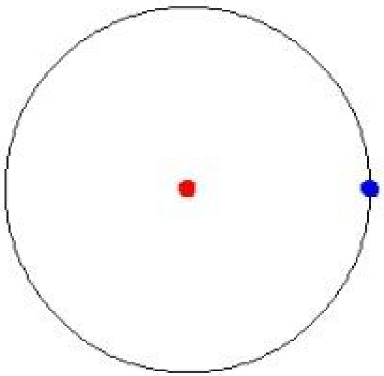
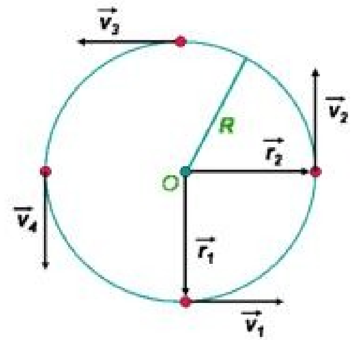
Circular Motion

Speed/Velocity in a Circle



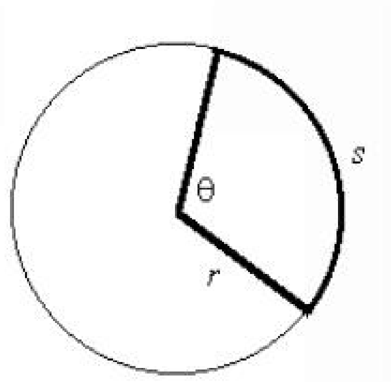
Consider an object moving in a circle around a specific origin. The DISTANCE the object covers in ONE REVOLUTION is called the ______________. The TIME that it takes to cover this distance is called the ______________.

Speed is the MAGNITUDE of the velocity. And while the speed may be constant, the VELOCITY is NOT. Since velocity is a vector with BOTH magnitude AND direction, we see that the direction o the velocity is ALWAYS changing.



We call this velocity, ________________ velocity as its direction is draw TANGENT to the circle.

Centripetal Acceleration



Suppose we had a circle with angle, θ, between 2 radaii. You may recall:

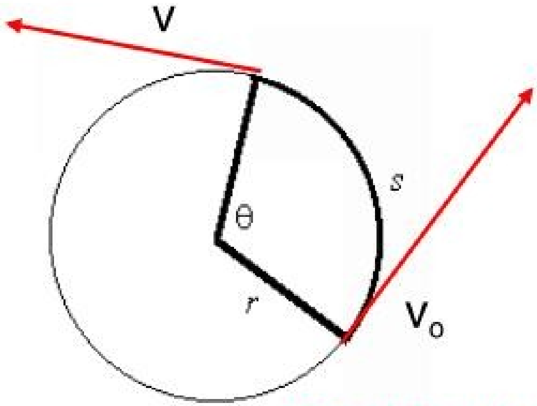
$$\theta = \frac{s}{r}$$

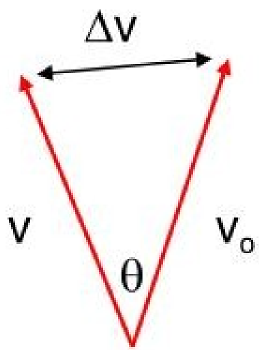
$s =$ arc length in meters

$$\theta = \frac{s}{r} = \frac{\Delta v}{v}$$

$$s = \Delta v t$$

$$\frac{\Delta v t}{r} = \frac{\Delta v}{v}$$





$$\frac{v^2}{r} = \frac{\Delta v}{t} = a_c$$

$a_c = centripetal\ acceleration$

Centripetal means “center seeking” so that means that the acceleration points towards the CENTER of the circle

Drawing the Directions correctly

So for an object traveling in a counter-clockwise path. The velocity would be drawn TANGENT to the circle and the acceleration would be drawn TOWARDS the CENTER.

To find the MAGNITUDES of each we have:

$$v_c = \frac{2\pi r}{T} \qquad a_c = \frac{v^2}{r}$$

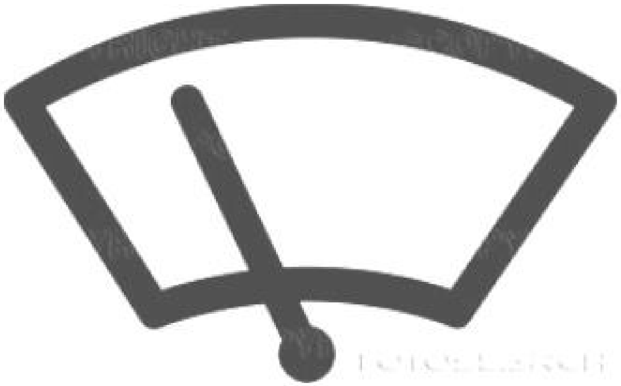
Circular Motion and N.S.L

Recall that according to Newton's Second Law, the acceleration is directly proportional to the Force. If this is true:

Since the acceleration and the force are directly related, the force must ALSO point towards the center. This is called ___________________.

