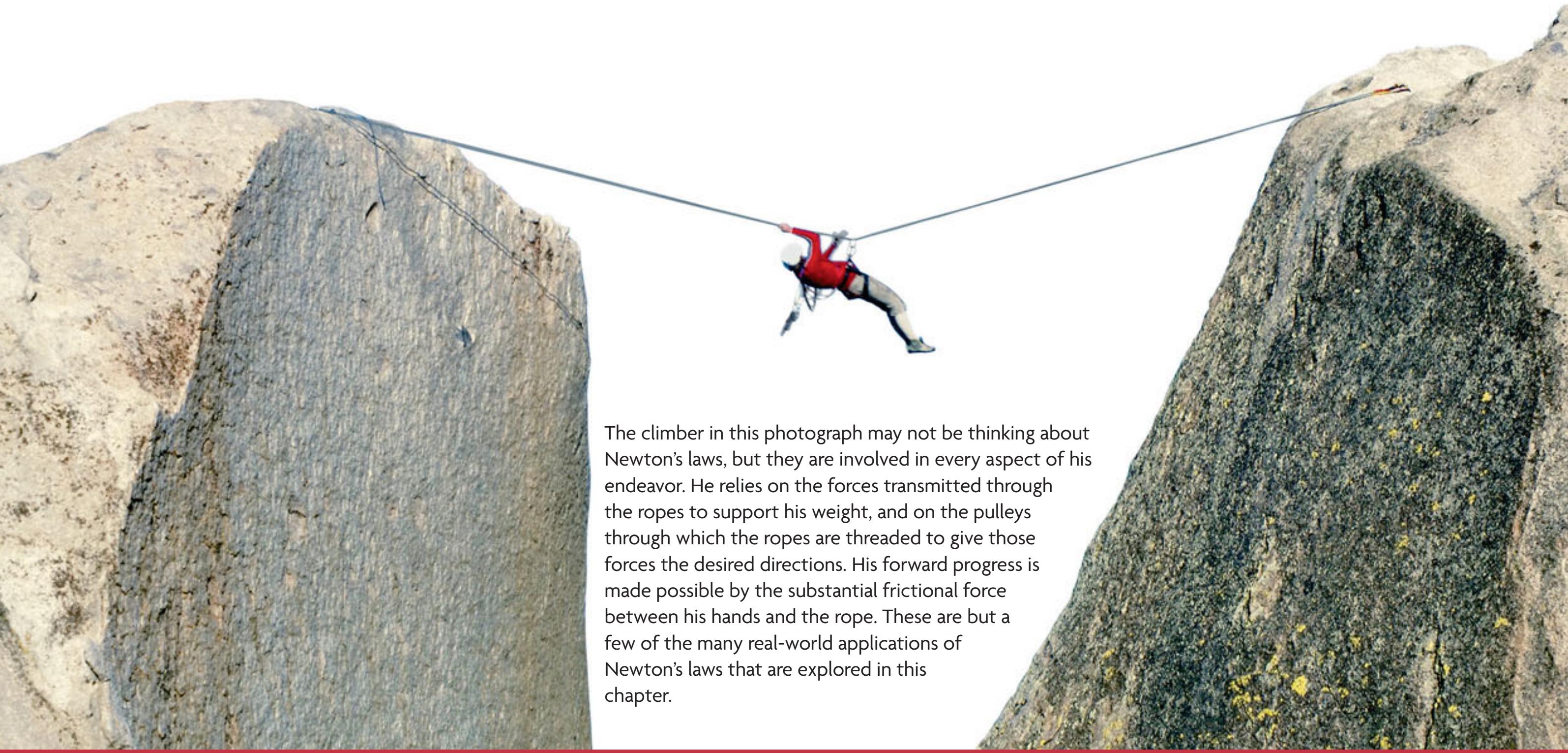
6 Applications of Newton's Laws

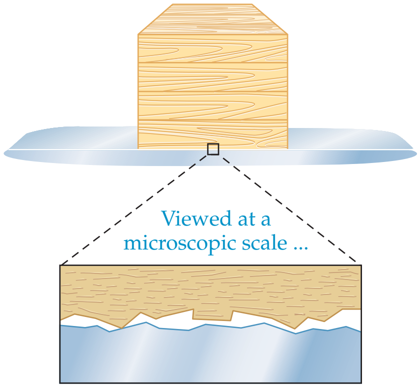


The climber in this photograph may not be thinking about Newton's laws, but they are involved in every aspect of his endeavor. He relies on the forces transmitted through the ropes to support his weight, and on the pulleys through which the ropes are threaded to give those forces the desired directions. His forward progress is made possible by the substantial frictional force between his hands and the rope. These are but a few of the many real-world applications of Newton's laws that are explored in this chapter.

Newton's laws of motion can be applied to an immense variety of systems, a sampling of which was discussed in Chapter 5. In this chapter we extend our discussion of Newton's laws by introducing new types of forces and by considering new classes of systems.

For example, we begin by considering the forces due to friction between two surfaces. As we shall see, the force of friction is different depending on whether the surfaces are in static contact, or are moving relative to one another—an important consideration in antilock braking systems. And though friction may seem like something that should be eliminated, we show that it is actually essential to life as we know it.

Next, we investigate the forces exerted by strings and springs, and show how these forces can safely suspend a mountain climber over a chasm, or cushion the ride of a locomotive. Finally, we consider the key role that force plays in making circular motion possible.



▲ **FIGURE 6–1 The origin of friction**

Even "smooth" surfaces have irregularities when viewed at the microscopic level. This type of roughness contributes to friction.

6–1 Frictional Forces

In Chapter 5 we always assumed that surfaces were smooth and that objects could slide without resistance to their motion. No surface is perfectly smooth, however. When viewed on the atomic level, even the "smoothest" surface is actually rough and jagged, as indicated in **Figure 6–1**. To slide one such surface across another requires a force large enough to overcome the resistance of microscopic hills and valleys bumping together. This is the origin of the force we call **friction.**

We often think of friction as something that should be reduced, or even eliminated if possible. For example, roughly 20% of the gasoline you buy does nothing but overcome friction within your car's engine. Clearly, reducing that friction would be most desirable.

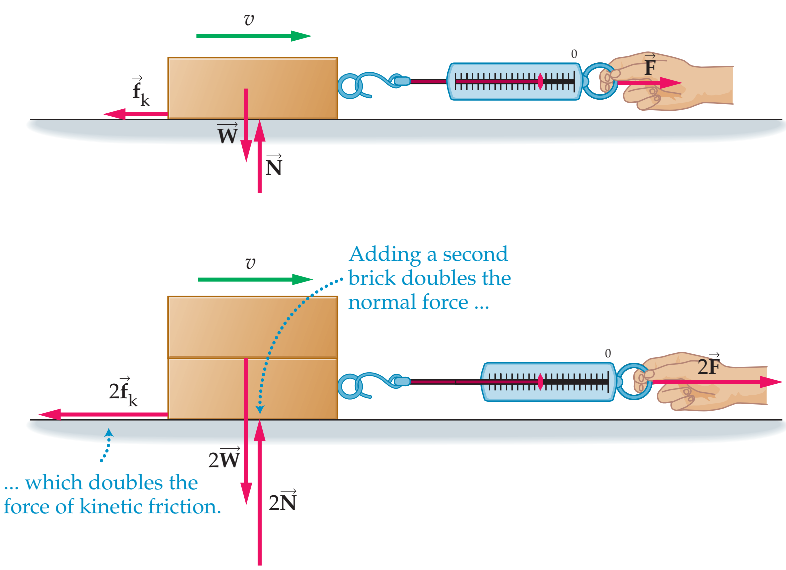
On the other hand, friction can be helpful—even indispensable—in other situations. Suppose, for example, that you are standing still and then decide to begin walking forward. The force that accelerates you is the force of friction between your shoes and the ground. We simply couldn't walk or run without friction—it's hard enough when friction is merely reduced, as on an icy sidewalk. Similarly, starting or stopping a car, or even turning a corner, all require friction. Friction is an important and common feature of everyday life.

Since friction is caused by the random, microscopic irregularities of a surface, and since it is greatly affected by other factors such as the presence of lubricants, there is no simple "law of nature" for friction. There are, however, some very useful rules of thumb that give us rather accurate, approximate results for calculating frictional forces. In what follows, we describe these rules of thumb for the two types of friction most commonly used in this text—kinetic friction and static friction.

Kinetic Friction

As its name implies, kinetic friction is the friction encountered when surfaces slide against one another with a finite relative speed. The force generated by this friction, which will be designated with the symbol f_k, acts to oppose the sliding motion at the point of contact between the surfaces.

A series of simple experiments illustrates the main characteristics of kinetic friction. First, imagine attaching a spring scale to a rough object, like a brick, and pulling it across a table, as shown in **Figure 6–2**. If the brick moves with constant velocity, Newton's second law tells us that the net force on the brick must be zero. Hence, the force read on the scale, F, has the same magnitude as the force of kinetic friction, f_k. Now, if we repeat the experiment, but this time put a second brick on top of the first, we find that the force needed to pull the brick with constant velocity is doubled, to $2F$.



▶ **FIGURE 6–2 The force of kinetic friction depends on the normal force**

In the top part of the figure, a force F is required to pull the brick with constant speed v. Thus the force of kinetic friction is $f_k = F$. In the bottom part of the figure, the normal force has been doubled, and so has the force of kinetic friction, to $f_k = 2F$.

From this experiment we see that when we double the normal force—by stacking up two bricks, for example—the force of kinetic friction is also doubled. In general, the force of kinetic friction is found to be proportional to the magnitude of the normal force, N. Stated mathematically, this observation can be written as follows:

$$f_k = \mu_k N \quad \text{6–1}$$

The constant of proportionality, μ_k (pronounced "mew sub k"), is referred to as the **coefficient of kinetic friction.** In Figure 6–2 the normal force is equal to the weight of the bricks, but this is a special case. The normal force is greater than the weight if someone pushes down on the bricks, and this would cause more friction, or less than the weight if the bricks are placed on an incline. The former case is considered in several homework problems, and the latter case is considered in Examples 6–2 and 6–3.

Since f_k and N are both forces, and hence have the same units, we see that μ_k is a dimensionless number. The coefficient of kinetic friction is always positive, and typical values range between 0 and 1, as indicated in Table 6–1. The interpretation of μ_k is simple: If $\mu_k = 0.1$, for example, the force of kinetic friction is one-tenth of the normal force. Simply put, the greater μ_k the greater the friction; the smaller μ_k the smaller the friction.

