frac{1}{2}h$, above $\frac{1}{2}h$, or below $\frac{1}{2}h$?

81. •• Bill steps off a 3.0-m-high diving board and drops to the water below. At the same time, Ted jumps upward with a speed of 4.2 m/s from a 1.0-m-high diving board. Choosing the origin to be at the water's surface, and upward to be the positive x direction, write x-versus-t equations of motion for both Bill and Ted.

82. •• Repeat the previous problem, this time with the origin 3.0 m above the water, and with downward as the positive *x* direction.

83. •• On a hot summer day in the state of Washington while kayaking, I saw several swimmers jump from a railroad bridge into the Snohomish River below. The swimmers stepped off the bridge, and I estimated that they hit the water 1.5 s later. **(a)** How high was the bridge? **(b)** How fast were the swimmers moving when they hit the water? **(c)** What would the swimmers' drop time be if the bridge were twice as high?

84. •• **Highest Water Fountain** The world's highest fountain of water is located, appropriately enough, in Fountain Hills, Arizona. The fountain rises to a height of 560 ft (5 feet higher than the Washington Monument). **(a)** What is the initial speed of the water? **(b)** How long does it take for water to reach the top of the fountain?

85. •• Wrongly called for a foul, an angry basketball player throws the ball straight down to the floor. If the ball bounces straight up and returns to the floor 2.8 s after first striking it, what was the ball's greatest height above the floor?

86. •• To celebrate a victory, a pitcher throws her glove straight upward with an initial speed of 6.0 m/s. **(a)** How long does it take for the glove to return to the pitcher? **(b)** How long does it take for the glove to reach its maximum height?

87. •• **IP** Standing at the edge of a cliff 32.5 m high, you drop a ball. Later, you throw a second ball downward with an initial speed of 11.0 m/s. **(a)** Which ball has the greater *increase* in speed when it reaches the base of the cliff, or do both balls speed up by the same amount? **(b)** Verify your answer to part (a) with a calculation.

88. •• You shoot an arrow into the air. Two seconds later (2.00 s) the arrow has gone straight upward to a height of 30.0 m above its launch point. **(a)** What was the arrow's initial speed? **(b)** How long did it take for the arrow to first reach a height of 15.0 m above its launch point?

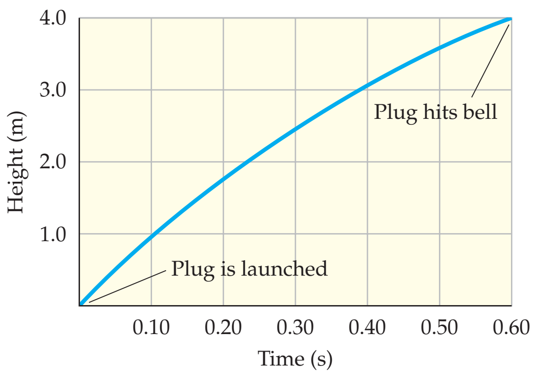
89. •• While riding on an elevator descending with a constant speed of 3.0 m/s, you accidentally drop a book from under your arm. **(a)** How long does it take for the book to reach the elevator floor, 1.2 m below your arm? **(b)** What is the book's speed relative to you when it hits the elevator floor?

90. •• A hot-air balloon is descending at a rate of 2.0 m/s when a passenger drops a camera. If the camera is 45 m above the ground when it is dropped, **(a)** how long does it take for the camera to reach the ground, and **(b)** what is its velocity just before it lands? Let upward be the positive direction for this problem.

91. •• **IP** Standing side by side, you and a friend step off a bridge at different times and fall for 1.6 s to the water below. Your friend goes first, and you follow after she has dropped a distance of 2.0 m. **(a)** When your friend hits the water, is the separation between the two of you 2.0 m, less than 2.0 m, or more than 2.0 m? **(b)** Verify your answer to part (a) with a calculation.

92. •• A model rocket blasts off and moves upward with an acceleration of 12 m/s^2 until it reaches a height of 26 m, at which point its engine shuts off and it continues its flight in free fall. **(a)** What is the maximum height attained by the rocket? **(b)** What is the speed of the rocket just before it hits the ground? **(c)** What is the total duration of the rocket's flight?

93. ••• **Hitting the "High Striker"** A young woman at a carnival steps up to the "high striker," a popular test of strength where the contestant hits one end of a lever with a mallet, propelling a small metal plug upward toward a bell. She gives the mallet a mighty swing and sends the plug to the top of the striker, where it rings the bell. **Figure 2–34** shows the corresponding position-versus-time plot for the plug. Using the information given in the plot, answer the following questions: **(a)** What is the average speed of the plug during its upward journey? **(b)** By how much does the speed of the plug decrease during its upward journey? **(c)** What is the initial speed of the plug? (Assume the plug to be in free fall during its upward motion, with no effects of air resistance or friction.)



▲ **FIGURE 2–34** Problem 93

94. ••• While sitting on a tree branch 10.0 m above the ground, you drop a chestnut. When the chestnut has fallen 2.5 m, you throw a second chestnut straight down. What initial speed must you give the second chestnut if they are both to reach the ground at the same time?

GENERAL PROBLEMS

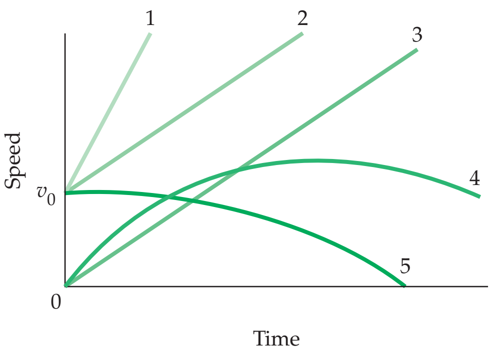
95. • In a well-known Jules Verne novel, Phileas Fogg travels around the world in 80 days. What was Mr. Fogg's approximate average speed during his adventure?

96. • An astronaut on the Moon drops a rock straight downward from a height of 1.25 m. If the acceleration of gravity on the Moon is 1.62 m/s^2, what is the speed of the rock just before it lands?

97. • You jump from the top of a boulder to the ground 1.5 m below. Estimate your deceleration on landing.

98. • **A Supersonic Waterfall** Geologists have learned of periods in the past when the Strait of Gibraltar closed off, and the Mediterranean Sea dried out and become a desert. Later, when the strait reopened, a massive saltwater waterfall was created. According to geologists, the water in this waterfall was supersonic; that is, it fell with speeds in excess of the speed of sound. Ignoring air resistance, what is the minimum height necessary to create a supersonic waterfall? (The speed of sound may be taken to be 340 m/s.)

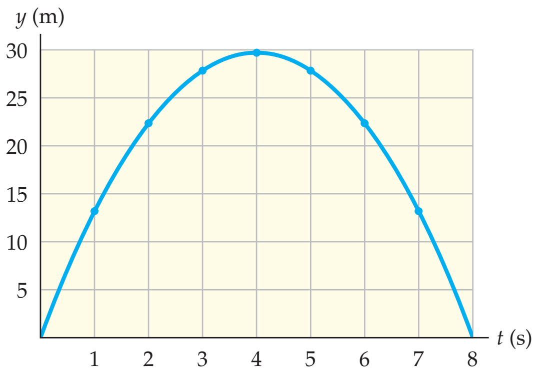
99. •• **CE** At the edge of a roof you drop ball A from rest, and then throw ball B downward with an initial velocity of v_0. Is the increase in speed just before the balls land more for ball A, more for ball B, or the same for each ball?



▲ **FIGURE 2–35** Problem 100

100. •• **CE** Suppose the two balls described in Problem 99 are released at the same time, with ball A dropped from rest and ball B thrown downward with an initial speed v_0. Identify which of the five plots shown in **Figure 2–35** corresponds to **(a)** ball A and **(b)** ball B.

101. •• Astronauts on a distant planet throw a rock straight upward and record its motion with a video camera. After digitizing their video, they are able to produce the graph of height, y, versus time, t, shown in **Figure 2–36.** **(a)** What is the acceleration of gravity on this planet? **(b)** What was the initial speed of the rock?



▲ **FIGURE 2–36** Problem 101

102. •• **Drop Tower** NASA operates a 2.2-second drop tower at the Glenn Research Center in Cleveland, Ohio. At this facility, experimental packages are dropped from the top of the tower, on the 8th floor of the building. During their 2.2 seconds of free fall, experiments experience a microgravity environment similar to that of a spacecraft in orbit. **(a)** What is the drop distance of a 2.2-s tower? **(b)** How fast are the experiments traveling when they hit the air bags at the bottom of the tower? **(c)** If the experimental package comes to rest over a distance of 0.75 m upon hitting the air bags, what is the average stopping acceleration?

103. •• **IP** A youngster bounces straight up and down on a trampoline. Suppose she doubles her initial speed from 2.0 m/s to 4.0 m/s. **(a)** By what factor does her time in the air increase? **(b)** By what factor does her maximum height increase? **(c)** Verify your answers to parts (a) and (b) with an explicit calculation.

104. •• At the 18th green of the U.S. Open you need to make a 20.5-ft putt to win the tournament. When you hit the ball, giving it an initial speed of 1.57 m/s, it stops 6.00 ft short of the hole. **(a)** Assuming the deceleration caused by the grass is constant, what should the initial speed have been to just make the putt? **(b)** What initial speed do you need to make the remaining 6.00-ft putt?

105. •• **IP** A popular entertainment at some carnivals is the blanket toss (see photo, p. 39). **(a)** If a person is thrown to a maximum height of 28.0 ft above the blanket, how long does she spend in the air? **(b)** Is the amount of time the person is above a height of 14.0 ft more than, less than, or equal to the amount of time the person is below a height of 14.0 ft? Explain. **(c)** Verify your answer to part (b) with a calculation.

106. •• Referring to Conceptual Checkpoint 2–5, find the separation between the rocks at **(a)** $t = 1.0$ s, **(b)** $t = 2.0$ s, and **(c)** $t = 3.0$ s, where time is measured from the instant the second rock is dropped. **(d)** Verify that the separation increases linearly with time.

107. •• **IP** A glaucous-winged gull, ascending straight upward at 5.20 m/s, drops a shell when it is 12.5 m above the ground. **(a)** What are the magnitude and direction of the shell's acceleration just after it is released? **(b)** Find the maximum height above the ground reached by the shell. **(c)** How long does it take for the shell to reach the ground? **(d)** What is the speed of the shell at this time?

108. •• A doctor, preparing to give a patient an injection, squirts a small amount of liquid straight upward from a syringe. If the liquid emerges with a speed of 1.5 m/s, **(a)** how long does it take for it to return to the level of the syringe? **(b)** What is the maximum height of the liquid above the syringe?

109. •• A hot-air balloon has just lifted off and is rising at the constant rate of 2.0 m/s. Suddenly one of the passengers realizes she has left her camera on the ground. A friend picks it up and tosses it straight upward with an initial speed of 13 m/s. If the passenger is 2.5 m above her friend when the camera is tossed, how high is she when the camera reaches her?

110. ••• In the previous problem, what is the minimum initial speed of the camera if it is to just reach the passenger? (*Hint:* When the camera is thrown with its minimum speed, its speed on reaching the passenger is the same as the speed of the passenger.)

111. ••• **Old Faithful** Watching Old Faithful erupt, you notice that it takes a time t for water to emerge from the base of the geyser and reach its maximum height. **(a)** What is the height of the geyser, and **(b)** what is the initial speed of the water? Evaluate your expressions for **(c)** the height and **(d)** the initial speed for a measured time of 1.65 s.

112. ••• **IP** A ball is thrown upward with an initial speed v_0. When it reaches the top of its flight, at a height h, a second ball is thrown upward with the same initial velocity. **(a)** Sketch an x-versus-t plot for each ball. **(b)** From your graph, decide whether the balls cross paths at $h/2$, above $h/2$, or below $h/2$. **(c)** Find the height where the paths cross.

113. ••• Weights are tied to each end of a 20.0-cm string. You hold one weight in your hand and let the other hang vertically a height h above the floor. When you release the weight in your hand, the two weights strike the ground one after the other with audible thuds. Find the value of h for which the time between release and the first thud is equal to the time between the first thud and the second thud.

114. ••• A ball, dropped from rest, covers three-quarters of the distance to the ground in the last second of its fall. **(a)** From what height was the ball dropped? **(b)** What was the total time of fall?

115. ••• A stalactite on the roof of a cave drips water at a steady rate to a pool 4.0 m below. As one drop of water hits the pool, a second drop is in the air, and a third is just detaching from the stalactite. **(a)** What are the position and velocity of the second drop when the first drop hits the pool? **(b)** How many drops per minute fall into the pool?

116. ••• You drop a ski glove from a height h onto fresh snow, and it sinks to a depth d before coming to rest. **(a)** In terms of g and h, what is the speed of the glove when it reaches the snow? **(b)** What are the magnitude and direction of the glove's acceleration as it moves through the snow, assuming it to be constant? Give your answer in terms of g, h, and d.

117. ••• To find the height of an overhead power line, you throw a ball straight upward. The ball passes the line on the way up after 0.75 s, and passes it again on the way down 1.5 s after it was tossed. What are the height of the power line and the initial speed of the ball?

118. ••• Suppose the first rock in Conceptual Checkpoint 2–5 drops through a height h before the second rock is released from rest. Show that the separation between the rocks, S, is given by the following expression:

$$S = h + (\sqrt{2gh})t$$

In this result, the time t is measured from the time the second rock is dropped.

119. ••• An arrow is fired with a speed of 20.0 m/s at a block of Styrofoam resting on a smooth surface. The arrow penetrates a certain distance into the block before coming to rest relative to it. During this process the arrow's deceleration has a magnitude of 1550 m/s^2 and the block's acceleration has a magnitude of 450 m/s^2. **(a)** How long does it take for the arrow to stop moving with respect to the block? **(b)** What is the common speed of the arrow and block when this happens? **(c)** How far into the block does the arrow penetrate?

120. ••• Sitting in a second-story apartment, a physicist notices a ball moving straight upward just outside her window. The ball is visible for 0.25 s as it moves a distance of 1.05 m from the bottom to the top of the window. **(a)** How long does it take before the ball reappears? **(b)** What is the greatest height of the ball above the top of the window?

121. ••• **The Quadratic Formula from Kinematics** In this problem we show how the kinematic equations of motion can be used to derive the quadratic formula. First, consider an object with an initial position x_0, an initial velocity v_0, and an acceleration a. To find the time when this object reaches the position $x = 0$ we can use the quadratic formula, or apply the following two-step procedure: **(a)** Use Equation 2–12 to show that the velocity of the object when it reaches $x = 0$ is given by $v = \pm\sqrt{v_0^2 - 2ax_0}$. **(b)** Use Equation 2–7 to show that the time corresponding to the velocity found in part (a) is $t = \dfrac{-v_0 \pm \sqrt{v_0^2 - 2ax_0}}{a}$. **(c)** To complete our derivation, show that the result of part (b) is the same as applying the quadratic formula to $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0$.

PASSAGE PROBLEMS

Bam!—*Apollo 15* Lands on the Moon

The first word spoken on the surface of the Moon after *Apollo 15* landed on July 30, 1971, was "Bam!" This was James Irwin's involuntary reaction to their rather bone-jarring touchdown. "We did hit harder than any of the other flights!" says Irwin. "And I was startled, obviously, when I said, 'Bam!'"

The reason for the "firm touchdown" of *Apollo 15,* as pilot David Scott later characterized it, was that the rocket engine was shut off a bit earlier than planned, when the lander was still 4.30 ft above the lunar surface and moving downward with a speed of 0.500 ft/s. From that point on the lander descended in lunar free fall, with an acceleration of 1.62 m/s^2. As a result, the landing speed of *Apollo 15* was by far the largest of any of the *Apollo* missions. In comparison, Neil Armstrong's landing speed on *Apollo 11* was the lowest at 1.7 ft/s—he didn't shut off the engine until the footpads were actually on the surface. *Apollos 12, 14,* and *17* all landed with speeds between 3.0 and 3.5 ft/s.

To better understand the descent of *Apollo 15,* we show its trajectory during the final stages of landing in **Figure 2–37 (a)**. In **Figure 2–37 (b)** we show a variety of speed-versus-time plots.

122. • How long did it take for the lander to drop the final 4.30 ft to the Moon's surface?

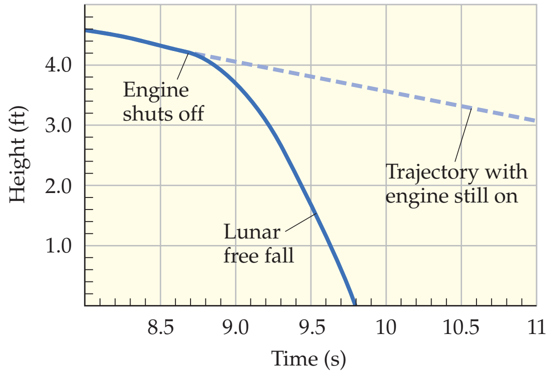
A. 1.18 s **B.** 1.37 s

C. 1.78 s **D.** 2.36 s

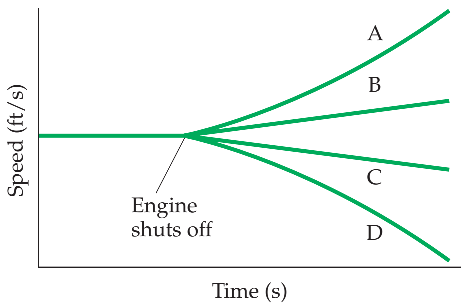
123. •• What was the impact speed of the lander when it touched down? Give your answer in feet per second (ft/s), the same units used by the astronauts.

A. 2.41 ft/s **B.** 6.78 ft/s

C. 9.95 ft/s **D.** 10.6 ft/s



(a)



(b)

▲ **FIGURE 2–37** Problems 122, 123, 124, and 125

124. • Which of the speed-versus-time plots in Figure 2–37 (b) correctly represents the speed of the *Apollo 15* lander?

A B C D

125. • Suppose, instead of shutting off the engine, the astronauts had increased its thrust, giving the lander a small, but constant, upward acceleration. Which speed-versus-time plot in Figure 2–37 (b) would describe this situation?

