7 Work and Kinetic Energy

We all know intuitively that motion, energy, and work are somehow related. For example, the chemical energy stored in this pitcher's muscles enables him to do work on a baseball. This means, basically, that he exerts a force on it over a distance. The work done on the ball appears as kinetic energy—the energy of motion—and when the ball is caught, its kinetic energy can in turn do work on the catcher. In this chapter we'll give precise definitions of the concepts of work, kinetic energy, and power, and explore the physical relationships among them.

The concept of force is one of the foundations of physics, as we have seen in the previous two chapters. Equally fundamental, though less obvious, is the idea that a force times the displacement through which it acts is also an important physical quantity. We refer to this quantity as the *work* done by a force.

Now, we all know what work means in everyday life: We get up in the morning and go to work, or we "work up a sweat" as we hike a mountain trail. Later in the day we eat lunch, which gives us the "energy" to continue working or to continue our hike. In this chapter we give a precise physical definition of work, and show how it is related to another important physical quantity—the energy of motion, or *kinetic energy*. When these concepts are extended in the next chapter, we are led to the rather sweeping observation that the total amount of energy in the universe remains constant at all times.

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7–1 Work Done by a Constant Force

In this section we define work—in the physics sense of the word—and apply our definition to a variety of physical situations. We start with the simplest case; namely, the work done when force and displacement are in the same direction. Later in the section we generalize our definition to include cases where the force and displacement are in arbitrary directions. We conclude with a discussion of the work done on an object when it is acted on by more than one force.

Force in the Direction of Displacement

When we push a shopping cart in a store or pull a suitcase through an airport, we do work. The greater the force, the greater the work; the greater the distance, the greater the work. These simple ideas form the basis for our definition of work.

To be specific, suppose we push a box with a constant force $\vec{\mathbf{F}}$, as shown in **Figure 7–1**. If we move the box *in the direction of* $\vec{\mathbf{F}}$ through a displacement $\vec{\mathbf{d}}$, the **work** W we have done is Fd:

Definition of Work, *W*, When a Constant Force Is in the Direction of Displacement

$$W = Fd \qquad \text{7–1}$$

SI unit: newton-meter $(\mathrm{N \cdot m})$ = joule, J

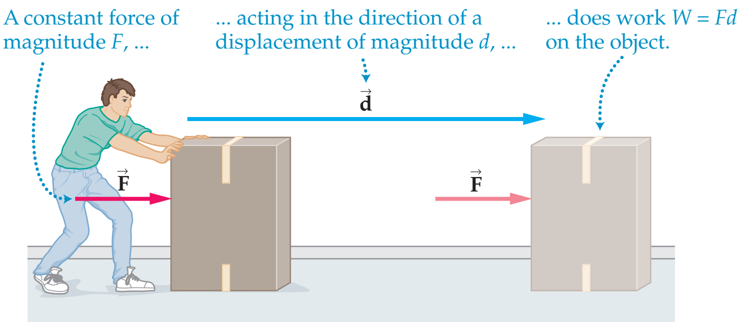


FIGURE 7–1 Work: constant force in the direction of motion

A constant force $\vec{\mathbf{F}}$ pushes a box through a displacement $\vec{\mathbf{d}}$. In this special case, where the force and displacement are in the *same* direction, the work done on the box by the force is $W = Fd$.

Note that work is the product of two magnitudes, and hence it is a scalar. In addition, notice that a small force acting over a large distance gives the same work as a large force acting over a small distance. For example, $W = (1\ \mathrm{N})(400\ \mathrm{m}) = (400\ \mathrm{N})(1\ \mathrm{m})$.

The dimensions of work are newtons (force) times meters (distance), or $\mathrm{N \cdot m}$. This combination of dimensions is called the **joule** (rhymes with "school," as commonly pronounced) in honor of James Prescott Joule (1818–1889), a dedicated physicist who is said to have conducted physics experiments even while on his honeymoon. We define a joule as follows:

Definition of the Joule, J

$$1\ \text{joule} = 1\ \mathrm{J} = 1\ \mathrm{N \cdot m} = 1(\mathrm{kg \cdot m/s^2}) \cdot \mathrm{m} = 1\ \mathrm{kg \cdot m^2/s^2} \qquad \text{7–2}$$

To get a better feeling for work and the associated units, suppose you exert a force of 82.0 N on the box in Figure 7–1 and move it in the direction of the force through a distance of 3.00 m. The work you have done is

$$W = Fd = (82.0\ \mathrm{N})(3.00\ \mathrm{m}) = 246\ \mathrm{N \cdot m} = 246\ \mathrm{J}$$

Similarly, if you do 5.00 J of work to lift a book through a vertical distance of 0.750 m, the force you exerted on the book is

$$F = \frac{W}{d} = \frac{5.00\ \mathrm{J}}{0.750\ \mathrm{m}} = \frac{5.00\ \mathrm{N \cdot m}}{0.750\ \mathrm{m}} = 6.67\ \mathrm{N}$$

TABLE 7–1 Typical Values of Work

Activity	Equivalent work (J)
Annual U.S. energy use	8×10^{19}
Mt. St. Helens eruption	10^{18}
Burning one gallon of gas	10^{8}
Human food intake/day	10^{7}
Melting an ice cube	10^{4}
Lighting a 100-W bulb for 1 minute	6000
Heartbeat	0.5
Turning page of a book	10^{-3}
Hop of a flea	10^{-7}
Breaking a bond in DNA	10^{-20}

EXERCISE 7–1

One species of Darwin's finch, *Geospiza magnirostris*, can exert a force of 205 N with its beak as it cracks open a *Tribulus* seed case. If its beak moves through a distance of 0.40 cm during this operation, how much work does the finch do to get the seed?

SOLUTION

$W = Fd = (205\ \mathrm{N})(0.0040\ \mathrm{m}) = 0.82\ \mathrm{J}$

Just how much work is a joule, anyway? Well, you do one joule of work when you lift a gallon of milk through a height of about an inch, or lift an apple a meter. One joule of work lights a 100-watt lightbulb for 0.01 seconds or heats a glass of water 0.00125 degrees Celsius. Clearly, a joule is a modest amount of work in everyday terms. Additional examples of work are listed in Table 7–1.

EXAMPLE 7–1 HEADING FOR THE ER

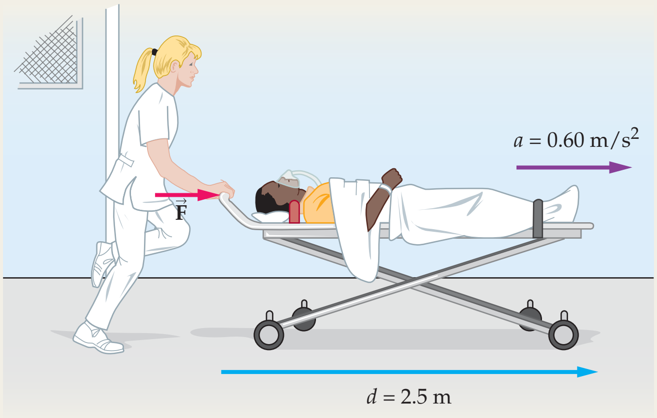
An intern pushes a 72-kg patient on a 15-kg gurney, producing an acceleration of $0.60\ \mathrm{m/s^2}$. **(a)** How much work does the intern do by pushing the patient and gurney through a distance of 2.5 m? Assume the gurney moves without friction. **(b)** How far must the intern push the gurney to do 140 J of work?

PICTURE THE PROBLEM

Our sketch shows the physical situation for this problem. Note that the force exerted by the intern is in the same direction as the displacement of the gurney; therefore, we know that $W = Fd$.

STRATEGY

We are not given the magnitude of the force F, so we cannot apply Equation 7–1 directly. However, we are given the mass and acceleration of the patient and gurney, from which we can calculate the force with $F = ma$. The work done by the intern is then $W = Fd$.



SOLUTION

Part (a)

1. First, find the force F exerted by the intern: $F = ma = (72\ \mathrm{kg} + 15\ \mathrm{kg})(0.60\ \mathrm{m/s^2}) = 52\ \mathrm{N}$

2. The work done by the intern, W, is the force times the distance: $W = Fd = (52\ \mathrm{N})(2.5\ \mathrm{m}) = 130\ \mathrm{J}$

Part (b)

3. Use $W = Fd$ to solve for the distance d: $W = Fd$ therefore $d = \dfrac{W}{F} = \dfrac{140\ \mathrm{J}}{52\ \mathrm{N}} = 2.7\ \mathrm{m}$

INSIGHT

You might wonder whether the work done by the intern depends on the speed of the gurney. The answer is no. The work done on an object, $W = Fd$, doesn't depend on whether the object moves through the distance d quickly or slowly. What does depend on the speed of the gurney is the *rate* at which work is done, as we discuss in detail in Section 7–4.

PRACTICE PROBLEM

If the total mass of the gurney plus patient is halved and the acceleration is doubled, does the work done by the intern increase, decrease, or remain the same? [**Answer:** The work remains the same.]

Some related homework problems: Problem 4, Problem 5

Before moving on, let's note an interesting point about our definition of work. It's clear from Equation 7–1 that *the work W is zero if the distance d is zero*—and this is true regardless of how great the force might be. For example, if you push against a solid wall you do no work on it, even though you may become tired from your efforts. Similarly, if you stand in one place holding a 50-pound suitcase in your hand, you do no work on the suitcase. The fact that we become tired when we push against a wall or hold a heavy object is due to the repeated contraction and expansion of individual cells within our muscles. Thus, even when we are "at rest," our muscles are doing mechanical work on the microscopic level.

The weightlifter at left does more work in raising 150 kilograms above her head than Atlas, who is supporting the entire world. Why?

Force at an Angle to the Displacement

In **Figure 7–2** we see a person pulling a suitcase on a level surface with a strap that makes an angle θ with the horizontal—in this case the force is at an angle to the direction of motion. How do we calculate the work now? Well, instead of force times distance, we say that work is the *component* of force in the *direction* of displacement times the magnitude of the displacement. In Figure 7–2, the component of force in the direction of the displacement is $F\cos\theta$ and the magnitude of the displacement is d. Therefore, the work is $F\cos\theta$ times d:

Definition of Work When the Angle Between a Constant Force and the Displacement Is θ

$$W = (F\cos\theta)d = Fd\cos\theta \qquad 7\text{–}3$$

SI unit: joule, J

Of course, in the case where the force is in the direction of motion, the angle θ is zero; then $W = Fd\cos\theta^\circ = Fd \cdot 1 = Fd$, in agreement with Equation 7–1.

Equally interesting is a situation in which the force and the displacement are at right angles to one another. In this case $\theta = 90^\circ$ and the work done by the force F is zero; $W = Fd\cos 90^\circ = 0$.

This result leads naturally to an alternative way to think about the expression $W = Fd\cos\theta$. In **Figure 7–3** we show the displacement and the force for the suitcase in Figure 7–2. Notice that the displacement is equivalent to a displacement in the

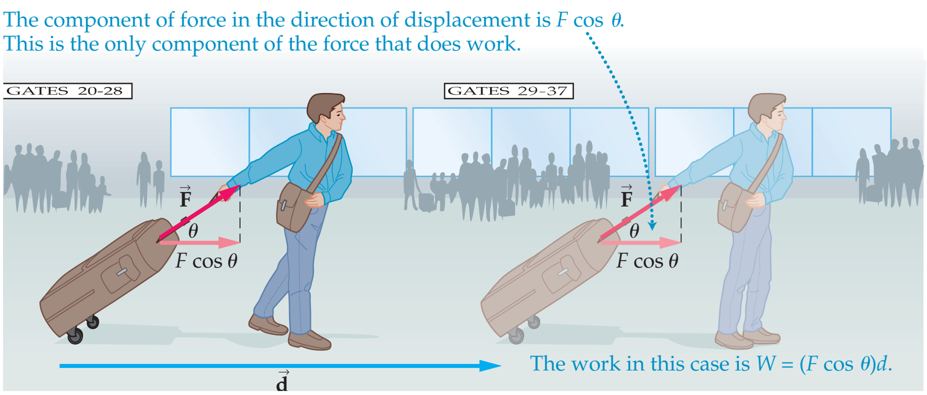


FIGURE 7–2 Work: force at an angle to the direction of motion

A person pulls a suitcase with a strap at an angle θ to the direction of motion. The component of force in the direction of motion is $F\cos\theta$, and the work done by the person is $W = (F\cos\theta)d$.

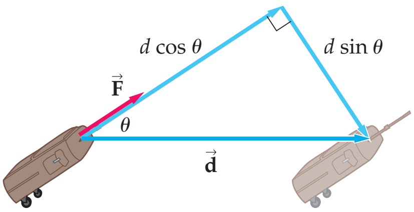


FIGURE 7–3 Force at an angle to direction of motion: another look

The displacement of the suitcase in Figure 7–2 is equivalent to a displacement of magnitude $d\cos\theta$ in the direction of the force $\vec{\mathbf{F}}$, plus a displacement of magnitude $d\sin\theta$ perpendicular to the force. Only the displacement parallel to the force results in nonzero work, hence the total work done is $F(d\cos\theta)$ as expected.

direction of the force of magnitude $(d\cos\theta)$ *plus* a displacement at right angles to the force of magnitude $(d\sin\theta)$. Since the displacement at right angles to the force corresponds to zero work and the displacement in the direction of the force corresponds to a work $W = F(d\cos\theta)$, it follows that the work done in this case is $Fd\cos\theta$, as given in Equation 7–3. Thus, the work done by a force can be thought of in the following two *equivalent* ways:

(i) *Work is the component of force in the direction of the displacement times the magnitude of the displacement.*

(ii) *Work is the component of displacement in the direction of the force times the magnitude of the force.*

In either of these interpretations, the mathematical expression for work is exactly the same, $W = Fd\cos\theta$, where θ is the angle between the force vector and the displacement vector when they are placed tail-to-tail. This definition of θ is illustrated in Figure 7–3.

Finally, we can also express work as the **dot product** between the vectors $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$; that is, $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta$. Note that the dot product, which is always a scalar, is simply the magnitude of one vector times the magnitude of the second vector times the cosine of the angle between them. We discuss the dot product in greater detail in Appendix A.

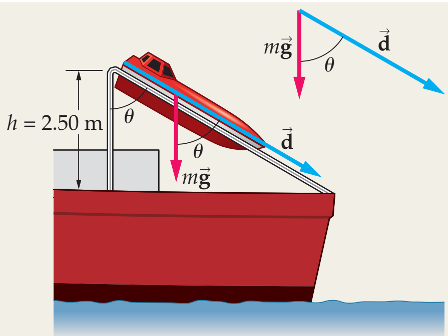
EXAMPLE 7–2 GRAVITY ESCAPE SYSTEM

REAL-WORLD PHYSICS

In a gravity escape system (GES), an enclosed lifeboat on a large ship is deployed by letting it slide down a ramp and then continuing in free fall to the water below. Suppose a 4970-kg lifeboat slides a distance of 5.00 m on a ramp, dropping through a vertical height of 2.50 m. How much work does gravity do on the boat?