TABLE 6–1 Typical Coefficients of Friction

Materials	Kinetic, μ_k	Static, μ_s
Rubber on concrete (dry)	0.80	1–4
Steel on steel	0.57	0.74
Glass on glass	0.40	0.94
Wood on leather	0.40	0.50
Copper on steel	0.36	0.53
Rubber on concrete (wet)	0.25	0.30
Steel on ice	0.06	0.10
Waxed ski on snow	0.05	0.10
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.003	0.01

As we know from everyday experience, the force of kinetic friction tends to oppose motion, as shown in Figure 6–2. Thus, $f_k = \mu_k N$ is not a vector equation, because N is perpendicular to the direction of motion. When doing calculations with the force of kinetic friction, we use $f_k = \mu_k N$ to find its magnitude, and we draw its direction so that it is opposite to the direction of motion.

There are two more friction experiments of particular interest. First, suppose that when we pull a brick, we initially pull it at the speed v, then later at the speed $2v$. What forces do we measure? It turns out that the force of kinetic friction is approximately the same in each case—it certainly does not double when we double the speed. Second, let's try standing the brick on end, so that it has a smaller area in contact with the table. If this smaller area is half the previous area, is the force halved? No, the force remains essentially the same, regardless of the area of contact.

We summarize these observations with the following three rules of thumb for kinetic friction:

Rules of Thumb for Kinetic Friction

The force of kinetic friction between two surfaces is:

1. Proportional to the magnitude of the normal force, N, between the surfaces:

$$f_k = \mu_k N$$

2. Independent of the relative speed of the surfaces.
3. Independent of the area of contact between the surfaces.

▲ Friction plays an important role in almost everything we do. Sometimes it is desirable to reduce friction; in other cases we want as much friction as possible. For example, it is more fun to ride on a water slide (upper) if the friction is low. Similarly, an engine operates more efficiently when it is oiled. When running, however, we need friction to help us speed up, slow down, and make turns. The sole of this running shoe (lower), like a car tire, is designed to maximize friction.

Again, these rules are useful and fairly accurate, though they are still only approximate. For simplicity, when we do calculations involving kinetic friction in this text, we will use these rules as if they were exact.

Before we show how to use f_k in calculations, we should make a comment regarding rule 3. This rule often seems rather surprising and counterintuitive. How is it that a larger area of contact doesn't produce a larger force? One way to think about this is to consider that when the area of contact is large, the normal force is spread out over a large area, giving a small force per area, F/A. As a result, the microscopic hills and valleys are not pressed too deeply against one another. On the other hand, if the area is small, the normal force is concentrated in a small region, which presses the surfaces together more firmly, due to the large force per area. The net effect is roughly the same in either case.

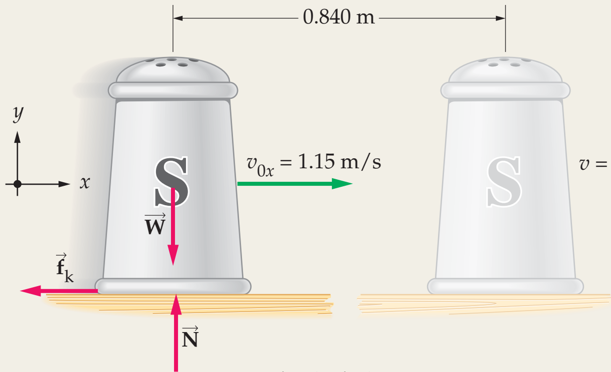
Now, let's consider a commonly encountered situation in which kinetic friction plays a decisive role.

EXAMPLE 6–1 PASS THE SALT—PLEASE

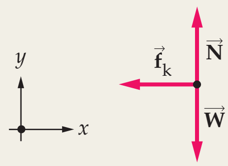
Someone at the other end of the table asks you to pass the salt. Feeling quite dashing, you slide the 50.0-g salt shaker in their direction, giving it an initial speed of 1.15 m/s. **(a)** If the shaker comes to rest with constant acceleration in 0.840 m, what is the coefficient of kinetic friction between the shaker and the table? **(b)** How much time is required for the shaker to come to rest if you slide it with an initial speed of 1.32 m/s?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion, and the positive y direction to be upward. Two forces act in the y direction; the shaker's weight, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$, and the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$. Only one force acts in the x direction: the force of kinetic friction, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$. Note that the shaker moves through a distance of 0.840 m with an initial speed $v_{0x} = 1.15$ m/s.



Physical picture



Free-body diagram

STRATEGY

a. Since the frictional force has a magnitude of $f_k = \mu_k N$, it follows that $\mu_k = f_k/N$. Therefore, we need to find the magnitudes of the frictional force, f_k, and the normal force, N. To find f_k we set $\sum F_x = ma_x$, and find a_x with the kinematic equation $v_x^2 = v_{0x}^2 + 2a_x\Delta x$. To find N we set $a_y = 0$ (since there is no motion in the y direction) and solve for N using $\sum F_y = ma_y = 0$.

b. The coefficient of kinetic friction is independent of the sliding speed, and hence the acceleration of the shaker is also independent of the speed. As a result, we can use the acceleration from part (a) in the relation $v_x = v_{0x} + a_x t$ to find the sliding time.

SOLUTION

Part (a)

1. Set $\Sigma F_x = ma_x$ to find f_k in terms of a_x:

$$\sum F_x = -f_k = ma_x \quad \text{or} \quad f_k = -ma_x$$

2. Determine a_x by using the kinematic equation relating velocity to position, $v_x^2 = v_{0x}^2 + 2a_x\Delta x$:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

$$a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x} = \frac{0 - (1.15\ \text{m/s})^2}{2(0.840\ \text{m})} = -0.787\ \text{m/s}^2$$

3. Set $\Sigma F_y = ma_y = 0$ to find the normal force, N:

$$\sum F_y = N + (-W) = ma_y = 0 \quad \text{or} \quad N = W = mg$$

4. Substitute $N = mg$ and $f_k = -ma_x$ (with $a_x = -0.787\ \text{m/s}^2$) into $\mu_k = f_k/N$ to find μ_k:

$$\mu_k = \frac{f_k}{N} = \frac{-ma_x}{mg} = \frac{-a_x}{g} = \frac{-(-0.787\ \text{m/s}^2)}{9.81\ \text{m/s}^2} = 0.0802$$

Part (b)

5. Use $a_x = -0.787\ \text{m/s}^2$, $v_{0x} = 1.32\ \text{m/s}$, and $v_x = 0$ in $v_x = v_{0x} + a_x t$ to solve for the time, t:

$$v_x = v_{0x} + a_x t \quad \text{or}$$

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - (1.32\ \text{m/s})}{-0.787\ \text{m/s}^2} = 1.68\ \text{s}$$

INSIGHT

Note that m canceled in Step 4, so our result for the coefficient of friction is independent of the shaker's mass. For example, if we were to slide a shaker with twice the mass, but with the same initial speed, it would slide the same distance. It is unlikely this independence would have been apparent if we had worked the problem numerically rather than symbolically. Part (b) shows that the same comments apply to the sliding time—it too is independent of the shaker's mass.

PRACTICE PROBLEM

Given the same initial speed and a coefficient of kinetic friction equal to 0.120, what are **(a)** the acceleration of the shaker, and **(b)** the distance it slides? [**Answer:** **(a)** $a_x = -1.18\ \text{m/s}^2$, **(b)** 0.560 m]

Some related homework problems: Problem 3, Problem 18

In the next Example we consider a system that is inclined at an angle θ relative to the horizontal. As a result, the normal force responsible for the kinetic friction is less than the weight of the object. To be very clear about how we handle the force vectors in such a case, we begin by resolving each vector into its x and y components.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Incline

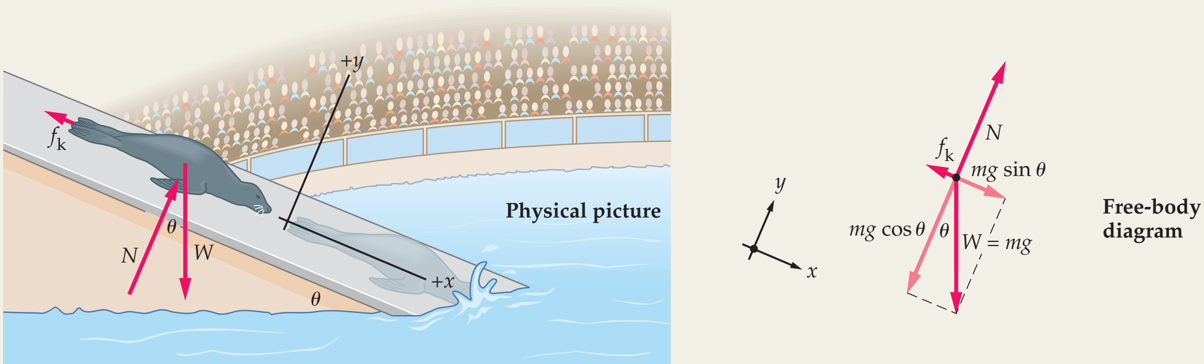
On an incline, align one axis (x) parallel to the surface, and the other axis (y) perpendicular to the surface. That way the motion is in the x direction. Since no motion occurs in the y direction, we know that $a_y = 0$.

EXAMPLE 6–2 MAKING A BIG SPLASH

A trained sea lion slides from rest with constant acceleration down a 3.0-m-long ramp into a pool of water. If the ramp is inclined at an angle of 23° above the horizontal and the coefficient of kinetic friction between the sea lion and the ramp is 0.26, how long does it take for the sea lion to make a splash in the pool?

PICTURE THE PROBLEM

As is usual with inclined surfaces, we choose one axis to be parallel to the surface and the other to be perpendicular to it. In our sketch, the sea lion accelerates in the positive x direction ($a_x > 0$), having started from rest, $v_{0x} = 0$. We are free to choose the initial position of the sea lion to be $x_0 = 0$. There is no motion in the y direction, and therefore $a_y = 0$. Finally, we note from the free-body diagram that $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$, and $\vec{\mathbf{W}} = (mg\sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.



STRATEGY

We can use the kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to find the time of the sea lion's slide. It will be necessary, however, to first determine the acceleration of the sea lion in the x direction, a_x.