A B C D

INTERACTIVE PROBLEMS

126. •• **Referring to Example 2–9** Suppose the speeder (red car) is traveling with a constant speed of 25 m/s, and that the maximum acceleration of the police car (blue car) is 3.8 m/s^2. If the police car is to start from rest and catch the speeder in 15 s or less, what is the maximum head-start distance the speeder can have? Measure time from the moment the police car starts.

127. •• **Referring to Example 2–9** The speeder passes the position of the police car with a constant speed of 15 m/s. The police car immediately starts from rest and pursues the speeder with constant acceleration. What acceleration must the police car have if it is to catch the speeder in 7.0 s? Measure time from the moment the police car starts.

128. •• **IP Referring to Example 2–12** **(a)** In Example 2–12, the bag of sand is released at 20.0 m and reaches a maximum height of 22 m. If the bag had been released at 30.0 m instead, with everything else remaining the same, would its maximum height be 32 m, greater than 32 m, or less than 32 m? **(b)** Find the speed of the bag just before it lands when it is released from 30.0 m.

129. •• **Referring to Example 2–12** Suppose the balloon is descending with a constant speed of 4.2 m/s when the bag of sand comes loose at a height of 35 m. **(a)** How long is the bag in the air? **(b)** What is the speed of the bag when it is 15 m above the ground?

3 Vectors in Physics

The points of the compass have long been used as a framework for indicating directions. The compass shown here was produced by Gowin Knight (1713–1772), whose improved designs were adopted by the Royal Navy in 1752. In physics, we more frequently indicate directions with x and y rather than N, S, E, and W. Either way, specifying a direction as well as a magnitude is essential to defining one of the physicist's basic tools, the vector.

Of all the mathematical tools used in this book, perhaps none is more important than the vector. In the next chapter, for example, we use vectors to extend our study of motion from one dimension to two dimensions. More generally, vectors are *indispensable* when a physical quantity has a direction associated with it. Suppose, for example, that a pilot wants to fly from Denver to Dallas. If the air is still, the pilot can simply head the plane toward the destination. If there is a wind blowing from west to east, however, the pilot must use vectors to determine the correct heading so that the plane and its passengers will arrive in Dallas and not Little Rock.

In this chapter we discuss what a vector is, how it differs from a scalar, and how it can represent a physical quantity. We also show how to find the components of a vector and how to add and subtract vectors. All of these techniques are used time and again throughout the book. Other useful aspects of vectors, such as how to multiply them, will be presented in later chapters when the need arises.

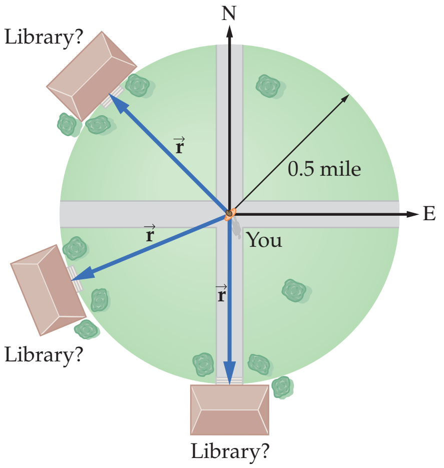
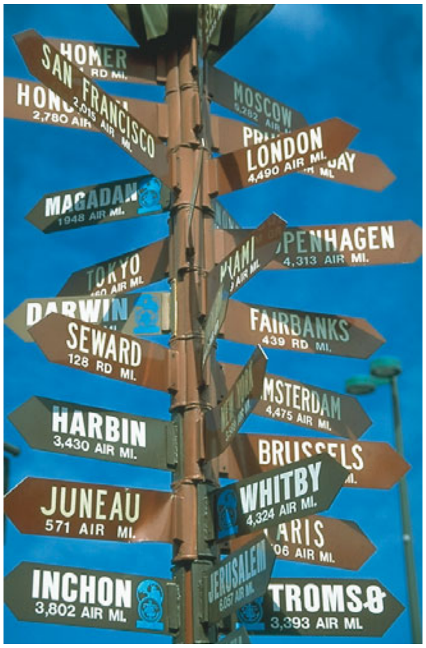


FIGURE 3–1 Distance and direction
If you know only that the library is 0.5 mi from you, it could lie anywhere on a circle of radius 0.5 mi. If, instead, you are told the library is 0.5 mi northwest, you know its precise location.



The information given by this sign includes both a distance and a direction for each city. In effect, the sign defines a displacement vector for each of these destinations.

3–1 Scalars Versus Vectors

Numbers can represent many quantities in physics. For example, a numerical value, together with the appropriate units, can specify the volume of a container, the temperature of the air, or the time of an event. In physics, a number with its units is referred to as a **scalar:**

- A scalar is a number with units. It can be positive, negative, or zero.

Sometimes, however, a scalar isn't enough to adequately describe a physical quantity—in many cases, a direction is needed as well. For example, suppose you're walking in an unfamiliar city and you want directions to the library. You ask a passerby, "Do you know where the library is?" If the person replies "Yes," and walks on, he hasn't been too helpful. If he says, "Yes, it is half a mile from here," that is more helpful, but you still don't know where it is. The library could be anywhere on a circle of radius one-half mile, as shown in **Figure 3–1**. To pin down the location, you need a reply such as, "Yes, the library is half a mile northwest of here." With both a distance *and* a direction, you know the location of the library.

Thus, if you walk northwest for half a mile you arrive at the library, as indicated by the upper left arrow in Figure 3–1. The arrow points in the direction traveled, and its **magnitude,** 0.5 mi in this case, represents the distance covered. In general, a quantity that is specified by both a *magnitude* and a *direction* is represented by a **vector:**

- A vector is a mathematical quantity with both a direction and a magnitude.

In the example of walking to the library, the vector corresponding to the trip is the displacement vector. Other examples of vector quantities are the velocity and the acceleration of an object. For example, the magnitude of a velocity vector is its speed, and its direction is the direction of motion, as we shall see later in this chapter.

When we indicate a vector on a diagram or a sketch, we draw an arrow, as in Figure 3–1. To indicate a vector with a written symbol, we use **boldface** for the vector itself, with a small arrow above it to remind us of its vector nature, and *italic* for its magnitude. Thus, for example, the upper-left vector in Figure 3–1 is designated by the symbol $\vec{\mathbf{r}}$**,** and its magnitude is $r = 0.5$ mi. (When we represent a vector in a graph, we sometimes label it with the corresponding boldface symbol, and sometimes with the appropriate magnitude.) It is common in handwritten material to draw a small arrow over the vector's symbol, which is very similar to the way vectors are represented in this text.

3–2 The Components of a Vector

When we discussed directions for finding a library in the previous section, we pointed out that knowing the magnitude and direction angle—0.5 mi northwest—gives its precise location. We left out one key element in actually *getting* to the library, however. In most cities it would not be possible to simply walk in a straight line for 0.5 mi directly to the library, since to do so would take you through buildings where there are no doors, through people's backyards, and through all kinds of other obstructions. In fact, if the city streets are laid out along north–south and east–west directions, you might instead walk west for a certain distance, then turn and proceed north an equal distance until you reach the library, as illustrated in **Figure 3–2**. What you have just done is "resolved" displacement vector $\vec{\mathbf{r}}$ between you and the library into east–west and north–south "components."

In general, to find the components of a vector we need to set up a coordinate system. In two dimensions we choose an origin, O, and a positive direction for both the x and the y axes, as in **Figure 3–3**. If the system were three-dimensional, we would also indicate a z axis.

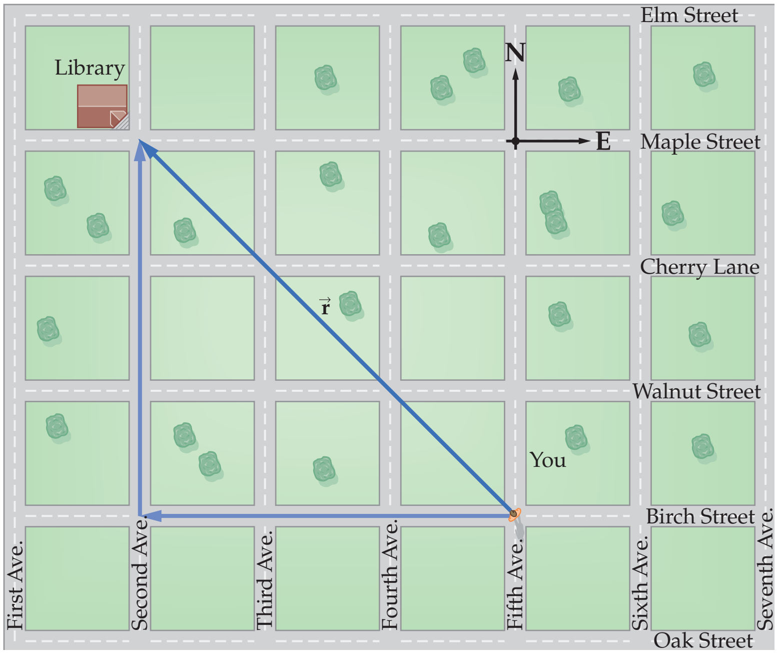


FIGURE 3–2 A walk along city streets to the library

By taking the indicated path, we have "resolved" the vector $\vec{\mathbf{r}}$ into east–west and north–south components.

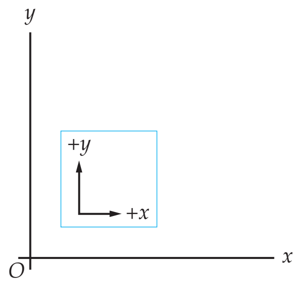


FIGURE 3–3 A two-dimensional coordinate system

The positive x and y directions are indicated in this shorthand form.

Now, a vector is defined by its magnitude (indicated by the length of the arrow representing the vector) and its direction. For example, suppose an ant leaves its nest at the origin and, after foraging for some time, is at the location given by the vector $\vec{\mathbf{r}}$ in **Figure 3–4 (a)**. This vector has a magnitude $r = 1.50$ m and points in a direction $\theta = 25.0°$ above the x axis. Equivalently, $\vec{\mathbf{r}}$ can be defined by saying that it extends a distance r_x in the x direction and a distance r_y in the y direction, as shown in **Figure 3–4 (b)**. The quantities r_x and r_y are referred to as the x and y **scalar components** of the vector $\vec{\mathbf{r}}$.

We can find r_x and r_y by using standard trigonometric relations, as summarized in the Problem-Solving Note on this page. Referring to Figure 3–4 (b), we see that

$$r_x = r\cos 25.0° = (1.50\text{ m})(0.906) = 1.36\text{ m}$$

and

$$r_y = r\sin 25.0° = (1.50\text{ m})(0.423) = 0.634\text{ m}$$

Thus, we can say that the ant's final displacement is equivalent to what it would be if the ant had simply walked 1.36 m in the x direction and then 0.634 m in the y direction.

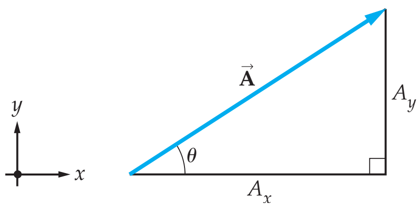
To show the equivalence of these two ways of describing a vector, let's start with the components of $\vec{\mathbf{r}}$, as determined previously, and use them to calculate the magnitude r and the angle θ. First, note that r_x, r_y, and r form a right triangle with r as the hypotenuse. Thus, we can use the Pythagorean theorem (Appendix A) to find r in terms of r_x and r_y. This gives

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.36\text{ m})^2 + (0.634\text{ m})^2} = \sqrt{2.25\text{ m}^2} = 1.50\text{ m}$$

PROBLEM-SOLVING NOTE

A Vector and Its Components

Given the magnitude and direction of a vector, find its components:

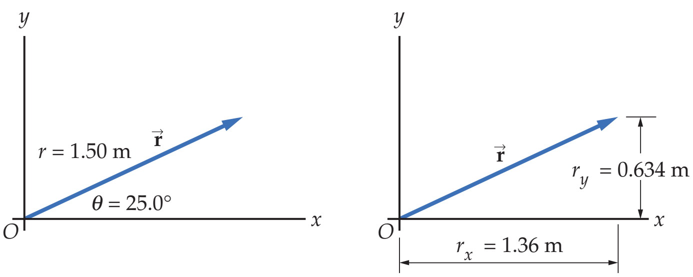


$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

Given the components of a vector, find its magnitude and direction:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$



(a) A vector defined in terms of its length and direction angle

(b) The same vector defined in terms of its x and y components

FIGURE 3–4 A vector and its scalar components

(a) The vector $\vec{\mathbf{r}}$ is defined by its length ($r = 1.50$ m) and its direction angle ($\theta = 25.0°$) measured counterclockwise from the positive x axis. **(b)** Alternatively, the vector $\vec{\mathbf{r}}$ can be defined by its x component, $r_x = 1.36$ m, and its y component, $r_y = 0.634$ m.

as expected. Second, we can use any two sides of the triangle to obtain the angle θ, as shown in the next three calculations:

$$\theta = \sin^{-1}\left(\frac{0.634\text{ m}}{1.50\text{ m}}\right) = \sin^{-1}(0.423) = 25.0°$$

$$\theta = \cos^{-1}\left(\frac{1.36\text{ m}}{1.50\text{ m}}\right) = \cos^{-1}(0.907) = 25.0°$$

$$\theta = \tan^{-1}\left(\frac{0.634\text{ m}}{1.36\text{ m}}\right) = \tan^{-1}(0.466) = 25.0°$$

In some situations we know a vector's magnitude and direction; in other cases we are given the vector's components. You will find it useful to be able to convert quickly and easily from one description of a vector to the other using trigonometric functions and the Pythagorean theorem.

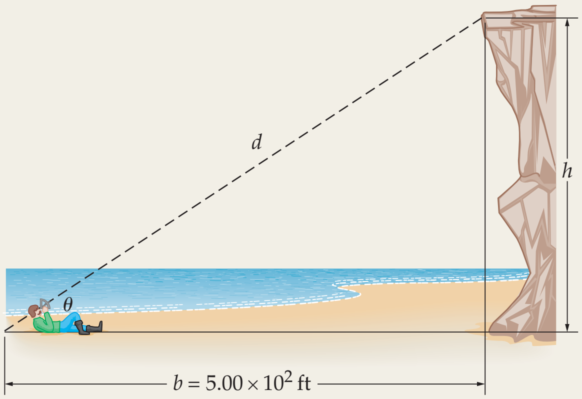
EXAMPLE 3–1 DETERMINING THE HEIGHT OF A CLIFF

REAL-WORLD PHYSICS

In the Jules Verne novel *Mysterious Island*, Captain Cyrus Harding wants to find the height of a cliff. He stands with his back to the base of the cliff, then marches straight away from it for 5.00×10^2 ft. At this point he lies on the ground and measures the angle from the horizontal to the top of the cliff. If the angle is 34.0°, **(a)** how high is the cliff? **(b)** What is the straight-line distance from Captain Harding to the top of the cliff?

PICTURE THE PROBLEM

Our sketch shows Cyrus Harding making his measurement of the angle, $\theta = 34.0°$, to the top of the cliff. The relevant triangle for this problem is also indicated. Note that the opposite side of the triangle is the height of the cliff, h; the adjacent side is the distance from the base of the cliff to Harding, $b = 5.00 \times 10^2$ ft; and finally, the hypotenuse is the distance, d, from Harding to the top of the cliff.



STRATEGY

The tangent of θ is the height of the triangle divided by the base: $\tan\theta = h/b$. Since we know both θ and the base, we can find the height using this relation. Similarly, the distance from Harding to the top of the cliff can be obtained by solving $\cos\theta = b/d$ for d.

SOLUTION

Part (a)

1. Use $\tan\theta = h/b$ to solve for the height of the cliff, h:

$$h = b\tan\theta = (5.00 \times 10^2\text{ ft})\tan 34.0° = 337\text{ ft}$$

Part (b)

2. Similarly, use $\cos\theta = b/d$ to solve for the distance d from Captain Harding to the top of the cliff:

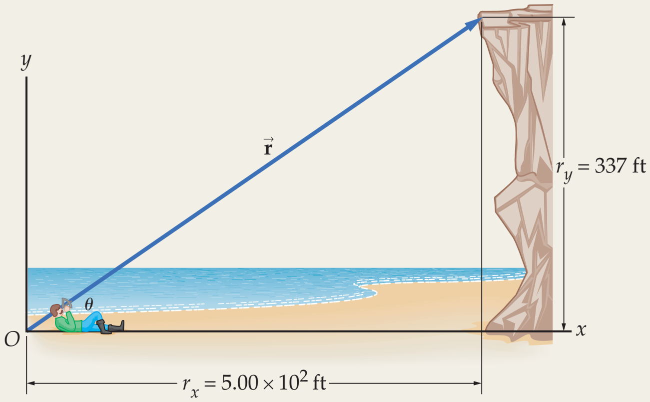
$$d = \frac{b}{\cos\theta} = \frac{5.00 \times 10^2\text{ ft}}{\cos 34.0°} = 603\text{ ft}$$

INSIGHT

An alternative way to solve part (b) is to use the Pythagorean theorem:

$$d = \sqrt{h^2 + b^2} = \sqrt{(337\text{ ft})^2 + (5.00 \times 10^2\text{ ft})^2} = 603\text{ ft}$$

Thus, if we let $\vec{\mathbf{r}}$ denote the vector from Cyrus Harding to the top of the cliff, as shown here, its magnitude is 603 ft and its direction is 34.0° above the x axis. Alternatively, the x component of $\vec{\mathbf{r}}$ is 5.00×10^2 ft and its y component is 337 ft.



PRACTICE PROBLEM

What angle would Cyrus Harding have found if he had walked 6.00×10^2 ft from the cliff to make his measurement? [**Answer:** $\theta = 29.3°$]

Some related homework problems: Problem 5, Problem 17

EXERCISE 3–1

a. Find A_x and A_y for the vector $\vec{\mathbf{A}}$ with magnitude and direction given by $A = 3.5$ m and $\theta = 66°$, respectively.

b. Find B and θ for the vector $\vec{\mathbf{B}}$ with components $B_x = 75.5$ m and $B_y = 6.20$ m.