PICTURE THE PROBLEM

From our sketch, we see that the force of gravity $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ are at an angle θ relative to one another when placed tail-to-tail, and that θ is also the angle the ramp makes with the vertical. In addition, we note that the vertical height of the ramp is $h = 2.50$ m and the length of the ramp is $d = 5.00$ m.



STRATEGY

By definition, the work done on the lifeboat by gravity is $W = Fd\cos\theta$, where $F = mg$, $d = 5.00$ m, and θ is the angle between $m\vec{\mathbf{g}}$ and $\vec{\mathbf{d}}$. We are not given θ in the problem statement, but from the right triangle that forms the ramp we see that $\cos\theta = h/d$. Once θ is determined from the geometry of our sketch, it is straightforward to calculate W.

SOLUTION

1. First, find the component of $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$ in the direction of motion:

$$F\cos\theta = (mg)\left(\frac{h}{d}\right)$$
$$= (4970 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{2.50 \text{ m}}{5.00 \text{ m}}\right) = 24{,}400 \text{ N}$$

2. Multiply by distance to find the work:

$$W = (F\cos\theta)d = (24{,}400 \text{ N})(5.00 \text{ m}) = 122{,}000 \text{ J}$$

3. Alternatively, cancel d algebraically before substituting numerical values:

$$W = Fd\cos\theta = (mg)(d)\left(\frac{h}{d}\right)$$
$$= mgh = (4970 \text{ kg})(9.81 \text{ m/s}^2)(2.50 \text{ m}) = 122{,}000 \text{ J}$$

INSIGHT

The work is simply $W = mgh$, exactly the same as if the lifeboat had fallen straight down through the height h.

Notice that working the problem symbolically, as in Step 3, results in two distinct advantages. First, it makes for a simpler expression for the work. Second, and more importantly, it shows that the distance d cancels; hence the work depends on the height h but not on d. Such a result is not apparent when we work solely with numbers, as in Steps 1 and 2.

PRACTICE PROBLEM

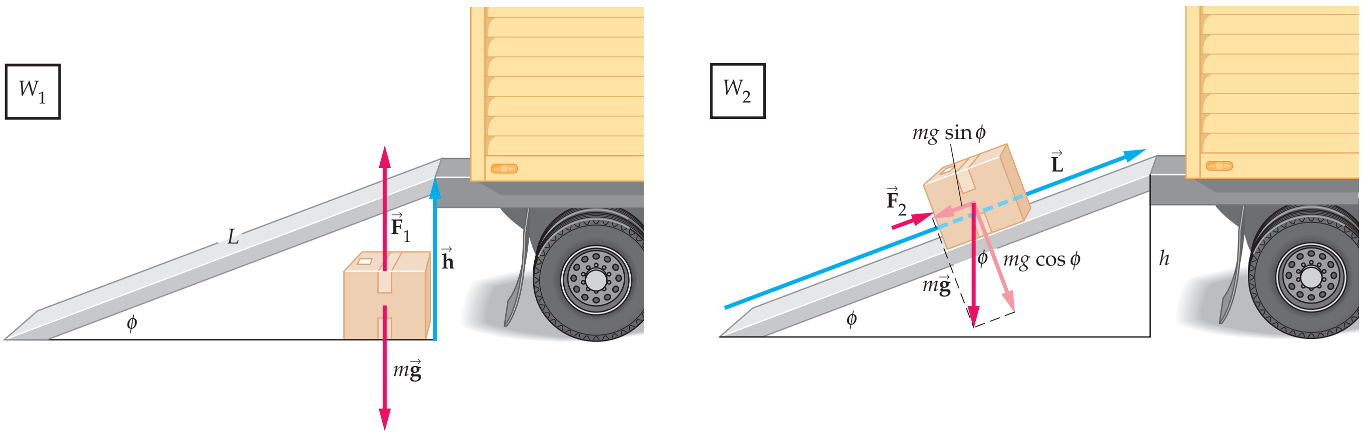
Suppose the lifeboat slides halfway to the water, gets stuck for a moment, and then starts up again and continues to the end of the ramp. What is the work done by gravity in this case? [**Answer:** The work done by gravity is exactly the same, $W = mgh$, independent of how the boat moves down the ramp.]

Some related homework problems: Problem 11, Problem 12

Next, we present a Conceptual Checkpoint that compares the work required to move an object along two different paths.

CONCEPTUAL CHECKPOINT 7–1 PATH DEPENDENCE OF WORK

You want to load a box into the back of a truck. One way is to lift it straight up through a height h, as shown, doing a work W_1. Alternatively, you can slide the box up a loading ramp a distance L, doing a work W_2. Assuming the box slides on the ramp without friction, which of the following is correct: **(a)** $W_1 < W_2$, **(b)** $W_1 = W_2$, **(c)** $W_1 > W_2$?



REASONING AND DISCUSSION

You might think that W_2 is less than W_1, since the force needed to slide the box up the ramp, F_2, is less than the force needed to lift it straight up. On the other hand, the distance up the ramp, L, is greater than the vertical distance, h, so perhaps W_2 should be greater than W_1. In fact, these two effects cancel exactly, giving $W_1 = W_2$.

To see this, we first calculate W_1. The force needed to lift the box with constant speed is $F_1 = mg$, and the height is h, therefore $W_1 = mgh$.

Next, the work to slide the box up the ramp with constant speed is $W_2 = F_2L$, where F_2 is the force required to push against the tangential component of gravity. In the figure we see that $F_2 = mg \sin \phi$. The figure also shows that $\sin \phi = h/L$; thus $W_2 = (mg \sin \phi)L = (mg)(h/L)L = mgh = W_1$.

Clearly, the ramp is a useful device—it reduces the *force* required to move the box upward from $F_1 = mg$ to $F_2 = mg(h/L)$. Even so, it doesn't decrease the amount of *work* we need to do. As we have seen, the reduced force on the ramp is offset by the increased distance.

ANSWER

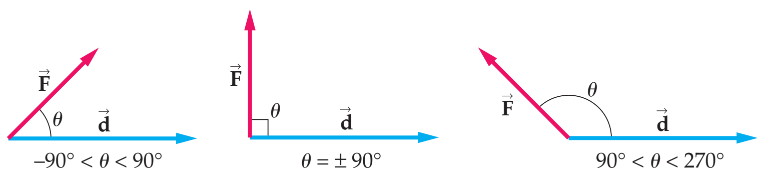
(b) $W_1 = W_2$

Negative Work and Total Work

Work depends on the angle between the force, $\vec{\mathbf{F}}$, and the displacement (or direction of motion), $\vec{\mathbf{d}}$. This dependence gives rise to three distinct possibilities, as shown in **Figure 7–4**:

(i) *Work is positive if the force has a component in the direction of motion* $(-90° < \theta < 90°)$.
(ii) *Work is zero if the force has no component in the direction of motion* $(\theta = \pm 90°)$.
(iii) *Work is negative if the force has a component opposite to the direction of motion* $(90° < \theta < 270°)$.

Thus, whenever we calculate work, we must be careful about its sign and not just assume it to be positive.



(a) Positive work **(b)** Zero work **(c)** Negative work

FIGURE 7–4 Positive, negative, and zero work

Work is positive when the force is in the same general direction as the displacement and is negative if the force is generally opposite to the displacement. Zero work is done if the force is at right angles to the displacement.

PROBLEM-SOLVING NOTE

Be Careful About the Angle θ

In calculating $W = Fd\cos\theta$ be sure that the angle you use in the cosine is the angle between the force and the displacement vectors when they are placed tail to tail. Sometimes θ may be used to label a different angle in a given problem. For example, θ is often used to label the angle of a slope, in which case it may have nothing to do with the angle between the force and the displacement. To summarize: Just because an angle is labeled θ doesn't mean it's automatically the correct angle to use in the work formula.

When more than one force acts on an object, the total work is the sum of the work done by each force separately. Thus, if force $\vec{\mathbf{F}}_1$ does work W_1, force $\vec{\mathbf{F}}_2$ does work W_2, and so on, the total work is

$$W_{\text{total}} = W_1 + W_2 + W_3 + \cdots = \sum W \qquad \text{7–4}$$

Equivalently, the total work can be calculated by first performing a vector sum of all the forces acting on an object to obtain $\vec{\mathbf{F}}_{\text{total}}$ and then using our basic definition of work:

$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}\,d\cos\theta \qquad \text{7–5}$$

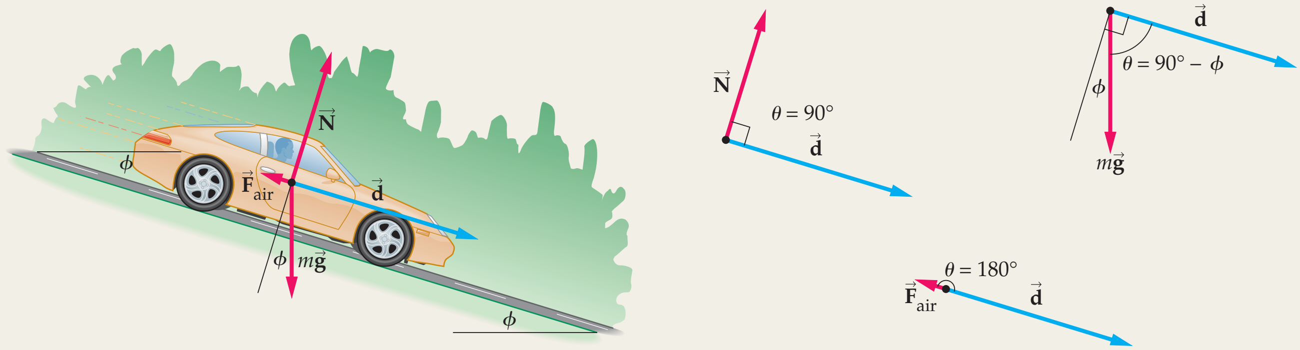
where θ is the angle between $\vec{\mathbf{F}}_{\text{total}}$ and the displacement $\vec{\mathbf{d}}$. In the next two Examples we calculate the total work in each of these ways.

EXAMPLE 7–3 A COASTING CAR I

A car of mass m coasts down a hill inclined at an angle ϕ below the horizontal. The car is acted on by three forces: (i) the normal force $\vec{\mathbf{N}}$ exerted by the road, (ii) a force due to air resistance, $\vec{\mathbf{F}}_{\text{air}}$, and (iii) the force of gravity, $m\vec{\mathbf{g}}$. Find the total work done on the car as it travels a distance d along the road.

PICTURE THE PROBLEM

Because ϕ is the angle the slope makes with the horizontal, it is also the angle between $m\vec{\mathbf{g}}$ and the downward normal direction, as was shown in Figure 5–15. It follows that the angle between $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ is $\theta = 90° - \phi$. Our sketch also shows that the angle between $\vec{\mathbf{N}}$ and $\vec{\mathbf{d}}$ is $\theta = 90°$, and the angle between $\vec{\mathbf{F}}_{\text{air}}$ and $\vec{\mathbf{d}}$ is $\theta = 180°$.



STRATEGY

For each force we calculate the work using $W = Fd\cos\theta$, where θ is the angle between that particular force and the displacement $\vec{\mathbf{d}}$. The total work is the sum of the work done by each of the three forces.

SOLUTION

1. We start with the work done by the normal force, $\vec{\mathbf{N}}$. From the figure we see that $\theta = 90°$ for this force: $\quad W_N = Nd\cos\theta = Nd\cos 90° = Nd(0) = 0$

2. For the force of air resistance, $\theta = 180°$: $\quad W_{\text{air}} = F_{\text{air}}d\cos 180° = F_{\text{air}}d(-1) = -F_{\text{air}}d$

3. For gravity the angle θ is $\theta = 90° - \phi$, as indicated in the figure. Recall that $\cos(90° - \phi) = \sin\phi$ (see Appendix A): $\quad W_{mg} = mgd\cos(90° - \phi) = mgd\sin\phi$

4. The total work is the sum of the individual works: $\quad W_{\text{total}} = W_N + W_{\text{air}} + W_{mg} = 0 - F_{\text{air}}d + mgd\sin\phi$

INSIGHT

The normal force is perpendicular to the motion of the car, and thus does no work. Air resistance points in a direction that opposes the motion, so it does negative work. On the other hand, gravity has a component in the direction of motion; therefore, its work is positive. The physical significance of positive, negative, and zero work will be discussed in detail in the next section.

PRACTICE PROBLEM

Calculate the total work done on a 1550-kg car as it coasts 20.4 m down a hill with $\phi = 5.00°$. Let the force due to air resistance be 15.0 N. **[Answer:** $W_{\text{total}} = W_N + W_{\text{air}} + W_{mg} = 0 - F_{\text{air}}d + mgd\sin\phi = 0 - 306\text{ J} + 2.70 \times 10^4\text{ J} = 2.67 \times 10^4\text{ J}$**]**

Some related homework problems: Problem 15, Problem 81

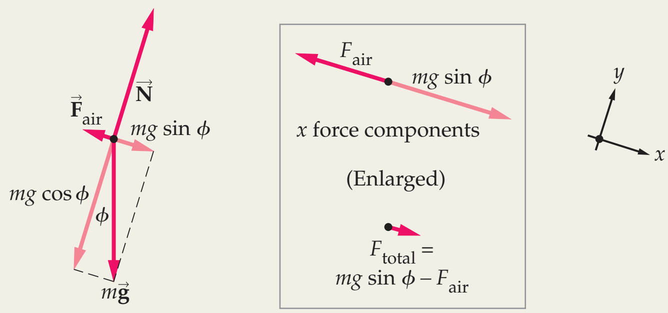
In the previous Example, we showed that the total work can be calculated by finding the work done by each force separately, and then summing the individual works. In the next Example, we take a different approach. We first sum the forces acting on the car to find F_{total}. Once the total force is determined, we calculate the total work using $W_{\text{total}} = F_{\text{total}} d \cos \theta$.

EXAMPLE 7–4 A COASTING CAR II

Consider the car described in Example 7–3. Calculate the total work done on the car using $W_{\text{total}} = F_{\text{total}} d \cos \theta$.

PICTURE THE PROBLEM

First, we choose the x axis to point down the slope, and the y axis to be at right angles to the slope. With this choice, there is no acceleration in the y direction, which means that the total force in that direction must be zero. As a result, the total force acting on the car is in the x direction. The magnitude of the total force is $mg \sin \phi - F_{\text{air}}$, as can be seen in our sketch.



STRATEGY

We begin by finding the x component of each force vector and then summing them to find the total force acting on the car. As can be seen from the figure, the total force points in the positive x direction; that is, in the same direction as the displacement. Therefore, the angle θ in $W = F_{\text{total}} d \cos \theta$ is zero.

SOLUTION

1. Referring to the figure above, we see that the *magnitude* of the total force is $mg \sin \phi$ minus F_{air}:

$$F_{\text{total}} = mg \sin \phi - F_{\text{air}}$$

2. The *direction* of $\vec{\mathbf{F}}_{\text{total}}$ is the same as the direction of $\vec{\mathbf{d}}$, thus $\theta = 0°$. We can now calculate W_{total}:

$$W_{\text{total}} = F_{\text{total}} d \cos \theta = (mg \sin \phi - F_{\text{air}}) d \cos 0° = mgd \sin \phi - F_{\text{air}} d$$

INSIGHT

Note that we were careful to calculate both the magnitude and the direction of the total force. The magnitude (which is always positive) gives F_{total} and the direction gives $\theta = 0°$, allowing us to use $W_{\text{total}} = F_{\text{total}} d \cos \theta$.