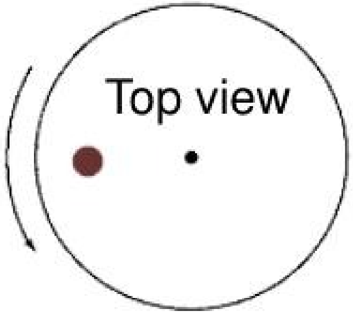
NOTE: The centripetal force is a NET FORCE. It could be represented by one or more forces. So NEVER draw it in an F.B.D.

Examples



The blade of a windshield wiper moves through an angle of 90 degrees in 0.28 seconds. The tip of the blade moves on the arc of a circle that has a radius of 0.76m. What is the magnitude of the centripetal acceleration of the tip of the blade?

Examples



What is the minimum coefficient of static friction necessary to allow a penny to rotate along a 33 1/3 rpm record (diameter= 0.300 m), when the penny is placed at the outer edge of the record?

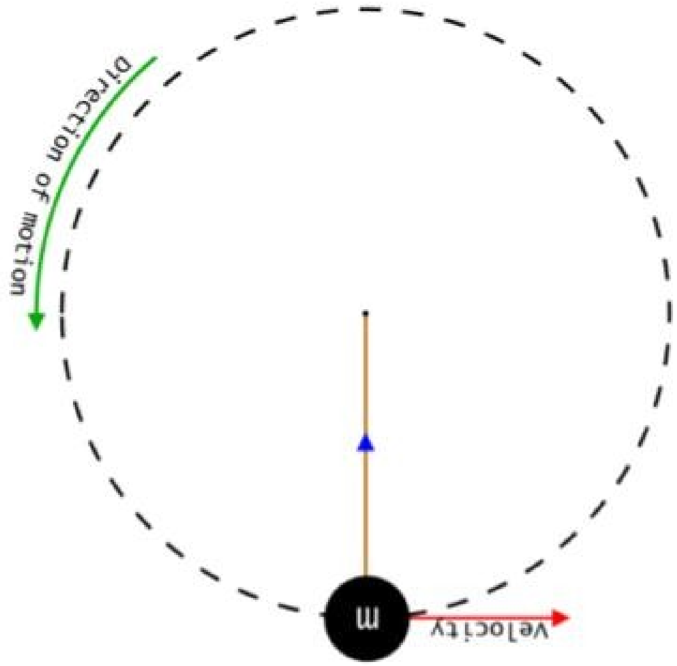
Side view

Examples

The maximum tension that a 0.50 m string can tolerate is 14 N. A 0.25-kg ball attached to this string is being whirled in a vertical circle. What is the maximum speed the ball can have (a) the top of the circle, (b) at the bottom of the circle?

Examples

At the bottom?



Friction as a centripetal force…

A 1200 kg car rounds a corner of radius r = 45 m. If the coefficient of static friction between the tires and the road is $\mu s = 0.82$, what is the greatest speed the car can have in the corner without skidding?