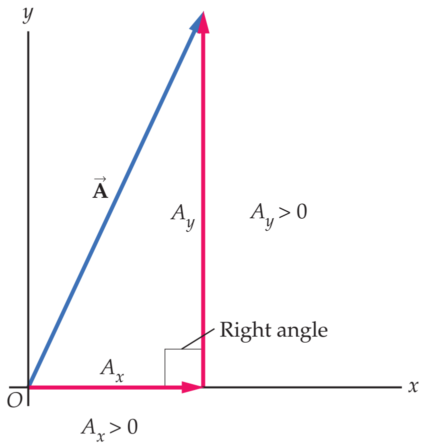
SOLUTION

a. $A_x = 1.4$ m, $A_y = 3.2$ m

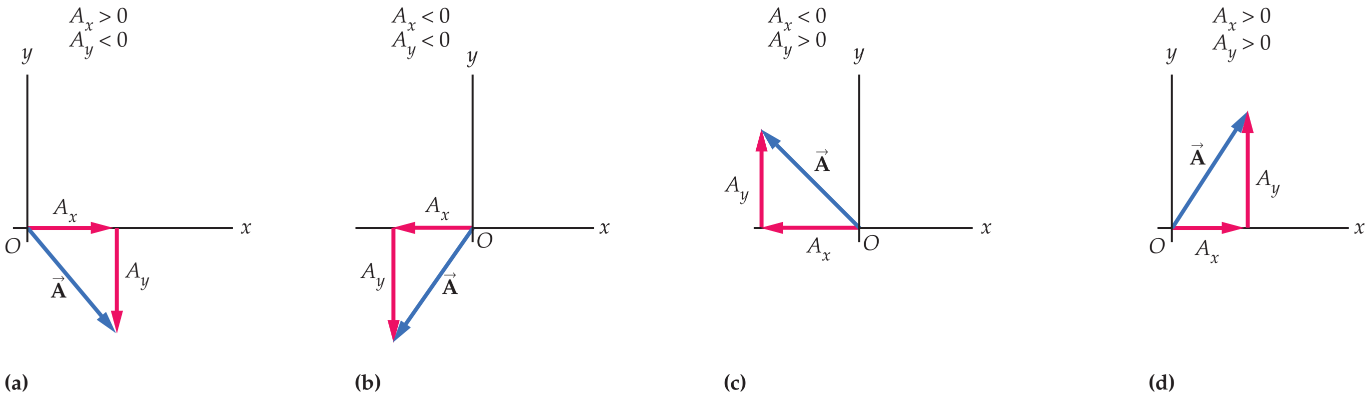
b. $B = 75.8$ m, $\theta = 4.69°$

Next, how do you determine the correct signs for the x and y components of a vector? This can be done by considering the right triangle formed by A_x, A_y, and $\vec{\mathbf{A}}$, as shown in **Figure 3–5**. To determine the sign of A_x, start at the tail of the vector and move along the x axis toward the right angle. If you are moving in the positive x direction, then A_x is positive $(A_x > 0)$; if you are moving in the negative x direction, then A_x is negative $(A_x < 0)$. For the y component, start at the right angle and move toward the tip of the arrow. A_y is positive or negative depending on whether you are moving in the positive or negative y direction.

For example, consider the vector shown in **Figure 3–6 (a)**. In this case, $A_x > 0$ and $A_y < 0$, as indicated in the figure. Similarly, the signs of A_x and A_y are given in **Figure 3–6 (b, c, d)** for the vectors shown there. Be sure to verify each of these cases by applying the rules just given. As we continue our study of physics, it is important to be able to find the components of a vector *and* to assign to them the correct signs.



▲ FIGURE 3–5 A vector whose *x* and *y* components are positive



▲ FIGURE 3–6 Examples of vectors with components of different signs

To determine the signs of a vector's components, it is only necessary to observe the direction in which they point. For example, in part (a) the x component points in the positive direction; hence $A_x > 0$. Similarly, the y component in part (a) points in the negative y direction; therefore $A_y < 0$.

EXERCISE 3–2

The vector $\vec{\mathbf{A}}$ has a magnitude of 7.25 m. Find its components for direction angles of

a. $\theta = 5.00°$ **c.** $\theta = 245°$

b. $\theta = 125°$ **d.** $\theta = 335°$

SOLUTION

a. $A_x = 7.22$ m, $A_y = 0.632$ m

b. $A_x = -4.16$ m, $A_y = 5.94$ m

c. $A_x = -3.06$ m, $A_y = -6.57$ m

d. $A_x = 6.57$ m, $A_y = -3.06$ m

Be careful when using your calculator to determine the direction angle, θ, because you may need to add 180° to get the correct angle, as measured counterclockwise from the positive x axis. For example, if $A_x = -0.50$ m and $A_y = 1.0$ m, your calculator will give the following result:

$$\theta = \tan^{-1}\left(\frac{1.0\text{ m}}{-0.50\text{ m}}\right) = \tan^{-1}(-2.0) = -63°$$

Does this angle correspond to the specified vector? The way to check is to sketch $\vec{\mathbf{A}}$. When you do, your drawing is similar to Figure 3–6 (c), and thus the direction angle of $\vec{\mathbf{A}}$ should be between 90° and 180°. To obtain the correct angle, add 180° to the calculator's result:

$$\theta = -63° + 180° = 117°$$

This, in fact, is the direction angle for the vector $\vec{\mathbf{A}}$.

EXERCISE 3–3

The vector $\vec{\mathbf{B}}$ has components $B_x = -2.10$ m and $B_y = -1.70$ m. Find the direction angle, θ, for this vector.

SOLUTION

$\tan^{-1}[(-1.70\text{ m})/(-2.10\text{ m})] = \tan^{-1}(1.70/2.10) = 39.0°, \theta = 39.0 + 180° = 219°$

Finally, in many situations the direction of a vector $\vec{\mathbf{A}}$ is given by the angle θ, measured relative to the x axis, as in **Figure 3–7 (a)**. In these cases we know that

$$A_x = A\cos\theta$$

and

$$A_y = A\sin\theta$$

On the other hand, we are sometimes given the angle between the vector and the y axis, as in **Figure 3–7 (b)**. If we call this angle θ', then it follows that

$$A_x = A\sin\theta'$$

and

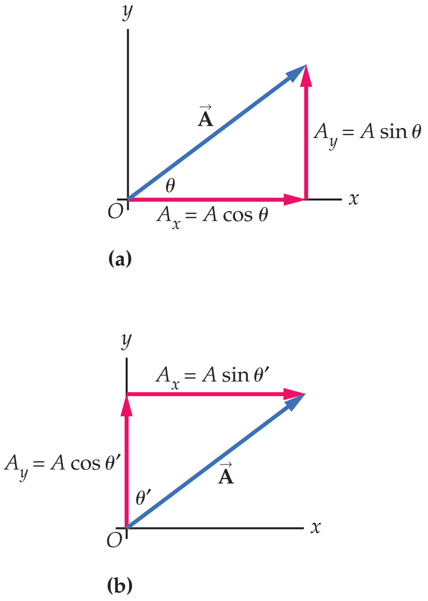
$$A_y = A\cos\theta'$$

These two seemingly different results are actually in complete agreement. Note that $\theta + \theta' = 90°$, or $\theta' = 90° - \theta$. If we use the trigonometric identities given in Appendix A, we find

$$A_x = A\sin\theta' = A\sin(90° - \theta) = A\cos\theta$$

and

$$A_y = A\cos\theta' = A\cos(90° - \theta) = A\sin\theta$$



▲ **FIGURE 3–7 Vector direction angles**
Vector $\vec{\mathbf{A}}$ and its components in terms of **(a)** the angle relative to the x axis and **(b)** the angle relative to the y axis.

EXERCISE 3–4

If a vector's direction angle relative to the x axis is 35°, then its direction angle relative to the y axis is 55°. Find the components of a vector $\vec{\mathbf{A}}$ of magnitude 5.2 m in terms of

a. its direction relative to the x axis, and

b. its direction relative to the y axis.

SOLUTION

a. $A_x = (5.2\text{ m})\cos 35° = 4.3\text{ m}$, $A_y = (5.2\text{ m})\sin 35° = 3.0\text{ m}$

b. $A_x = (5.2\text{ m})\sin 55° = 4.3\text{ m}$, $A_y = (5.2\text{ m})\cos 55° = 3.0\text{ m}$

3–3 Adding and Subtracting Vectors

One important reason for determining the components of a vector is that they are useful in adding and subtracting vectors. In this section we begin by defining vector addition graphically, and then show how the same addition can be performed more concisely and accurately with components.

Adding Vectors Graphically

One day you open an old chest in the attic and find a treasure map inside. To locate the treasure, the map says that you must "Go to the sycamore tree in the backyard, march 5 paces north, then 3 paces east." If these two displacements are represented by the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in **Figure 3–8**, the total displacement from the tree to the treasure is given by the vector $\vec{\mathbf{C}}$. We say that $\vec{\mathbf{C}}$ is the *vector sum* of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$; that is, $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. In general, vectors are added graphically according to the following rule:

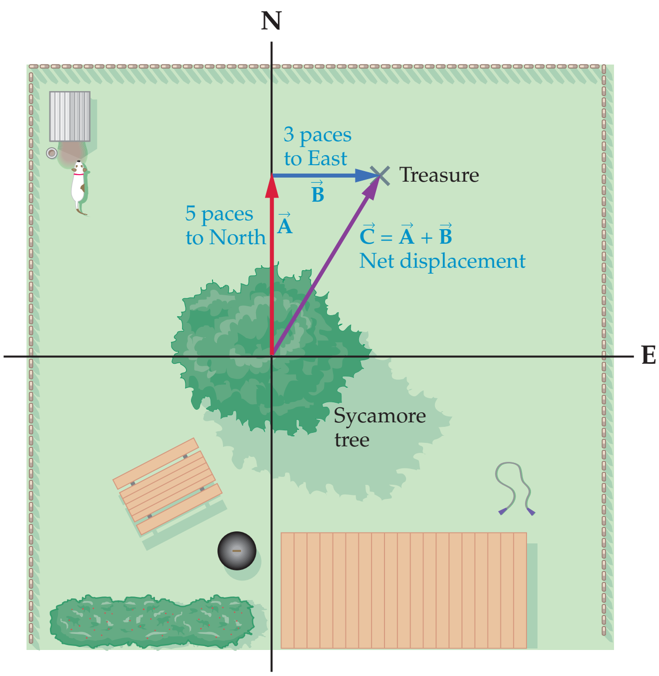
- To add the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, place the tail of $\vec{\mathbf{B}}$ at the head of $\vec{\mathbf{A}}$. The sum, $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, is the vector extending from the tail of $\vec{\mathbf{A}}$ to the head of $\vec{\mathbf{B}}$.

If the instructions to find the treasure were a bit more complicated—5 paces north, 3 paces east, then 4 paces southeast, for example—the path from the sycamore tree to the treasure would be like that shown in **Figure 3–9**. In this case, the total displacement, $\vec{\mathbf{D}}$, is the sum of the three vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$; that is, $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. It follows that to add more than two vectors, we just keep placing the vectors head-to-tail, head-to-tail, and then draw a vector from the tail of the first vector to the head of the last vector, as in Figure 3–9.

In order to place a given pair of vectors head-to-tail, it may be necessary to move the corresponding arrows. This is fine, as long as you don't change their length or their direction. After all, a vector is defined by its length and direction; if these are unchanged, so is the vector.

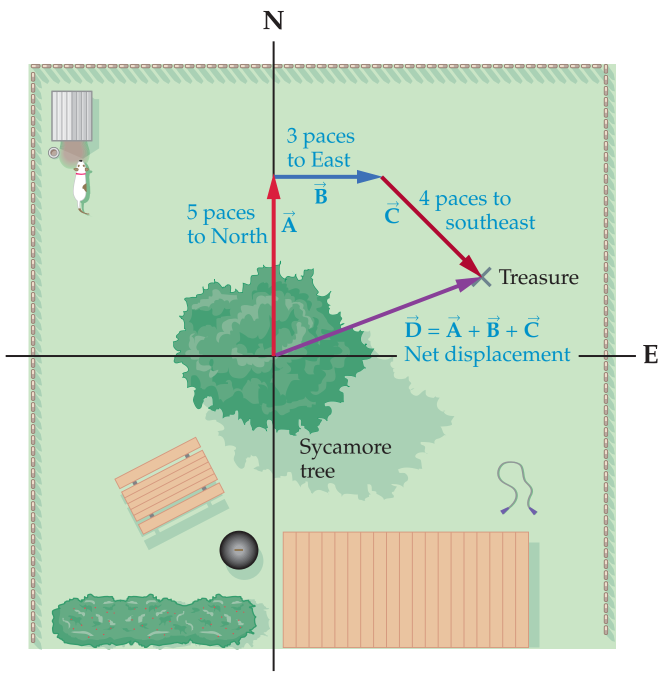
- A vector is defined by its magnitude and direction, regardless of its location.

▲ To a good approximation, these snow geese are all moving in the same direction with the same speed. As a result, their velocity vectors are equal, even though their positions are different.



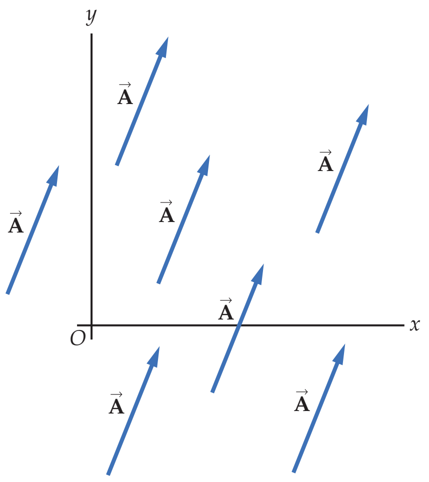
▲ **FIGURE 3–8 The sum of two vectors**

To go from the sycamore tree to the treasure, one must first go 5 paces north ($\vec{\mathbf{A}}$) and then 3 paces east ($\vec{\mathbf{B}}$). The net displacement from the tree to the treasure is $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.



▲ **FIGURE 3–9 Adding several vectors**

Searching for a treasure that is 5 paces north ($\vec{\mathbf{A}}$), 3 paces east ($\vec{\mathbf{B}}$), and 4 paces southeast ($\vec{\mathbf{C}}$) of the sycamore tree. The net displacement from the tree to the treasure is $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$.



▲ **FIGURE 3–10 Identical vectors $\vec{\mathbf{A}}$ at different locations**

A vector is defined by its direction and length; its location is immaterial.

For example, in **Figure 3–10** all of the vectors are the same, even though they are at different locations on the graph.

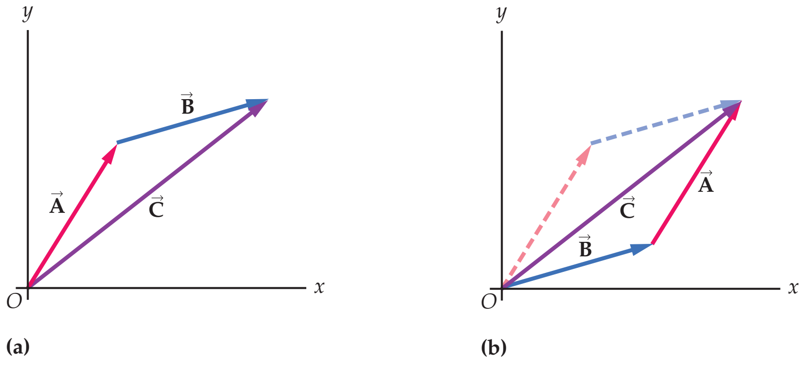
As an example of moving vectors, consider two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, and their vector sum, $\vec{\mathbf{C}}$:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

as illustrated in **Figure 3–11 (a)**. By moving the arrow representing $\vec{\mathbf{B}}$ so that its tail is at the origin, and moving the arrow for $\vec{\mathbf{A}}$ so that its tail is at the head of $\vec{\mathbf{B}}$, we obtain the construction shown in **Figure 3–11 (b)**. From this graph we see that $\vec{\mathbf{C}}$, which is $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, is also equal to $\vec{\mathbf{B}} + \vec{\mathbf{A}}$:

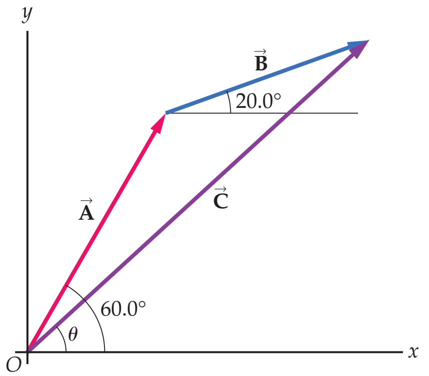
$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$

That is, the sum of vectors is independent of the order in which the vectors are added.



▲ **FIGURE 3–11 $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$**

The vector $\vec{\mathbf{C}}$ is equal to **(a)** $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and **(b)** $\vec{\mathbf{B}} + \vec{\mathbf{A}}$. Note also that $\vec{\mathbf{C}}$ is the diagonal of the parallelogram formed by the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. For this reason, this method of vector addition is referred to as the "parallelogram method."



▲ **FIGURE 3–12 Graphical addition of vectors**

The vector $\vec{\mathbf{A}}$ has a magnitude of 5.00 m and a direction angle of 60.0°; the vector $\vec{\mathbf{B}}$ has a magnitude of 4.00 m and a direction angle of 20.0°. The magnitude and direction of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ can be measured on the graph with a ruler and a protractor.

Now, suppose that $\vec{\mathbf{A}}$ has a magnitude of 5.00 m and a direction angle of 60.0° above the x axis, and that $\vec{\mathbf{B}}$ has a magnitude of 4.00 m and a direction angle of 20.0° above the x axis. These two vectors and their sum, $\vec{\mathbf{C}}$, are shown in **Figure 3–12**. The question is: What are the length and direction angle of $\vec{\mathbf{C}}$?