PRACTICE PROBLEM

Suppose the total work done on a 1620-kg car as it coasts 25.0 m down a hill with $\phi = 6.00°$ is $W_{\text{total}} = 3.75 \times 10^4$ J. Find the magnitude of the force due to air resistance. [**Answer:** $F_{\text{air}} d = -W_{\text{total}} + mgd \sin \phi = 4030$ J, thus $F_{\text{air}} = (4030 \text{ J})/d = 161$ N]

Some related homework problems: Problem 15, Problem 81

The full significance of positive versus negative work is seen in the next section, where we relate the work done on an object to the change in its speed.

7–2 Kinetic Energy and the Work–Energy Theorem

Suppose you drop an apple. As it falls, gravity does positive work on it, as indicated in **Figure 7–5**, and its speed increases. If you toss the apple upward, gravity does negative work, and the apple slows down. In general, whenever the total work done on an object is positive, its speed increases; when the total work is negative, its speed decreases. In this section we derive an important result, the **work–energy theorem,** which makes this connection between work and change in speed precise.

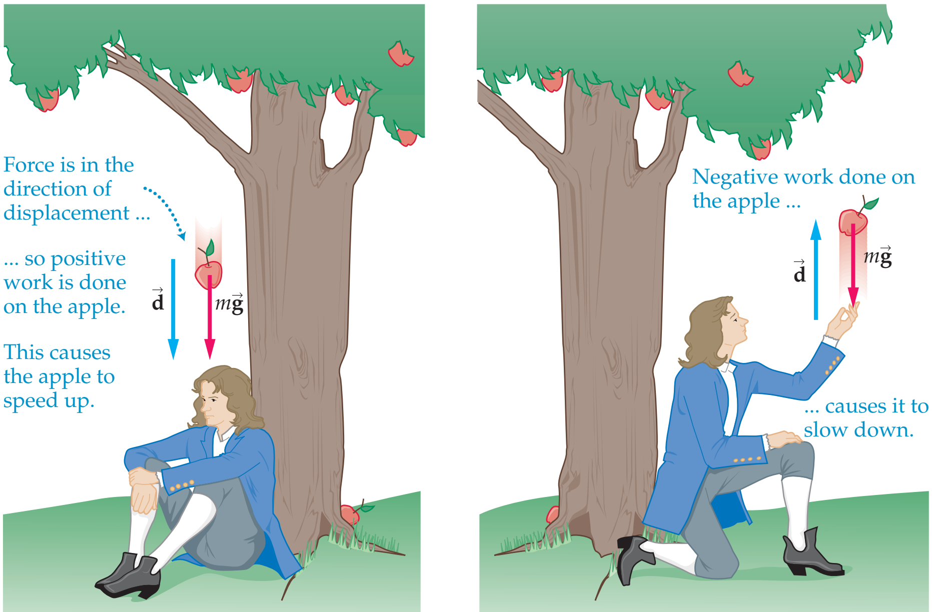
To begin, consider an apple of mass m falling through the air, and suppose that two forces act on the apple—gravity, $m\vec{\mathbf{g}}$, and the average force of air resistance, $\vec{\mathbf{F}}_{\text{air}}$. The total force acting on the apple, $\vec{\mathbf{F}}_{\text{total}}$, gives the apple a constant downward acceleration of magnitude

$$a = F_{\text{total}}/m$$

Since the total force is downward and the motion is downward, the work done on the apple is positive.

FIGURE 7–5 Gravitational work

The work done by gravity on an apple that moves downward is positive. If the apple is in free fall, this positive work will result in an increase in speed. On the other hand, the work done by gravity on an apple that moves upward is negative. If the apple is in free fall, the negative work done by gravity will result in a decrease of speed.



Apple falling: $W > 0$, speed increases   Apple tossed upward: $W < 0$, speed decreases

Now, suppose the initial speed of the apple is v_i, and that after falling a distance d its speed increases to v_f. The apple falls with constant acceleration a, hence constant-acceleration kinematics (Equation 2–12) gives

$$v_f^2 = v_i^2 + 2ad$$

or, with a slight rearrangement,

$$2ad = v_f^2 - v_i^2$$

Next, substitute $a = F_{total}/m$ into this equation:

$$2\left(\frac{F_{total}}{m}\right)d = v_f^2 - v_i^2$$

Multiplying both sides by m and dividing by 2 yields

$$F_{total}d = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

where $F_{total}d$ is simply the total work done on the apple. Thus we find

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

showing that total work is directly related to change in speed, as just mentioned. Note that $W_{total} > 0$ means $v_f > v_i$, $W_{total} < 0$ means $v_f < v_i$, and $W_{total} = 0$ implies that $v_f = v_i$.

The quantity $\frac{1}{2}mv^2$ in the equation for W_{total} has a special significance in physics, as we shall see. We call it the **kinetic energy,** K:

PROBLEM-SOLVING NOTE

Work Can Be Positive, Negative, or Zero

When you calculate work, be sure to keep track of whether it is positive or negative. The distinction is important, since positive work increases speed, whereas negative work decreases speed. Zero work, of course, has no effect on speed.

Definition of Kinetic Energy, K

$$K = \tfrac{1}{2}mv^2 \qquad 7\text{–}6$$

SI unit: $\text{kg} \cdot \text{m}^2/\text{s}^2 = \text{joule, J}$

In general, the kinetic energy of an object is the energy due to its motion. We measure kinetic energy in joules, the same units as work, and both kinetic energy and work are scalars. Unlike work, however, kinetic energy is never negative. Instead, K is always greater than or equal to zero, independent of the direction of motion or the direction of any forces.

To get a feeling for typical values of kinetic energy, consider your kinetic energy when jogging. Assuming a mass of about 62 kg and a speed of 2.5 m/s, your kinetic energy is $K = \frac{1}{2}(62\text{ kg})(2.5\text{ m/s})^2 = 190\text{ J}$. Additional examples of kinetic energy are given in Table 7–2.

TABLE 7–2 Typical Kinetic Energies

Source	Approximate kinetic energy (J)
Jet aircraft at 500 mi/h	10^9
Car at 60 mi/h	10^6
Home-run baseball	10^3
Person at walking speed	50
Housefly in flight	10^{-3}

EXERCISE 7–2

A truck moving at 15 m/s has a kinetic energy of 4.2×10^5 J. **(a)** What is the mass of the truck? **(b)** By what multiplicative factor does the kinetic energy of the truck increase if its speed is doubled?

SOLUTION

(a) $K = \frac{1}{2}mv^2$; therefore $m = 2K/v^2 = 3700$ kg. **(b)** Kinetic energy depends on the speed squared, and hence doubling the speed increases the kinetic energy by a factor of four.

In terms of kinetic energy, the work–energy theorem can be stated as follows:

Work–Energy Theorem

The total work done on an object is equal to the change in its kinetic energy:

$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad 7\text{–}7$$

Thus, the work–energy theorem says that when a force acts on an object over a distance—doing work on it—the result is a change in the speed of the object, and hence a change in its energy of motion. Equation 7–7 is the quantitative expression of this connection.

Finally, though we have derived the work–energy theorem for a force that is constant in direction and magnitude, it is valid for any force, as can be shown using the methods of calculus. In fact, the work–energy theorem is completely general, making it one of the more important and fundamental results in physics. It is also a very handy tool for problem solving, as we shall see many times throughout this text.

PROBLEM-SOLVING NOTE

Starts from Rest Means $v_i = 0$

A problem statement that uses a phrase like "starts from rest" or "is raised from rest" is telling you that $v_i = 0$.

EXERCISE 7–3

How much work is required for a 74-kg sprinter to accelerate from rest to 2.2 m/s?

SOLUTION

Since $v_i = 0$, we have $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 = \frac{1}{2}(74 \text{ kg})(2.2 \text{ m/s})^2 = 180$ J.

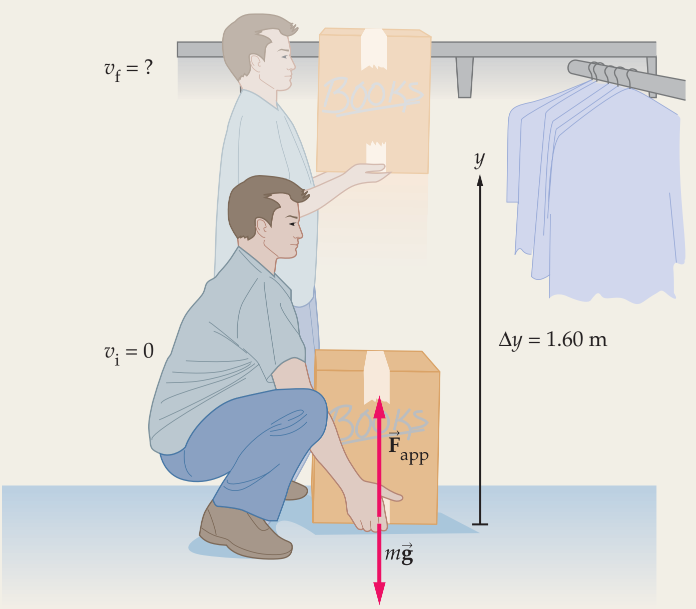
We now present a variety of Examples showing how the work–energy theorem is used in practical situations.

EXAMPLE 7–5 HIT THE BOOKS

A 4.10-kg box of books is lifted vertically from rest a distance of 1.60 m with a constant, upward applied force of 52.7 N. Find **(a)** the work done by the applied force, **(b)** the work done by gravity, and **(c)** the final speed of the box.

PICTURE THE PROBLEM

Our sketch shows that the direction of motion of the box is upward. In addition, we see that the applied force, $\vec{\mathbf{F}}_{\text{app}}$, is upward and the force of gravity, $m\vec{\mathbf{g}}$, is downward. Finally, the box is lifted from rest ($v_i = 0$) through a distance $\Delta y = 1.60$ m.



STRATEGY

The applied force is in the direction of motion, so the work it does, W_{app}, is positive. Gravity is opposite in direction to the motion; thus its work, W_g, is negative. The total work is the sum of W_{app} and W_g, and the final speed of the box is found by applying the work–energy theorem, $W_{\text{total}} = \Delta K$.

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SOLUTION

Part (a)

1. First we find the work done by the applied force. In this case, $\theta = 0°$ and the distance is $\Delta y = 1.60$ m:

$$W_{app} = F_{app} \cos 0° \, \Delta y = (52.7 \text{ N})(1)(1.60 \text{ m}) = 84.3 \text{ J}$$

Part (b)

2. Next, we calculate the work done by gravity. The distance is $\Delta y = 1.60$ m, as before, but now $\theta = 180°$:

$$W_g = mg \cos 180° \, \Delta y$$
$$= (4.10 \text{ kg})(9.81 \text{ m/s}^2)(-1)(1.60 \text{ m}) = -64.4 \text{ J}$$

Part (c)

3. The total work done on the box, W_{total}, is the sum of W_{app} and W_g:

$$W_{total} = W_{app} + W_g = 84.3 \text{ J} - 64.4 \text{ J} = 19.9 \text{ J}$$

4. To find the final speed, v_f, we apply the work–energy theorem. Recall that the box started at rest, thus $v_i = 0$:

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2W_{total}}{m}} = \sqrt{\frac{2(19.9 \text{ J})}{4.10 \text{ kg}}} = 3.12 \text{ m/s}$$

INSIGHT

As a check on our result, we can find v_f in a completely different way. First, calculate the acceleration of the box with the result $a = (F_{app} - mg)/m = 3.04 \text{ m/s}^2$. Next, use this result in the kinematic equation $v^2 = v_0^2 + 2a\Delta y$. With $v_0 = 0$ and $\Delta y = 1.60$ m, we find $v = 3.12$ m/s, in agreement with the results using the work–energy theorem.

PRACTICE PROBLEM

If the box is lifted only a quarter of the distance, is the final speed 1/8, 1/4, or 1/2 of the value found in Step 4? Calculate v_f in this case as a check on your answer. [**Answer:** Since work depends linearly on Δy, and v_f depends on the square root of the work, it follows that the final speed is $\sqrt{1/4} = \frac{1}{2}$ the value in Step 4. Letting $\Delta y = (1.60 \text{ m})/4 = 0.400$ m, we find $v_f = \frac{1}{2}(3.12 \text{ m/s}) = 1.56$ m/s.]

Some related homework problems: Problem 19, Problem 24, Problem 25

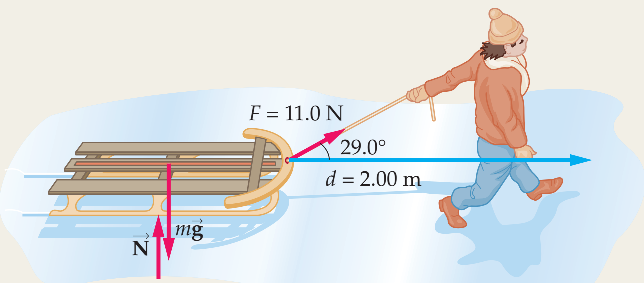
In the previous Example the initial speed was zero. This is not always the case, of course. The next Example illustrates how to use the work–energy theorem when the initial velocity is nonzero.

EXAMPLE 7–6 PULLING A SLED

A boy exerts a force of 11.0 N at 29.0° above the horizontal on a 6.40-kg sled. Find **(a)** the work done by the boy and **(b)** the final speed of the sled after it moves 2.00 m, assuming the sled starts with an initial speed of 0.500 m/s and slides horizontally without friction.

PICTURE THE PROBLEM

Our sketch shows the direction of motion and the directions of each of the forces. Note that the normal force and the force due to gravity are vertical, whereas the displacement is horizontal. The force exerted by the boy has both a vertical component, $F \sin \theta$, and a horizontal component, $F \cos \theta$.



STRATEGY

a. The forces $\vec{N}$ and $m\vec{g}$ do no work because they are at right angles to the horizontal displacement. The force exerted by the boy, however, has a horizontal component that does positive work on the sled. Therefore, the total work is simply the work done by the boy.

b. After calculating this work, we find v_f by applying the work–energy theorem with $v_i = 0.500$ m/s.

SOLUTION

Part (a)

1. The work done by the boy is $(F \cos \theta)d$, where $\theta = 29.0°$. This is also the total work done on the sled:

$$W_{boy} = (F \cos \theta)d$$
$$= (11.0 \text{ N})(\cos 29.0°)(2.00 \text{ m}) = 19.2 \text{ J} = W_{total}$$

Part (b)

2. Use the work–energy theorem to solve for the final speed:

$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\tfrac{1}{2}mv_f^2 = W_{\text{total}} + \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{\frac{2W_{\text{total}}}{m} + v_i^2}$$

3. Substitute numerical values to get the final answer:

$$v_f = \sqrt{\frac{2(19.2\ \text{J})}{6.40\ \text{kg}} + (0.500\ \text{m/s})^2}$$

$$= 2.50\ \text{m/s}$$

INSIGHT

If the sled had started from rest, instead of with an initial speed of 0.500 m/s, would its final speed be 2.50 m/s − 0.500 m/s = 2.00 m/s?