Translational Motion vs. Rotational Motion

Translational motion ___________
____
____

Example: motion of a bullet fired from a gun

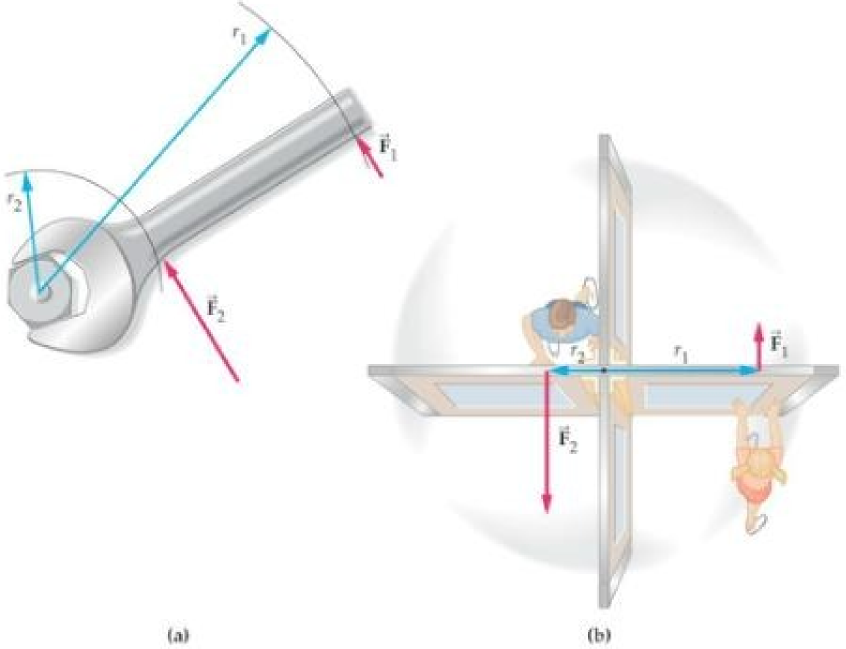
Rotational motion deals only with **rigid bodies**__________
____
____
____
____

Example: a wheel and rotor of a motor

Circular motion is a common type of rotational motion.

Torque, τ (tau)

- analogous to force in that force produces linear acceleration and torque produces rotational, or angular acceleration

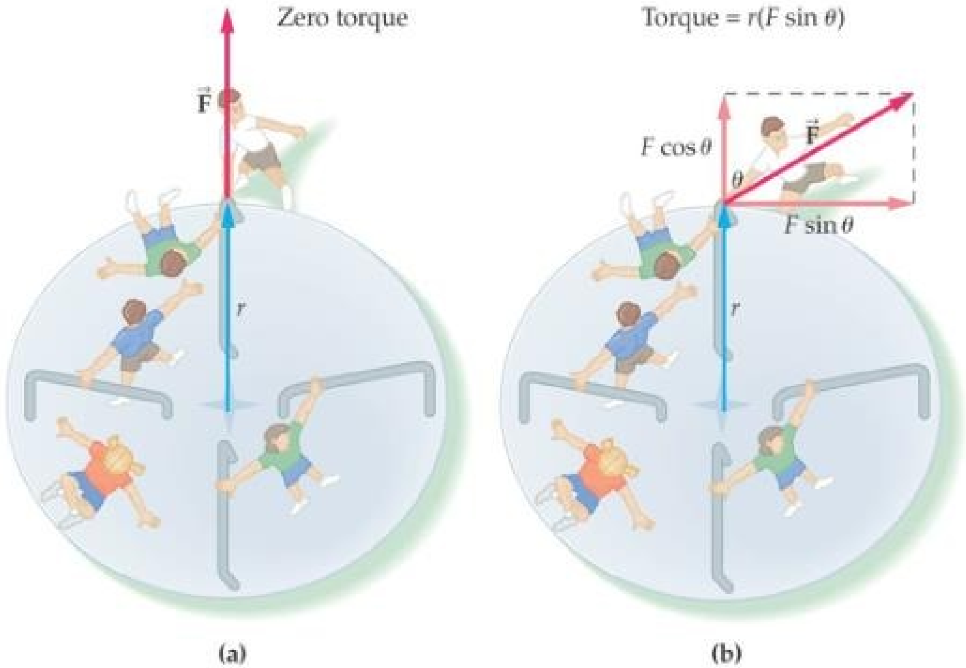


torque =

Line of action – extended line collinear with the force

Lever arm – distance *l* between the line of action and the axis of rotation, measured on the line perpendicular to both.

The "sin θ" term comes from the fact that only forces tangential to the circle (of radius r centered on the axis of rotation) cause torque:



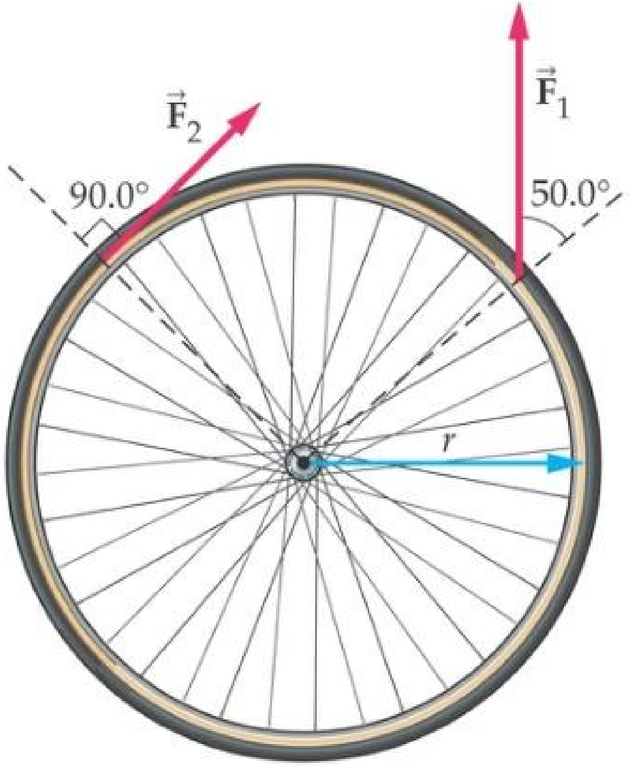
Thus, radial forces do not cause torque.