A graphical way to answer this question is to simply measure the length and direction of $\vec{\mathbf{C}}$ in Figure 3–12. With a ruler, we find the length of $\vec{\mathbf{C}}$ to be approximately 1.75 times the length of $\vec{\mathbf{A}}$, which means that $\vec{\mathbf{C}}$ is roughly 1.75 (5.00 m) = 8.75 m. Similarly, with a protractor we measure the angle θ to be about 45.0° above the x axis.

Adding Vectors Using Components

The graphical method of adding vectors yields approximate results, limited by the accuracy with which the vectors can be drawn and measured. In contrast, exact results can be obtained by adding $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in terms of their components. To see how this is done, consider **Figure 3–13 (a)**, which shows the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, and **Figure 3–13 (b)**, which shows the components of $\vec{\mathbf{C}}$. Clearly,

$$C_x = A_x + B_x$$

and

$$C_y = A_y + B_y$$

Thus, to add vectors, you simply add the components.

Returning to our example in Figure 3–12, the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are

$$A_x = (5.00\text{ m})\cos 60.0° = 2.50\text{ m} \quad A_y = (5.00\text{ m})\sin 60.0° = 4.33\text{ m}$$

and

$$B_x = (4.00\text{ m})\cos 20.0° = 3.76\text{ m} \quad B_y = (4.00\text{ m})\sin 20.0° = 1.37\text{ m}$$

Adding component by component yields the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$:

$$C_x = A_x + B_x = 2.50\text{ m} + 3.76\text{ m} = 6.26\text{ m}$$

and

$$C_y = A_y + B_y = 4.33\text{ m} + 1.37\text{ m} = 5.70\text{ m}$$

With these results, we can now find *precise* values for C, the magnitude of vector $\vec{\mathbf{C}}$, and its direction angle θ. In particular,

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(6.26\text{ m})^2 + (5.70\text{ m})^2} = \sqrt{71.7\text{ m}^2} = 8.47\text{ m}$$

and

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{5.70\text{ m}}{6.26\text{ m}}\right) = \tan^{-1}(0.911) = 42.3°$$

Note that these exact values are in rough agreement with the approximate results found by graphical addition.

In the future, we will always add vectors using components—graphical addition is useful primarily as a rough check on the results obtained with components.

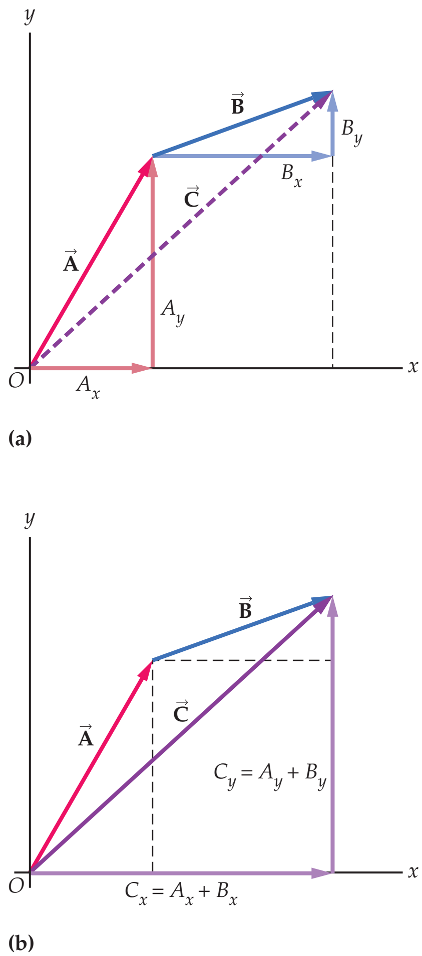


FIGURE 3–13 Component addition of vectors
(a) The x and y components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. **(b)** The x and y components of $\vec{\mathbf{C}}$. Notice that $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

ACTIVE EXAMPLE 3–1 TREASURE HUNT: FIND THE DIRECTION AND MAGNITUDE

What are the magnitude and direction of the total displacement for the treasure hunt illustrated in Figure 3–9? Assume each pace is 0.750 m in length.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

To define a convenient notation, let the first 5 paces be represented by $\vec{\mathbf{A}}$, the next 3 paces by $\vec{\mathbf{B}}$, and the final 4 paces by $\vec{\mathbf{C}}$. The total displacement, then, is $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$.

1. Find the components of $\vec{\mathbf{A}}$:	$A_x = 0$, $A_y = 3.75$ m
2. Find the components of $\vec{\mathbf{B}}$:	$B_x = 2.25$ m, $B_y = 0$
3. Find the components of $\vec{\mathbf{C}}$:	$C_x = 2.12$ m, $C_y = -2.12$ m
4. Sum the components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ to find the components of $\vec{\mathbf{D}}$:	$D_x = 4.37$ m, $D_y = 1.63$ m
5. Determine D and θ:	$D = 4.66$ m, $\theta = 20.5°$

YOUR TURN

If the length of each pace is decreased by a factor of two, to 0.375 m, by what factors do you expect D and θ to change? Verify your answers with a numerical calculation.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Subtracting Vectors

Next, how do we subtract vectors? Suppose, for example, that we would like to determine the vector $\vec{\mathbf{D}}$, where

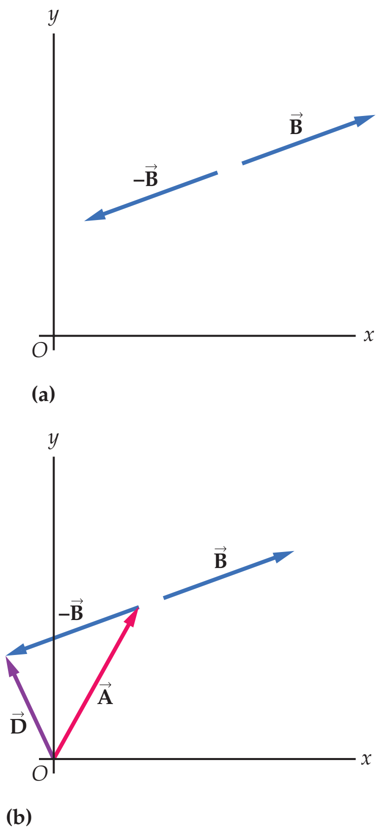
$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$$

and $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are the vectors shown in Figure 3–12. To find $\vec{\mathbf{D}}$, we start by rewriting it as follows:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$$

That is, $\vec{\mathbf{D}}$ is the sum of $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$. Now the negative of a vector has a very simple graphical interpretation:

- The negative of a vector is represented by an arrow of the same length as the original vector, but pointing in the opposite direction. That is, multiplying a vector by minus one *reverses its direction.*



▲ **FIGURE 3–14 Vector subtraction**
(a) The vector $\vec{\mathbf{B}}$ and its negative $-\vec{\mathbf{B}}$.
(b) A vector construction for $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$.

For example, the vectors $\vec{\mathbf{B}}$ and $-\vec{\mathbf{B}}$ are indicated in **Figure 3–14 (a)**. Thus, to subtract $\vec{\mathbf{B}}$ from $\vec{\mathbf{A}}$, simply reverse the direction of $\vec{\mathbf{B}}$ and add it to $\vec{\mathbf{A}}$, as indicated in **Figure 3–14 (b)**.

In terms of components, you subtract vectors by simply subtracting the components. For example, if

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$$

then

$$D_x = A_x - B_x$$

and

$$D_y = A_y - B_y$$

Once the components of $\vec{\mathbf{D}}$ are found, its magnitude and direction angle can be calculated as usual.

EXERCISE 3–5

a. For the vectors given in Figure 3–12, find the components of $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$.
b. Find D and θ and compare with the vector $\vec{\mathbf{D}}$ shown in Figure 3–14 (b).

SOLUTION

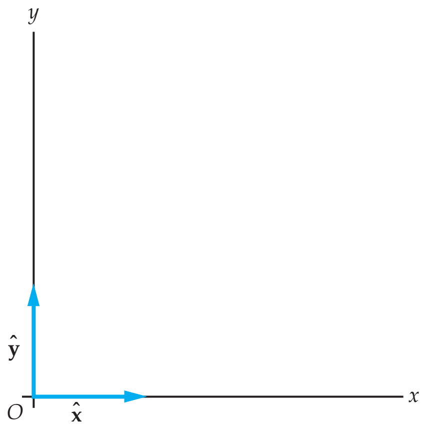
a. $D_x = -1.26$ m, $D_y = 2.96$ m
b. $D = 3.22$ m, $\theta = -66.9° + 180° = 113°$. In Figure 3–14 (b) we see that $\vec{\mathbf{D}}$ is shorter than $\vec{\mathbf{B}}$, which has a magnitude of 4.00 m, and its direction angle is somewhat greater than 90°, in agreement with our numerical results.

3–4 Unit Vectors

Unit vectors provide a convenient way of expressing an arbitrary vector in terms of its components, as we shall see. But first, let's define what we mean by a unit vector. In particular, the unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are defined to be dimensionless vectors of unit magnitude pointing in the positive x and y directions, respectively:

- The x unit vector, $\hat{\mathbf{x}}$, is a dimensionless vector of unit length pointing in the positive x direction.
- The y unit vector, $\hat{\mathbf{y}}$, is a dimensionless vector of unit length pointing in the positive y direction.

Figure 3–15 shows $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ on a two-dimensional coordinate system. Since unit vectors have no physical dimensions—like mass, length, or time—they are used to specify direction only.



▲ **FIGURE 3–15 Unit vectors**
The unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ point in the positive x and y directions, respectively.

Multiplying Unit Vectors by Scalars

To see the utility of unit vectors, consider the effect of multiplying a vector by a scalar. For example, multiplying a vector by 3 increases its magnitude by a factor of 3, but does not change its direction, as shown in **Figure 3–16**. Multiplying by -3 increases the magnitude by a factor of 3 *and* reverses the direction of the vector. This is also shown in Figure 3–16. In the case of unit vectors—which have a magnitude of 1 and are dimensionless—multiplication by a scalar results in a vector with the same magnitude and dimensions as the scalar.

For example, if a vector $\vec{\mathbf{A}}$ has the scalar components $A_x = 5$ m and $A_y = 3$ m, we can write it as

$$\vec{\mathbf{A}} = (5\text{ m})\hat{\mathbf{x}} + (3\text{ m})\hat{\mathbf{y}}$$

We refer to the quantities $(5\text{ m})\hat{\mathbf{x}}$ and $(3\text{ m})\hat{\mathbf{y}}$ as the x and y **vector components** of the vector $\vec{\mathbf{A}}$. In general, an arbitrary two-dimensional vector $\vec{\mathbf{A}}$ can always be written as the sum of a vector component in the x direction and a vector component in the y direction:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

This is illustrated in **Figure 3–17 (a)**. An equivalent way of representing the vector components of a vector is illustrated in **Figure 3–17 (b)**. In this case we see that the vector components are the *projection* of a vector onto the x and y axes. The sign of the vector components is positive if they point in the positive x or y direction, and negative if they point in the negative x or y direction. This is how vector components will generally be shown in later chapters.

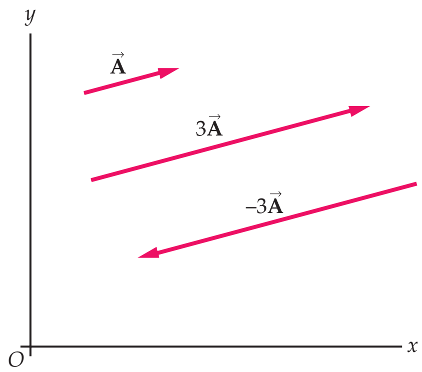
Finally, note that vector addition and subtraction are straightforward with unit vector notation:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}}$$

and

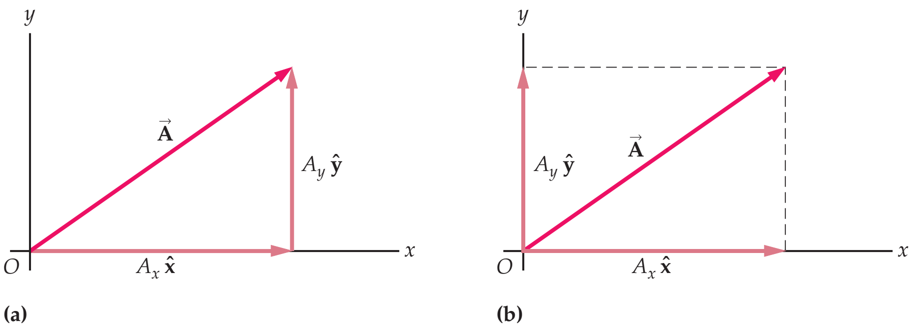
$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = (A_x - B_x)\hat{\mathbf{x}} + (A_y - B_y)\hat{\mathbf{y}}$$

Clearly, unit vectors provide a useful way to keep track of the x and y components of a vector.



▲ **FIGURE 3–16 Multiplying a vector by a scalar**

Multiplying a vector by a positive scalar different from 1 will change the length of the vector but leave its direction the same. If the vector is multiplied by a negative scalar its direction is reversed.



◀ **FIGURE 3–17 Vector components**

(a) A vector $\vec{\mathbf{A}}$ can be written in terms of unit vectors as $\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$. **(b)** Vector components can be thought of as the projection of the vector onto the x and y axes. This method of representing vector components will be used frequently in subsequent chapters.

3–5 Position, Displacement, Velocity, and Acceleration Vectors

In Chapter 2 we discussed four different one-dimensional vectors: position, displacement, velocity, and acceleration. Each of these quantities had a direction associated with it, indicated by its sign; positive meant in the positive direction, negative meant in the negative direction. Now we consider these vectors again, this time in two dimensions, where the possibilities for direction are not so limited.

Position Vectors

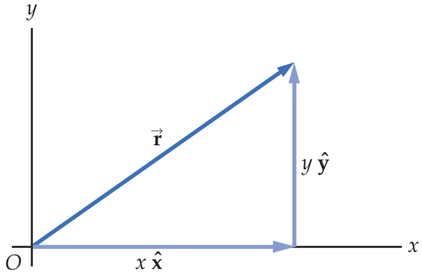
To begin, imagine a two-dimensional coordinate system, as in **Figure 3–18**. Position is indicated by a vector from the origin to the location in question. We refer to the position vector as $\vec{\mathbf{r}}$; its units are meters, m.

Definition: Position Vector, $\vec{\mathbf{r}}$

position vector $= \vec{\mathbf{r}}$ 3–1

SI unit: meter, m

In terms of unit vectors, the position vector is simply $\vec{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$.



▲ **FIGURE 3–18 Position vector**

The position vector $\vec{\mathbf{r}}$ points from the origin to the current location of an object. The x and y vector components of $\vec{\mathbf{r}}$ are $x\hat{\mathbf{x}}$ and $y\hat{\mathbf{y}}$, respectively.

▲ A map can be used to determine the direction and magnitude of the displacement vector from your initial position to your destination.

Displacement Vectors

Now, suppose that initially you are at the location indicated by the position vector $\vec{\mathbf{r}}_\mathrm{i}$, and that later you are at the final position represented by the position vector $\vec{\mathbf{r}}_\mathrm{f}$. Your displacement vector, $\Delta\vec{\mathbf{r}}$, is the change in position:

Definition: Displacement Vector, $\Delta\vec{\mathbf{r}}$

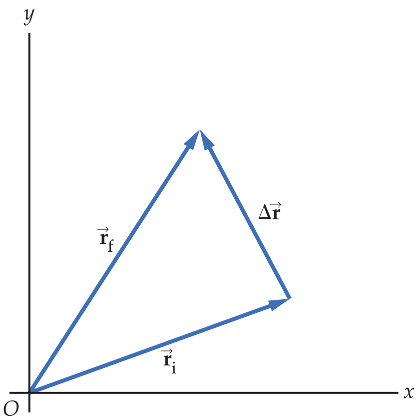
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \qquad 3\text{–}2$$

SI unit: meter, m

Rearranging this definition slightly, we see that

$$\vec{\mathbf{r}}_\mathrm{f} = \vec{\mathbf{r}}_\mathrm{i} + \Delta\vec{\mathbf{r}}$$

That is, the final position is equal to the initial position plus the change in position. This is illustrated in **Figure 3–19**, where we see that $\Delta\vec{\mathbf{r}}$ extends from the head of $\vec{\mathbf{r}}_\mathrm{i}$ to the head of $\vec{\mathbf{r}}_\mathrm{f}$.



▲ **FIGURE 3–19 Displacement vector**

The displacement vector $\Delta\vec{\mathbf{r}}$ is the change in position. It points from the head of the initial position vector $\vec{\mathbf{r}}_\mathrm{i}$ to the head of the final position vector $\vec{\mathbf{r}}_\mathrm{f}$. Thus $\vec{\mathbf{r}}_\mathrm{f} = \vec{\mathbf{r}}_\mathrm{i} + \Delta\vec{\mathbf{r}}$ or $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i}$.

Velocity Vectors

Next, the average velocity vector is defined as the displacement vector $\Delta\vec{\mathbf{r}}$ divided by the elapsed time Δt.

Definition: Average Velocity Vector, $\vec{\mathbf{v}}_\mathrm{av}$

$$\vec{\mathbf{v}}_\mathrm{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad 3\text{–}3$$

SI unit: meter per second, m/s

Since $\Delta\vec{\mathbf{r}}$ is a vector, it follows that $\vec{\mathbf{v}}_\mathrm{av}$ is also a vector; it is the vector $\Delta\vec{\mathbf{r}}$ times the scalar $(1/\Delta t)$. Thus $\vec{\mathbf{v}}_\mathrm{av}$ is parallel to $\Delta\vec{\mathbf{r}}$ and has the units m/s.

EXERCISE 3–6

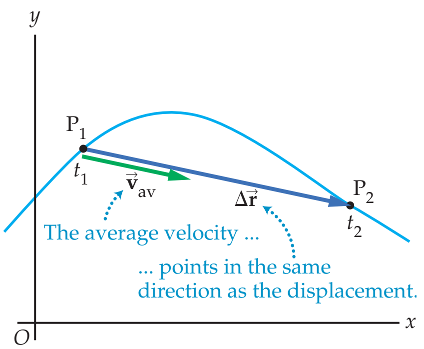
A dragonfly is observed initially at the position $\vec{\mathbf{r}}_\mathrm{i} = (2.00\text{ m})\hat{\mathbf{x}} + (3.50\text{ m})\hat{\mathbf{y}}$. Three seconds later it is at the position $\vec{\mathbf{r}}_\mathrm{f} = (-3.00\text{ m})\hat{\mathbf{x}} + (5.50\text{ m})\hat{\mathbf{y}}$. What was the dragonfly's average velocity during this time?