No. If the initial speed is zero, then $v_f = \sqrt{\frac{2W_{\text{total}}}{m}} = \sqrt{\frac{2(19.2\ \text{J})}{6.40\ \text{kg}}} = 2.45$ m/s. Why don't the speeds add and subtract in a straightforward way? The reason is that the work–energy theorem depends on the *square* of the speeds rather than on v_i and v_f directly.

PRACTICE PROBLEM

Suppose the sled starts with a speed of 0.500 m/s and has a final speed of 2.50 m/s after the boy pulls it through a distance of 3.00 m. What force did the boy exert on the sled? [**Answer:** $F = W_{\text{total}}/(d\cos\theta) = \Delta K/(d\cos\theta) = 7.32$ N]

Some related homework problems: Problem 28, Problem 61

The final speeds in the previous Examples could have been found using Newton's laws and the constant-acceleration kinematics of Chapter 2, as indicated in the Insight following Example 7–5. The work–energy theorem provides an alternative method of calculation that is often much easier to apply than Newton's laws. We return to this point in Chapter 8.

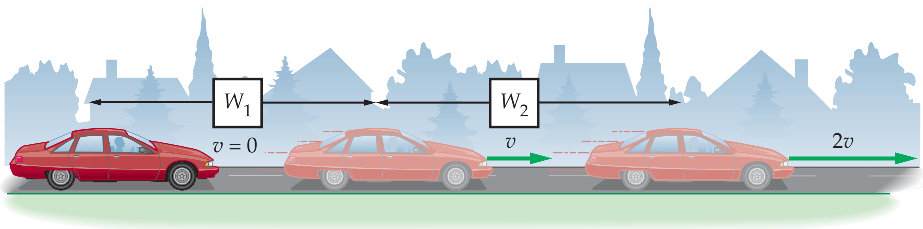
PROBLEM-SOLVING NOTE

Be Careful About Linear Reasoning

Though some relations are linear—if you *double* the mass, you *double* the kinetic energy—others are not. For example, if you *double* the speed, you *quadruple* the kinetic energy. Be careful not to jump to conclusions based on linear reasoning.

CONCEPTUAL CHECKPOINT 7–2 COMPARE THE WORK

To accelerate a certain car from rest to the speed v requires the work W_1. The work needed to accelerate the car from v to $2v$ is W_2. Which of the following is correct: **(a)** $W_2 = W_1$, **(b)** $W_2 = 2\,W_1$, **(c)** $W_2 = 3W_1$, **(d)** $W_2 = 4W_1$?



REASONING AND DISCUSSION

A common mistake is to reason that since we increase the speed by the same amount in each case, the work required is the same. It is not, and the reason is that work depends on the speed squared rather than on the speed itself.

To see how this works, first calculate W_1, the work needed to go from rest to a speed v. From the work–energy theorem, with $v_i = 0$ and $v_f = v$, we find $W_1 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv^2$. Similarly, the work needed to go from rest, $v_i = 0$, to a speed $v_f = 2v$, is simply $\tfrac{1}{2}m(2v)^2 = 4\left(\tfrac{1}{2}mv^2\right) = 4W_1$. Therefore, the work needed to increase the speed from v to $2v$ is the difference: $W_2 = 4W_1 - W_1 = 3W_1$.

ANSWER

(c) $W_2 = 3W_1$

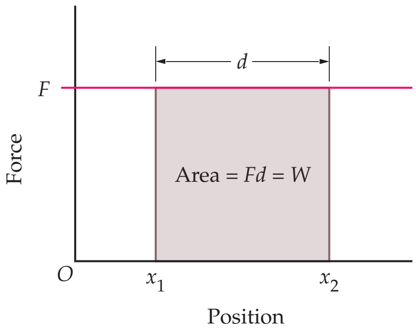


FIGURE 7–6 Graphical representation of the work done by a constant force

A constant force F acting through a distance d does a work $W = Fd$. Note that Fd is also equal to the shaded area between the force line and the x axis.

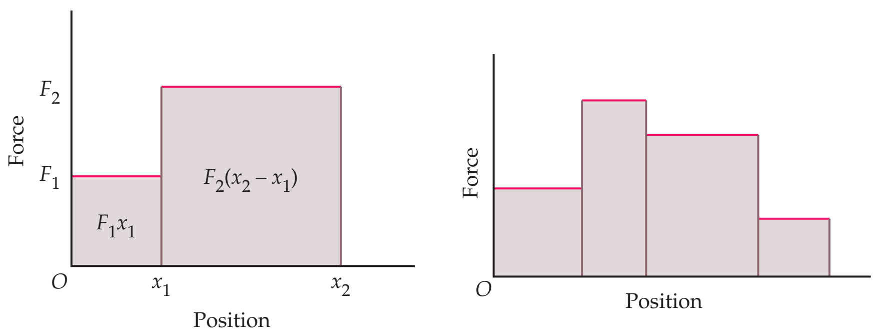
7–3 Work Done by a Variable Force

Thus far we have calculated work only for constant forces, yet most forces in nature vary with position. For example, the force exerted by a spring depends on how far the spring is stretched, and the force of gravity between planets depends on their separation. In this section we show how to calculate the work for a force that varies with position.

First, let's review briefly the case of a constant force, and develop a graphical interpretation of work. **Figure 7–6** shows a constant force plotted versus position, x. If the force acts in the positive x direction and moves an object a distance d, from x_1 to x_2, the work it does is $W = Fd = F(x_2 - x_1)$. Referring to the figure, we see that the work is equal to the shaded area[1] between the force line and the x axis.

Next, consider a force that has the value F_1 from $x = 0$ to $x = x_1$ and a different value F_2 from $x = x_1$ to $x = x_2$, as in **Figure 7–7 (a)**. The work in this case is the sum of the works done by F_1 and F_2. Therefore, $W = F_1x_1 + F_2(x_2 - x_1)$ which, again, is the area between the force lines and the x axis. Clearly, this type of calculation can be extended to a force with any number of different values, as indicated in **Figure 7–7 (b)**.

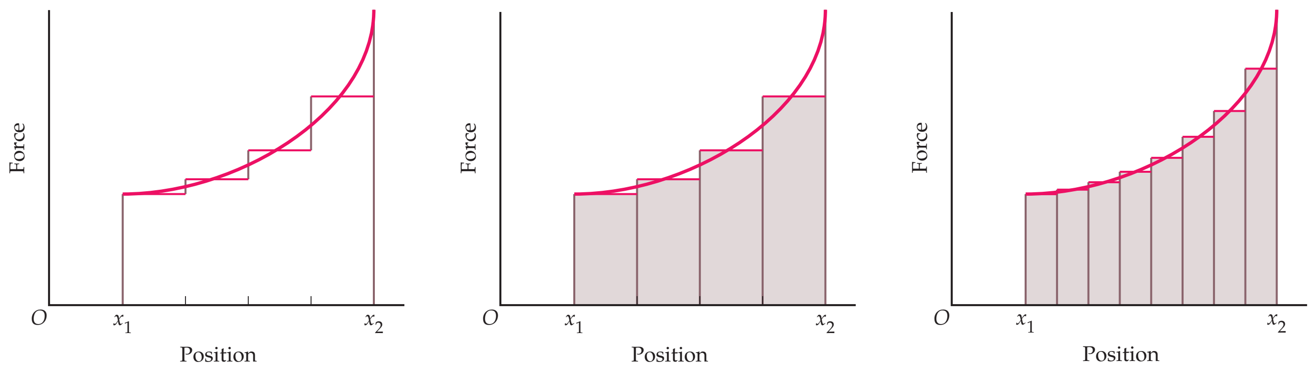
If a force varies continuously with position, we can approximate it with a series of constant values that follow the shape of the curve, as shown in **Figure 7–8 (a)**. It follows that the work done by the continuous force is approximately equal to the area of the corresponding rectangles, as **Figure 7–8 (b)** shows. The approximation can be made better by using more rectangles, as illustrated in **Figure 7–8 (c)**. In the



(a) **(b)**

FIGURE 7–7 Work done by a nonconstant force

(a) A force with a value F_1 from 0 to x_1 and a value F_2 from x_1 to x_2 does the work $W = F_1x_1 + F_2(x_2 - x_1)$. This is simply the area of the two shaded rectangles. **(b)** If a force takes on a number of different values, the work it does is still the total area between the force lines and the x axis, just as in part (a).



(a) Approximating a continuous force

(b) Approximating the work done by a continuous force

(c) A better approximation

FIGURE 7–8 Work done by a continuously varying force

(a) A continuously varying force can be approximated by a series of constant values that follow the shape of the curve. **(b)** The work done by the continuous force is approximately equal to the area of the small rectangles corresponding to the constant values of force shown in part (a). **(c)** In the limit of an infinite number of vanishingly small rectangles, we see that the work done by the force is equal to the area between the force curve and the x axis.

[1]Usually, area has the dimensions of (length) × (length), or length^2. In this case, however, the vertical axis is force and the horizontal axis is distance. As a result, the dimensions of area are (force) × (distance), which in SI units is $\mathrm{N \cdot m = J}$.

limit of an infinite number of vanishingly small rectangles, the area of the rectangles becomes identical to the area under the force curve. Hence this area is the work done by the continuous force. To summarize:

The work done by a force in moving an object from x_1 to x_2 is equal to the corresponding area between the force curve and the x axis.

A case of particular interest is that of a spring. Since the force exerted by a spring is given by $F_x = -kx$ (Section 6–2), it follows that the force we must exert to hold it at the position x is $+kx$. This is illustrated in **Figure 7–9**, where we also show that the corresponding force curve is a straight line extending from the origin. Therefore, the work we do in stretching a spring from $x = 0$ (equilibrium) to the general position x is the shaded, triangular area shown in **Figure 7–10**. This area is equal to $\frac{1}{2}$(base)(height), where in this case the base is x and the height is kx. As a result, the work is $\frac{1}{2}(x)(kx) = \frac{1}{2}kx^2$. Similar reasoning shows that the work needed to compress a spring a distance x is also $\frac{1}{2}kx^2$. Therefore,

Work to Stretch or Compress a Spring a Distance *x* from Equilibrium

$$W = \frac{1}{2}kx^2 \qquad 7\text{–}8$$

SI unit: joule, J

We can get a feeling for the amount of work required to compress a typical spring in the following Exercise.

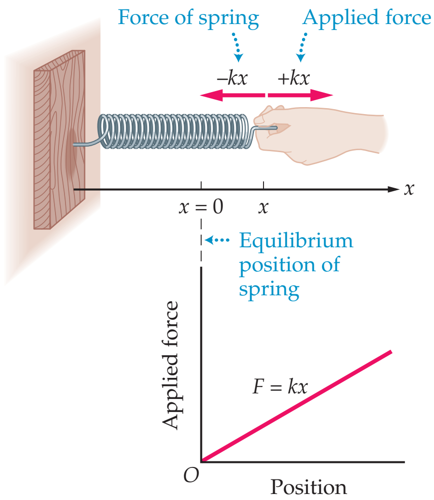


FIGURE 7–9 Stretching a spring
The force we must exert on a spring to stretch it a distance x is $+kx$. Thus, applied force versus position for a spring is a straight line of slope k.

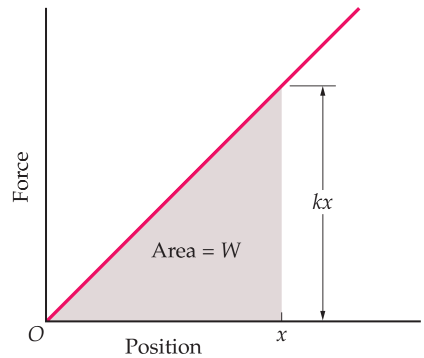


FIGURE 7–10 Work needed to stretch a spring a distance *x*
The work done is equal to the shaded area, which is a right triangle. The area of the triangle is $\frac{1}{2}(x)(kx) = \frac{1}{2}kx^2$.

EXERCISE 7–4

The spring in a pinball launcher has a force constant of 405 N/m. How much work is required to compress the spring a distance of 3.00 cm?

SOLUTION

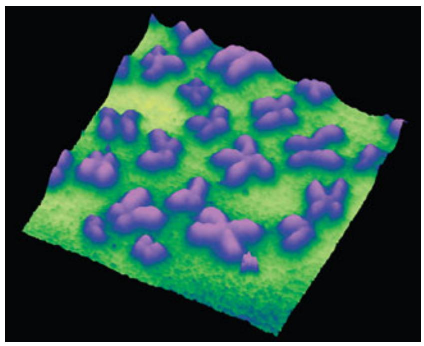
$$W = \frac{1}{2}kx^2 = \frac{1}{2}(405\ \text{N/m})(0.0300\ \text{m})^2 = 0.182\ \text{J}$$

Note that the work done in compressing or expanding a spring varies with the second power of x, the displacement from equilibrium. The consequences of this dependence are explored throughout the rest of this section.

Before we consider a specific example, however, recall that the results for a spring apply to more than just the classic case of a helical coil of wire. In fact, any flexible structure satisfies the relations $F_x = -kx$ and $W = \frac{1}{2}kx^2$, given the appropriate value of the force constant, k, and small enough displacements, x. Several examples were mentioned in Section 6–2.

Here we consider an example from the field of nanotechnology; namely, the cantilevers used in **atomic-force microscopy** (AFM). As we show in Example 7–7, a typical atomic-force cantilever is basically a thin silicon bar about 250 μm in length, supported at one end like a diving board, with a sharp, hanging point at the other end. When the point is pulled across the surface of a material—like an old-fashioned phonograph needle in the groove of a record—individual atoms on the surface cause the point to move up and down, deflecting the cantilever. These deflections, which can be measured by reflecting a laser beam from the top of the cantilever, are then converted into an atomic-level picture of the surface, as shown in the accompanying photograph.

A typical force constant for an AFM cantilever is on the order of 1 N/m, much smaller than the 100–500 N/m force constant of a common lab spring. The implications of this are discussed in the following Example.



Human chromosomes, as imaged by an atomic-force microscope.

EXAMPLE 7–7 FLEXING AN AFM CANTILEVER

The work required to deflect a typical AFM cantilever by 0.10 nm is 1.2×10^{-20} J. **(a)** What is the force constant of the cantilever, treating it as an ideal spring? **(b)** How much work is required to increase the deflection of the cantilever from 0.10 nm to 0.20 nm?

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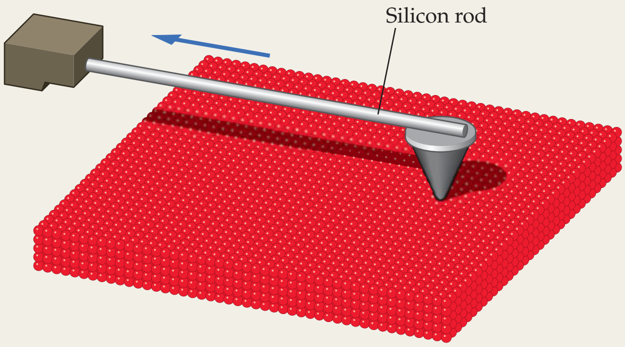
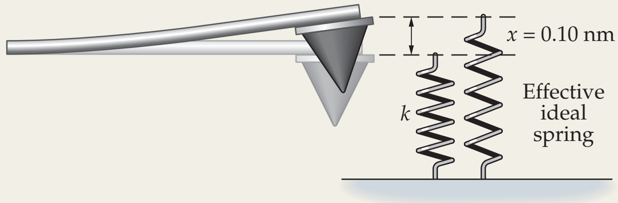
PICTURE THE PROBLEM

The sketch on the left shows the cantilever and its sharp point being dragged across the surface of a material. In the sketch to the right, we show an exaggerated view of the cantilever's deflection, and indicate that it is equivalent to the stretch of an "effective" ideal spring with a force constant k.