To find a_x we apply Newton's second law to the sea lion. First, we can find N by setting $\Sigma F_y = ma_y$ equal to zero (since $a_y = 0$). It is important to start by finding N because we need it to find the force of kinetic friction, $f_k = \mu_k N$. Using f_k in the sum of forces in the x direction, $\Sigma F_x = ma_x$, allows us to solve for a_x and, finally, for the time.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

1. We begin by resolving each of the three force vectors into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{k,x} = -f_k = -\mu_k N \qquad f_{k,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$ to find N:

$$\sum F_y = N - mg\cos\theta = ma_y = 0$$

We see that N is less than the weight, mg:

$$N = mg\cos\theta$$

3. Next, set $\Sigma F_x = ma_x$:

$$\sum F_x = mg\sin\theta - \mu_k$$

Note that the mass cancels in this equation:

$$= mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

4. Solve for the acceleration in the x direction, a_x:

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$
$$= (9.81\ \text{m/s}^2)[\sin 23° - (0.26)\cos 23°]$$
$$= 1.5\ \text{m/s}^2$$

5. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the time when the sea lion reaches the bottom. We choose $x_0 = 0$, and we are given that $v_{0x} = 0$, hence we set $x = \frac{1}{2}a_x t^2 = 3.0$ m and solve for t:

$$x = \tfrac{1}{2}a_x t^2$$
$$t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2(3.0\ \text{m})}{1.5\ \text{m/s}^2}} = 2.0\ \text{s}$$

INSIGHT

Note that we don't need the sea lion's mass to find the time. On the other hand, if we wanted the magnitude of the force of kinetic friction, $f_k = \mu_k N = \mu_k mg\cos\theta$, the mass would be needed.

It is useful to compare the sliding salt shaker in Example 6–1 with the sliding sea lion in this Example. In the case of the salt shaker, friction is the only force acting along the direction of motion (opposite to the direction of motion, in fact), and it brings the object to rest. Because of the slope on which the sea lion slides, however, it experiences both a component of its weight in the forward direction and the friction force opposite to the motion. Since the component of the weight is the larger of the two forces, the sea lion accelerates down the slope—friction only acts to slow its progress.

PRACTICE PROBLEM

How long would it take the sea lion to reach the water if there were no friction in this system? [**Answer:** 1.3 s]

Some related homework problems: Problem 11, Problem 72

Static Friction

Static friction tends to keep two surfaces from moving relative to one another. It, like kinetic friction, is due to the microscopic irregularities of surfaces that are in contact. In fact, static friction is typically stronger than kinetic friction because when surfaces are in static contact, their microscopic hills and valleys can nestle down deeply into one another, thus forming a strong connection between the surfaces that may even include molecular bonding. In kinetic friction, the surfaces bounce along relative to one another and don't become as firmly enmeshed.

As we did with kinetic friction, let's use the results of some simple experiments to determine the rules of thumb for static friction. We start with a brick at rest on a table, with no horizontal force pulling on it, as in **Figure 6–3**. Of course, in this case the force of static friction is zero; no force is needed to keep the brick from sliding.

Next, attach a spring scale to the brick and pull with a small force of magnitude F_1, a force small enough that the brick doesn't move. Since the brick is still at rest, it follows that the force of static friction, f_s, is equal in magnitude to the applied force; that is, $f_s = F_1$. Now, increase the applied force to a new value, F_2, which is still small enough that the brick stays at rest. In this case, the force of static friction has also increased so that $f_s = F_2$. If we continue increasing the applied force, we eventually reach a value beyond which the brick starts to move and kinetic friction takes over, as shown in the figure. Thus, there is an upper limit to the force that can be exerted by static friction, and we call this upper limit $f_{s,max}$.

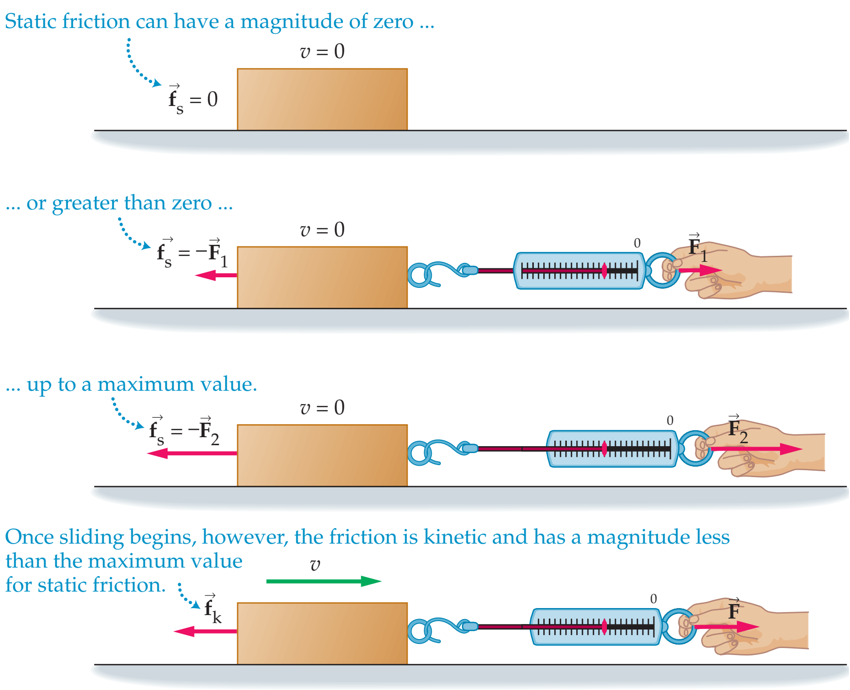


FIGURE 6–3 The maximum limit of static friction

As the force applied to an object increases, so does the force of static friction—up to a certain point. Beyond this maximum value, static friction can no longer hold the object, and it begins to slide. Now kinetic friction takes over.

To summarize, the force of static friction, f_s, can have any value between zero and $f_{s,max}$. This can be written mathematically as follows:

$$0 \le f_s \le f_{s,max} \quad \text{6–2}$$

Imagine repeating the experiment, only now with a second brick on top of the first. This doubles the normal force and it also doubles the maximum force of static friction. Thus, the maximum force is proportional to the magnitude of the normal force, or

$$f_{s,max} = \mu_s N \quad \text{6–3}$$

The constant of proportionality is called μ_s (pronounced "mew sub s"), the **coefficient of static friction.** Note that μ_s, like μ_k, is dimensionless. Typical values are given in Table 6–1. In most cases, μ_s is greater than μ_k, indicating that the force of static friction is greater than the force of kinetic friction, as mentioned. In fact, it is not uncommon for μ_s to be greater than 1, as in the case of rubber in contact with dry concrete.

Finally, two additional comments regarding the nature of static friction: (i) Experiments show that static friction, like kinetic friction, is independent of the area of contact. (ii) The force of static friction is not in the direction of the normal force, thus $f_{s,max} = \mu_s N$ is not a vector relation. The direction of f_s is parallel to the surface of contact, and opposite to the direction the object would move if there were no friction.

These observations are summarized in the following rules of thumb:

Rules of Thumb for Static Friction

The force of static friction between two surfaces has the following properties:

1. It takes on any value between zero and the maximum possible force of static friction, $f_{s,max} = \mu_s N$:

$$0 \le f_s \le \mu_s N$$

2. It is independent of the area of contact between the surfaces.
3. It is parallel to the surface of contact, and in the direction that opposes relative motion.

▲ The coefficient of static friction between two surfaces depends on many factors, including whether the surfaces are dry or wet. On the desert floor of Death Valley, California, occasional rains can reduce the friction between rocks and the sandy ground to such an extent that strong winds can move the rocks over considerable distances. This results in linear "rock trails," which record the direction of the winds at different times.

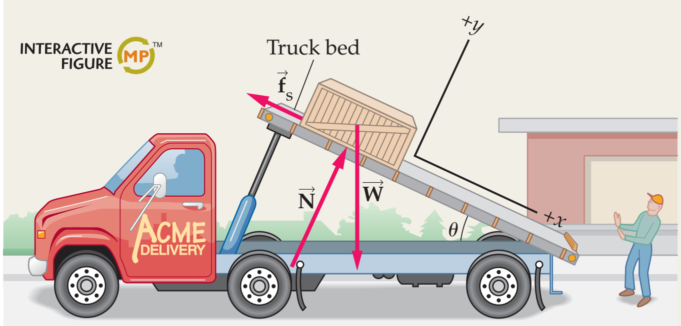
Next, we consider a practical method of determining the coefficient of static friction. As with the last Example, we begin by resolving all relevant force vectors into their x and y components.

EXAMPLE 6–3 SLIGHTLY TILTED

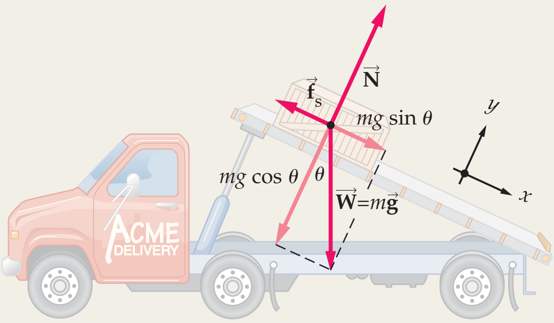
A flatbed truck slowly tilts its bed upward to dispose of a 95.0-kg crate. For small angles of tilt the crate stays put, but when the tilt angle exceeds 23.2°, the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate?

PICTURE THE PROBLEM

We align our coordinate system with the incline, and choose the positive x direction to point down the slope. Note that three forces act on the crate: the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, the force of static friction, $\vec{\mathbf{f}}_s = -\mu_s N\hat{\mathbf{x}}$, and the weight, $\vec{\mathbf{W}} = (mg\sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.



Physical picture



Free-body diagram

STRATEGY

When the crate is on the verge of slipping, but has not yet slipped, its acceleration is zero in both the x and y directions. In addition, "verge of slipping" means that the magnitude of the static friction is at its maximum value, $f_s = f_{s,max} = \mu_s N$. Thus, we set $\Sigma F_y = ma_y = 0$ to find N, then use $\Sigma F_x = ma_x = 0$ to find μ_s.

