Kirchoffs Circuit Law

We saw in the Resistors tutorial that a single equivalent resistance, (R_T) can be found when two or more resistors are connected together in either series, parallel or combinations of both, and that these circuits obey Ohm's Law.

However, sometimes in complex circuits such as bridge or T networks, we can not simply use Ohm's Law alone to find the voltages or currents circulating within the circuit. For these types of calculations we need certain rules which allow us to obtain the circuit equations and for this we can use **Kirchoffs Circuit Law**.

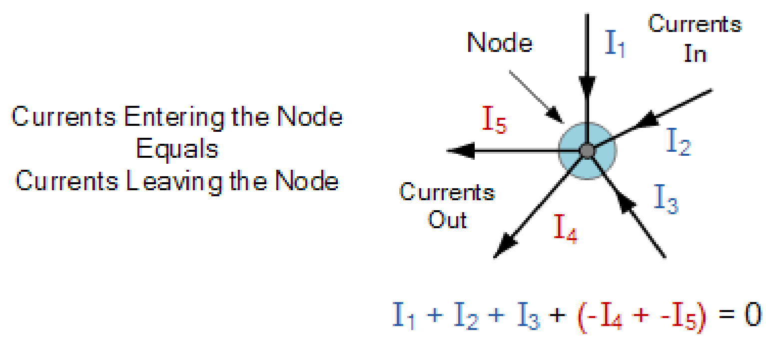
In 1845, a German physicist, **Gustav Kirchoff** developed a pair or set of rules or laws which deal with the conservation of current and energy within Electrical Circuits

These two rules are commonly known as: *Kirchoffs Circuit Laws* with one of Kirchoffs laws dealing with the current flowing around a closed circuit, **Kirchoffs Current Law, (KCL)** while the other law deals with the voltage sources present in a closed circuit, **Kirchoffs Voltage Law, (KVL)**.

Kirchoffs First Law – The Current Law, (KCL)

Kirchoffs Current Law or KCL, states that the "*total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node*". In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero, $I_{(exiting)} + I_{(entering)} = 0$. This idea by Kirchoff is commonly known as the **Conservation of Charge**.

Kirchoffs Current Law



Here, the 3 currents entering the node, I_1, I_2, I_3 are all positive in value and the 2 currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;

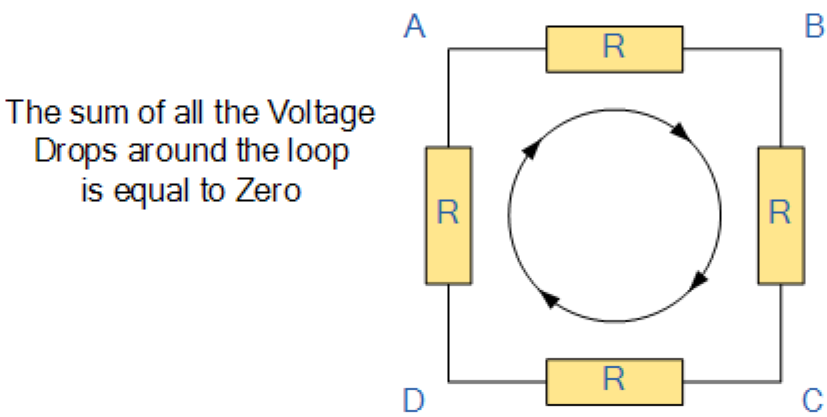
$I_1 + I_2 + I_3 - I_4 - I_5 = 0$

The term **Node** in an electrical circuit generally refers to a connection or junction of two or more current carrying paths or elements such as cables and components. Also for current to flow either in or out of a node a closed circuit path must exist. We can use Kirchoff's current law when analysing parallel circuits.

Kirchoffs Second Law – The Voltage Law, (KVL)

Kirchoffs Voltage Law or KVL, states that "*in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop*" which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchoff is known as the **Conservation of Energy**.

Kirchoffs Voltage Law



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Starting at any point in the loop continue in the **same direction** noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero. We can use Kirchoff's voltage law when analysing series circuits.