STRATEGY

a. Given that $W = 1.2 \times 10^{-20}$ J for a deflection of $x = 0.10$ nm, we can find the effective force constant k using $W = \frac{1}{2}kx^2$.

b. To find the work required to deflect from $x = 0.10$ nm to $x = 0.20$ nm, $W_{1\to 2}$, we calculate the work to deflect from $x = 0$ to $x = 0.20$ nm, $W_{0\to 2}$, and then subtract the work needed to deflect from $x = 0$ to $x = 0.10$ nm, $W_{0\to 1}$. (Note that we *cannot* simply assume the work to go from $x = 0.10$ nm to $x = 0.20$ nm is the same as the work to go from $x = 0$ to $x = 0.10$ nm.)

SOLUTION

Part (a)

1. Solve $W = \frac{1}{2}kx^2$ for the force constant k:

$$k = \frac{2W}{x^2} = \frac{2(1.2 \times 10^{-20}\ \text{J})}{(0.10 \times 10^{-9}\ \text{m})^2} = 2.4\ \text{N/m}$$

Part (b)

2. First, calculate the work needed to deflect the cantilever from $x = 0$ to $x = 0.20$ nm:

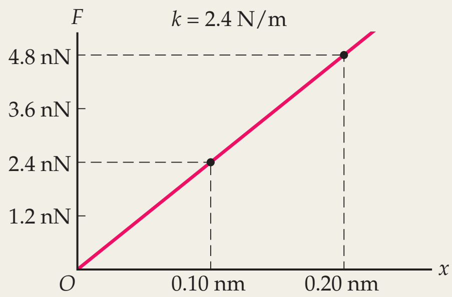
$$W_{0\to 2} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(2.4\ \text{N/m})(0.2 \times 10^{-9}\ \text{m})^2 = 4.8 \times 10^{-20}\ \text{J}$$

3. Subtract from the above result the work to deflect from $x = 0$ to $x = 0.10$ nm, which the problem statement gives as 1.2×10^{-20} J:

$$W_{1\to 2} = W_{0\to 2} - W_{0\to 1} = 4.8 \times 10^{-20}\ \text{J} - 1.2 \times 10^{-20}\ \text{J} = 3.6 \times 10^{-20}\ \text{J}$$

INSIGHT

Our results show that more energy is needed to deflect the cantilever the second 0.10 nm than to deflect it the first 0.10 nm. Why? The reason is that the force of the cantilever increases with distance; thus, the average force over the second 0.10 nm is greater than the average force over the first 0.10 nm. In fact, we can see from the adjacent figure that the average force between 0.10 nm and 0.20 nm (3.6 nN) is three times the average force between 0 and 0.10 nm (1.2 nN). It follows, then, that the work required for the second 0.10 nm is three times the work required for the first 0.10 nm.



PRACTICE PROBLEM

A second cantilever has half the force constant of the cantilever in this Example. Is the work required to deflect the second cantilever by 0.20 nm greater than, less than, or equal to the work required to deflect the cantilever in this Example by 0.10 nm? **[Answer:** Halving the force constant halves the work, but doubling the deflection quadruples the work. The net effect is that the work increases by a factor of two, to 2.4×10^{-20} J.]

Some related homework problems: Problem 32, Problem 38

An equivalent way to calculate the work for a variable force is to multiply the average force, F_{av}, by the distance, d:

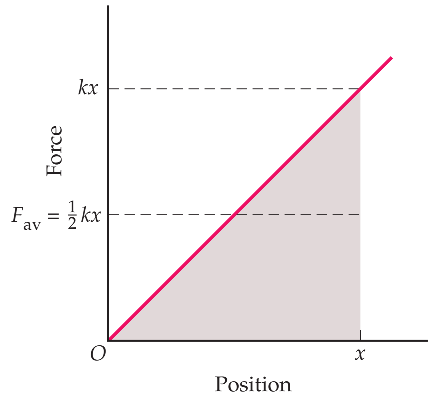
$$W = F_{av}d \qquad 7\text{–}9$$

For a spring that is stretched a distance x from equilibrium the force varies linearly from 0 to kx. Thus, the average force is $F_{av} = \frac{1}{2}kx$, as indicated in **Figure 7–11**. Therefore, the work is

$$W = \tfrac{1}{2}kx(x) = \tfrac{1}{2}kx^2$$

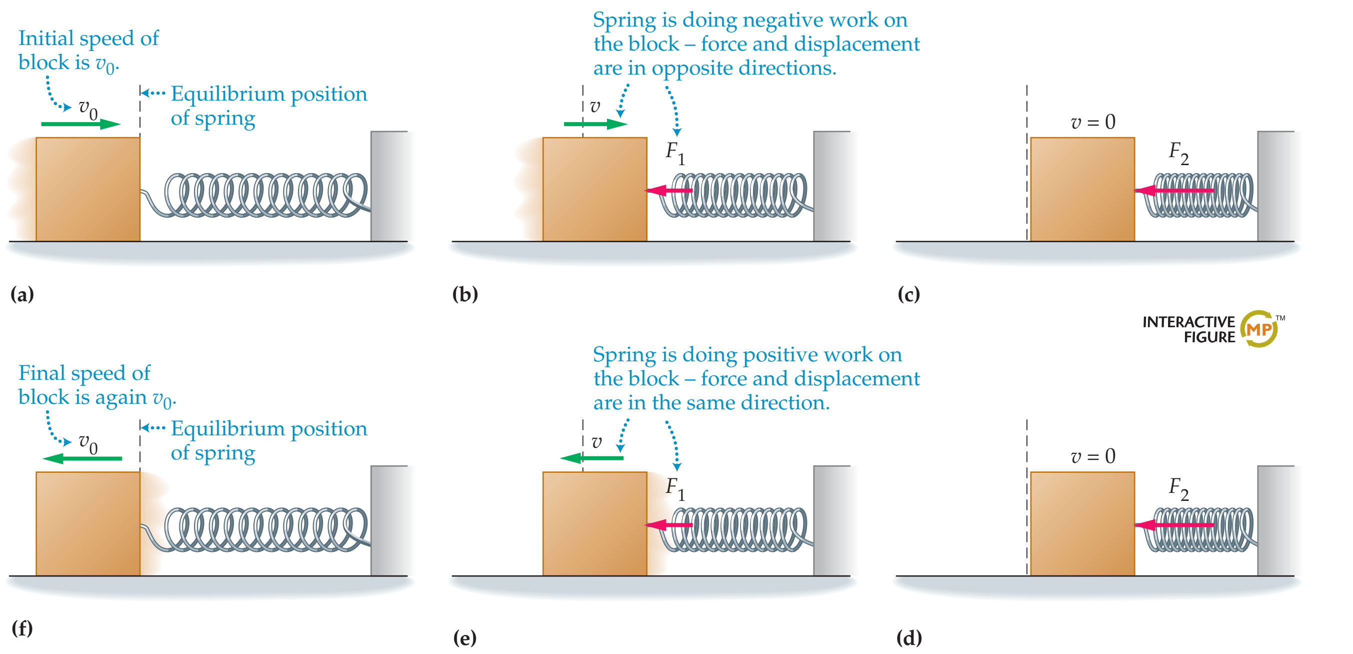
As expected, our result agrees with Equation 7–8.

Finally, when you stretch or compress a spring from its equilibrium position, the work you do is always positive. The work done *by* a spring, however, may be positive or negative, depending on the situation. For example, consider a block sliding to the right with an initial speed v_0 on a smooth, horizontal surface, as shown in **Figure 7–12 (a)**. When the block begins to compress the spring, as in **Figure 7–12 (b)**, the spring exerts a force on the block to the left—that is, opposite to the block's direction of motion. As a result, the spring does *negative* work on the block, which causes the block's speed to decrease. Eventually the negative work done by the spring, $W = -\frac{1}{2}kx^2$, is equal in magnitude to the initial kinetic energy of the block. At this point, **Figure 7–12 (c)**, the block comes to rest momentarily, and $W = \Delta K = K_f - K_i = 0 - K_i = -K_i = -\frac{1}{2}mv_0^2 = -\frac{1}{2}kx^2$. We apply this result in Active Example 7–1.



▲ **FIGURE 7–11 Work done in stretching a spring: average force**

The average force of a spring from $x = 0$ to x is $F_{av} = \frac{1}{2}kx$, and the work done is $W = F_{av}d = \frac{1}{2}kx(x) = \frac{1}{2}kx^2$.



▲ **FIGURE 7–12 The work done by a spring can be positive or negative**

(a) A block slides to the right on a frictionless surface with a speed v_0 until it encounters a spring. **(b)** The spring now exerts a force to the left—opposite to the block's motion—and hence it does negative work on the block. This causes the block's speed to decrease. **(c)** The negative work done by the spring eventually is equal in magnitude to the block's initial kinetic energy, at which point the block comes to rest momentarily. As the spring expands, **(d)** and **(e)**, it does positive work on the block and increases its speed. **(f)** When the block leaves the spring its speed is again equal to v_0.

ACTIVE EXAMPLE 7–1 A BLOCK COMPRESSES A SPRING

Suppose the block in Figure 7–12 (a) has a mass of 1.5 kg and moves with an initial speed of $v_0 = 2.2$ m/s. Find the compression of the spring, whose force constant is 475 N/m, when the block momentarily comes to rest.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the initial and final kinetic energies of the block:	$K_i = 3.6$ J, $K_f = 0$
2. Calculate the change in kinetic energy of the block:	$\Delta K = -3.6$ J
3. Set the negative work done by the spring equal to the change in kinetic energy of the block:	$-\frac{1}{2}kx^2 = \Delta K = -3.6$ J
4. Solve for the compression, x, and substitute numerical values:	$x = 0.12$ m

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INSIGHT

After the block comes to rest, the spring expands back to its equilibrium position, as shown in **Figures 7–12 (d)–(f)**. During this expansion the force exerted by the spring is in the same direction as the block's motion, and hence it does *positive* work in the amount $W = \frac{1}{2}kx^2$. As a result, the block leaves the spring with the same speed it had initially.

YOUR TURN

Find the compression of the spring for the case where the mass of the block is doubled to 3.0 kg.

(Answers to **Your Turn** *problems are given in the back of the book.)*

7–4 Power

Power is a measure of how *quickly* work is done. To be precise, suppose the work W is performed in the time t. The average power delivered during this time is defined as follows:

Definition of Average Power, P

$$P = \frac{W}{t} \qquad 7\text{–}10$$

SI unit: J/s = watt, W

For simplicity of notation we drop the usual subscript av for an average quantity and simply understand that the power P refers to an average power unless stated otherwise.

Note that the dimensions of power are joules (work) per second (time). We define one joule per second to be a watt (W), after James Watt (1736–1819), the Scottish engineer and inventor who played a key role in the development of practical steam engines:

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} \qquad 7\text{–}11$$

Of course, the watt is the unit of power used to rate the output of lightbulbs. Another common unit of power is the horsepower (hp), which is used to rate the output of car engines. It is defined as follows:

$$1 \text{ horsepower} = 1 \text{ hp} = 746 \text{ W} \qquad 7\text{–}12$$

Though it sounds like a horse should be able to produce one horsepower, in fact, a horse can generate only about 2/3 hp for sustained periods. The reason for the discrepancy is that when James Watt defined the horsepower—as a way to characterize the output of his steam engines—he purposely chose a unit that was overly generous to the horse, so that potential investors couldn't complain he was overstating the capability of his engines.

To get a feel for the magnitude of the watt and the horsepower, consider the power you might generate when walking up a flight of stairs. Suppose, for example, that an 80.0-kg person walks up a flight of stairs in 20.0 s, and that the altitude gain is 12.0 ft (3.66 m). Referring to Example 7–2 and Conceptual Checkpoint 7–1, we find that the work done by the person is $W = mgh = (80.0 \text{ kg})(9.81 \text{ m/s}^2)(3.66 \text{ m}) = 2870 \text{ J}$. To find the power, we simply divide by the time: $P = W/t = (2870 \text{ J})/(20.0 \text{ s}) = 144 \text{ W} = 0.193 \text{ hp}$. Thus, a leisurely stroll up the stairs requires about 1/5 hp or 150 W. Similarly, the power produced by a sprinter bolting out of the starting blocks is about 1 hp, and the greatest power most people can produce for sustained periods of time is roughly 1/3 to 1/2 hp. Further examples of power are given in Table 7–3.

Human-powered flight is a feat just barely within our capabilities, since the most efficient human-powered airplanes require a steady power output of about 1/3 hp. On August 23, 1977, the *Gossamer Condor*, designed by Paul MacCready and flown by Bryan Allen, became the first human-powered airplane to complete a prescribed one-mile, figure-eight course and claim the Kremer Prize of £50,000. Allen, an accomplished bicycle racer, used bicycle-like pedals to spin the pro-

TABLE 7–3 Typical Values of Power

Source	Approximate power (W)
Hoover Dam	1.34×10^9
Car moving at 40 mi/h	7×10^4
Home stove	1.2×10^4
Sunlight falling on one square meter	1380
Refrigerator	615
Television	200
Person walking up stairs	150
Human brain	20

REAL-WORLD PHYSICS: BIO

Human power output and flight

▲ The *Gossamer Albatross* on its record-breaking flight across the English Channel in 1979. On two occasions the aircraft actually touched the surface of the water, but the pilot was able to maintain control and complete the 22.25-mile flight.

peller. Controlling the slow-moving craft while pedaling at full power was no easy task. Allen also piloted the *Gossamer Albatross*, which, in 1979, became the first (and so far the only) human-powered aircraft to fly across the English Channel. This 22.25-mile flight—from Folkestone, England, to Cap Gris-Nez, France—took 2 hours 49 minutes and required a total energy output roughly equivalent to climbing to the top of the Empire State Building 10 times.

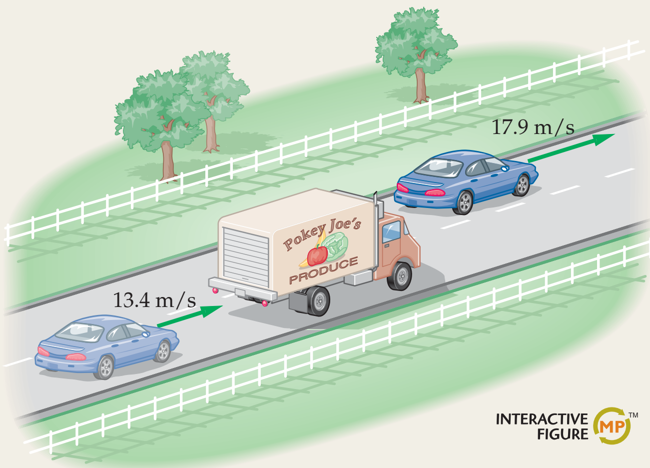
Power output is also an important factor in the performance of a car. For example, suppose it takes a certain amount of work, W, to accelerate a car from 0 to 60 mi/h. If the average power provided by the engine is P, then according to Equation 7–10 the amount of time required to reach 60 mi/h is $t = W/P$. Clearly, the greater the power P, the less the time required to accelerate. Thus, in a loose way of speaking, we can say that the power of a car is a measure of "how fast it can go fast."