SOLUTION

1. Resolve the three force vectors acting on the crate into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{s,x} = -f_{s,max} = -\mu_s N \qquad f_{s,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$, since $a_y = 0$.

$$\sum F_y = N_y + f_{s,y} + W_y = N + 0 - mg\cos\theta = ma_y = 0$$

Solve for the normal force, N:

$$N = mg\cos\theta$$

3. Set $\Sigma F_x = ma_x = 0$, since the crate is at rest, and use the result for N obtained in Step 2:

$$\sum F_x = N_x + f_{s,x} + W_x = ma_x = 0$$
$$= 0 - \mu_s N + mg\sin\theta$$
$$= 0 - \mu_s mg\cos\theta + mg\sin\theta$$

4. Solve the expression for the coefficient of static friction, μ_s:

$$\mu_s mg\cos\theta = mg\sin\theta$$
$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 23.2° = 0.429$$

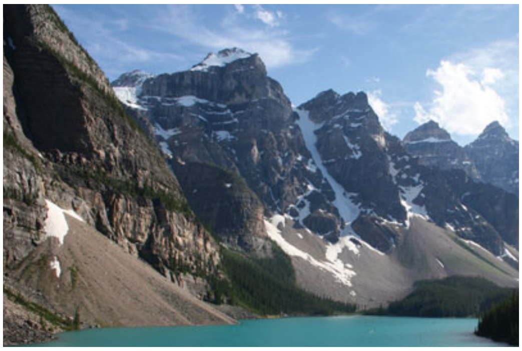
INSIGHT

In general, if an object is on the verge of slipping when the surface on which it rests is tilted at an angle θ_c, the coefficient of static friction between the object and the surface is $\mu_s = \tan\theta_c$. Note that this result is independent of the mass of the object. In particular, the critical angle for this crate is precisely the same whether it is filled with feathers or lead bricks.

PRACTICE PROBLEM

Find the magnitude of the force of static friction acting on the crate. [**Answer:** $f_{s,max} = \mu_s N = 367$ N]

Some related homework problems: Problem 12, Problem 82



The angle that the sloping sides of a sand pile (left) make with the horizontal is determined by the coefficient of static friction between grains of sand, in much the same way that static friction determines the angle at which the crate in Example 6–3 begins to slide. The same basic mechanism determines the angle of the cone-shaped mass of rock debris at the base of a cliff, known as a talus slope (right).

Recall that static friction can have magnitudes less than its maximum possible value. This point is emphasized in the following Active Example.

ACTIVE EXAMPLE 6–1 THE FORCE OF STATIC FRICTION

In the previous Example, what is the magnitude of the force of static friction acting on the crate when the truck bed is tilted at an angle of 20.0°?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Sum the x components of force acting on the crate: $\sum F_x = 0 - f_s + mg\sin\theta$
2. Set this sum equal to zero (since $a_x = 0$) and solve for the magnitude of the static friction force, f_s: $f_s = mg\sin\theta$
3. Substitute numerical values, including $\theta = 20.0°$: $f_s = 319\text{ N}$

INSIGHT

Notice that the force of static friction in this case has a magnitude (319 N) that is less than the value of 367 N found in the Practice Problem of Example 6–3, even though the coefficient of static friction is precisely the same.

YOUR TURN

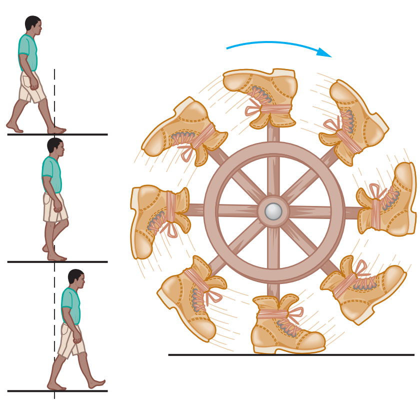
At what tilt angle will the force of static friction have a magnitude of 225 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Finally, friction often enters into problems dealing with vehicles with rolling wheels. In Conceptual Checkpoint 6–1, we consider which type of friction is appropriate in such cases.

CONCEPTUAL CHECKPOINT 6–1 FRICTION FOR ROLLING TIRES

A car drives with its tires rolling freely. Is the friction between the tires and the road **(a)** kinetic or **(b)** static?



REASONING AND DISCUSSION

A reasonable-sounding answer is that because the car is moving, the friction between its tires and the road must be kinetic friction—but this is not the case.

Actually, the friction is static because the bottom of the tire is in static contact with the road. To understand this, watch your feet as you walk. Even though you are moving, each foot is in static contact with the ground once you step down on it. Your foot doesn't move again until you lift it up and move it forward for the next step. A tire can be thought of as a succession of feet arranged in a circle, each of which is momentarily in static contact with the ground.

ANSWER

(b) The friction between the tires and the road is static friction.

REAL-WORLD PHYSICS

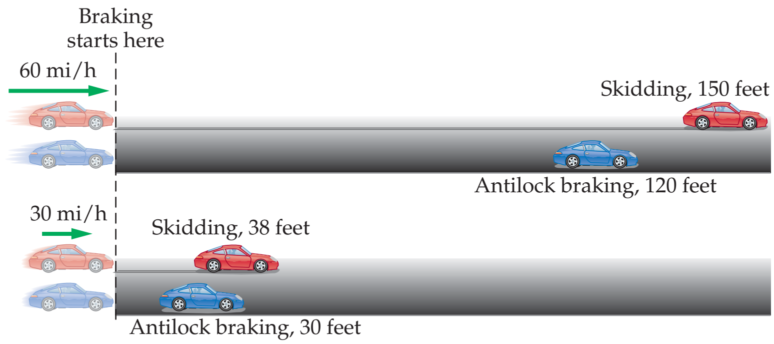
Antilock braking systems

To summarize, if a car skids, the friction acting on it is kinetic; if its wheels are rolling, the friction is static. Since static friction is generally greater than kinetic friction, it follows that a car can be stopped in less distance if its wheels are rolling (static friction) than if its wheels are locked up (kinetic friction). This is the idea behind the antilock braking systems (ABS) that are available on many cars. When the brakes are applied in a car with ABS, an electronic rotation sensor at each wheel detects whether the wheel is about to start skidding. To prevent skidding, a small computer automatically begins to modulate the hydraulic pressure in the brake lines in short bursts, causing the brakes to release and then reapply in rapid succession. This allows the wheels to continue rotating, even in an emergency stop, and for static friction to determine the stopping distance. **Figure 6–4** shows a comparison of braking distances for cars with and without ABS. An added benefit of ABS is that a driver is better able to steer and control a braking car if its wheels are rotating.

(a) Front wheels locked; rear wheels free to turn

(b) Rear wheels locked; front wheels free to turn

▲ **Static Versus Kinetic Friction** Each of the two photos above shows five images of a toy car as it slides down an inclined surface. **(a)** In this photo the front wheels are locked, and skid on the surface, but the rear wheels roll without slipping. This means the front wheels experience kinetic friction and the rear wheels experience static friction. Because the force of kinetic friction is usually less than the force of static friction, the front wheels go down the incline first, pulling the rear wheels behind. **(b)** The situation is reversed in this photo, and the rear wheels are the ones that skid and experience a smaller frictional force. As a result, the rear wheels slide down the incline more quickly than the front wheels, causing the car to spin around. This change in behavior, which could be dangerous in a real-life situation, illustrates the significant differences between static and kinetic friction.



▲ **FIGURE 6–4 Stopping distance with and without ABS**

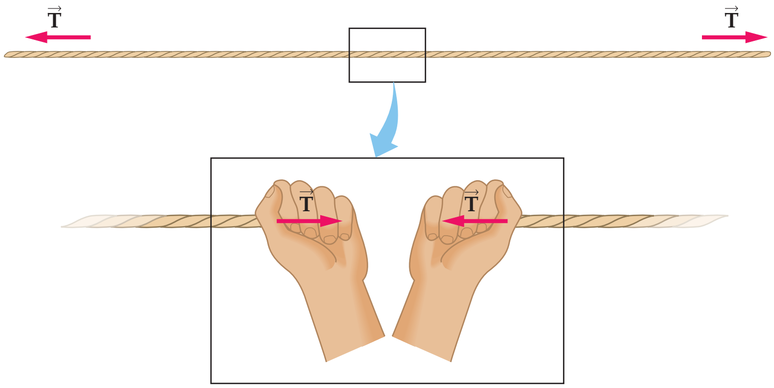
Antilock braking systems (ABS) allow a car to stop with static friction rather than kinetic friction—even in a case where a person slams on the brakes. As a result, the braking distance is reduced, due to the fact that μ_s is typically greater than μ_k. Professional drivers can beat the performance of ABS by carefully adjusting the force they apply to the brake pedal during a stop, but ABS provides essentially the same performance—within a few percent—for a person who simply pushes the brake pedal to the floor and holds it there.

6–2 Strings and Springs

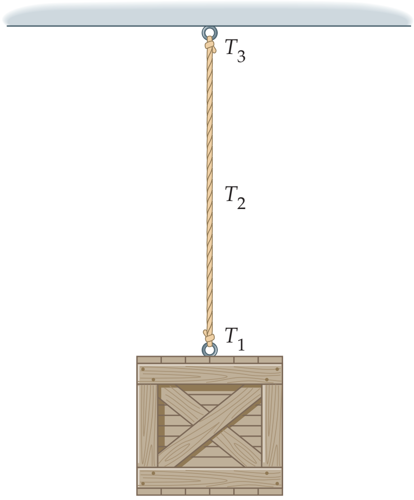
A common way to exert a force on an object is to pull on it with a string, a rope, a cable, or a wire. Similarly, you can push or pull on an object if you attach it to a spring. In this section we discuss the basic features of strings and springs and how they transmit forces.

Strings and Tension

Imagine picking up a light string and holding it with one end in each hand. If you pull to the right with your right hand with a force T and to the left with your left hand with a force T, the string becomes taut. In such a case, we say that there is a **tension** T in the string. To be more specific, if your friend were to cut the string at some point, the tension T is the force pulling the ends apart, as illustrated in **Figure 6–5**—that is, T is the force your friend would have to exert with each hand to hold the cut ends together. Note that at any given point, the tension pulls equally to the right and to the left.



▲ **FIGURE 6–5 Tension in a string**
A string, pulled from either end, has a tension, T. If the string were to be cut at any point, the force required to hold the ends together is T.



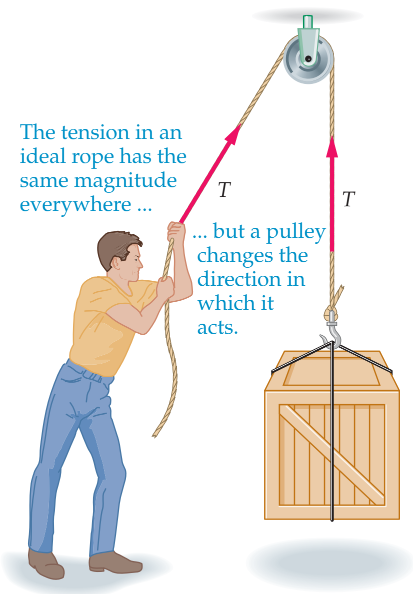
▲ **FIGURE 6–6 Tension in a heavy rope**
Because of the weight of the rope, the tension is noticeably different at points 1, 2, and 3. As the rope becomes lighter, however, the difference in tension decreases. In the limit of a rope of zero mass, the tension is the same throughout the rope.