When analysing either DC circuits or AC circuits using **Kirchoffs Circuit Laws** a number of definitions and terminologies are used to describe the parts of the circuit being analysed such as: node, paths, branches, loops and meshes. These terms are used frequently in circuit analysis so it is important to understand them.

Common DC Circuit Theory Terms:

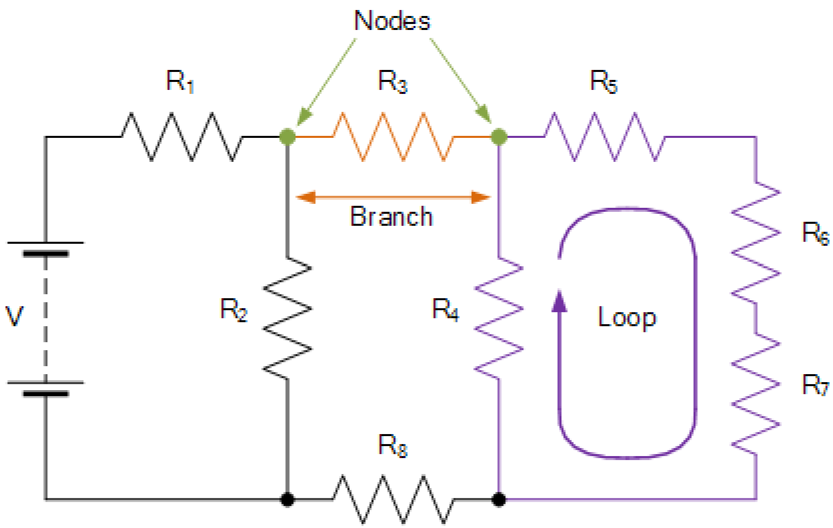
- • Circuit – a circuit is a closed loop conducting path in which an electrical current flows.
- • Path – a single line of connecting elements or sources.
- • Node – a node is a junction, connection or terminal within a circuit were two or more circuit elements are connected or joined together giving a connection point between two or more branches. A node is indicated by a dot.
- • Branch – a branch is a single or group of components such as resistors or a source which are connected between two nodes.
- • Loop – a loop is a simple closed path in a circuit in which no circuit element or node is encountered more than once.
- • Mesh – a mesh is a single open loop that does not have a closed path. There are no components inside a mesh.

Note that:

Components are said to be connected in Series if the same current flows through component.

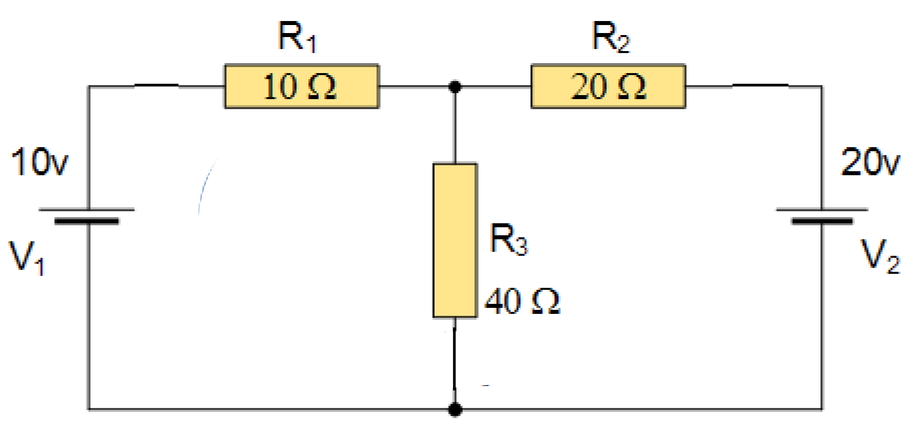
Components are said to be connected in Parallel if the same voltage is applied across them.