EXAMPLE 7–8 PASSING FANCY

To pass a slow-moving truck, you want your fancy 1.30×10^3-kg car to accelerate from 13.4 m/s (30.0 mi/h) to 17.9 m/s (40.0 mi/h) in 3.00 s. What is the minimum power required for this pass?

PICTURE THE PROBLEM

Our sketch shows the car accelerating from an initial speed of $v_i = 13.4$ m/s to a final speed of $v_f = 17.9$ m/s. We assume the road is level, so that no work is done against gravity, and that friction and air resistance may be ignored.



STRATEGY

Power is work divided by time, and work is equal to the change in kinetic energy as the car accelerates. We can determine the change in kinetic energy from the given mass of the car and its initial and final speeds. With this information at hand, we can determine the power with the relation $P = W/t = \Delta K/t$.

SOLUTION

1. First, calculate the change in kinetic energy:

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1.30 \times 10^3\text{ kg})(17.9\text{ m/s})^2 - \tfrac{1}{2}(1.30 \times 10^3\text{ kg})(13.4\text{ m/s})^2 = 9.16 \times 10^4\text{ J}$$

2. Divide by time to find the minimum power. (The actual power would have to be greater to overcome frictional losses.):

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{9.16 \times 10^4\text{ J}}{3.00\text{ s}} = 3.05 \times 10^4\text{ W} = 40.9\text{ hp}$$

INSIGHT

Suppose that your fancy car continues to produce the same 3.05×10^4 W of power as it accelerates from $v = 17.9$ m/s (40.0 mi/h) to $v = 22.4$ ms (50.0 mi/h). Is the time required more than, less than, or equal to 3.00 s? It will take more than 3.00 s. The reason is that ΔK is greater for a change in speed from 40.0 mi/h to 50.0 mi/h than for a change in speed from 30.0 mi/h to 40.0 mi/h, because K depends on speed squared. Since ΔK is greater, the time $t = \Delta K/P$ is also greater.

PRACTICE PROBLEM

Find the time required to accelerate from 40.0 mi/h to 50.0 mi/h with 3.05×10^4 W of power. [**Answer:** First, $\Delta K = 1.18 \times 10^5$ J. Second, $P = \Delta K/t$ can be solved for time to give $t = \Delta K/P$. Thus, $t = 3.87$ s.]

Some related homework problems: Problem 44, Problem 59

Finally, consider a system in which a car, or some other object, is moving with a constant speed v. For example, a car might be traveling uphill on a road inclined at an angle θ above the horizontal. To maintain a constant speed, the engine must exert a constant force F equal to the combined effects of friction, gravity, and air

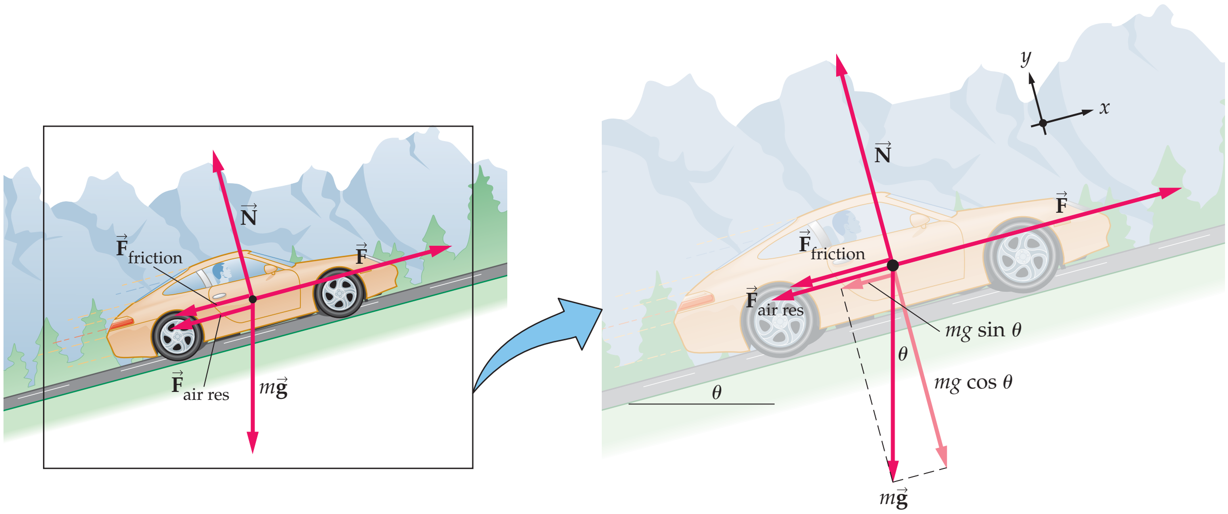


FIGURE 7–13 Driving up a hill
A car traveling uphill at constant speed requires a constant force, F, of magnitude $mg \sin \theta + F_{air\ res} + F_{friction}$, applied in the direction of motion.

resistance, as indicated in **Figure 7–13**. Now, as the car travels a distance d, the work done by the engine is $W = Fd$, and the power it delivers is

$$P = \frac{W}{t} = \frac{Fd}{t}$$

Since the car has a constant speed, $v = d/t$, it follows that

$$P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv \qquad 7\text{–}13$$

Note that power is directly proportional to both the force and the speed. For example, suppose you push a heavy shopping cart with a force F. You produce twice as much power when you push at 2 m/s than when you push at 1 m/s, even though you are pushing no harder. It's just that the amount of work you do in a given time period is doubled.

ACTIVE EXAMPLE 7–2 FIND THE MAXIMUM SPEED

It takes a force of 1280 N to keep a 1500-kg car moving with constant speed up a slope of 5.00°. If the engine delivers 50.0 hp to the drive wheels, what is the maximum speed of the car?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Convert the power of 50.0 hp to watts: $P = 3.73 \times 10^4$ W
2. Solve Equation 7–13 for the speed v: $v = P/F$
3. Substitute numerical values for the power and force: $v = 29.1$ m/s

INSIGHT
Thus, the maximum speed of the car on this slope is approximately 65 mi/h.

YOUR TURN
How much power is required for a maximum speed of 32.0 m/s?

(Answers to **Your Turn** *problems are given in the back of the book.)*

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Even though work and kinetic energy are scalar quantities, the idea of vectors, and vector components in particular (Chapter 3), was used in the definition of work in Section 7–1.

The kinematic equations of motion for constant acceleration (Chapters 2 and 4) were used in the derivation of kinetic energy in Section 7–2. In particular, we used the relation between the speed of an object and the distance through which it accelerates.

The basic concepts of force, mass, and acceleration (Chapters 5 and 6) were used throughout this chapter. One particular force, the force exerted by a spring (Chapter 6), played a key role in Section 7–3.

LOOKING AHEAD

In Chapter 8 we introduce the concept of potential energy. The combination of kinetic and potential energy is referred to as the mechanical energy, which will play a central role in our discussion of the conservation of energy.

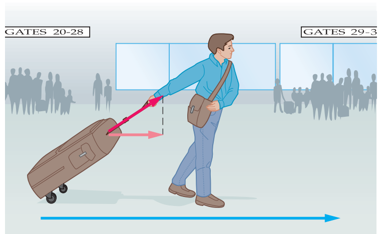
Collisions are studied in Chapter 9. As we shall see, the kinetic energy before and after a collision is an important characterizing feature. Look for the discussion of elastic versus inelastic collision in particular.

The concept of kinetic energy plays a significant role in many areas of physics. Look for it to reappear when we study rotational motion in Chapter 10, and in Section 10–5 in particular. Kinetic energy is also important when we study ideal gases in Chapter 17—in fact, Section 17–2 is titled Kinetic Theory.

CHAPTER SUMMARY

7–1 WORK DONE BY A CONSTANT FORCE

A force exerted through a distance performs mechanical work.



Force in Direction of Motion
In this, the simplest case, work is force times distance:

$$W = Fd \qquad \text{7–1}$$

Force at an Angle θ to Motion
Work is the component of force in the direction of motion, $F \cos\theta$, times distance, d:

$$W = (F\cos\theta)d = Fd\cos\theta \qquad \text{7–3}$$

Negative and Total Work
Work is negative if the force opposes the motion; that is, if $\theta > 90°$. If more than one force does work, the total work is the sum of the works done by each force separately:

$$W_{\text{total}} = W_1 + W_2 + W_3 + \cdots \qquad \text{7–4}$$

Equivalently, sum the forces first to find F_{total}, then

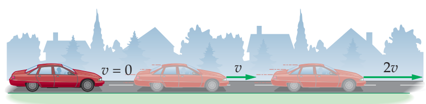
$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}\, d\cos\theta \qquad \text{7–5}$$

Units
The SI unit of work and energy is the joule, J:

$$1\ \text{J} = 1\ \text{N}\cdot\text{m} \qquad \text{7–2}$$

7–2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM

Total work is equal to the change in kinetic energy:



$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \text{7–7}$$

Note: To apply this theorem correctly, you must use the *total* work. Kinetic energy is one-half mass times speed squared:

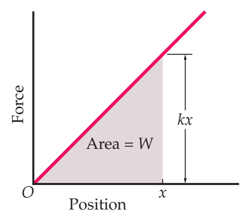
$$K = \tfrac{1}{2}mv^2 \qquad \text{7–6}$$

It follows that kinetic energy is always positive or zero.

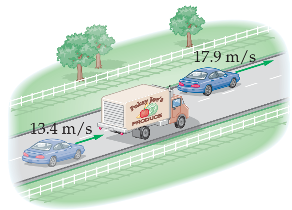
7–3 WORK DONE BY A VARIABLE FORCE

Work is equal to the area between the force curve and the displacement on the x axis. For the case of a spring force, the work to stretch or compress a distance x from equilibrium is

$$W = \tfrac{1}{2}kx^2 \qquad \text{7–8}$$



7–4 POWER



Average power is work divided by the time required to do the work:

$$P = \frac{W}{t} \qquad 7\text{–}10$$

Equivalently, power is force times speed:

$$P = Fv \qquad 7\text{–}13$$

Units

The SI unit of power is the watt, W:

$$1\ \text{W} = 1\ \text{J/s} \qquad 7\text{–}11$$

$$746\ \text{W} = 1\ \text{hp} \qquad 7\text{–}12$$

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Find the work done by a constant force.	Work is defined as force times displacement, $W = Fd$, when F is in the direction of motion. Use $W = (F\cos\theta)d$ when there is an angle θ between the force and the direction of motion.	Examples 7–1 through 7–6
Calculate the change in speed.	The change in kinetic energy is given by the work–energy theorem, $W_{\text{total}} = \Delta K$. From this, the change in speed can be found by recalling that $K = \frac{1}{2}mv^2$. Be sure W_{total} is the total work and that it has the correct sign.	Examples 7–5, 7–6
Calculate the power.	Find the work done, then divide by time: $P = W/t$. Alternatively, find the force, then multiply by the speed: $P = Fv$.	Example 7–8 Active Example 7–2

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

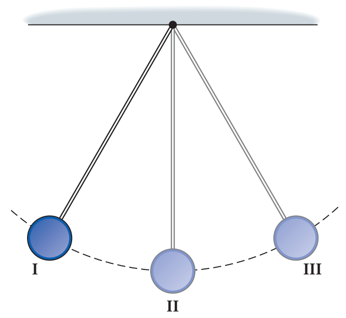
1. Is it possible to do work on an object that remains at rest?
2. A friend makes the statement, "Only the total force acting on an object can do work." Is this statement true or false? If it is true, state why; if it is false, give a counterexample.
3. A friend makes the statement, "A force that is always perpendicular to the velocity of a particle does no work on the particle." Is this statement true or false? If it is true, state why; if it is false, give a counterexample.
4. The net work done on a certain object is zero. What can you say about its speed?
5. To get out of bed in the morning, do you have to do work? Explain.
6. Give an example of a frictional force doing negative work.
7. Give an example of a frictional force doing positive work.
8. A ski boat moves with constant velocity. Is the net force acting on the boat doing work? Explain.
9. A package rests on the floor of an elevator that is rising with constant speed. The elevator exerts an upward normal force on the package, and hence does positive work on it. Why doesn't the kinetic energy of the package increase?
10. An object moves with constant velocity. Is it safe to conclude that no force acts on the object? Why, or why not?
11. Engine 1 does twice the work of engine 2. Is it correct to conclude that engine 1 produces twice as much power as engine 2? Explain.
12. Engine 1 produces twice the power of engine 2. Is it correct to conclude that engine 1 does twice as much work as engine 2? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 7–1 WORK DONE BY A CONSTANT FORCE

1. • **CE** The International Space Station orbits the Earth in an approximately circular orbit at a height of $h = 375$ km above the Earth's surface. In one complete orbit, is the work done by the Earth on the space station positive, negative, or zero? Explain.
2. • **CE** A pendulum bob swings from point I to point II along the circular arc indicated in **Figure 7–14**. **(a)** Is the work done on the bob by gravity positive, negative, or zero? Explain. **(b)** Is the work done on the bob by the string positive, negative, or zero? Explain.
3. • **CE** A pendulum bob swings from point II to point III along the circular arc indicated in Figure 7–14. **(a)** Is the work done on



▲ **FIGURE 7–14** Problems 2 and 3

the bob by gravity positive, negative, or zero? Explain. **(b)** Is the work done on the bob by the string positive, negative, or zero? Explain.

4. • A farmhand pushes a 26-kg bale of hay 3.9 m across the floor of a barn. If she exerts a horizontal force of 88 N on the hay, how much work has she done?

5. • Children in a tree house lift a small dog in a basket 4.70 m up to their house. If it takes 201 J of work to do this, what is the combined mass of the dog and basket?

6. • Early one October, you go to a pumpkin patch to select your Halloween pumpkin. You lift the 3.2-kg pumpkin to a height of 1.2 m, then carry it 50.0 m (on level ground) to the check-out stand. **(a)** Calculate the work you do on the pumpkin as you lift it from the ground. **(b)** How much work do you do on the pumpkin as you carry it from the field?

7. • The coefficient of kinetic friction between a suitcase and the floor is 0.272. If the suitcase has a mass of 71.5 kg, how far can it be pushed across the level floor with 642 J of work?

8. •• You pick up a 3.4-kg can of paint from the ground and lift it to a height of 1.8 m. **(a)** How much work do you do on the can of paint? **(b)** You hold the can stationary for half a minute, waiting for a friend on a ladder to take it. How much work do you do during this time? **(c)** Your friend decides against the paint, so you lower it back to the ground. How much work do you do on the can as you lower it?