As an example, consider a rope that is attached to the ceiling at one end, and to a box with a weight of 105 N at the other end, as shown in **Figure 6–6**. In addition, suppose the rope is uniform, and that it has a total weight of 2.00 N. What is the tension in the rope (i) where it attaches to the box, (ii) at its midpoint, and (iii) where it attaches to the ceiling?

First, the rope holds the box at rest; thus, the tension where the rope attaches to the box is simply the weight of the box, $T_1 = 105\text{ N}$. At the midpoint of the rope, the tension supports the weight of the box, plus the weight of half the rope. Thus, $T_2 = 105\text{ N} + \frac{1}{2}(2.00\text{ N}) = 106\text{ N}$. Similarly, at the ceiling the tension supports the box plus all of the rope, giving a tension of $T_3 = 107\text{ N}$. Note that the tension pulls down on the ceiling but pulls up on the box.

From this discussion, we can see that the tension in the rope changes slightly from top to bottom because of the mass of the rope. If the rope had less mass, the difference in tension between its two ends would also be less. In particular, if the rope's mass were to be vanishingly small, the difference in tension would vanish as well. In this text, we will assume that all ropes, strings, wires, and so on are practically massless—unless specifically stated otherwise—and, hence, that the tension is the same throughout their length.

Pulleys are often used to redirect a force exerted by a string, as indicated in **Figure 6–7**. In the ideal case, a pulley has no mass and no friction in its bearings. Thus, *an ideal pulley simply changes the direction of the tension in a string, without changing its magnitude*. If a system contains more than one pulley, however, it is possible to arrange them in such a way as to "magnify a force," even if each pulley itself merely redirects the tension in a string. The traction device considered in the next Example shows one way this can be accomplished in a system that uses three ideal pulleys.



▲ **FIGURE 6–7 A pulley changes the direction of a tension**

EXAMPLE 6–4 A BAD BREAK: SETTING A BROKEN LEG WITH TRACTION

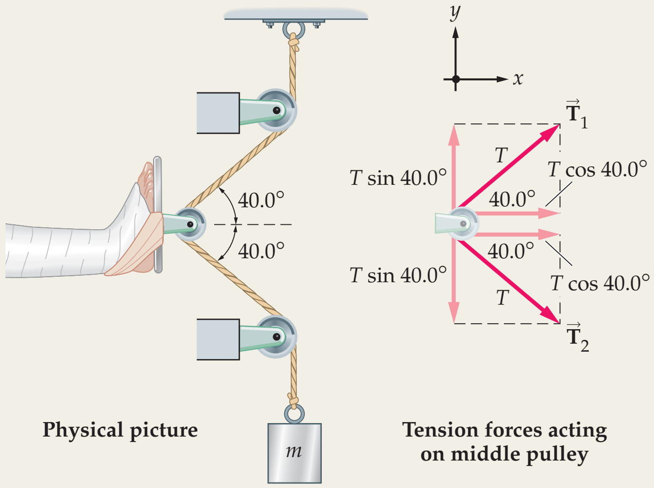
A traction device employing three pulleys is applied to a broken leg, as shown in the sketch. The middle pulley is attached to the sole of the foot, and a mass m supplies the tension in the ropes. Find the value of the mass m if the force exerted on the sole of the foot by the middle pulley is to be 165 N.

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

PICTURE THE PROBLEM

Our sketch shows the physical picture as well as the tension forces acting on the middle pulley. Notice that on the upper portion of the rope the tension is $\vec{\mathbf{T}}_1 = (T\cos 40.0°)\hat{\mathbf{x}} + (T\sin 40.0°)\hat{\mathbf{y}}$; on the lower portion it is $\vec{\mathbf{T}}_2 = (T\cos 40.0°)\hat{\mathbf{x}} + (-T\sin 40.0°)\hat{\mathbf{y}}$.



Physical picture

Tension forces acting on middle pulley

STRATEGY

We begin by noting that the rope supports the hanging mass m. As a result, the tension in the rope, T, must be equal in magnitude to the weight of the mass: $T = mg$.

Next, the pulleys simply change the direction of the tension without changing its magnitude. Therefore, the net force exerted on the sole of the foot is the sum of the tension T at 40.0° above the horizontal plus the tension T at 40.0° below the horizontal. We will calculate the net force component by component.

Once we calculate the net force acting on the foot, we set it equal to 165 N and solve for the tension T. Finally, we find the mass using the relation $T = mg$.

SOLUTION

1. First, consider the tension that acts upward and to the right on the middle pulley. Resolve this tension into x and y components:

$$T_{1,x} = T\cos 40.0° \qquad T_{1,y} = T\sin 40.0°$$

2. Next, consider the tension that acts downward and to the right on the middle pulley. Resolve this tension into x and y components. Note the minus sign in the y component:

$$T_{2,x} = T\cos 40.0° \qquad T_{2,y} = -T\sin 40.0°$$

3. Sum the x and y components of force acting on the middle pulley. We see that the net force acts only in the x direction, as one might expect from symmetry:

$$\sum F_x = T\cos 40.0° + T\cos 40.0° = 2T\cos 40.0°$$

$$\sum F_y = T\sin 40.0° - T\sin 40.0° = 0$$

4. Step 3 shows that the net force acting on the middle pulley is $2T\cos 40.0°$. Set this force equal to 165 N and solve for T:

$$2T\cos 40.0° = 165\ \text{N}$$

$$T = \frac{165\ \text{N}}{2\cos 40.0°} = 108\ \text{N}$$

5. Solve for the mass, m, using $T = mg$:

$$T = mg$$

$$m = \frac{T}{g} = \frac{108\ \text{N}}{9.81\ \text{m/s}^2} = 11.0\ \text{kg}$$

INSIGHT

As pointed out earlier, this pulley arrangement "magnifies the force" in the sense that a 108-N weight attached to the rope produces a 165-N force exerted on the foot by the middle pulley. Note that the tension in the rope always has the same value—$T = 108$ N—as expected with ideal pulleys, but because of the arrangement of the pulleys the force applied to the foot by the rope is $2T\cos 40.0° > T$.

In addition, notice that the force exerted on the foot by the middle pulley produces an opposing force in the leg that acts in the direction of the head (a cephalad force), as desired to set a broken leg and keep it straight as it heals.

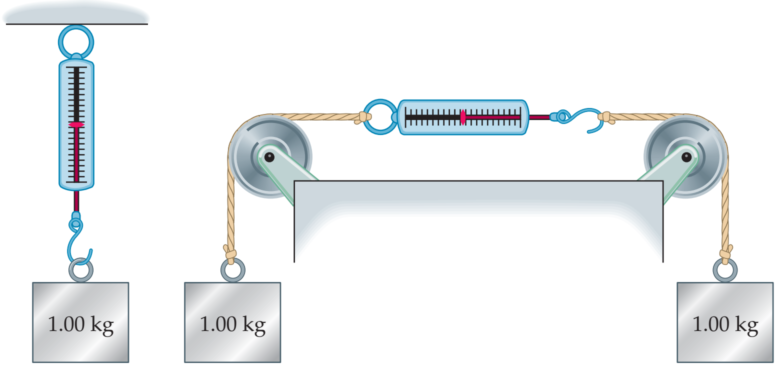
PRACTICE PROBLEM

(a) Would the required mass m increase or decrease if the angles in this device were changed from 40.0° to 30.0°? **(b)** Find the mass m for an angle of 30.0°. [**Answer:** **(a)** The required mass m would decrease. **(b)** 9.71 kg]

Some related homework problems: Problem 23, Problem 26, Problem 36

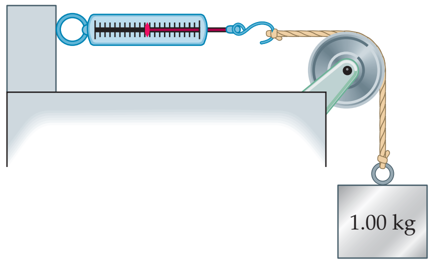
CONCEPTUAL CHECKPOINT 6–2 COMPARE THE READINGS ON THE SCALES

The scale at left reads 9.81 N. Is the reading of the scale at right **(a)** greater than 9.81 N, **(b)** equal to 9.81 N, or **(c)** less than 9.81 N?



REASONING AND DISCUSSION

Since a pulley simply changes the direction of the tension in a string without changing its magnitude, it is clear that the scale attached to the ceiling reads the same as the scale shown in the figure to the right.



There is no difference, however, between attaching the top end of the scale to something rigid and attaching it to another 1.00-kg hanging mass. In either case, the fact that the scale is at rest means that a force of 9.81 N must be exerted to the left on the top of the scale to balance the 9.81-N force exerted on the lower end of the scale. As a result, the two scales read the same.

ANSWER

(b) The reading of the scale at right is equal to 9.81 N.

Springs and Hooke's Law

Suppose you take a spring of length L, as shown in **Figure 6–8 (a)**, and attach it to a block. If you pull on the spring, causing it to stretch to a length $L + x$, the spring pulls on the block with a force of magnitude F. If you increase the length of the spring to $L + 2x$, the force exerted by the spring increases to $2F$. Similarly, if you compress the spring to a length $L - x$, the spring pushes on the block with a force of magnitude F, where F is the same force given previously. As you might expect, compression to a length $L - 2x$ results in a push of magnitude $2F$.