SOLUTION

$$\vec{\mathbf{v}}_\mathrm{av} = (\vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i})/\Delta t = [(-5.00\text{ m})\hat{\mathbf{x}} + (2.00\text{ m})\hat{\mathbf{y}}]/(3.00\text{ s})$$
$$= (-1.67\text{ m/s})\hat{\mathbf{x}} + (0.667\text{ m/s})\hat{\mathbf{y}}$$

To help visualize $\vec{\mathbf{v}}_\mathrm{av}$, imagine a particle moving in two dimensions along the blue path shown in **Figure 3–20**. If the particle is at point P_1 at time t_1, and at P_2 at time t_2, its displacement is indicated by the vector $\Delta\vec{\mathbf{r}}$. The average velocity is parallel to $\Delta\vec{\mathbf{r}}$, as indicated in Figure 3–20. It makes sense physically that $\vec{\mathbf{v}}_\mathrm{av}$ is parallel to $\Delta\vec{\mathbf{r}}$; after all, on *average* you have moved in the direction of $\Delta\vec{\mathbf{r}}$ during the time from t_1 to t_2. To put it another way, a particle that starts at P_1 at the time t_1 and moves with the velocity $\vec{\mathbf{v}}_\mathrm{av}$ until the time t_2 will arrive in precisely the same location as the particle that follows the blue path.

By considering smaller and smaller time intervals, as in **Figure 3–21**, it is possible to calculate the instantaneous velocity vector:



▲ **FIGURE 3–20 Average velocity vector**

The average velocity, $\vec{\mathbf{v}}_\mathrm{av}$, points in the same direction as the displacement, $\Delta\vec{\mathbf{r}}$, for any given interval of time.

Definition: Instantaneous Velocity Vector, $\vec{\mathbf{v}}$

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad 3\text{–}4$$

SI unit: meter per second, m/s

As can be seen in Figure 3–21, the instantaneous velocity at a given time is tangential to the path of the particle at that time. In addition, the magnitude of the velocity vector is the speed of the particle. Thus, the instantaneous velocity vector tells you both how fast a particle is moving and in what direction.

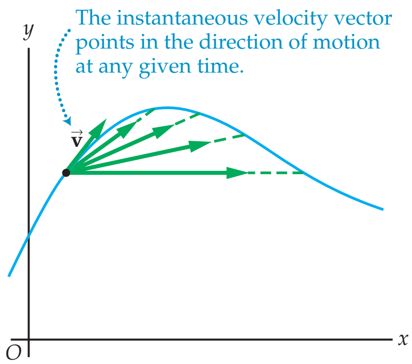


FIGURE 3–21 Instantaneous velocity vector

The instantaneous velocity vector $\vec{\mathbf{v}}$ is obtained by calculating the average velocity vector over smaller and smaller time intervals. In the limit of vanishingly small time intervals, the average velocity approaches the instantaneous velocity, which points in the direction of motion.

EXERCISE 3–7

Find the speed and direction of motion for a rainbow trout whose velocity is $\vec{\mathbf{v}} = (3.7\ \mathrm{m/s})\hat{\mathbf{x}} + (-1.3\ \mathrm{m/s})\hat{\mathbf{y}}$.

SOLUTION

speed $= v = \sqrt{(3.7\ \mathrm{m/s})^2 + (-1.3\ \mathrm{m/s})^2} = 3.9\ \mathrm{m/s}$, $\theta = \tan^{-1}\left(\dfrac{-1.3\ \mathrm{m/s}}{3.7\ \mathrm{m/s}}\right) = -19°$,

that is, 19° below the x axis.

Acceleration Vectors

Finally, the average acceleration vector over an interval of time, Δt, is defined as the change in the velocity vector, $\Delta\vec{\mathbf{v}}$, divided by the scalar Δt.

Definition: Average Acceleration Vector, $\vec{\mathbf{a}}_{\mathrm{av}}$

$$\vec{\mathbf{a}}_{\mathrm{av}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad 3\text{–}5$$

SI unit: meter per second per second, $\mathrm{m/s^2}$

An example is given in **Figure 3–22**, where we show the initial and final velocity vectors corresponding to two different times. Since the change in velocity is defined as

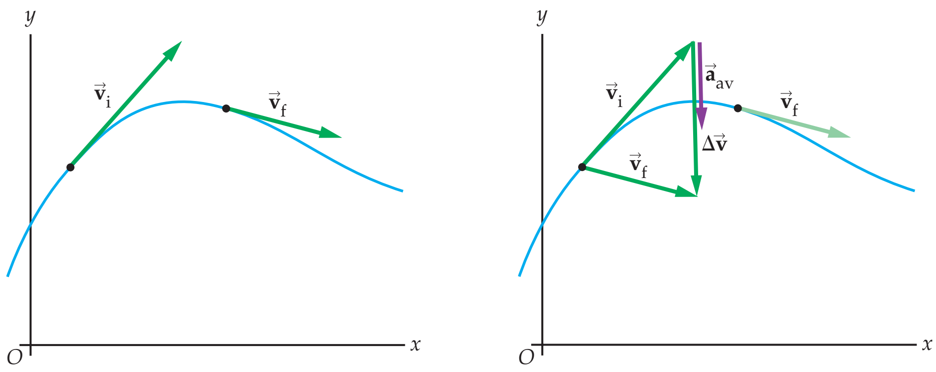
$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_{\mathrm{f}} - \vec{\mathbf{v}}_{\mathrm{i}}$$

it follows that

$$\vec{\mathbf{v}}_{\mathrm{f}} = \vec{\mathbf{v}}_{\mathrm{i}} + \Delta\vec{\mathbf{v}}$$

as indicated in Figure 3–22. Thus, $\Delta\vec{\mathbf{v}}$ is the vector extending from the head of $\vec{\mathbf{v}}_{\mathrm{i}}$ to the head of $\vec{\mathbf{v}}_{\mathrm{f}}$, just as $\Delta\vec{\mathbf{r}}$ extends from the head of $\vec{\mathbf{r}}_{\mathrm{i}}$ to the head of $\vec{\mathbf{r}}_{\mathrm{f}}$ in Figure 3–19. The direction of $\vec{\mathbf{a}}_{\mathrm{av}}$ is the direction of $\Delta\vec{\mathbf{v}}$, as shown in **Figure 3–22(b)**.

Can an object accelerate if its speed is constant? Absolutely—if its direction changes. Consider a car driving with a constant speed on a circular track, as

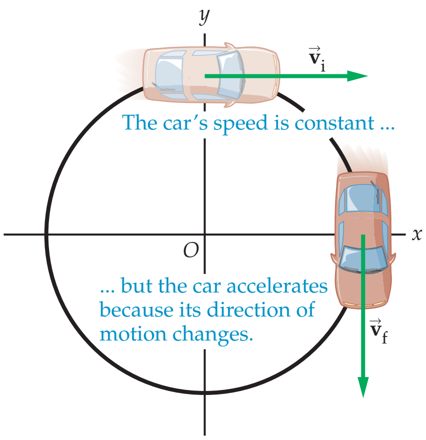


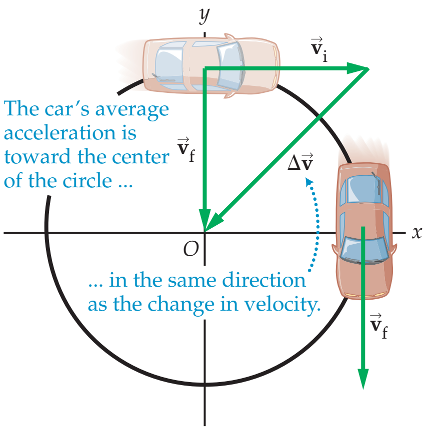
(a) The instantaneous velocity at two different times

(b) The average acceleration points in the same direction as the change in velocity

FIGURE 3–22 Average acceleration vector

(a) As a particle moves along the blue path its velocity changes in magnitude and direction. At the time t_{i} the velocity is $\vec{\mathbf{v}}_{\mathrm{i}}$; at the time t_{f} the velocity is $\vec{\mathbf{v}}_{\mathrm{f}}$. **(b)** The average acceleration vector $\vec{\mathbf{a}}_{\mathrm{av}} = \Delta\vec{\mathbf{v}}/\Delta t$ points in the direction of the change in velocity vector $\Delta\vec{\mathbf{v}}$. We obtain $\Delta\vec{\mathbf{v}}$ by moving $\vec{\mathbf{v}}_{\mathrm{f}}$ so that its tail coincides with the tail of $\vec{\mathbf{v}}_{\mathrm{i}}$, and then drawing the arrow that connects the head of $\vec{\mathbf{v}}_{\mathrm{i}}$ to the head of $\vec{\mathbf{v}}_{\mathrm{f}}$. Note that $\vec{\mathbf{a}}_{\mathrm{av}}$ need not point in the direction of motion, and in general it doesn't.





▲ **FIGURE 3–23 Average acceleration for a car traveling in a circle with constant speed**
Although the speed of this car never changes, it is still accelerating—due to the change in its direction of motion. For the time interval depicted, the car's average acceleration is in the direction of $\Delta\vec{v}$, which is toward the center of the circle. (As we shall see in Chapter 6, the car's acceleration is always toward the center of the circle.)

shown in **Figure 3–23**. Suppose that the initial velocity of the car is $\vec{v}_i = (12 \text{ m/s})\hat{x}$, and that 10.0 s later its final velocity is $\vec{v}_f = (-12 \text{ m/s})\hat{y}$. Note that the speed is 12 m/s in each case, but the velocity is different because the *direction* has changed. Calculating the average acceleration, we find a nonzero acceleration:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{10.0 \text{ s}}$$
$$= \frac{(-12 \text{ m/s})\hat{y} - (12 \text{ m/s})\hat{x}}{10.0 \text{ s}} = (-1.2 \text{ m/s}^2)\hat{x} + (-1.2 \text{ m/s}^2)\hat{y}$$

Thus, a change in direction is just as important as a change in speed in producing an acceleration. We shall study circular motion in detail in Chapter 6.

Finally, by going to infinitesimally small time intervals, $\Delta t \to 0$, we can define the instantaneous acceleration:

Definition: Instantaneous Acceleration Vector, $\vec{a}$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} \qquad 3\text{–}6$$

SI unit: meter per second per second, m/s^2

ACTIVE EXAMPLE 3–2 FIND THE AVERAGE ACCELERATION

A car is traveling northwest at 9.00 m/s. Eight seconds later it has rounded a corner and is now heading north at 15.0 m/s. What are the magnitude and direction of its average acceleration during those 8.00 seconds?

Let the positive x direction be east, and the positive y direction be north.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write out $\vec{v}_i$:	$\vec{v}_i = (-6.36 \text{ m/s})\hat{x} + (6.36 \text{ m/s})\hat{y}$
2. Write out $\vec{v}_f$:	$\vec{v}_f = (15.0 \text{ m/s})\hat{y}$
3. Calculate $\Delta\vec{v}$:	$\Delta\vec{v} = (6.36 \text{ m/s})\hat{x} + (8.64 \text{ m/s})\hat{y}$
4. Find $\vec{a}_{av}$:	$\vec{a}_{av} = (0.795 \text{ m/s}^2)\hat{x} + (1.08 \text{ m/s}^2)\hat{y}$
5. Determine a_{av} and θ:	$a_{av} = 1.34 \text{ m/s}^2, \theta = 53.6°$ north of east

YOUR TURN

Find the magnitude and direction of the average acceleration if the same change in velocity occurs in 4.00 s rather than 8.00 s.

(Answers to **Your Turn** *problems are given in the back of the book.)*

▶ The velocities of these cyclists change in both magnitude and direction as they slow to negotiate a series of sharp curves and then speed up again. Both kinds of velocity change involve an acceleration.

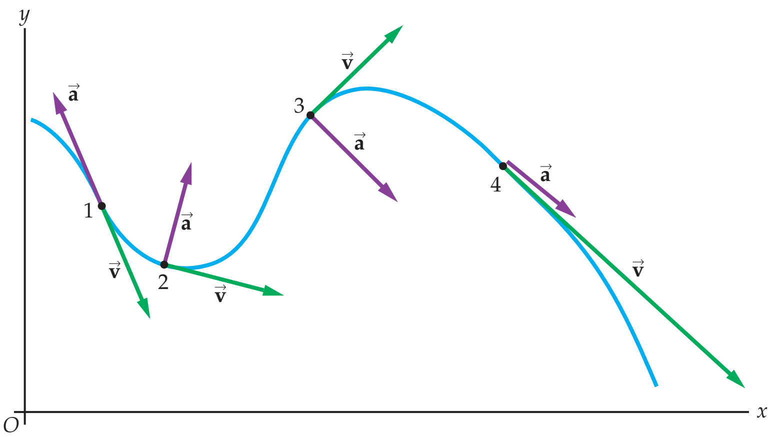


FIGURE 3–24 Velocity and acceleration vectors for a particle moving along a winding path
The acceleration of a particle need not point in the direction of motion. At point (1) the particle is slowing down, at (2) it is turning to the left, at (3) it is turning to the right, and, finally, at point (4) it is speeding up.

Note carefully the following critical distinctions between the velocity vector and the acceleration vector:

- The velocity vector, $\vec{\mathbf{v}}$, is always in the direction of a particle's motion.
- The acceleration vector, $\vec{\mathbf{a}}$, can point in directions other than the direction of motion, and in general it does.

An example of a particle's motion, showing the velocity and acceleration vectors at various times, is presented in **Figure 3–24**.

Note that in all cases the velocity is tangential to the motion, though the acceleration points in various directions. When the acceleration is perpendicular to the velocity of an object, as at points (2) and (3) in Figure 3–24, its speed remains constant while its direction of motion changes. At points (1) and (4) in Figure 3–24 the acceleration is antiparallel (opposite) or parallel to the velocity of the object, respectively. In such cases, the direction of motion remains the same while the speed changes. Throughout the next chapter we shall see further examples of motion in which the velocity and acceleration are in different directions.

3–6 Relative Motion

A good example of the use of vectors is in the description of relative motion. Suppose, for example, that you are standing on the ground as a train goes by at 15.0 m/s, as shown in **Figure 3–25**. Inside the train, a free-riding passenger is walking in the forward direction at 1.2 m/s relative to the train. How fast is the passenger moving relative to you? Clearly, the answer is 1.2 m/s + 15.0 m/s = 16.2 m/s. What if the passenger had been walking with the same speed, but toward the back of the train? In this case, you would see the passenger going by with a speed of −1.2 m/s + 15.0 m/s = 13.8 m/s.

Let's generalize these results. Call the velocity of the *t*rain relative to the *g*round $\vec{\mathbf{v}}_{tg}$, the velocity of the *p*assenger relative to the *t*rain $\vec{\mathbf{v}}_{pt}$, and the velocity of the *p*assenger relative to the *g*round $\vec{\mathbf{v}}_{pg}$. As we saw in the previous paragraph, the velocity of the *p*assenger relative to the *g*round is

$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg} \qquad 3\text{–}7$$

This vector addition is illustrated in **Figure 3–26** for the two cases we discussed.

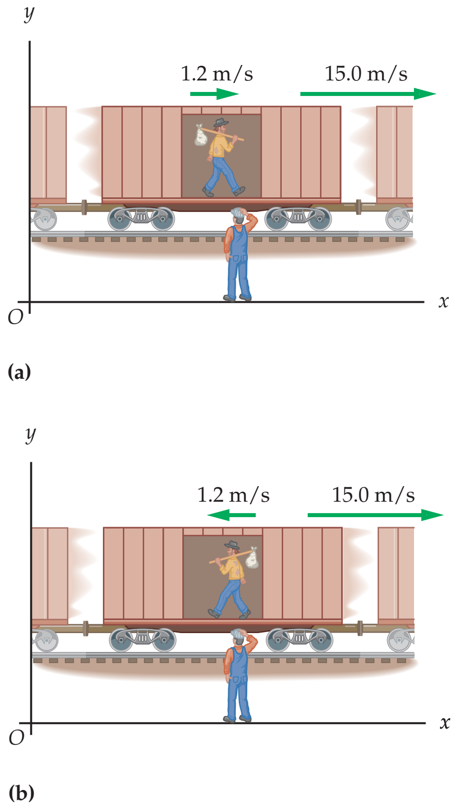
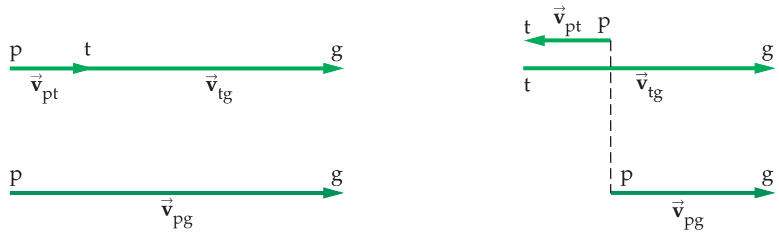
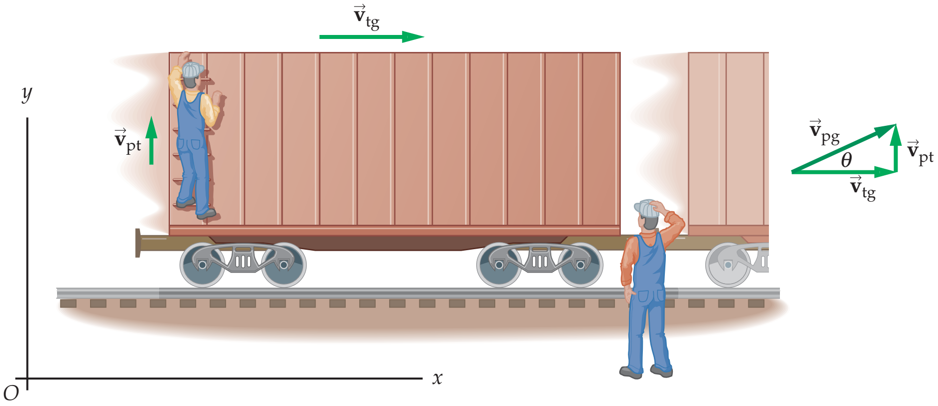


FIGURE 3–25 Relative velocity of a passenger on a train with respect to a person on the ground
(a) The passenger walks toward the front of the train. **(b)** The passenger walks toward the rear of the train.