9. •• **IP** A tow rope, parallel to the water, pulls a water skier directly behind the boat with constant velocity for a distance of 65 m before the skier falls. The tension in the rope is 120 N. **(a)** Is the work done on the skier by the rope positive, negative, or zero? Explain. **(b)** Calculate the work done by the rope on the skier.

10. •• **IP** In the situation described in the previous problem, **(a)** is the work done on the boat by the rope positive, negative, or zero? Explain. **(b)** Calculate the work done by the rope on the boat.

11. •• A child pulls a friend in a little red wagon with constant speed. If the child pulls with a force of 16 N for 10.0 m, and the handle of the wagon is inclined at an angle of 25° above the horizontal, how much work does the child do on the wagon?

12. •• A 51-kg packing crate is pulled with constant speed across a rough floor with a rope that is at an angle of 43.5° above the horizontal. If the tension in the rope is 115 N, how much work is done on the crate to move it 8.0 m?

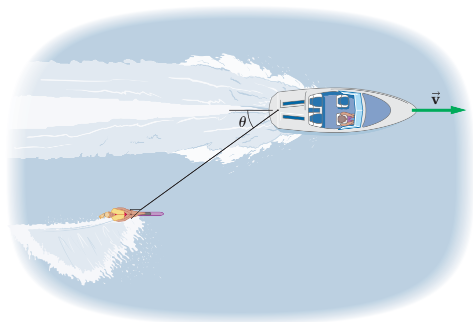
13. •• **IP** To clean a floor, a janitor pushes on a mop handle with a force of 50.0 N. **(a)** If the mop handle is at an angle of 55° above the horizontal, how much work is required to push the mop 0.50 m? **(b)** If the angle the mop handle makes with the horizontal is increased to 65°, does the work done by the janitor increase, decrease, or stay the same? Explain.

14. •• A small plane tows a glider at constant speed and altitude. If the plane does 2.00×10^5 J of work to tow the glider 145 m and the tension in the tow rope is 2560 N, what is the angle between the tow rope and the horizontal?

15. •• A young woman on a skateboard is pulled by a rope attached to a bicycle. The velocity of the skateboarder is $\vec{\mathbf{v}} = (4.1 \text{ m/s})\hat{\mathbf{x}}$ and the force exerted on her by the rope is $\vec{\mathbf{F}} = (17 \text{ N})\hat{\mathbf{x}} + (12 \text{ N})\hat{\mathbf{y}}$. **(a)** Find the work done on the skateboarder by the rope in 25 seconds. **(b)** Assuming the velocity of the bike is the same as that of the skateboarder, find the work the rope does on the bicycle in 25 seconds.

16. •• To keep her dog from running away while she talks to a friend, Susan pulls gently on the dog's leash with a constant force given by $\vec{\mathbf{F}} = (2.2 \text{ N})\hat{\mathbf{x}} + (1.1 \text{ N})\hat{\mathbf{y}}$. How much work does she do on the dog if its displacement is **(a)** $\vec{\mathbf{d}} = (0.25 \text{ m})\hat{\mathbf{x}}$, **(b)** $\vec{\mathbf{d}} = (0.25 \text{ m})\hat{\mathbf{y}}$, or **(c)** $\vec{\mathbf{d}} = (-0.50 \text{ m})\hat{\mathbf{x}} + (-0.25 \text{ m})\hat{\mathbf{y}}$?

17. •• Water skiers often ride to one side of the center line of a boat, as shown in **Figure 7–15**. In this case, the ski boat is traveling at 15 m/s and the tension in the rope is 75 N. If the boat does 3500 J of work on the skier in 50.0 m, what is the angle θ between the tow rope and the center line of the boat?



▲ **FIGURE 7–15** Problems 17 and 69

SECTION 7–2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM

18. • **CE** A pitcher throws a ball at 90 mi/h and the catcher stops it in her glove. **(a)** Is the work done on the ball by the pitcher positive, negative, or zero? Explain. **(b)** Is the work done on the ball by the catcher positive, negative, or zero? Explain.

19. • How much work is needed for a 73-kg runner to accelerate from rest to 7.7 m/s?

20. • **Skylab's Reentry** When Skylab reentered the Earth's atmosphere on July 11, 1979, it broke into a myriad of pieces. One of the largest fragments was a 1770-kg lead-lined film vault, and it landed with an estimated speed of 120 m/s. What was the kinetic energy of the film vault when it landed?

21. • **IP** A 9.50-g bullet has a speed of 1.30 km/s. **(a)** What is its kinetic energy in joules? **(b)** What is the bullet's kinetic energy if its speed is halved? **(c)** If its speed is doubled?

22. •• **CE** **Predict/Explain** The work W_0 accelerates a car from 0 to 50 km/h. **(a)** Is the work required to accelerate the car from 50 km/h to 150 km/h equal to $2W_0$, $3W_0$, $8W_0$, or $9W_0$? **(b)** Choose the *best explanation* from among the following:

I. The work to accelerate the car depends on the speed squared.

II. The final speed is three times the speed that was produced by the work W_0.

III. The increase in speed from 50 km/h to 150 km/h is twice the increase in speed from 0 to 50 km/h.

23. •• **CE** Jogger A has a mass m and a speed v, jogger B has a mass $m/2$ and a speed $3v$, jogger C has a mass $3m$ and a speed $v/2$, and jogger D has a mass $4m$ and a speed $v/2$. Rank the joggers in order of increasing kinetic energy. Indicate ties where appropriate.

24. •• **IP** A 0.14-kg pinecone falls 16 m to the ground, where it lands with a speed of 13 m/s. **(a)** With what speed would the pinecone have landed if there had been no air resistance? **(b)** Did air resistance do positive work, negative work, or zero work on the pinecone? Explain.

25. •• In the previous problem, **(a)** how much work was done on the pinecone by air resistance? **(b)** What was the average force of air resistance exerted on the pinecone?

26. •• At $t = 1.0$ s, a 0.40-kg object is falling with a speed of 6.0 m/s. At $t = 2.0$ s, it has a kinetic energy of 25 J. **(a)** What is the kinetic energy of the object at $t = 1.0$ s? **(b)** What is the speed of the object at $t = 2.0$ s? **(c)** How much work was done on the object between $t = 1.0$ s and $t = 2.0$ s?

27. •• After hitting a long fly ball that goes over the right fielder's head and lands in the outfield, the batter decides to keep going past second base and try for third base. The 62.0-kg player begins sliding 3.40 m from the base with a speed of 4.35 m/s. If the player comes to rest at third base, **(a)** how much work was done on the player by friction? **(b)** What was the coefficient of kinetic friction between the player and the ground?

28. •• **IP** A 1100-kg car coasts on a horizontal road with a speed of 19 m/s. After crossing an unpaved, sandy stretch of road 32 m long, its speed decreases to 12 m/s. **(a)** Was the net work done on the car positive, negative, or zero? Explain. **(b)** Find the magnitude of the average net force on the car in the sandy section.

29. •• **IP** **(a)** In the previous problem, the car's speed decreased by 7.0 m/s as it coasted across a sandy section of road 32 m long. If the sandy portion of the road had been only 16 m long, would the car's speed have decreased by 3.5 m/s, more than 3.5 m/s, or less than 3.5 m/s? Explain. **(b)** Calculate the change in speed in this case.

30. •• A 65-kg bicyclist rides his 8.8-kg bicycle with a speed of 14 m/s. **(a)** How much work must be done by the brakes to bring the bike and rider to a stop? **(b)** How far does the bicycle travel if it takes 4.0 s to come to rest? **(c)** What is the magnitude of the braking force?

SECTION 7–3 WORK DONE BY A VARIABLE FORCE

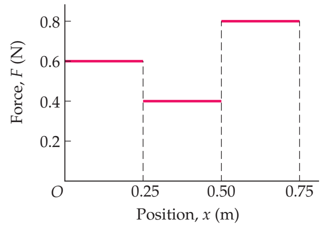
31. • **CE** A block of mass m and speed v collides with a spring, compressing it a distance Δx. What is the compression of the spring if the force constant of the spring is increased by a factor of four?

32. • A spring with a force constant of 3.5×10^4 N/m is initially at its equilibrium length. **(a)** How much work must you do to stretch the spring 0.050 m? **(b)** How much work must you do to compress it 0.050 m?

33. • A 1.2-kg block is held against a spring of force constant 1.0×10^4 N/m, compressing it a distance of 0.15 m. How fast is the block moving after it is released and the spring pushes it away?

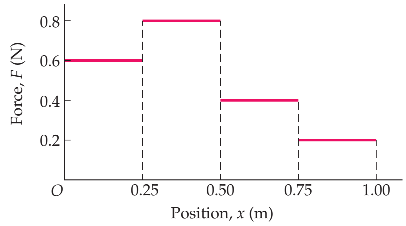
34. • Initially sliding with a speed of 2.2 m/s, a 1.8-kg block collides with a spring and compresses it 0.31 m before coming to rest. What is the force constant of the spring?

35. • The force shown in **Figure 7–16** moves an object from $x = 0$ to $x = 0.75$ m. **(a)** How much work is done by the force? **(b)** How much work is done by the force if the object moves from $x = 0.15$ m to $x = 0.60$ m?



▲ **FIGURE 7–16** Problem 35

36. • An object is acted on by the force shown in **Figure 7–17**. What is the final position of the object if its initial position is $x = 0.40$ m and the work done on it is equal to **(a)** 0.21 J, or **(b)** -0.19 J?



▲ **FIGURE 7–17** Problems 36 and 40

37. •• **CE** A block of mass m and speed v collides with a spring, compressing it a distance Δx. What is the compression of the spring if the mass of the block is halved and its speed is doubled?

38. •• To compress spring 1 by 0.20 m takes 150 J of work. Stretching spring 2 by 0.30 m requires 210 J of work. Which spring is stiffer?

39. •• **IP** It takes 180 J of work to compress a certain spring 0.15 m. **(a)** What is the force constant of this spring? **(b)** To compress the spring an additional 0.15 m, does it take 180 J, more than 180 J, or less than 180 J? Verify your answer with a calculation.

40. •• The force shown in Figure 7–17 acts on a 1.7-kg object whose initial speed is 0.44 m/s and initial position is $x = 0.27$ m. **(a)** Find the speed of the object when it is at the location $x = 0.99$ m. **(b)** At what location would the object's speed be 0.32 m/s?

41. ••• A block is acted on by a force that varies as $(2.0 \times 10^4\ \mathrm{N/m})x$ for $0 \le x \le 0.21$ m, and then remains constant at 4200 N for larger x. How much work does the force do on the block in moving it **(a)** from $x = 0$ to $x = 0.30$ m, or **(b)** from $x = 0.10$ m to $x = 0.40$ m?

SECTION 7–4 POWER

42. • **CE** Force F_1 does 5 J of work in 10 seconds, force F_2 does 3 J of work in 5 seconds, force F_3 does 6 J of work in 18 seconds, and force F_4 does 25 J of work in 125 seconds. Rank these forces in order of increasing power they produce. Indicate ties where appropriate.

43. • **BIO** **Climbing the Empire State Building** A new record for running the stairs of the Empire State Building was set on February 3, 2003. The 86 flights, with a total of 1576 steps, was run in 9 minutes and 33 seconds. If the height gain of each step was 0.20 m, and the mass of the runner was 70.0 kg, what was his average power output during the climb? Give your answer in both watts and horsepower.

44. • How many joules of energy are in a kilowatt-hour?

45. • Calculate the power output of a 1.4-g fly as it walks straight up a windowpane at 2.3 cm/s.

46. • An ice cube is placed in a microwave oven. Suppose the oven delivers 105 W of power to the ice cube and that it takes 32,200 J to melt it. How long does it take for the ice cube to melt?

47. • You raise a bucket of water from the bottom of a deep well. If your power output is 108 W, and the mass of the bucket and the water in it is 5.00 kg, with what speed can you raise the bucket? Ignore the weight of the rope.

48. •• In order to keep a leaking ship from sinking, it is necessary to pump 12.0 lb of water each second from below deck up a height of 2.00 m and over the side. What is the minimum horsepower motor that can be used to save the ship?

49. •• **IP** A kayaker paddles with a power output of 50.0 W to maintain a steady speed of 1.50 m/s. **(a)** Calculate the resistive force exerted by the water on the kayak. **(b)** If the kayaker doubles her power output, and the resistive force due to the water remains the same, by what factor does the kayaker's speed change?

50. •• **BIO Human-Powered Flight** Human-powered aircraft require a pilot to pedal, as in a bicycle, and produce a sustained power output of about 0.30 hp. The *Gossamer Albatross* flew across the English Channel on June 12, 1979, in 2 h 49 min. **(a)** How much energy did the pilot expend during the flight? **(b)** How many Snickers candy bars (280 Cal per bar) would the pilot have to consume to be "fueled up" for the flight? [*Note:* The nutritional calorie, 1 Cal, is equivalent to 1000 calories (1000 cal) as defined in physics. In addition, the conversion factor between calories and joules is as follows: 1 Cal = 1000 cal = 1 kcal = 4186 J.]

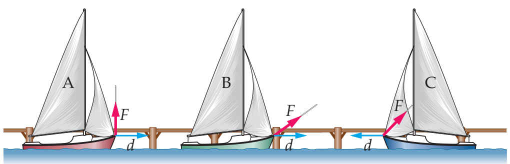
51. •• **IP** A grandfather clock is powered by the descent of a 4.35-kg weight. **(a)** If the weight descends through a distance of 0.760 m in 3.25 days, how much power does it deliver to the clock? **(b)** To increase the power delivered to the clock, should the time it takes for the mass to descend be increased or decreased? Explain.

52. •• **BIO The Power You Produce** Estimate the power you produce in running up a flight of stairs. Give your answer in horsepower.

53. ••• **IP** A certain car can accelerate from rest to the speed v in T seconds. If the power output of the car remains constant, **(a)** how long does it take for the car to accelerate from v to $2v$? **(b)** How fast is the car moving at $2T$ seconds after starting?

GENERAL PROBLEMS

54. • **CE** As the three small sailboats shown in **Figure 7–18** drift next to a dock, because of wind and water currents, students pull on a line attached to the bow and exert forces of equal magnitude F. Each boat drifts through the same distance d. Rank the three boats (A, B, and C) in order of increasing work done on the boat by the force F. Indicate ties where appropriate.



▲ **FIGURE 7–18** Problem 54

55. • **CE** A youngster rides on a skateboard with a speed of 2 m/s. After a force acts on the youngster, her speed is 3 m/s. Was the work done by the force positive, negative, or zero? Explain.

56. • **CE Predict/Explain** A car is accelerated by a constant force, F. The distance required to accelerate the car from rest to the speed v is Δx. **(a)** Is the distance required to accelerate the car from the speed v to the speed $2v$ equal to Δx, $2\Delta x$, $3\Delta x$, or $4\Delta x$? **(b)** Choose the *best explanation* from among the following:
   I. The final speed is twice the initial speed.
   II. The increase in speed is the same in each case.
   III. Work is force times distance, and work depends on the speed squared.

57. • **CE** Car 1 has four times the mass of car 2, but they both have the same kinetic energy. If the speed of car 2 is v, is the speed of car 1 equal to $v/4$, $v/2$, $2v$, or $4v$? Explain.