As a result of these experiments, we can say that a spring exerts a force that is proportional to the amount, x, by which it is stretched or compressed. Thus, if F is the magnitude of the spring force, we can say that

$$F = kx$$

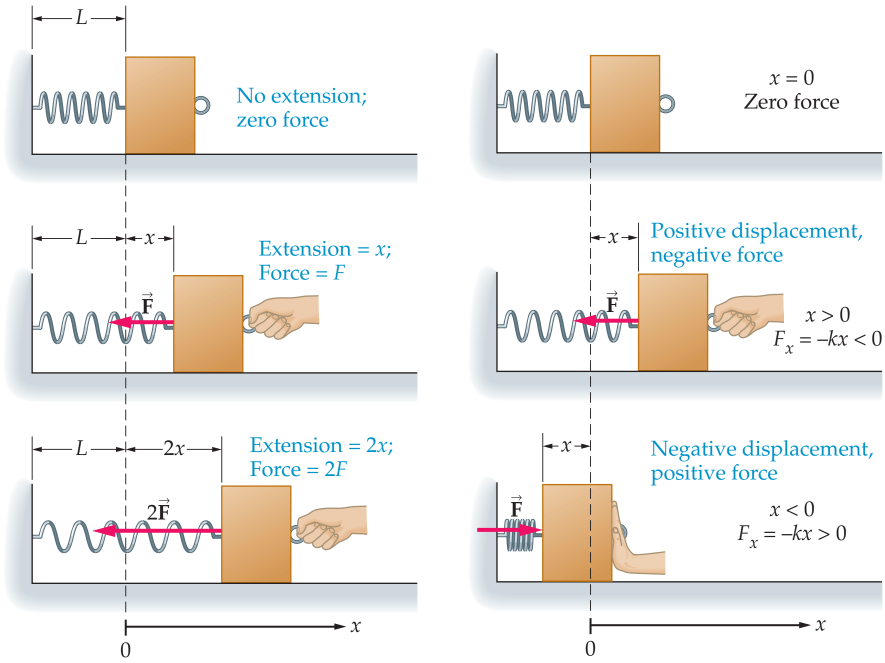
In this expression, k is a constant of proportionality, referred to as the **force constant,** or, equivalently, as the spring constant. Since F has units of newtons and x has units of meters, it follows that k has units of newtons per meter, or N/m. The larger the value of k, the stiffer the spring.

To be more precise, consider the spring shown in **Figure 6–8 (b)**. Note that we have placed the origin of the x axis at the equilibrium length of the spring—that is, at the position of the spring when no force acts on it. Now, if we stretch the spring so that the end of the spring is at a positive value of x ($x > 0$), we find that the spring exerts a force of magnitude kx in the negative x direction. Thus, the spring force (which has only an x component) can be written as

$$F_x = -kx$$

Similarly, consider compressing the spring so that its end is at a negative value of x ($x < 0$). In this case, the force exerted by the spring is of magnitude kx, and points in the positive x direction, as is shown in Figure 6–8 (b). Again, we can write the spring force as

$$F_x = -kx$$



(a) Doubling the extension doubles the force.

(b) The spring force is opposite to the displacement from equilibrium.

FIGURE 6–8 The force exerted by a spring

When dealing with a spring, it is convenient to choose the origin at the equilibrium (zero force) position. In the cases shown here, the force is strictly in the x direction, and is given by $F_x = -kx$. Note that the minus sign means that the force is opposite to the displacement; that is, the force is restoring.

To see that this is correct—that is, that F_x is positive in this case—recall that x is negative, which means that $(-x)$ is positive.

This result for the force of a spring is known as Hooke's law, after Robert Hooke (1635–1703). It is really just a good rule of thumb rather than a law of nature. Clearly, it can't work for any amount of stretching. For example, we know that if we stretch a spring far enough it will be permanently deformed, and will never return to its original length. Still, for small stretches or compressions, Hooke's law is quite accurate.

▲ Springs come in a variety of sizes and shapes. The large springs on a railroad car (top) are so stiff and heavy that you can't compress or stretch them by hand. Still, three of them are needed to smooth the ride of this car. In contrast, the delicate spiral spring inside a watch (bottom) flexes with even the slightest touch. It exerts enough force, however, to power the equally delicate mechanism of the watch.

Rules of Thumb for Springs (Hooke's Law)

A spring stretched or compressed by the amount x from its equilibrium length exerts a force whose x component is given by

$$F_x = -kx \text{ (gives magnitude and direction)} \qquad 6\text{–}4$$

If we are interested only in the magnitude of the force associated with a given stretch or compression, we use the somewhat simpler form of Hooke's law:

$$F = kx \text{ (gives magnitude only)} \qquad 6\text{–}5$$

In this text, we consider only **ideal springs**—that is, springs that are massless, and that are assumed to obey Hooke's law exactly.

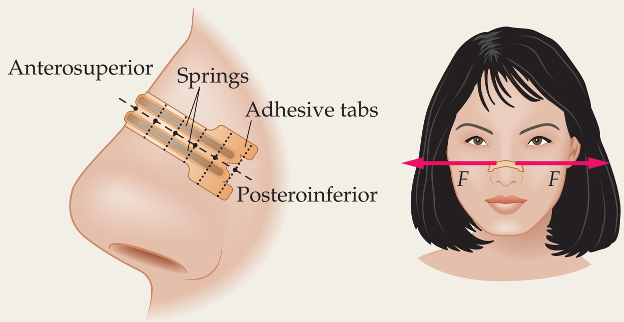
Since the stretch of a spring and the force it exerts are proportional, we can now see how a spring scale operates. In particular, pulling on the two ends of a scale stretches the spring inside it by an amount proportional to the applied force. Once the scale is calibrated—by stretching the spring with a known, or reference, force—we can use it to measure other unknown forces.

Finally, it is useful to note that Hooke's law, which we've introduced in the context of ideal springs, is particularly important in physics because it applies to so much more than just springs. For example, the forces that hold atoms together are often modeled by Hooke's law—that is, as "interatomic springs"—and these are the forces that are ultimately responsible for the normal force (Chapter 5), vibrations and oscillations (Chapter 13), wave motion (Chapter 14), and even the thermal expansion of solids (Chapter 16). And this just scratches the surface—Hooke's law comes up in one form or another in virtually every field of physics. In the following Active Example, we present a biomedical application of Hooke's law.

ACTIVE EXAMPLE 6–2 NASAL STRIPS

REAL-WORLD PHYSICS: BIO

An increasingly popular device for improving air flow through nasal passages is the nasal strip, which consists of two flat, polyester springs enclosed by an adhesive tape covering. Measurements show that a nasal strip can exert an outward force of 0.22 N on the nose, causing it to expand by 3.5 mm. **(a)** Treating the nose as an ideal spring, find its force constant in newtons per meter. **(b)** How much force would be required to expand the nose by 4.0 mm?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Solve the magnitude form of Hooke's law, $F = kx$, for the force constant, k:	$k = F/x$
2. Substitute numerical values for F and x:	$k = 62\ \mathrm{N/m}$

Part (b)

3. Use $F = kx$ to find the required force:	$F = 0.25\ \mathrm{N}$

INSIGHT

Even though the human nose is certainly not an ideal spring, Hooke's law is still a useful way to model its behavior when dealing with forces and the stretches they cause.

YOUR TURN

Suppose a new nasal strip comes on the market that exerts an outward force of 0.32 N. What expansion of the nose will be caused by this strip?

(Answers to **Your Turn** *problems are given in the back of the book.)*

6–3 Translational Equilibrium

When we say that an object is in **translational equilibrium,** we mean that the net force acting on it is zero:

$$\sum \vec{\mathbf{F}} = 0 \qquad 6\text{–}6$$

From Newton's second law, this is equivalent to saying that the object's acceleration is zero. In two-dimensional systems, translational equilibrium implies two independent conditions: $\Sigma F_x = 0$ and $\Sigma F_y = 0$. In one dimension, only one of these conditions will apply.

Later, in Chapters 10 and 11, we will study objects that have both rotational and linear motions. In such cases, rotational equilibrium will be as important as translation equilibrium. For now, however, when we say equilibrium, we simply mean translational equilibrium.

As a first example, consider the one-dimensional situation illustrated in **Figure 6–9**. Here we see a person lifting a bucket of water from a well by pulling down on a rope that passes over a pulley. If the bucket's mass is m, and it is rising with constant speed v, what is the tension T_1 in the rope attached to the bucket? In addition, what is the tension T_2 in the chain that supports the pulley?

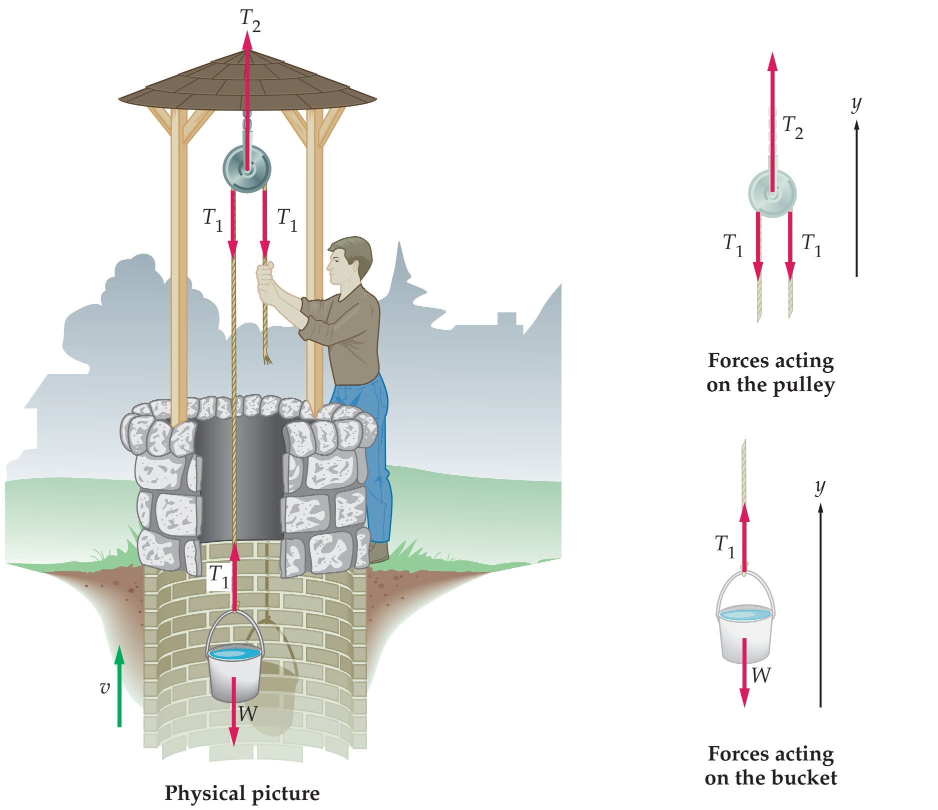


FIGURE 6–9 Raising a bucket
A person lifts a bucket of water from the bottom of a well with a constant speed, v. Because the speed is constant, the net force acting on the bucket must be zero.

To answer these questions, we first note that both the bucket and the pulley are in equilibrium; that is, they both have zero acceleration. As a result, the net force on each of them must be zero.

Let's start with the bucket. In Figure 6–9, we see that just two forces act on the bucket: (i) its weight $W = mg$ downward, and (ii) the tension in the rope, T_1 upward. If we take upward to be the positive direction, we can write $\Sigma F_y = 0$ for the bucket as follows:

$$T_1 - mg = 0$$

Therefore, the tension in the rope is $T_1 = mg$. Note that this is also the force the person must exert downward on the rope, as expected.