(a) **(b)**

FIGURE 3–26 Adding velocity vectors
Vector addition to find the velocity of the passenger with respect to the ground for **(a)** Figure 3–25 (a) and **(b)** Figure 3–25 (b).



▲ **FIGURE 3–27 Relative velocity in two dimensions**
A person climbs up a ladder on a moving train with velocity $\vec{\mathbf{v}}_{pt}$ relative to the train. If the train moves relative to the ground with a velocity $\vec{\mathbf{v}}_{tg}$, the velocity of the person on the train relative to the ground is $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$.

Though this example dealt with one-dimensional motion, Equation 3–7 is valid for velocity vectors pointing in arbitrary directions. For example, instead of walking on the car's floor, the passenger might be climbing a ladder to the roof of the car, as in **Figure 3–27**. In this case $\vec{\mathbf{v}}_{pt}$ is vertical, $\vec{\mathbf{v}}_{tg}$ is horizontal, and $\vec{\mathbf{v}}_{pg}$ is simply the vector sum $\vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$.

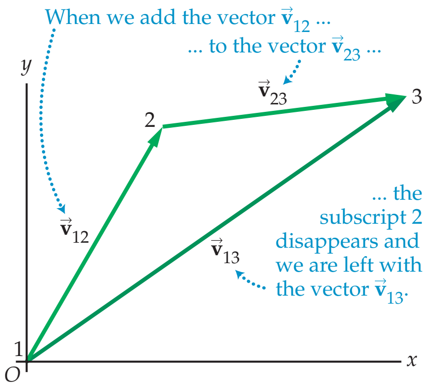
EXERCISE 3–8

Suppose the passenger in Figure 3–27 is climbing a vertical ladder with a speed of 0.20 m/s, and the train is slowly coasting forward at 0.70 m/s. Find the speed and direction of the passenger relative to the ground.

SOLUTION

$$\vec{\mathbf{v}}_{pg} = (0.70\ \mathrm{m/s})\hat{\mathbf{x}} + (0.20\ \mathrm{m/s})\hat{\mathbf{y}};\ \text{thus}$$

$$v_{pg} = \sqrt{(0.70\ \mathrm{m/s})^2 + (0.20\ \mathrm{m/s})^2} = 0.73\ \mathrm{m/s},\ \theta = \tan^{-1}(0.20/0.70) = 16^\circ$$



▲ **FIGURE 3–28 Vector addition used to determine relative velocity**

Note that the subscripts in Equation 3–7 follow a definite pattern. On the left-hand side of the equation we have the subscripts pg. On the right-hand side we have two sets of subscripts, pt and tg; note that a pair of t's has been inserted between the p and the g. This pattern always holds for any relative motion problem, though the subscripts will be different when referring to different objects. Thus, we can say quite generally that

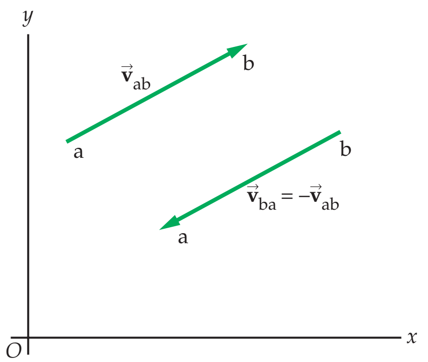
$$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23} \qquad 3\text{–}8$$

where, in the train example, we can identify 1 as the *p*assenger, 2 as the *t*rain, and 3 as the *g*round.

The vector addition in Equation 3–8 is shown in **Figure 3–28**. For convenience in seeing how the subscripts are ordered in the equation, we have labeled the tail of each vector with its first subscript and the head of each vector with its second subscript.

One final note about velocities and their subscripts: Reversing the subscripts reverses the velocity. This is indicated in **Figure 3–29**, where we see that

$$\vec{\mathbf{v}}_{ba} = -\vec{\mathbf{v}}_{ab}$$



▲ **FIGURE 3–29 Reversing the subscripts of a velocity reverses the corresponding velocity vector**

Physically, what we are saying is that if you are riding in a car due *north* at 20 m/s relative to the ground, then the ground, relative to you, is moving due *south* at 20 m/s.

Let's apply these results to a two-dimensional example.

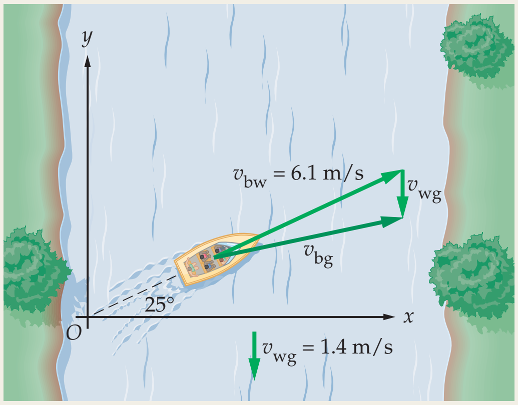
EXAMPLE 3–2 CROSSING A RIVER

REAL-WORLD PHYSICS

You are riding in a boat whose speed relative to the water is 6.1 m/s. The boat points at an angle of 25° upstream on a river flowing at 1.4 m/s. **(a)** What is your velocity relative to the ground? **(b)** Suppose the speed of the boat relative to the water remains the same, but the direction in which it points is changed. What angle is required for the boat to go straight across the river?

PICTURE THE PROBLEM

We choose the x axis to be perpendicular to the river, and the y axis to point upstream. With these choices the velocity of the boat relative to the water is 25° above the x axis. In addition, the velocity of the water relative to the ground has a magnitude of 1.4 m/s and points in the negative y direction.



INTERACTIVE FIGURE

STRATEGY

If the water were still, the boat would move in the direction in which it is pointed. With the water flowing downstream, as shown, the boat will move in a direction closer to the x axis. **(a)** To find the velocity of the boat we use $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with 1 referring to the boat (b), 2 referring to the water (w), and 3 referring to the ground (g). **(b)** To go "straight across the river" means that the velocity of the boat relative to the ground should be in the x direction. Thus, we choose the angle θ that cancels the y component of velocity.

SOLUTION

Part (a)

1. Rewrite $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with $1 \rightarrow$ b, $2 \rightarrow$ w, and $3 \rightarrow$ g: $\quad \vec{\mathbf{v}}_{bg} = \vec{\mathbf{v}}_{bw} + \vec{\mathbf{v}}_{wg}$
2. From our sketch we see that the water flows at 1.4 m/s in the negative y direction relative to the ground: $\quad \vec{\mathbf{v}}_{wg} = (-1.4\ \text{m/s})\hat{\mathbf{y}}$
3. The velocity of the boat relative to the water is given in the problem statement: $$\vec{\mathbf{v}}_{bw} = (6.1\ \text{m/s})\cos 25°\ \hat{\mathbf{x}} + (6.1\ \text{m/s})\sin 25°\ \hat{\mathbf{y}} = (5.5\ \text{m/s})\hat{\mathbf{x}} + (2.6\ \text{m/s})\hat{\mathbf{y}}$$
4. Carry out the vector sum in Step 1 to find $\vec{\mathbf{v}}_{bg}$: $$\vec{\mathbf{v}}_{bg} = (5.5\ \text{m/s})\hat{\mathbf{x}} + (2.6\ \text{m/s} - 1.4\ \text{m/s})\hat{\mathbf{y}} = (5.5\ \text{m/s})\hat{\mathbf{x}} + (1.2\ \text{m/s})\hat{\mathbf{y}}$$

Part (b)

5. To cancel the y component of $\vec{\mathbf{v}}_{bg}$, we choose the angle θ that gives 1.4 m/s for the y component of $\vec{\mathbf{v}}_{bw}$: $\quad (6.1\ \text{m/s})\sin\theta = 1.4\ \text{m/s}$
6. Solve for θ. With this angle, we see that the y component of $\vec{\mathbf{v}}_{bg}$ in Step 4 will be zero: $\quad \theta = \sin^{-1}(1.4/6.1) = 13°$

INSIGHT

(a) Note that the speed of the boat relative to the ground is $\sqrt{(5.5\ \text{m/s})^2 + (1.2\ \text{m/s})^2} = 5.6\ \text{m/s}$, and the direction angle is $\theta = \tan^{-1}(1.2/5.5) = 12°$ upstream. **(b)** The speed of the boat in this case is equal to the x component of its velocity, since the y component is zero. Therefore, its speed is $(6.1\ \text{m/s})\cos 13° = 5.9\ \text{m/s}$.

PRACTICE PROBLEM

Find the speed and direction of the boat relative to the ground if the river flows at 4.5 m/s. [**Answer:** $v_{bg} = 5.8\ \text{m/s}$, $\theta = -19°$. In this case, a person on the ground sees the boat going slowly downstream, even though the boat itself points upstream.]

Some related homework problems: Problem 50, Problem 53, Problem 55

Suppose the problem had been to find the velocity of the boat relative to the water so that it goes straight across the river at 5.0 m/s. That is, we want to find $\vec{\mathbf{v}}_{bw}$ such that $\vec{\mathbf{v}}_{bg} = (5.0\ \text{m/s})\hat{\mathbf{x}}$. One approach is to simply solve $\vec{\mathbf{v}}_{bg} = \vec{\mathbf{v}}_{bw} + \vec{\mathbf{v}}_{wg}$ for $\vec{\mathbf{v}}_{bw}$, which gives

$$\vec{\mathbf{v}}_{bw} = \vec{\mathbf{v}}_{bg} - \vec{\mathbf{v}}_{wg} \qquad 3\text{–}9$$

Another approach is to go back to our general relation, $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ and choose 1 to be the *boat*, 2 to be the *ground*, and 3 to be the *water*. With these substitutions we find

$$\vec{\mathbf{v}}_{bw} = \vec{\mathbf{v}}_{bg} + \vec{\mathbf{v}}_{gw}$$

This is the same as Equation 3–9, since $\vec{\mathbf{v}}_{gw} = -\vec{\mathbf{v}}_{wg}$. In either case, the desired velocity of the boat relative to the water is

$$\vec{\mathbf{v}}_{bw} = (5.0\ \mathrm{m/s})\hat{\mathbf{x}} + (1.4\ \mathrm{m/s})\hat{\mathbf{y}}$$

which corresponds to a speed of 5.2 m/s and a direction angle of 16° upstream.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In Chapter 2 we indicated direction with + and – signs, since only two directions were possible. With the results from this chapter we can now deal with quantities that point in any direction at all.

The vector quantities we have considered so far are position, displacement, velocity, and acceleration. These quantities are important throughout our study of mechanics.

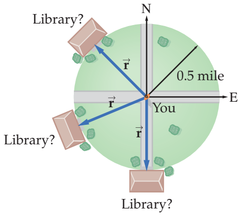
LOOKING AHEAD

In Chapter 4 we will consider kinematics in two dimensions. As we shall see, the vectors developed in this chapter will play a key role in that study. In particular, vectors will allow us to analyze two-dimensional motion as a combination of two completely independent one-dimensional motions.

In Chapter 5 we will introduce one of the most important concepts in all of physics—force. It is a vector quantity. Other important vector quantities to be introduced in later chapters include linear momentum (Chapter 9), angular momentum (Chapter 11), electric field (Chapter 19), and magnetic field (Chapter 22).

CHAPTER SUMMARY

3–1 SCALARS VERSUS VECTORS



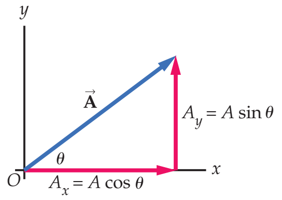
Scalar

A number with appropriate units. Examples of scalar quantities include time and length.

Vector

A quantity with both a magnitude and a direction. Examples include displacement, velocity, and acceleration.

3–2 THE COMPONENTS OF A VECTOR



x Component of Vector $\vec{\mathbf{A}}$

$A_x = A\cos\theta$, where θ is measured relative to the x axis.

y Component of Vector $\vec{\mathbf{A}}$

$A_y = A\sin\theta$, where θ is measured relative to the x axis.

Sign of the Components

A_x is positive if $\vec{\mathbf{A}}$ points in the positive x direction, and negative if it points in the negative x direction. Similar remarks apply to A_y.

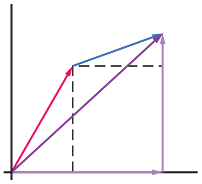
Magnitude of Vector $\vec{\mathbf{A}}$

The magnitude of $\vec{\mathbf{A}}$ is $A = \sqrt{A_x^2 + A_y^2}$.

Direction Angle of Vector $\vec{\mathbf{A}}$

The direction angle of $\vec{\mathbf{A}}$ is $\theta = \tan^{-1}(A_y/A_x)$, where θ is measured relative to the x axis.

3–3 ADDING AND SUBTRACTING VECTORS

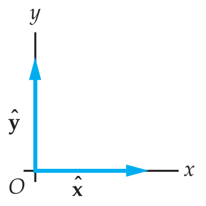


Graphical Method
To add $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, place them so that the tail of $\vec{\mathbf{B}}$ is at the head of $\vec{\mathbf{A}}$. The sum $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the arrow from the tail of $\vec{\mathbf{A}}$ to the head of $\vec{\mathbf{B}}$. See Figure 3–8.

To find $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, place $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$ head-to-tail and draw an arrow from the tail of $\vec{\mathbf{A}}$ to the head of $-\vec{\mathbf{B}}$. See Figure 3–14.

Component Method
If $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. If $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, then $C_x = A_x - B_x$ and $C_y = A_y - B_y$.

3–4 UNIT VECTORS

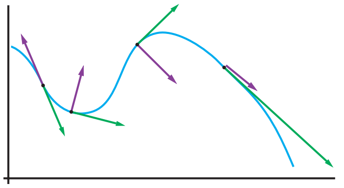


***x* Unit Vector**
Written $\hat{\mathbf{x}}$, the x unit vector is a dimensionless vector of unit length in the positive x direction.

***y* Unit Vector**
Written $\hat{\mathbf{y}}$, the y unit vector is a dimensionless vector of unit length in the positive y direction.

Vector Addition
$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}}$

3–5 POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS



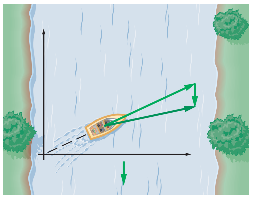
Position Vector
The position vector $\vec{\mathbf{r}}$ points from the origin to a particle's location.

Displacement Vector
The displacement vector $\Delta\vec{\mathbf{r}}$ is the change in position; $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$.

Velocity Vector
The velocity vector $\vec{\mathbf{v}}$ points in the direction of motion and has a magnitude equal to the speed.

Acceleration Vector
The acceleration vector $\vec{\mathbf{a}}$ indicates how quickly and in what direction the velocity is changing. It need not point in the direction of motion.

3–6 RELATIVE MOTION



Velocity of Object 1 Relative to Object 3
$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$, where object 2 can be anything.

Reversing the Subscripts on a Velocity
$\vec{\mathbf{v}}_{12} = -\vec{\mathbf{v}}_{21}$.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Add or subtract vectors.	Resolve the vectors into x and y components, then add or subtract the components.	Active Example 3–1 Exercise 3–5
Calculate the average velocity.	Divide the displacement, $\Delta\vec{\mathbf{r}}$, by the elapsed time, Δt.	Exercise 3–6
Calculate the average acceleration.	Divide the change in velocity, $\Delta\vec{\mathbf{v}}$, by the elapsed time, Δt.	Active Example 3–2
Find the relative velocity of object 1 with respect to object 3.	Use $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with the appropriate choices for 1, 2, and 3.	Example 3–2 Exercise 3–8

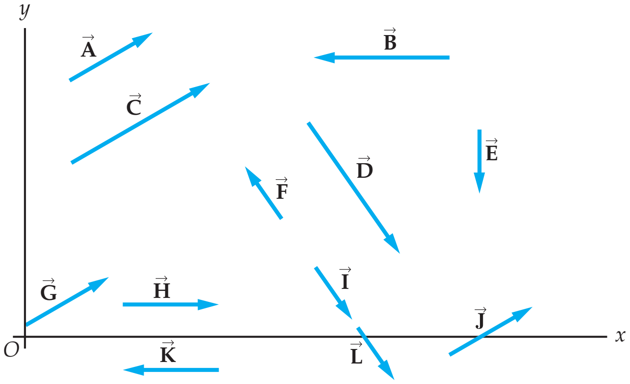
CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. For the following quantities, indicate which is a scalar and which is a vector: **(a)** the time it takes for you to run the 100-yard dash; **(b)** your displacement after running the 100-yard dash; **(c)** your average velocity while running; **(d)** your average speed while running.

2. Which, if any, of the vectors shown in **Figure 3–30** are equal?



▲ **FIGURE 3–30** Conceptual Question 2

3. Given that $\vec{A} + \vec{B} = 0$, **(a)** how does the magnitude of $\vec{B}$ compare with the magnitude of $\vec{A}$? **(b)** How does the direction of $\vec{B}$ compare with the direction of $\vec{A}$?

4. Can a component of a vector be greater than the vector's magnitude?

5. Suppose that $\vec{A}$ and $\vec{B}$ have nonzero magnitude. Is it possible for $\vec{A} + \vec{B}$ to be zero?

6. Can a vector with zero magnitude have one or more components that are nonzero? Explain.

7. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A^2 + B^2 = C^2$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?

8. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A + B = C$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?

9. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A - B = C$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?

10. Vector $\vec{A}$ has x and y components of equal magnitude. What can you say about the possible directions of $\vec{A}$?

11. The components of a vector $\vec{A}$ satisfy the relation $A_x = -A_y \neq 0$. What are the possible directions of $\vec{A}$?

12. Use a sketch to show that two vectors of unequal magnitude cannot add to zero, but that three vectors of unequal magnitude can.