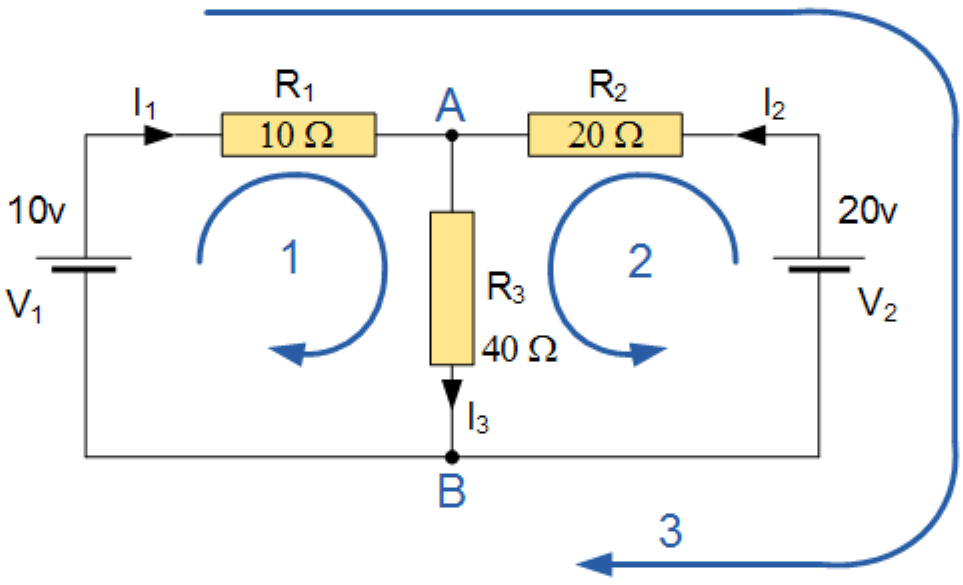
A Typical DC Circuit



Kirchoffs Circuit Law Example No1

Find the current flowing in the 40Ω Resistor, R_3





The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using **Kirchoffs Current Law**, **KCL** the equations are given as;

At node A : $I_1 + I_2 = I_3$

At node B : $I_3 = I_1 + I_2$

Using **Kirchoffs Voltage Law**, **KVL** the equations are given as;

Loop 1 is given as : $10 = R_1 \text{ x } I_1 + R_3 \text{ x } I_3 = 10I_1 + 40I_3$

Loop 2 is given as : $20 = R_2 \text{ x } I_2 + R_3 \text{ x } I_3 = 20I_2 + 40I_3$

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$

As I_3 is the sum of $I_1 + I_2$ we can rewrite the equations as;

Eq. No 1 : $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$

Eq. No 2 : $20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$

We now have two “**Simultaneous Equations**” that can be reduced to give us the values of I_1 and I_2

Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amps

Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amps

As : $I_3 = I_1 + I_2$

The current flowing in resistor R_3 is given as : -0.143 + 0.429 = 0.286 Amps

and the voltage across the resistor R_3 is given as : 0.286 x 40 = 11.44 volts

The negative sign for I_1 means that the direction of current flow initially chosen was wrong, but never the less still valid. In fact, the 20v battery is charging the 10v battery.

Application of Kirchoffs Circuit Laws

These two laws enable the Currents and Voltages in a circuit to be found, ie, the circuit is said to be "Analysed", and the basic procedure for using **Kirchoff's Circuit Laws** is as follows:

- **1.** Assume all voltages and resistances are given. (If not label them V1, V2,… R1, R2, etc.)
- **2.** Label each branch with a branch current. (I1, I2, I3 etc.)
- **3.** Find Kirchoff's first law equations for each node.
- **4.** Find Kirchoff's second law equations for each of the independent loops of the circuit.
- **5.** Use Linear simultaneous equations as required to find the unknown currents.

As well as using **Kirchoffs Circuit Law** to calculate the various voltages and currents circulating around a linear circuit, we can also use loop analysis to calculate the currents in each independent loop which helps to reduce the amount of mathematics required by using just Kirchoff's laws. In the next tutorial about DC Theory we will look at Mesh Current Analysis to do just that.