Next, we consider the pulley. In Figure 6–9, we see that three forces act on it: (i) the tension in the chain, T_2 upward, (ii) the tension in the part of the rope leading to the bucket, T_1 downward, and (iii) the tension in the part of the rope leading to the person, T_1 downward. Note that we don't include the weight of the pulley since we consider it to be ideal; that is, massless and frictionless. If we again take upward to be positive, the statement that the net force acting on the pulley is zero $(\Sigma F_y = 0)$ can be written

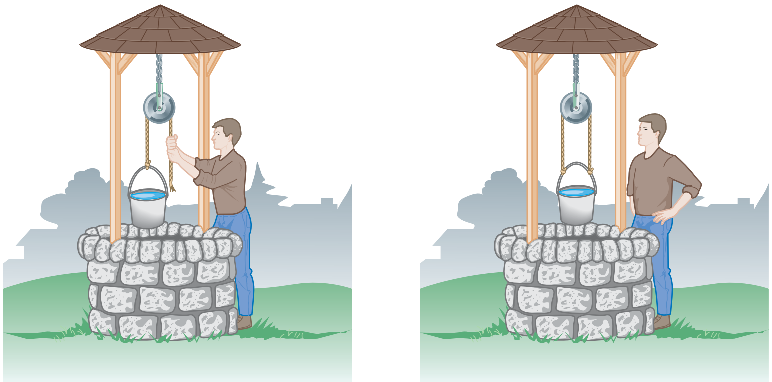
$$T_2 - T_1 - T_1 = 0$$

It follows that the tension in the chain is $T_2 = 2T_1 = 2mg$, twice the weight of the bucket of water!

In the next Conceptual Checkpoint we consider a slight variation of this situation.

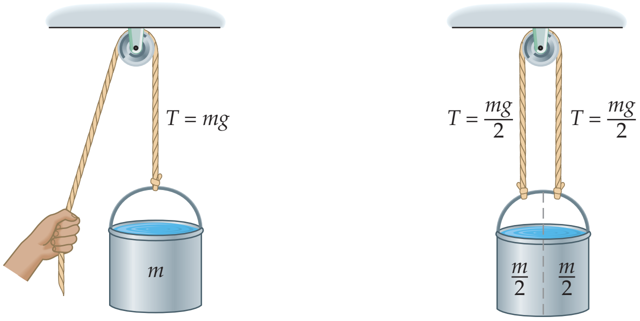
CONCEPTUAL CHECKPOINT 6–3 COMPARING TENSIONS

A person hoists a bucket of water from a well and holds the rope, keeping the bucket at rest, as at left. A short time later, the person ties the rope to the bucket so that the rope holds the bucket in place, as at right. In this case, is the tension in the rope **(a)** greater than, **(b)** less than, or **(c)** equal to the tension in the first case?



REASONING AND DISCUSSION

In the first case (left), the only upward force exerted on the bucket is the tension in the rope. Since the bucket is at rest, the tension must be equal in magnitude to the weight of the bucket. In the second case (right), the two ends of the rope exert equal upward forces on the bucket, hence the tension in the rope is only half the weight of the bucket. To see this more clearly, imagine cutting the bucket in half so that each end of the rope supports half the weight, as indicated in the accompanying diagram.



ANSWER

(b) The tension in the second case is less than in the first.

In the next two Examples, we consider two-dimensional systems in which forces act at various angles with respect to one another. Hence, our first step is to resolve the relevant vectors into their x and y components. Following that, we apply the conditions for translational equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

EXAMPLE 6–5 SUSPENDED VEGETATION

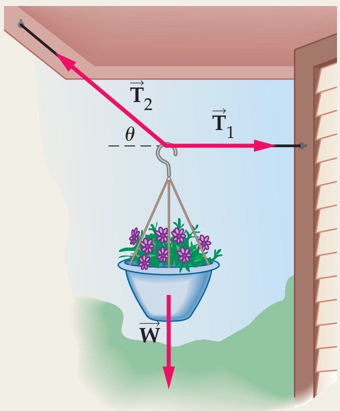
To hang a 6.20-kg pot of flowers, a gardener uses two wires—one attached horizontally to a wall, the other sloping upward at an angle of $\theta = 40.0°$ and attached to the ceiling. Find the tension in each wire.

PICTURE THE PROBLEM

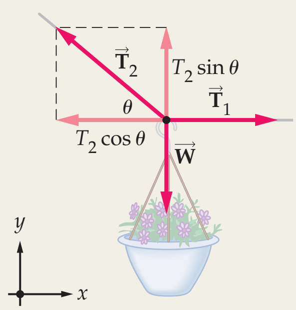
We choose a typical coordinate system, with the positive x direction to the right and the positive y direction upward. With this choice, tension 1 is in the positive x direction, $\vec{\mathbf{T}}_1 = T_1\hat{\mathbf{x}}$, the weight is in the negative y direction, $\vec{\mathbf{W}} = -mg\,\hat{\mathbf{y}}$, and tension 2 has a negative x component and a positive y component, $\vec{\mathbf{T}}_2 = (-T_2 \cos\theta)\hat{\mathbf{x}} + (T_2 \sin\theta)\hat{\mathbf{y}}$.

STRATEGY

The pot is at rest, and therefore the net force acting on it is zero. As a result, we can say that (i) $\Sigma F_x = 0$ and (ii) $\Sigma F_y = 0$. These two conditions allow us to determine the magnitude of the two tensions, T_1 and T_2.



Physical picture



Free-body diagram

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

SOLUTION

1. First, resolve each of the forces acting on the pot into x and y components:

$$T_{1,x} = T_1 \qquad T_{1,y} = 0$$
$$T_{2,x} = -T_2 \cos\theta \qquad T_{2,y} = T_2 \sin\theta$$
$$W_x = 0 \qquad W_y = -mg$$

2. Now, set $\Sigma F_x = 0$. Note that this condition gives a relation between T_1 and T_2:

$$\sum F_x = T_{1,x} + T_{2,x} + W_x = T_1 + (-T_2 \cos\theta) + 0 = 0$$
$$T_1 = T_2 \cos\theta$$

3. Next, set $\Sigma F_y = 0$. This time, the resulting condition determines T_2 in terms of the weight, mg:

$$\sum F_y = T_{1,y} + T_{2,y} + W_y = 0 + T_2 \sin\theta + (-mg) = 0$$
$$T_2 \sin\theta = mg$$

4. Use the relation obtained in Step 3 to find T_2:

$$T_2 = \frac{mg}{\sin\theta} = \frac{(6.20\ \text{kg})(9.81\ \text{m/s}^2)}{\sin 40.0°} = 94.6\ \text{N}$$

5. Finally, use the connection between the two tensions (obtained from $\Sigma F_x = 0$) to find T_1:

$$T_1 = T_2 \cos\theta = (94.6\ \text{N}) \cos 40.0° = 72.5\ \text{N}$$

INSIGHT

Notice that even though two wires suspend the pot, they both have tensions *greater* than the pot's weight, $mg = 60.8$ N. This is an important point for architects and engineers to consider when designing structures.

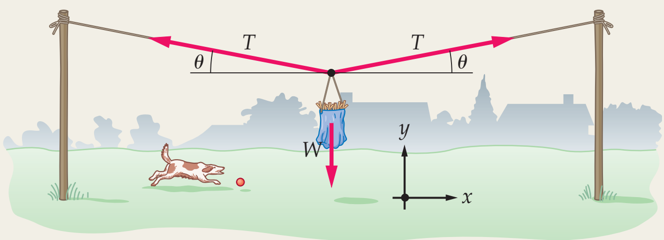
PRACTICE PROBLEM

Find T_1 and T_2 if the second wire slopes upward at the angle **(a)** $\theta = 20°$, **(b)** $\theta = 60.0°$, or **(c)** $\theta = 90.0°$. **[Answer: (a)** $T_1 = 167$ N, $T_2 = 178$ N **(b)** $T_1 = 35.1$ N, $T_2 = 70.2$ N **(c)** $T_1 = 0$, $T_2 = mg = 60.8$ N]

Some related homework problems: Problem 34, Problem 37

ACTIVE EXAMPLE 6–3 THE FORCES IN A LOW-TECH LAUNDRY

A 1.84-kg bag of clothespins hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50°$. Find the tension, T, in the clothesline.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the y component for each tension: $T_y = T \sin\theta$

2. Find the y component of the weight: $W_y = -mg$

3. Set $\Sigma F_y = 0$: $T \sin\theta + T \sin\theta - mg = 0$

4. Solve for T: $T = mg/(2 \sin\theta) = 148$ N

INSIGHT

Note that we only considered the y components of force in our calculation. This is because forces in the x direction automatically balance, due to the symmetry of the system.

YOUR TURN

At what sag angle, θ, will the tension in the clothesline have a magnitude of 175 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

At 148 N, the tension in the clothesline is quite large, especially when you consider that the weight of the clothespin bag itself is only 18.1 N. The reason for such a large value is that the vertical component of the two tensions is $2T \sin\theta$, which, for $\theta = 3.50°$, is $(0.122)T$. If $(0.122)T$ is to equal the weight of the bag, it is clear that T must be roughly eight times the bag's weight.

If you and a friend were to pull on the two ends of the clothesline, in an attempt to straighten it out, you would find that no matter how hard you pulled, the line would still sag. You may be able to reduce θ to quite a small value, but as you do so the corresponding tension increases rapidly. In principle, it would take an infinite force to completely straighten the line and reduce θ to zero.

On the other hand, if θ were 90°, so that the two halves of the clothesline were vertical, the tension would be $T = mg/(2 \sin 90°) = mg/2$. In this case, each side of the line supports half the weight of the bag, as expected.

▲ Like the bag of clothespins in Active Example 6–3, this mountain climber is in static equilibrium. Since the ropes suspending the climber are nearly horizontal, the tension in them is significantly greater than the climber's weight.

6–4 Connected Objects

Interesting applications of Newton's laws arise when we consider accelerating objects that are tied together. Suppose, for example, that a force of magnitude F pulls two boxes—connected by a string—along a frictionless surface, as in **Figure 6–10**. In such a case, the string has a certain tension, T, and the two boxes have the same acceleration, a. Given the masses of the boxes and the applied force F, we would like to determine both the tension in the string and the acceleration of the boxes.

