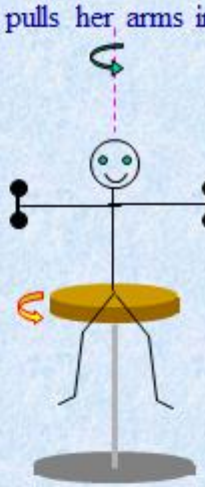
Spinning Ice Skater

Why does a spinning ice skater speed up when she pulls her arms in?

Suppose Dr. Smith (our fearless leader) is sitting on a stool that swivels holding a pair of dumbbells. Her axis of rotation is vertical. With the weights far from that axis, his moment of inertia is large. When she pulls her arms in she's spinning, the weights are closer to the axis, so her moment of inertia gets much smaller. Since $L = I\omega$ and L is conserved, the product of I and ω is a constant. So, when she pulls her arms in, I goes down, ω goes up, and she starts spinning much faster.



$$I\omega = L = I\omega$$

Comparison: Linear & Angular Momentum

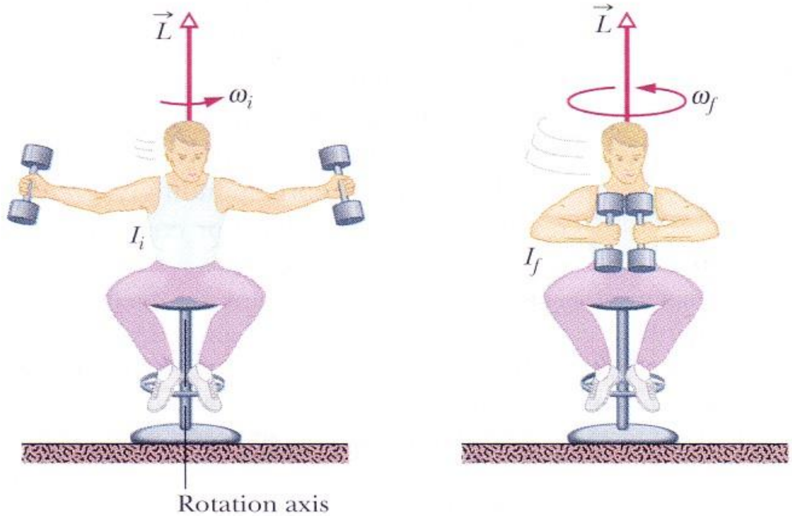
Linear Momentum, ***p***	Angular Momentum, ***L***
• Tendency for a mass to continue moving in a straight line.	• Tendency for a mass to continue rotating.
• Parallel to ***v***.	• Perpendicular to both ***v*** and ***r***.
• A conserved, vector quantity.	• A conserved, vector quantity.
• Magnitude is inertia (mass) times speed.	• Magnitude is rotational inertia times angular speed.
• Net force required to change it.	• Net torque required to change it.
• The greater the mass, the greater the force needed to change momentum.	• The greater the moment of inertia, the greater the torque needed to change angular momentum.

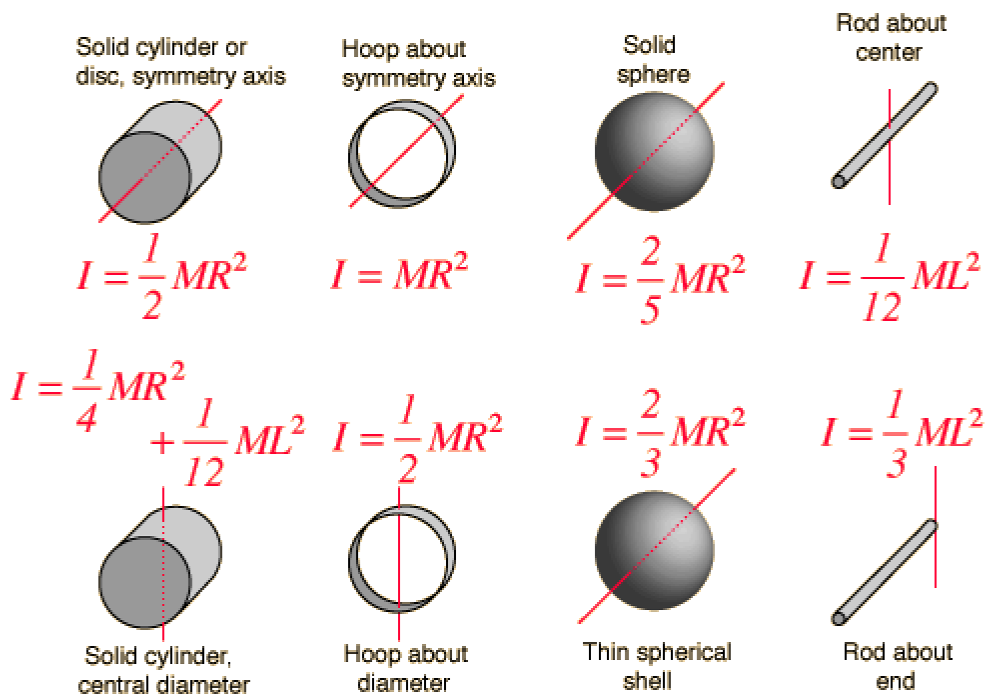
$$\vec{L} = \text{a constant}$$

$$\vec{L}_i = \vec{L}_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$





Rotational-Linear Parallels

	Linear Motion	Rotational Motion	
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

Comparison of linear and rotational motion

Quantity	**Linear Motion**	**Rotational Motion**
displacement	$\boldsymbol{x}$	θ
velocity	$\boldsymbol{v}$	ω
acceleration	$\boldsymbol{a}$	α
inertia	m	$I \sim (\text{constant}) mr^2$
kinetic energy	$K_{trans} = 1/2\ mv^2$	$K_{rot} = 1/2\ I\omega^2$
momentum	$\boldsymbol{p} = m\boldsymbol{v}$	$\boldsymbol{L} = I\omega$
2^{nd} Law (dynamics)	$\Sigma\ \boldsymbol{F} = d\boldsymbol{p}/dt$	$\Sigma\ \tau = dL/dt$
work	$W = F_{\parallel}\ \Delta x$	$W = \tau\ \Delta\theta$
conservation law	$\Delta \boldsymbol{p} = 0$ if $\Sigma \boldsymbol{F}_{ext} = 0$	$\Delta \boldsymbol{L} = 0$ if $\Sigma \tau_{ext} = 0$
impulse	$\boldsymbol{F} \Delta t = \Delta \boldsymbol{p}$	$\tau\ \Delta t = \Delta \boldsymbol{L}$