13. Rain is falling vertically downward and you are running for shelter. To keep driest, should you hold your umbrella vertically, tilted forward, or tilted backward? Explain.

14. When sailing, the wind feels stronger when you sail upwind ("beating") than when you are sailing downwind ("running"). Explain.

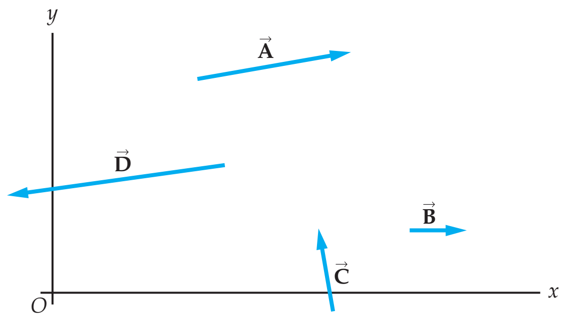
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 3–2 THE COMPONENTS OF A VECTOR

1. • **CE** Suppose that each component of a certain vector is doubled. **(a)** By what multiplicative factor does the magnitude of the vector change? **(b)** By what multiplicative factor does the direction angle of the vector change?

2. • **CE** Rank the vectors in **Figure 3–31** in order of increasing magnitude.



▲ **FIGURE 3–31** Problems 2, 3, and 4

3. • **CE** Rank the vectors in Figure 3–31 in order of increasing value of their x component.

4. • **CE** Rank the vectors in Figure 3–31 in order of increasing value of their y component.

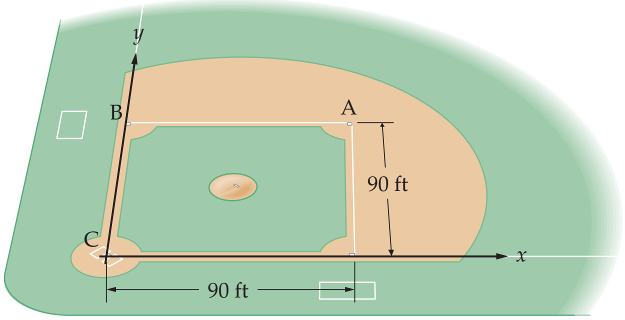
5. • The press box at a baseball park is 32.0 ft above the ground. A reporter in the press box looks at an angle of 15.0° below the horizontal to see second base. What is the horizontal distance from the press box to second base?

6. • You are driving up a long, inclined road. After 1.2 miles you notice that signs along the roadside indicate that your elevation has increased by 530 ft. **(a)** What is the angle of the road above the horizontal? **(b)** How far do you have to drive to gain an additional 150 ft of elevation?

7. • **A One-Percent Grade** A road that rises 1 ft for every 100 ft traveled horizontally is said to have a 1% grade. Portions of the Lewiston grade, near Lewiston, Idaho, have a 6% grade. At what angle is this road inclined above the horizontal?

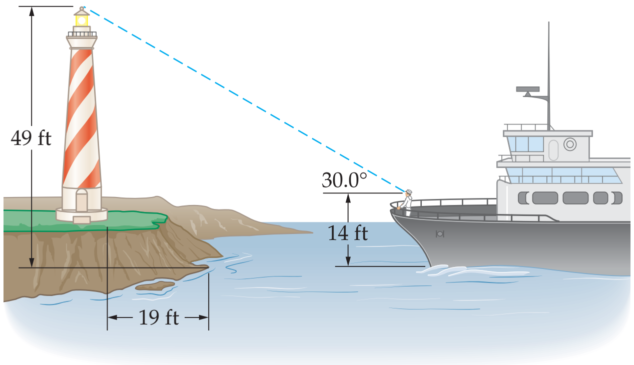
8. • Find the x and y components of a position vector $\vec{r}$ of magnitude $r = 75$ m, if its angle relative to the x axis is **(a)** 35.0° and **(b)** 65.0°.

9. • A baseball "diamond" **(Figure 3–32)** is a square with sides 90 ft in length. If the positive x axis points from home plate to first base, and the positive y axis points from home plate to third base, find the displacement vector of a base runner who has just hit **(a)** a double, **(b)** a triple, or **(c)** a home run.



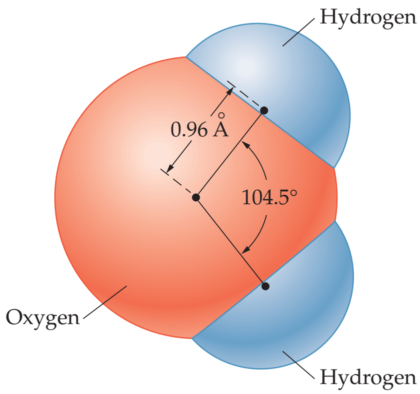
▲ **FIGURE 3–32** Problem 9

10. •• A lighthouse that rises 49 ft above the surface of the water sits on a rocky cliff that extends 19 ft from its base, as shown in **Figure 3–33**. A sailor on the deck of a ship sights the top of the lighthouse at an angle of 30.0° above the horizontal. If the sailor's eye level is 14 ft above the water, how far is the ship from the rocks?



▲ **FIGURE 3–33** Problem 10

11. •• H_2O A water molecule is shown schematically in **Figure 3–34**. The distance from the center of the oxygen atom to the center of a hydrogen atom is 0.96 Å, and the angle between the hydrogen atoms is 104.5°. Find the center-to-center distance between the hydrogen atoms. (1 Å $= 10^{-10}$ m.)



▲ **FIGURE 3–34** Problem 11

12. •• **IP** The x and y components of a vector $\vec{r}$ are $r_x = 14$ m and $r_y = -9.5$ m, respectively. Find **(a)** the direction and **(b)** the magnitude of the vector $\vec{r}$. **(c)** If both r_x and r_y are doubled, how do your answers to parts (a) and (b) change?

13. •• **IP The Longitude Problem** In 1755, John Harrison (1693–1776) completed his fourth precision chronometer, the H4, which eventually won the celebrated Longitude Prize. (For the human drama behind the Longitude Prize, see *Longitude*, by Dava Sobel.) When the minute hand of the H4 indicated 10 minutes past the hour, it extended 3.0 cm in the horizontal direction. **(a)** How long was the H4's minute hand? **(b)** At 10 minutes past the hour, was the extension of the minute hand in the vertical direction more than, less than, or equal to 3.0 cm? Explain. **(c)** Calculate the vertical extension of the minute hand at 10 minutes past the hour.

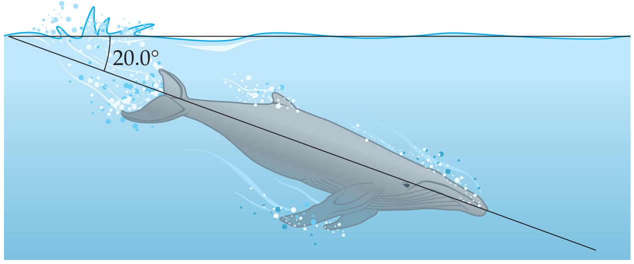
Not just a watch! The Harrison H4. (Problem 13)

14. •• You drive a car 680 ft to the east, then 340 ft to the north. **(a)** What is the magnitude of your displacement? **(b)** Using a sketch, estimate the direction of your displacement. **(c)** Verify your estimate in part (b) with a numerical calculation of the direction.

15. •• Vector $\vec{A}$ has a magnitude of 50 units and points in the positive x direction. A second vector, $\vec{B}$, has a magnitude of 120 units and points at an angle of 70° below the x axis. Which vector has **(a)** the greater x component, and **(b)** the greater y component?

16. •• A treasure map directs you to start at a palm tree and walk due north for 15.0 m. You are then to turn 90° and walk 22.0 m; then turn 90° again and walk 5.00 m. Give the distance from the palm tree, and the direction relative to north, for each of the four possible locations of the treasure.

17. •• A whale comes to the surface to breathe and then dives at an angle of 20.0° below the horizontal (**Figure 3–35**). If the whale continues in a straight line for 150 m, **(a)** how deep is it, and **(b)** how far has it traveled horizontally?



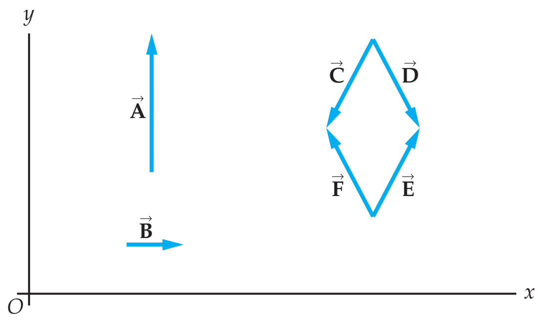
▲ **FIGURE 3–35** Problem 17

SECTION 3–3 ADDING AND SUBTRACTING VECTORS

18. • **CE** Consider the vectors $\vec{A}$ and $\vec{B}$ shown in **Figure 3–36**. Which of the other four vectors in the figure ($\vec{C}$, $\vec{D}$, $\vec{E}$, and $\vec{F}$) best represents the *direction* of **(a)** $\vec{A} + \vec{B}$, **(b)** $\vec{A} - \vec{B}$, and **(c)** $\vec{B} - \vec{A}$,?

19. • **CE** Refer to **Figure 3–36** for the following questions: **(a)** Is the magnitude of $\vec{A} + \vec{D}$ greater than, less than, or equal to the magnitude of $\vec{A} + \vec{E}$? **(b)** Is the magnitude of $\vec{A} + \vec{E}$ greater than, less than, or equal to the magnitude of $\vec{A} + \vec{F}$?

20. • A vector $\vec{A}$ has a magnitude of 40.0 m and points in a direction 20.0° below the positive x axis. A second vector, $\vec{B}$, has a magnitude of 75.0 m and points in a direction 50.0° above the positive x axis. **(a)** Sketch the vectors $\vec{A}$, $\vec{B}$, and $\vec{C} = \vec{A} + \vec{B}$. **(b)** Using the component method of vector addition, find the magnitude and direction of the vector $\vec{C}$.



▲ **FIGURE 3–36** Problems 18 and 19

21. • An air traffic controller observes two airplanes approaching the airport. The displacement from the control tower to plane 1 is given by the vector $\vec{A}$, which has a magnitude of 220 km and points in a direction 32° north of west. The displacement from the control tower to plane 2 is given by the vector $\vec{B}$, which has a magnitude of 140 km and points 65° east of north. **(a)** Sketch the vectors $\vec{A}$, $-\vec{B}$, and $\vec{D} = \vec{A} - \vec{B}$. Notice that $\vec{D}$ is the displacement from plane 2 to plane 1. **(b)** Find the magnitude and direction of the vector $\vec{D}$.

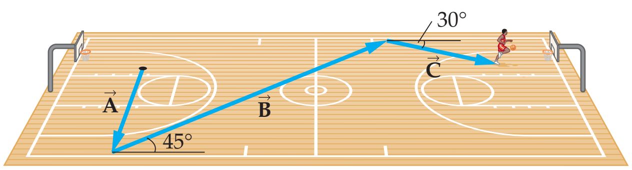
22. • The initial velocity of a car, $\vec{v}_i$, is 45 km/h in the positive x direction. The final velocity of the car, $\vec{v}_f$, is 66 km/h in a direction that points 75° above the positive x axis. **(a)** Sketch the vectors $-\vec{v}_i$, $\vec{v}_f$, and $\Delta\vec{v} = \vec{A}_f - \vec{A}_i$. **(b)** Find the magnitude and direction of the change in velocity, $\Delta\vec{v}$.

23. •• Vector $\vec{A}$ points in the positive x direction and has a magnitude of 75 m. The vector $\vec{C} = \vec{A} + \vec{B}$ points in the positive y direction and has a magnitude of 95 m. **(a)** Sketch $\vec{A}$, $\vec{B}$, and $\vec{C}$. **(b)** Estimate the magnitude and direction of the vector $\vec{B}$. **(c)** Verify your estimate in part (b) with a numerical calculation.

24. •• Vector $\vec{A}$ points in the negative x direction and has a magnitude of 22 units. The vector $\vec{B}$ points in the positive y direction. **(a)** Find the magnitude of $\vec{B}$ if $\vec{A} + \vec{B}$ has a magnitude of 37 units. **(b)** Sketch $\vec{A}$ and $\vec{B}$.

25. •• Vector $\vec{A}$ points in the negative y direction and has a magnitude of 5 units. Vector $\vec{B}$ has twice the magnitude and points in the positive x direction. Find the direction and magnitude of **(a)** $\vec{A} + \vec{B}$, **(b)** $\vec{A} - \vec{B}$, and **(c)** $\vec{B} - \vec{A}$.

26. •• A basketball player runs down the court, following the path indicated by the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in **Figure 3–37**. The magnitudes of these three vectors are $A = 10.0$ m, $B = 20.0$ m, and $C = 7.0$ m. Find the magnitude and direction of the net displacement of the player using **(a)** the graphical method and **(b)** the component method of vector addition. Compare your results.



▲ **FIGURE 3–37** Problem 26

SECTION 3–4 UNIT VECTORS

27. • A particle undergoes a displacement $\Delta\vec{r}$ of magnitude 54 m in a direction 42° below the x axis. Express $\Delta\vec{r}$ in terms of the unit vectors $\hat{x}$ and $\hat{y}$.

28. • A vector has a magnitude of 3.50 m and points in a direction that is 145° counterclockwise from the x axis. Find the x and y components of this vector.

29. • A vector $\vec{A}$ has a length of 6.1 m and points in the negative x direction. Find **(a)** the x component and **(b)** the magnitude of the vector $-3.7\,\vec{A}$.

30. • The vector $-5.2\,\vec{A}$ has a magnitude of 34 m and points in the positive x direction. Find **(a)** the x component and **(b)** the magnitude of the vector $\vec{A}$.

31. • Find the direction and magnitude of the vectors.

(a) $\vec{A} = (5.0\text{ m})\hat{x} + (-2.0\text{ m})\hat{y}$,

(b) $\vec{B} = (-2.0\text{ m})\hat{x} + (5.0\text{ m})\hat{y}$, and **(c)** $\vec{A} + \vec{B}$.

32. • Find the direction and magnitude of the vectors.

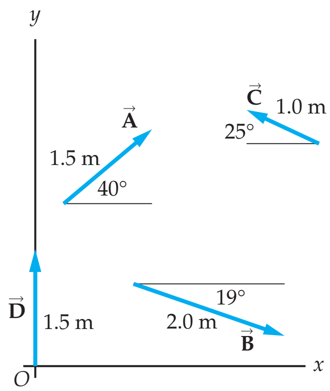
(a) $\vec{A} = (25\text{ m})\hat{x} + (-12\text{ m})\hat{y}$,

(b) $\vec{B} = (2.0\text{ m})\hat{x} + (15\text{ m})\hat{y}$, and **(c)** $\vec{A} + \vec{B}$.

33. • For the vectors given in Problem 32, express **(a)** $\vec{A} - \vec{B}$ and **(b)** $\vec{B} - \vec{A}$ in unit vector notation.

34. • Express each of the vectors in **Figure 3–38** in unit vector notation.

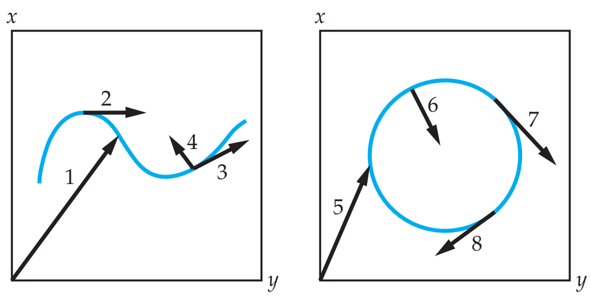
35. •• Referring to the vectors in Figure 3–38, express the sum $\vec{A} + \vec{B} + \vec{C}$ in unit vector notation.



▲ **FIGURE 3–38** Problems 34 and 35

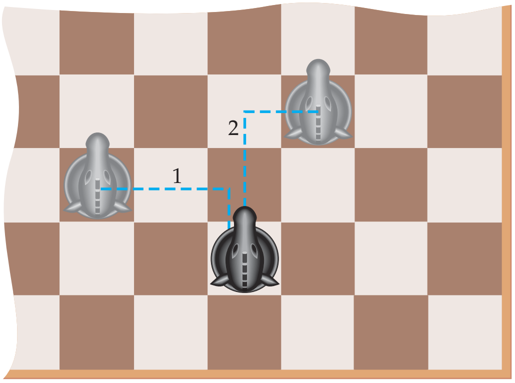
SECTION 3–5 POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

36. • **CE** The blue curves shown in **Figure 3–39** display the constant-speed motion of two different particles in the x-y plane. For each of the eight vectors in Figure 3–39, state whether it is **(a)** a position vector, **(b)** a velocity vector, or **(c)** an acceleration vector for the particles.



▲ **FIGURE 3–39** Problem 36

37. • **IP Moving the Knight** Two of the allowed chess moves for a knight are shown in **Figure 3–40**. **(a)** Is the magnitude of displacement 1 greater than, less than, or equal to the magnitude of displacement 2? Explain. **(b)** Find the magnitude and direction of the knight's displacement for each of the two moves. Assume that the checkerboard squares are 3.5 cm on a side.



▲ **FIGURE 3–40** Problem 37

38. • **IP** In its daily prowl of the neighborhood, a cat makes a displacement of 120 m due north, followed by a 72-m displacement due west. **(a)** Find the magnitude and direction of the displacement required for the cat to return home. **(b)** If, instead, the cat had first prowled 72 m west and then 120 m north, how would this affect the displacement needed to bring it home? Explain.

39. • If the cat in Problem 38 takes 45 minutes to complete the 120-m displacement and 17 minutes to complete the 72-m displacement, what are the magnitude and direction of its average velocity during this 62-minute period of time?

40. • What are the direction and magnitude of your total displacement if you have traveled due west with a speed of 27 m/s for 125 s, then due south at 14 m/s for 66 s?

41. •• You drive a car 1500 ft to the east, then 2500 ft to the north. If the trip took 3.0 minutes, what were the direction and magnitude of your average velocity?

42. •• **IP** A jogger runs with a speed of 3.25 m/s in a direction 30.0° above the x axis. **(a)** Find the x and y components of the jogger's velocity. **(b)** How will the velocity components found in part (a) change if the jogger's speed is halved?