Direction: the torque is positive if the force tends to produce a counterclockwise rotation about the axis, and negative if the force tends to produce a clockwise rotation.

Units: Nm (Newton-meters)

Example:

- Two forces act on a wheel, as shown below. The wheel is free to rotate without friction, has a radius of 0.42 m, and is initially at rest. Given that F_1 = 12 N and F_2 = 9.5 N, find (a) the torque caused by F_1 and (b) the torque caused by F_2. (c) In which direction does the wheel turn as a result of these two forces?



Equilibrium

- If a rigid body is in equilibrium, its motion does not change (meaning both linear and rotational motion). Thus it has no acceleration of any kind and the net force acting on the object is zero. Also, the net torque is zero.

Conditions for equilibrium of a rigid body:

$$\Sigma F = 0 \text{ and } \Sigma \tau = 0$$

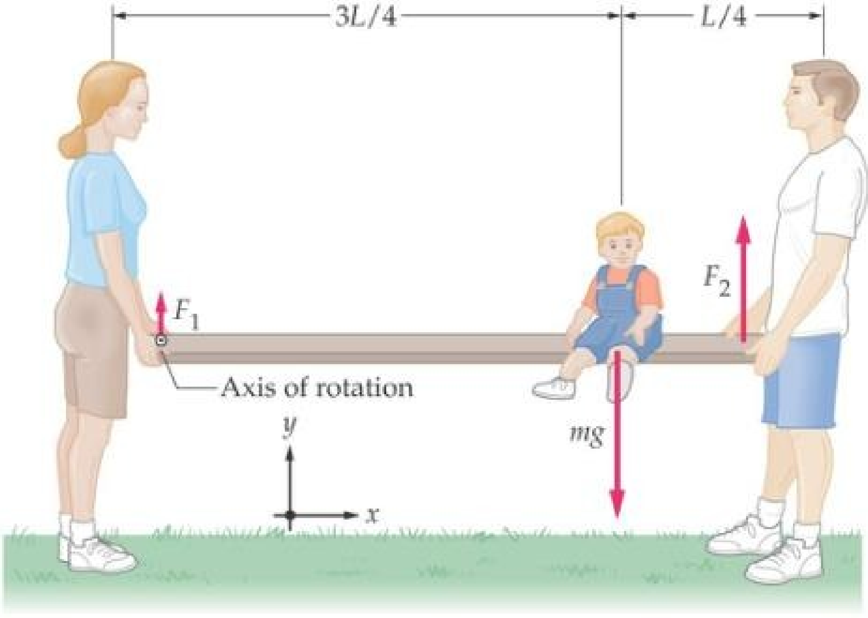
The **sum of the forces** equal to zero is not enough. The **sum of the torques** must also be zero.

Examples of objects in **static equilibrium:** bridges, buildings, playground structures, or sawhorses.

The **torque is always taken about an axis of rotation**. The axis can be placed at any location, but once placed, it must stay put for the rest of the problem. All torques in the problem must be computed about the same axis.

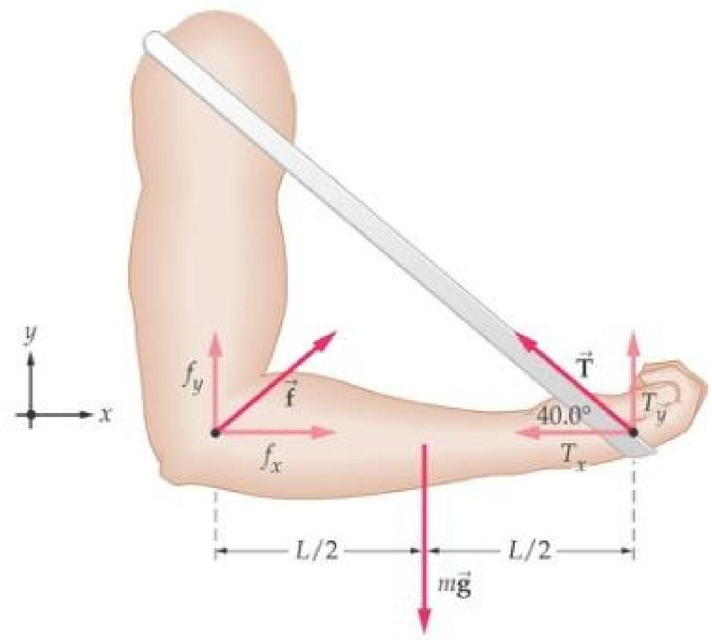
Example:

- A child of mass m is supported on a light plank by his parents, who exert the forces F1 and F2 as indicated. Find the forces required to keep the plank in static equilibrium. Use the right end of the plank as the axis of rotation.



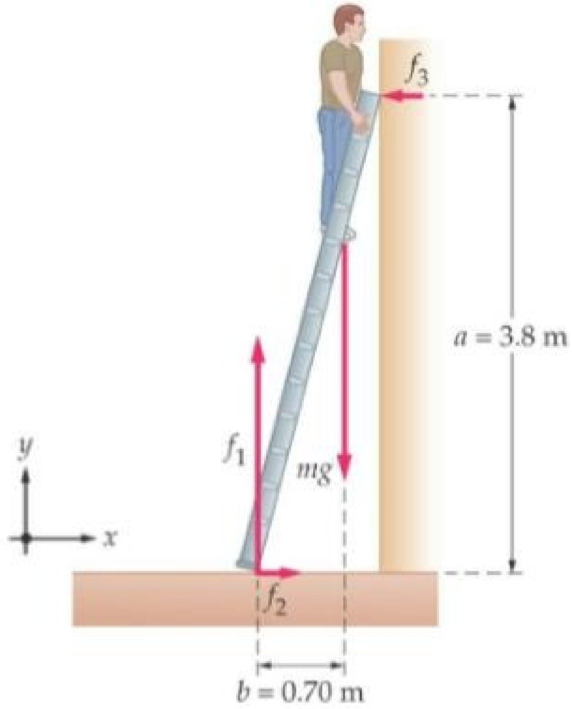
Example:

- A hiker who has broken his forearm rigs a temporary sling using a cord stretching from his shoulder to his hand. The cord holds the forearm level and makes an angle of 40° with the horizontal where it attaches to the hand. Considering the forearm and the hand to be uniform, with a total mass of 1.31 kg and a length of .300 m, find (a) the tension in the cord and (b) the horizontal and vertical components of the force, F, exerted by the humerus (the bone of the upper arm) on the radius and ulna (the bones of the forearm).



Example:

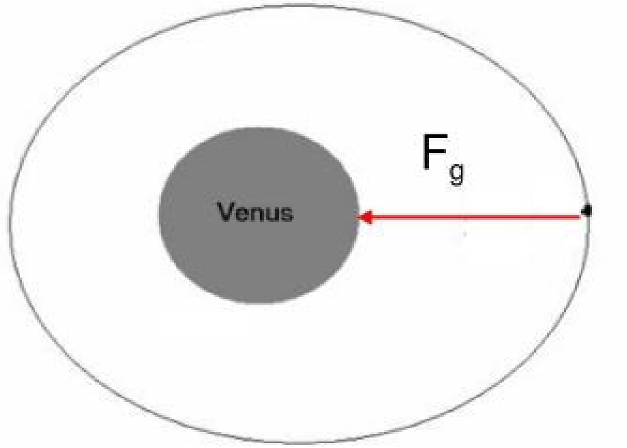
- An 85 kg person stands on a lightweight ladder, as shown. The floor is rough; hence it exerts both a normal force, F1, and a friction force, F2, on the ladder. The wall, on the other hand, is frictionless; it exerts only a normal force, F3. Using the dimensions given in the figure, find the magnitudes of F1, F2, and F3.



Practice Time…

Examples

Venus rotates slowly about its axis, the period being 243 days. The mass of Venus is 4.87 x 10^{24} kg. Determine the radius for a synchronous satellite in orbit around Venus. (assume circular orbit)



$$F_g = F_c \qquad G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r} = v^2 \qquad v_c = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \rightarrow r^3 = \frac{GMT^2}{4\pi^2} \rightarrow r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(6.67x10^{-11})(4.87x10^{24})(2.1x10^{7})^2}{4\pi^2}} = \quad 1.54x10^9 \text{ m}$$