58. • **BIO Muscle Cells** Biological muscle cells can be thought of as nanomotors that use the chemical energy of ATP to produce mechanical work. Measurements show that the active proteins within a muscle cell (such as myosin and actin) can produce a force of about 7.5 pN and displacements of 8.0 nm. How much work is done by such proteins?

59. • When you take a bite out of an apple, you do about 19 J of work. Estimate **(a)** the force and **(b)** the power produced by your jaw muscles during the bite.

60. • A Mountain bar has a mass of 0.045 kg and a calorie rating of 210 Cal. What speed would this candy bar have if its kinetic energy were equal to its metabolic energy? [See the note following Problem 50.]

61. • A small motor runs a lift that raises a load of bricks weighing 836 N to a height of 10.7 m in 23.2 s. Assuming that the bricks are lifted with constant speed, what is the minimum power the motor must produce?

62. • You push a 67-kg box across a floor where the coefficient of kinetic friction is $\mu_k = 0.55$. The force you exert is horizontal. **(a)** How much power is needed to push the box at a speed of 0.50 m/s? **(b)** How much work do you do if you push the box for 35 s?

63. • **BIO The Beating Heart** The average power output of the human heart is 1.33 watts. **(a)** How much energy does the heart produce in a day? **(b)** Compare the energy found in part (a) with the energy required to walk up a flight of stairs. Estimate the height a person could attain on a set of stairs using nothing more than the daily energy produced by the heart.

64. • **The Atmos Clock** The Atmos clock (the so-called perpetual motion clock) gets its name from the fact that it runs off pressure variations in the atmosphere, which drive a bellows containing a mixture of gas and liquid ethyl chloride. Because the power to drive these clocks is so limited, they must be very efficient. In fact, a single 60.0-W lightbulb could power 240 million Atmos clocks simultaneously. Find the amount of energy, in joules, required to run an Atmos clock for one day.

65. •• **CE** The work W_0 is required to accelerate a car from rest to the speed v_0. How much work is required to accelerate the car **(a)** from rest to the speed $v_0/2$ and **(b)** from $v_0/2$ to v_0?

66. •• **CE** A work W_0 is required to stretch a certain spring 2 cm from its equilibrium position. **(a)** How much work is required to stretch the spring 1 cm from equilibrium? **(b)** Suppose the spring is already stretched 2 cm from equilibrium. How much additional work is required to stretch it to 3 cm from equilibrium?

67. •• After a tornado, a 0.55-g straw was found embedded 2.3 cm into the trunk of a tree. If the average force exerted on the straw by the tree was 65 N, what was the speed of the straw when it hit the tree?

68. •• You throw a glove straight upward to celebrate a victory. Its initial kinetic energy is K and it reaches a maximum height h. What is the kinetic energy of the glove when it is at the height $h/2$?

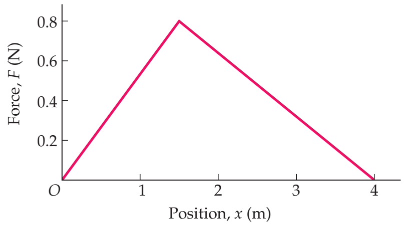
69. •• The water skier in Figure 7–15 is at an angle of 35° with respect to the center line of the boat, and is being pulled at a constant speed of 14 m/s. If the tension in the tow rope is 90.0 N, **(a)** how much work does the rope do on the skier in 10.0 s? **(b)** How much work does the resistive force of water do on the skier in the same time?

70. •• **IP** A sled with a mass of 5.80 kg is pulled along the ground through a displacement given by $\vec{\mathbf{d}} = (4.55\text{ m})\hat{\mathbf{x}}$. (Let the x axis be horizontal and the y axis be vertical.) **(a)** How much work is done on the sled when the force acting on it is $\vec{\mathbf{F}} = (2.89\text{ N})\hat{\mathbf{x}} + (0.131\text{ N})\hat{\mathbf{y}}$? **(b)** How much work is done on the sled when the force acting on it is $\vec{\mathbf{F}} = (2.89\text{ N})\hat{\mathbf{x}} + (0.231\text{ N})\hat{\mathbf{y}}$? **(c)** If the mass of the sled is increased, does the work done by the forces in parts (a) and (b) increase, decrease, or stay the same? Explain.

71. •• **IP** A 0.19-kg apple falls from a branch 3.5 m above the ground. **(a)** Does the power delivered to the apple by gravity increase, decrease, or stay the same during the time the apple falls to the ground? Explain. Find the power delivered by gravity to the apple when the apple is **(b)** 2.5 m and **(c)** 1.5 m above the ground.

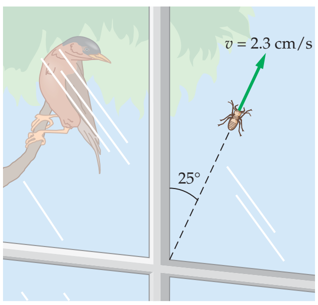
72. •• A juggling ball of mass m is thrown straight upward from an initial height h with an initial speed v_0. How much work has gravity done on the ball **(a)** when it reaches its greatest height, h_{max}, and **(b)** when it reaches ground level? **(c)** Find an expression for the kinetic energy of the ball as it lands.

73. •• The force shown in **Figure 7–19** acts on an object that moves along the x axis. How much work is done by the force as the object moves from **(a)** $x = 0$ to $x = 2.0$ m, **(b)** $x = 1.0$ m to $x = 4.0$ m, and **(c)** $x = 3.5$ m to $x = 1.2$ m?



▲ **FIGURE 7–19** Problem 73

74. •• Calculate the power output of a 1.8-g spider as it walks up a windowpane at 2.3 cm/s. The spider walks on a path that is at 25° to the vertical, as illustrated in **Figure 7–20**.



▲ **FIGURE 7–20** Problem 74

75. •• The motor of a ski boat produces a power of 36,600 W to maintain a constant speed of 14.0 m/s. To pull a water skier at the same constant speed, the motor must produce a power of 37,800 W. What is the tension in the rope pulling the skier?

76. •• **Cookie Power** To make a batch of cookies, you mix half a bag of chocolate chips into a bowl of cookie dough, exerting a 21-N force on the stirring spoon. Assume that your force is always in the direction of motion of the spoon. **(a)** What power is needed to move the spoon at a speed of 0.23 m/s? **(b)** How much work do you do if you stir the mixture for 1.5 min?

77. •• **IP** A pitcher accelerates a 0.14-kg hardball from rest to 42.5 m/s in 0.060 s. **(a)** How much work does the pitcher do on the ball? **(b)** What is the pitcher's power output during the pitch? **(c)** Suppose the ball reaches 42.5 m/s in less than 0.060 s. Is the power produced by the pitcher in this case more than, less than, or the same as the power found in part (b)? Explain.

78. •• **Catapult Launcher** A catapult launcher on an aircraft carrier accelerates a jet from rest to 72 m/s. The work done by the catapult during the launch is 7.6×10^7 J. **(a)** What is the mass of the jet? **(b)** If the jet is in contact with the catapult for 2.0 s, what is the power output of the catapult?

79. •• **BIO Brain Power** The human brain consumes about 22 W of power under normal conditions, though more power may be required during exams. **(a)** How long can one Snickers bar (see the note following Problem 50) power the normally functioning brain? **(b)** At what rate must you lift a 3.6-kg container of milk (one gallon) if the power output of your arm is to be 22 W? **(c)** How long does it take to lift the milk container through a distance of 1.0 m at this rate?

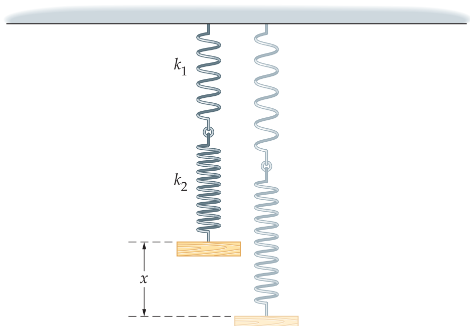
80. •• **IP** A 1300-kg car delivers a constant 49 hp to the drive wheels. We assume the car is traveling on a level road and that all frictional forces may be ignored. **(a)** What is the acceleration of this car when its speed is 14 m/s? **(b)** If the speed of the car is doubled, does its acceleration increase, decrease, or stay the same? Explain. **(c)** Calculate the car's acceleration when its speed is 28 m/s.

81. •• **Meteorite** On October 9, 1992, a 27-pound meteorite struck a car in Peekskill, NY, creating a dent about 22 cm deep. If the initial speed of the meteorite was 550 m/s, what was the average force exerted on the meteorite by the car?

An interplanetary fender bender (Problem 81)

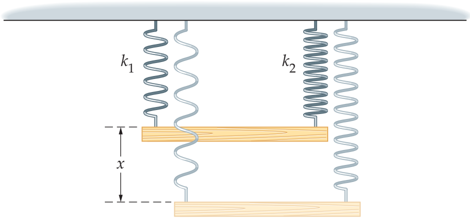
82. ••• **BIO Powering a Pigeon** A pigeon in flight experiences a force of air resistance given approximately by $F = bv^2$, where v is the flight speed and b is a constant. **(a)** What are the units of the constant b? **(b)** What is the largest possible speed of the pigeon if its maximum power output is P? **(c)** By what factor does the largest possible speed increase if the maximum power is doubled?

83. ••• **Springs in Series** Two springs, with force constants k_1 and k_2, are connected in series, as shown in **Figure 7–21**. How much work is required to stretch this system a distance x from the equilibrium position?



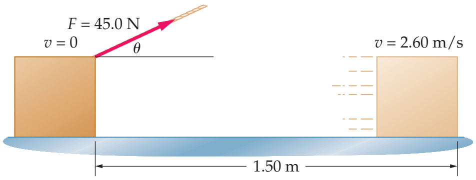
▲ **FIGURE 7–21** Problem 83

84. ••• **Springs in Parallel** Two springs, with force constants k_1 and k_2, are connected in parallel, as shown in **Figure 7–22**. How much work is required to stretch this system a distance x from the equilibrium position?



▲ **FIGURE 7–22** Problem 84

85. ••• A block rests on a horizontal frictionless surface. A string is attached to the block, and is pulled with a force of 45.0 N at an angle θ above the horizontal, as shown in **Figure 7–23**. After the block is pulled through a distance of 1.50 m, its speed is 2.60 m/s, and 50.0 J of work has been done on it. **(a)** What is the angle θ? **(b)** What is the mass of the block?



▲ **FIGURE 7–23** Problem 85

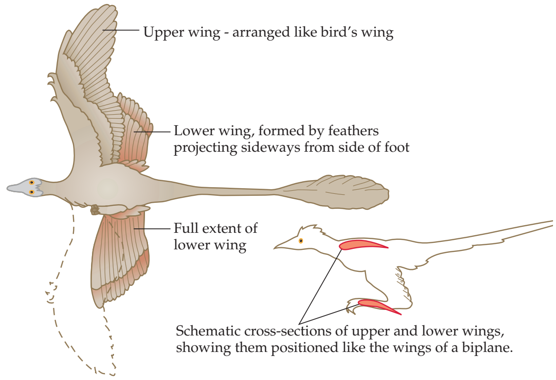
PASSAGE PROBLEMS

BIO *Microraptor gui*: The Biplane Dinosaur

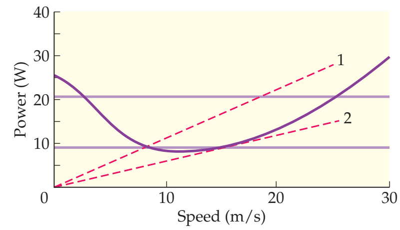
The evolution of flight is a subject of intense interest in paleontology. Some subscribe to the "cursorial" (or ground-up) hypothesis, in which flight began with ground-dwelling animals running and jumping after prey. Others favor the "arboreal" (or trees-down) hypothesis, in which tree-dwelling animals, like modern-day flying squirrels, developed flight as an extension of gliding from tree to tree.

A recently discovered fossil from the Cretaceous period in China supports the arboreal hypothesis and adds a new element—it suggests that feathers on both the wings and the lower legs and feet allowed this dinosaur, *Microraptor gui*, to glide much like a biplane, as shown in **Figure 7–24 (a)**. Researchers have produced a detailed computer simulation of *Microraptor*, and with its help have obtained the power-versus-speed plot presented in **Figure 7–24 (b)**. This curve shows how much power is required for flight at speeds between 0 and 30 m/s. Notice that the power increases at high speeds, as expected, but is also high for low speeds, where the dinosaur is almost hovering. A minimum of 8.1 W is needed for flight at 10 m/s. The lower horizontal line shows the estimated 9.8-W power output of *Microraptor*, indicating the small range of speeds for which flight would be possible. The upper horizontal line shows the wider range of flight speeds that would be available if *Microraptor* were able to produce 20 W of power.

Also of interest are the two dashed, straight lines labeled 1 and 2. These lines represent constant ratios of power to speed; that is, a constant value for P/v. Referring to Equation 7–13, we see that $P/v = Fv/v = F$, so the lines 1 and 2 correspond to lines of constant force. Line 2 is interesting in that it has the smallest slope that still touches the power-versus-speed curve.



(a) Possible reconstruction of *Microraptor gui* in flight



(b)

▲ **FIGURE 7–24** Problems 86, 87, 88, and 89

86. • Estimate the range of flight speeds for *Microraptor gui* if its power output is 9.8 W.

A. 0–7.7 m/s **B.** 7.7–15 m/s **C.** 15–30 m/s **D.** 0–15 m/s

87. • What approximate range of flight speeds would be possible if *Microraptor gui* could produce 20 W of power?

A. 0–25 m/s **B.** 25–30 m/s **C.** 2.5–25 m/s **D.** 0–2.5 m/s

88. •• How much energy would *Microraptor* have to expend to fly with a speed of 10 m/s for 1.0 minute?

A. 8.1 J **B.** 81 J **C.** 490 J **D.** 600 J

89. • Estimate the minimum force that *Microraptor* must exert to fly.

A. 0.65 N **B.** 1.3 N **C.** 1.0 N **D.** 10 N

INTERACTIVE PROBLEMS

90. •• **Referring to Figure 7–12** Suppose the block has a mass of 1.4 kg and an initial speed of 0.62 m/s. **(a)** What force constant must the spring have if the maximum compression is to be 2.4 cm? **(b)** If the spring has the force constant found in part (a), find the maximum compression if the mass of the block is doubled *and* its initial speed is halved.

91. •• **IP Referring to Figure 7–12** In the situation shown in Figure 7–12 (d), a spring with a force constant of 750 N/m is compressed by 4.1 cm. **(a)** If the speed of the block in Figure 7–12 (f) is 0.88 m/s, what is its mass? **(b)** If the mass of the block is doubled, is the final speed greater than, less than, or equal to 0.44 m/s? **(c)** Find the final speed for the case described in part (b).

92. •• **IP Referring to Example 7–8** Suppose the car has a mass of 1400 kg and delivers 48 hp to the wheels. **(a)** How long does it take for the car to increase its speed from 15 m/s to 25 m/s? **(b)** Would the time required to increase the speed from 5.0 m/s to 15 m/s be greater than, less than, or equal to the time found in part (a)? **(c)** Determine the time required to accelerate from 5.0 m/s to 15 m/s.