43. •• You throw a ball upward with an initial speed of 4.5 m/s. When it returns to your hand 0.92 s later, it has the same speed in the downward direction (assuming air resistance can be ignored). What was the average acceleration vector of the ball?

44. •• A skateboarder rolls from rest down an inclined ramp that is 15.0 m long and inclined above the horizontal at an angle of $\theta = 20.0°$. When she reaches the bottom of the ramp 3.00 s later her speed is 10.0 m/s. Show that the average acceleration of the skateboarder is $g \sin \theta$, where $g = 9.81\ \text{m/s}^2$.

45. •• Consider a skateboarder who starts from rest at the top of a ramp that is inclined at an angle of 17.5° to the horizontal. Assuming that the skateboarder's acceleration is $g \sin 17.5°$, find his speed when he reaches the bottom of the ramp in 3.25 s.

46. ••• **IP The Position of the Moon** Relative to the center of the Earth, the position of the Moon can be approximated by

$$\vec{\mathbf{r}} = (3.84 \times 10^8\ \text{m})\,\{\cos[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{x}} + \sin[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{y}}\}$$

where t is measured in seconds. **(a)** Find the magnitude and direction of the Moon's average velocity between $t = 0$ and $t = 7.38$ days. (This time is one-quarter of the 29.5 days it takes the Moon to complete one orbit.) **(b)** Is the instantaneous speed of the Moon greater than, less than, or the same as the average speed found in part (a)? Explain.

47. ••• **The Velocity of the Moon** The velocity of the Moon relative to the center of the Earth can be approximated by

$$\vec{\mathbf{v}} = (945\ \text{m/s})\,\{-\sin[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{x}} + \cos[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{y}}\}$$

where t is measured in seconds. To approximate the instantaneous acceleration of the Moon at $t = 0$, calculate the magnitude and direction of the average acceleration between the times **(a)** $t = 0$ and $t = 0.100$ days and **(b)** $t = 0$ and $t = 0.0100$ days. (The time required for the Moon to complete one orbit is 29.5 days.)

SECTION 3–6 RELATIVE MOTION

48. • **CE** The accompanying photo shows a KC-10A Extender using a boom to refuel an aircraft in flight. If the velocity of the KC-10A is 125 m/s due east relative to the ground, what is the velocity of the aircraft being refueled relative to **(a)** the ground, and **(b)** the KC-10A?

Air-to-air refueling. (Problem 48)

49. • As an airplane taxies on the runway with a speed of 16.5 m/s, a flight attendant walks toward the tail of the plane with a speed of 1.22 m/s. What is the flight attendant's speed relative to the ground?

50. • Referring to part (a) of Example 3–2, find the time it takes for the boat to reach the opposite shore if the river is 35 m wide.

51. •• As you hurry to catch your flight at the local airport, you encounter a moving walkway that is 85 m long and has a speed of 2.2 m/s relative to the ground. If it takes you 68 s to cover 85 m when walking on the ground, how long will it take you to cover the same distance on the walkway? Assume that you walk with the same speed on the walkway as you do on the ground.

52. •• In Problem 51, how long would it take you to cover the 85-m length of the walkway if, once you get on the walkway, you immediately turn around and start walking in the opposite direction with a speed of 1.3 m/s relative to the walkway?

53. •• **IP** The pilot of an airplane wishes to fly due north, but there is a 65-km/h wind blowing toward the east. **(a)** In what direction should the pilot head her plane if its speed relative to the air is 340 km/h? **(b)** Draw a vector diagram that illustrates your result in part (a). **(c)** If the pilot decreases the air speed of the plane, but still wants to head due north, should the angle found in part (a) be increased or decreased?

54. •• A passenger walks from one side of a ferry to the other as it approaches a dock. If the passenger's velocity is 1.50 m/s due north relative to the ferry, and 4.50 m/s at an angle of 30.0° west of north relative to the water, what are the direction and magnitude of the ferry's velocity relative to the water?

55. •• You are riding on a Jet Ski at an angle of 35° upstream on a river flowing with a speed of 2.8 m/s. If your velocity relative to the ground is 9.5 m/s at an angle of 20.0° upstream, what is the speed of the Jet Ski relative to the water? (*Note*: Angles are measured relative to the x axis shown in Example 3–2.)

56. •• **IP** In Problem 55, suppose the Jet Ski is moving at a speed of 12 m/s relative to the water. **(a)** At what angle must you point the Jet Ski if your velocity relative to the ground is to be perpendicular to the shore of the river? **(b)** If you increase the speed of the Jet Ski relative to the water, does the angle in part (a) increase, decrease, or stay the same? Explain. (*Note:* Angles are measured relative to the x axis shown in Example 3–2.)

57. ••• **IP** Two people take identical Jet Skis across a river, traveling at the same speed relative to the water. Jet Ski A heads directly across the river and is carried downstream by the current before reaching the opposite shore. Jet Ski B travels in a direction that is 35° upstream and arrives at the opposite shore directly across from the starting point. **(a)** Which Jet Ski reaches the opposite shore in the least amount of time? **(b)** Confirm your answer to part (a) by finding the ratio of the time it takes for the two Jet Skis to cross the river. (*Note:* Angles are measured relative to the x axis shown in Example 3–2.)

GENERAL PROBLEMS

58. • **CE Predict/Explain** Consider the vectors $\vec{\mathbf{A}} = (1.2\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (-3.4\text{ m})\hat{\mathbf{x}}$. **(a)** Is the magnitude of vector $\vec{\mathbf{A}}$ greater than, less than, or equal to the magnitude of vector $\vec{\mathbf{B}}$? **(b)** Choose the *best explanation* from among the following:
I. The number 3.4 is greater than the number 1.2.
II. The component of $\vec{\mathbf{B}}$ is negative.
III. The vector $\vec{\mathbf{A}}$ points in the positive x direction.

59. • **CE Predict/Explain** Two vectors are defined as follows: $\vec{\mathbf{A}} = (-2.2\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (1.4\text{ m})\hat{\mathbf{y}}$. **(a)** Is the magnitude of $1.4\,\vec{\mathbf{A}}$ greater than, less than, or equal to the magnitude of $2.2\,\vec{\mathbf{B}}$? **(b)** Choose the *best explanation* from among the following:
I. The vector $\vec{\mathbf{A}}$ has a negative component.
II. A number and its negative have the same magnitude.
III. The vectors $1.4\,\vec{\mathbf{A}}$ and $2.2\,\vec{\mathbf{B}}$ point in opposite directions.

60. • You slide a box up a loading ramp that is 10.0 ft long. At the top of the ramp the box has risen a height of 3.00 ft. What is the angle of the ramp above the horizontal?

61. • Find the direction and magnitude of the vector $2\vec{\mathbf{A}} + \vec{\mathbf{B}}$, where $\vec{\mathbf{A}} = (12.1\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (-32.2\text{ m})\hat{\mathbf{y}}$.

62. •• **CE** The components of a vector $\vec{\mathbf{A}}$ satisfy $A_x < 0$ and $A_y < 0$. Is the direction angle of $\vec{\mathbf{A}}$ between 0° and 90°, between 90° and 180°, between 180° and 270°, or between 270° and 360°?

63. •• **CE** The components of a vector $\vec{\mathbf{B}}$ satisfy $B_x > 0$ and $B_y < 0$. Is the direction angle of $\vec{\mathbf{B}}$ between 0° and 90°, between 90° and 180°, between 180° and 270°, or between 270° and 360°?

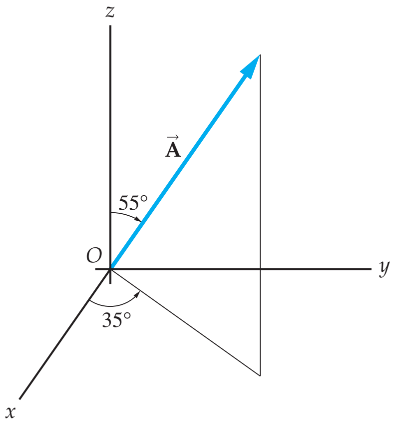
64. •• It is given that $\vec{\mathbf{A}} - \vec{\mathbf{B}} = (-51.4\text{ m})\hat{\mathbf{x}}$, $\vec{\mathbf{C}} = (62.2\text{ m})\hat{\mathbf{x}}$, and $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (13.8\text{ m})\hat{\mathbf{x}}$. Find the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

65. •• **IP** Two students perform an experiment with a train and a ball. Michelle rides on a flatcar pulled at 8.35 m/s by a train on a straight, horizontal track; Gary stands at rest on the ground near the tracks. When Michelle throws the ball with an initial angle of 65.0° above the horizontal, from her point of view, Gary sees the ball rise straight up and back down above a fixed point on the ground. **(a)** Did Michelle throw the ball toward the front of the train or toward the rear of the train? Explain. **(b)** What was the initial speed of Michelle's throw? **(c)** What was the initial speed of the ball as seen by Gary?

66. •• An off-roader explores the open desert in her Hummer. First she drives 25° west of north with a speed of 6.5 km/h for 15 minutes, then due east with a speed of 12 km/h for 7.5 minutes. She completes the final leg of her trip in 22 minutes. What are the direction and speed of travel on the final leg? (Assume her speed is constant on each leg, and that she returns to her starting point at the end of the final leg.)

67. •• Find the x, y, and z components of the vector $\vec{\mathbf{A}}$ shown in **Figure 3–41**, given that $A = 65$ m.

68. •• A football is thrown horizontally with an initial velocity of $(16.6\text{ m/s})\hat{\mathbf{x}}$. Ignoring air resistance, the average acceleration of the football over any period of time is $(-9.81\text{ m/s}^2)\hat{\mathbf{y}}$. **(a)** Find the velocity vector of the ball 1.75 s after it is thrown. **(b)** Find the magnitude and direction of the velocity at this time.

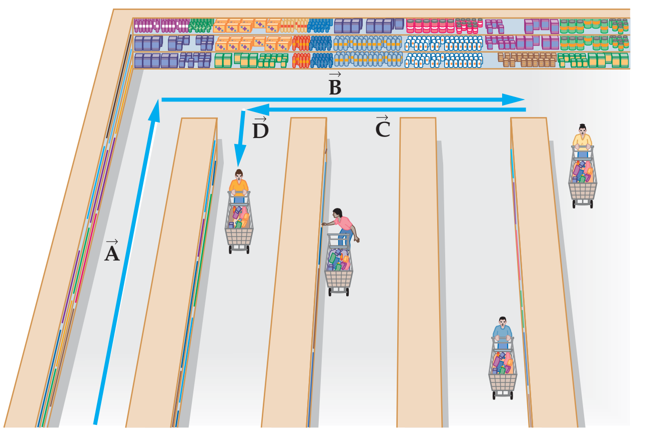


▲ **FIGURE 3–41** Problem 67

69. •• As a function of time, the velocity of the football described in Problem 68 can be written as $\vec{\mathbf{v}} = (16.6\text{ m/s})\hat{\mathbf{x}} - [(9.81\text{ m/s}^2)t]\hat{\mathbf{y}}$. Calculate the average acceleration vector of the football for the time periods **(a)** $t = 0$ to $t = 1.00$ s, **(b)** $t = 0$ to $t = 2.50$ s, and **(c)** $t = 0$ to $t = 5.00$ s. (If the acceleration of an object is constant, its average acceleration is the same for all time periods.)

70. •• Two airplanes taxi as they approach the terminal. Plane 1 taxies with a speed of 12 m/s due north. Plane 2 taxies with a speed of 7.5 m/s in a direction 20° north of west. **(a)** What are the direction and magnitude of the velocity of plane 1 relative to plane 2? **(b)** What are the direction and magnitude of the velocity of plane 2 relative to plane 1?

71. •• A shopper at the supermarket follows the path indicated by vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$, and $\vec{\mathbf{D}}$ in **Figure 3–42**. Given that the



▲ **FIGURE 3–42** Problem 71

vectors have the magnitudes $A = 51$ ft, $B = 45$ ft, $C = 35$ ft, and $D = 13$ ft, find the total displacement of the shopper using **(a)** the graphical method and **(b)** the component method of vector addition. Give the direction of the displacement relative to the direction of vector $\vec{\mathbf{A}}$.

72. •• Initially, a particle is moving at 4.10 m/s at an angle of 33.5° above the horizontal. Two seconds later, its velocity is 6.05 m/s at an angle of 59.0° below the horizontal. What was the particle's average acceleration during these 2.00 seconds?

73. •• A passenger on a stopped bus notices that rain is falling vertically just outside the window. When the bus moves with constant velocity, the passenger observes that the falling raindrops are now making an angle of 15° with respect to the vertical. **(a)** What is the ratio of the speed of the raindrops to the speed of the bus? **(b)** Find the speed of the raindrops, given that the bus is moving with a speed of 18 m/s.

74. •• **A Big Clock** The clock that rings the bell known as Big Ben has an hour hand that is 9.0 feet long and a minute hand that is 14 feet long, where the distance is measured from the center of the clock to the tip of each hand. What is the tip-to-tip distance between these two hands when the clock reads 12 minutes after four o'clock?

75. •• **IP** Suppose we orient the x axis of a two-dimensional coordinate system along the beach at Waikiki. Waves approaching the beach have a velocity relative to the shore given by $\vec{\mathbf{v}}_{\mathrm{ws}} = (1.3 \text{ m/s})\hat{\mathbf{y}}$. Surfers move more rapidly than the waves, but at an angle to the beach. The angle is chosen so that the surfers approach the shore with the same speed as the waves. **(a)** If a surfer has a speed of 7.2 m/s relative to the water, what is her direction of motion relative to the positive x axis? **(b)** What is the surfer's velocity relative to the wave? **(c)** If the surfer's speed is increased, will the angle in part (a) increase or decrease? Explain.

76. ••• **IP** Referring to Example 3–2, **(a)** what heading must the boat have if it is to land directly across the river from its starting point? **(b)** How much time is required for this trip if the river is 25.0 m wide? **(c)** Suppose the speed of the boat is increased, but it is still desired to land directly across from the starting point. Should the boat's heading be more upstream, more downstream, or the same as in part (a)? Explain.

77. ••• Vector $\vec{\mathbf{A}}$ points in the negative x direction. Vector $\vec{\mathbf{B}}$ points at an angle of 30.0° above the positive x axis. Vector $\vec{\mathbf{C}}$ has a magnitude of 15 m and points in a direction 40.0° below the positive x axis. Given that $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$, find the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

78. ••• As two boats approach the marina, the velocity of boat 1 relative to boat 2 is 2.15 m/s in a direction 47.0° east of north. If boat 1 has a velocity that is 0.775 m/s due north, what is the velocity (magnitude and direction) of boat 2?

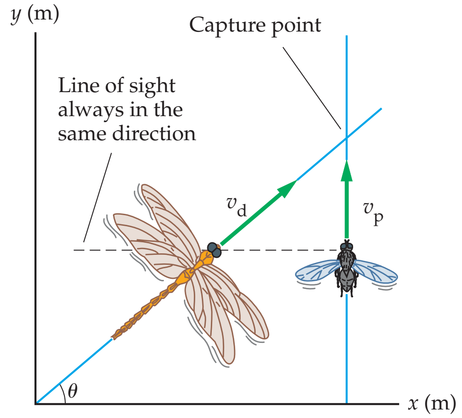
PASSAGE PROBLEMS

BIO Motion Camouflage in Dragonflies

Dragonflies, whose ancestors were once the size of hawks, have prowled the skies in search of small flying insects for over 250 million years. Faster and more maneuverable than any other insect, they even fold their front two legs in flight and tuck them behind their head to be as streamlined as possible. They also employ an intriguing stalking strategy known as "motion camouflage" to approach their prey almost undetected.

The basic idea of motion camouflage is for the dragonfly to move in such a way that the line of sight from the prey to the dragonfly is always in the same direction. Moving in this way, the dragonfly appears almost motionless to its prey, as if it were an object at infinity. Eventually the prey notices the dragonfly has grown in size and is therefore closer, but by that time it's too late for it to evade capture.

A typical capture scenario is shown in **Figure 3–43**, where the prey moves in the positive y direction with the constant speed $v_{\mathrm{p}} = 0.750$ m/s, and the dragonfly moves at an angle $\theta = 48.5°$ to the x axis with the constant speed v_{d}. If the dragonfly chooses its speed correctly, the line of sight from the prey to the dragonfly will always be in the same direction—parallel to the x axis in this case.



▲ **FIGURE 3–43** Problems 79, 80, 81, and 82

79. • What speed must the dragonfly have if the line of sight, which is parallel to the x axis initially, is to remain parallel to the x axis?

A. 0.562 m/s **B.** 0.664 m/s
C. 1.00 m/s **D.** 1.13 m/s

80. • Suppose the dragonfly now approaches its prey along a path with $\theta > 48.5°$, but it still keeps the line of sight parallel to the x axis. Is the speed of the dragonfly in this new case greater than, less than, or equal to its speed in Problem 79?

81. • What is the correct "motion camouflage" speed of approach for a dragonfly pursuing its prey at the angle $\theta = 68.5°$?

A. 0.295 m/s **B.** 0.698 m/s
C. 0.806 m/s **D.** 2.05 m/s

82. •• If the dragonfly approaches its prey with a speed of 0.950 m/s, what angle θ is required to maintain a constant line of sight parallel to the x axis?

A. 37.9° **B.** 38.3°
C. 51.7° **D.** 52.1°

INTERACTIVE PROBLEMS

83. •• **IP Referring to Example 3–2** Suppose the speed of the boat relative to the water is 7.0 m/s. **(a)** At what angle to the x axis must the boat be headed if it is to land directly across the river from its starting position? **(b)** If the speed of the boat relative to the water is increased, will the angle needed to go directly across the river increase, decrease, or stay the same? Explain.

84. ••• **Referring to Example 3–2** Suppose the boat has a speed of 6.7 m/s relative to the water, and that the dock on the opposite shore of the river is at the location $x = 55$ m and $y = 28$ m relative to the starting point of the boat. **(a)** At what angle relative to the x axis must the boat be pointed in order to reach the other dock? **(b)** With the angle found in part (a), what is the speed of the boat relative to the ground?