First, sketch the free-body diagram for each box. Box 1 has two horizontal forces acting on it: (i) the tension T to the left, and (ii) the force F to the right. Box 2 has only a single horizontal force, the tension T to the right. If we take the positive direction to be to the right, Newton's second law for the two boxes can be written as follows:

$$\begin{aligned} F - T &= m_1 a_1 = m_1 a \qquad \text{box 1} \\ T &= m_2 a_2 = m_2 a \qquad \text{box 2} \end{aligned} \tag{6–7}$$

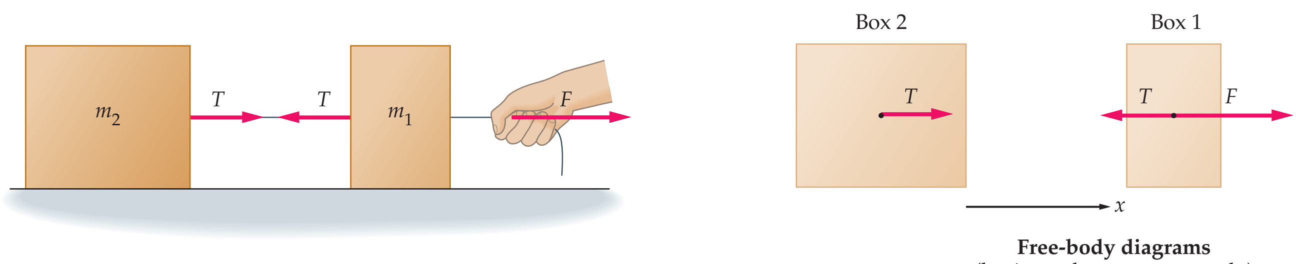
Since the boxes have the same acceleration, a, we have set $a_1 = a_2 = a$.

Next, we can eliminate the tension T by adding the two equations:

$$\begin{aligned} F - T &= m_1 a \\ T &= m_2 a \\ \hline F &= (m_1 + m_2)a \end{aligned}$$

With this result, it is straightforward to solve for the acceleration in terms of the applied force F:

$$a = \frac{F}{m_1 + m_2} \tag{6–8}$$



Physical picture

Free-body diagrams (horizontal components only)

▲ **FIGURE 6–10 Two boxes connected by a string**

The string ensures that the two boxes have the same acceleration. This physical connection results in a mathematical connection, as shown in Equation 6–7. Note that in this case we treat each box as a separate system.

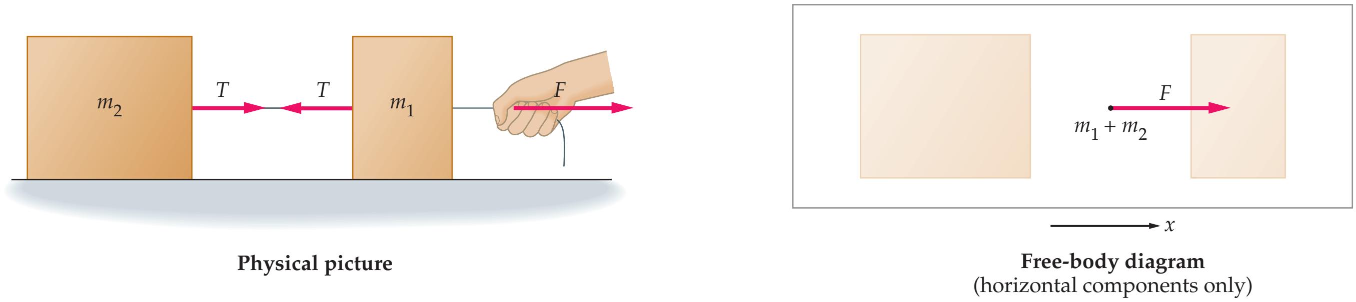
Finally, substitute this expression for a into either of the second-law equations to find the tension. The algebra is simpler if we use the equation for box 2. We find

$$T = m_2 a = \left(\frac{m_2}{m_1 + m_2}\right)F \quad \text{6–9}$$

It is left as an exercise to show that the equation for box 1 gives the same expression for T.

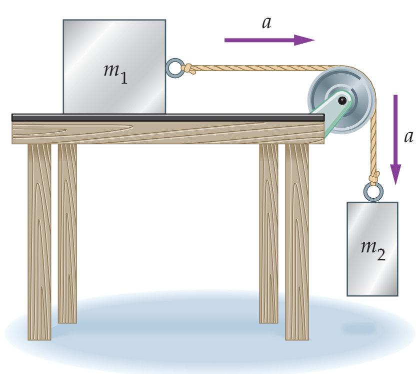
A second way to approach this problem is to treat both boxes together as a single system with a mass $m_1 + m_2$, as shown in **Figure 6–11**. The only *external* horizontal force acting on this system is the applied force F—the two tension forces are now *internal* to the system, and internal forces are not included when applying Newton's second law. As a result, the horizontal acceleration is simply $F/(m_1 + m_2)$, as given in Equation 6–8. This is certainly a quick way to find the acceleration a, but to find the tension T we must still use one of the relations given in Equations 6–7.

In general, we are always free to choose the "system" any way we like—we can choose any individual object, as when we considered box 1 and box 2 separately, or we can choose all the objects together. The important point is that Newton's second law is equally valid no matter what choice we make for the system, as long as we remember to include only forces *external* to *that system* in the corresponding free-body diagram.



▲ **FIGURE 6–11 Two boxes, one system**
In this case we consider the two boxes together as a single system of mass $m_1 + m_2$. The only external horizontal force acting on this system is $\vec{\mathbf{F}}$; hence the horizontal acceleration of the system is $a = F/(m_1 + m_2)$, in agreement with Equation 6–8.

CONCEPTUAL CHECKPOINT 6–4 TENSION IN THE STRING



Two masses, m_1 and m_2, are connected by a string that passes over a pulley. Mass m_1 slides without friction on a horizontal tabletop, and mass m_2 falls vertically downward. Both masses move with a constant acceleration of magnitude a. Is the tension in the string **(a)** greater than, **(b)** equal to, or **(c)** less than $m_2 g$?

REASONING AND DISCUSSION

First, note that m_2 accelerates downward, which means that the net force acting on it is downward. Only two forces act on m_2, however: the tension in the string (upward) and its weight (downward). Since the net force is downward, the tension in the string must be less than the weight, $m_2 g$.

A common misconception is that since m_2 has to pull m_1 behind it, the tension in the string must be greater than $m_2 g$. Certainly, attaching the string to m_1 has an effect on the tension. If the string were not attached, for example, its tension would be zero. Hence, m_2 pulling on m_1 increases the tension to a value greater than zero, though still less than $m_2 g$.

ANSWER

(c) The tension in the string is less than $m_2 g$.

In the next Example, we verify the qualitative conclusions given in the Conceptual Checkpoint with a detailed calculation. But first, a note about choosing a coordinate system for a problem such as this. Rather than apply the same coordinate system to both masses, it is useful to take into consideration the fact that a pulley simply changes the direction of the tension in a string. With this in mind, we choose a set of axes that "follow the motion" of the string, so that both masses accelerate in the positive x direction with accelerations of equal magnitude. Example 6–6 illustrates the use of this type of coordinate system.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Connected Objects

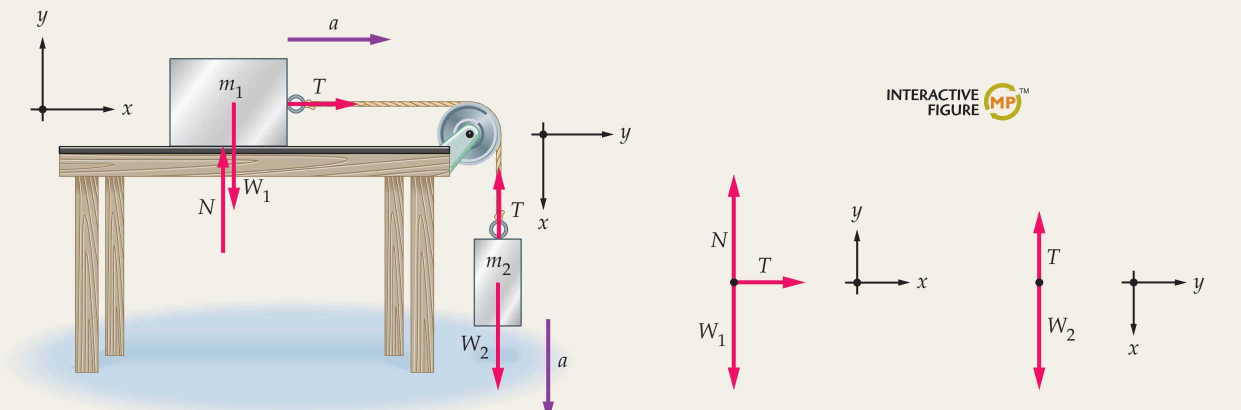
If two objects are connected by a string passing over a pulley, let the coordinate system follow the direction of the string. With this choice, both objects have accelerations of the same magnitude and in the same coordinate direction.

EXAMPLE 6–6 CONNECTED BLOCKS

A block of mass m_1 slides on a frictionless tabletop. It is connected to a string that passes over a pulley and suspends a mass m_2. Find **(a)** the acceleration of the masses and **(b)** the tension in the string.

PICTURE THE PROBLEM

Our coordinate system follows the motion of the string so that both masses move in the positive x direction. Since the masses are connected, their accelerations have the same magnitude. Thus, $a_{1,x} = a_{2,x} = a$. In addition, note that the tension, $\vec{\mathbf{T}}$, is in the positive x direction for mass 1, but in the negative x direction for mass 2. Its magnitude, T, is the same for each mass, however. Finally, the weight of mass 2, W_2, acts in the positive x direction, whereas the weight of mass 1 is offset by the normal force, N.



Physical picture — **Free-body diagram for m_1** — **Free-body diagram for m_2**

STRATEGY

Applying Newton's second law to the two masses yields the following relations: For mass 1, $\Sigma F_{1,x} = T = m_1 a_{1,x} = m_1 a$ and for mass 2, $\Sigma F_{2,x} = m_2 g - T = m_2 a_{2,x} = m_2 a$. These two equations can be solved for the two unknowns, a and T.

SOLUTION

Part (a)

1. First, write $\Sigma F_{1,x} = m_1 a$. Note that the only force acting on m_1 in the x direction is T:

$$\sum F_{1,x} = T = m_1 a$$

$$T = m_1 a$$

2. Next, write $\Sigma F_{2,x} = m_2 a$. In this case, two forces act in the x direction: $W_2 = m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$

$$m_2 g - T = m_2 a$$

3. Sum the two relations obtained to eliminate T:

$$\begin{aligned} T &= m_1 a \\ m_2 g - T &= m_2 a \\ \hline m_2 g &= (m_1 + m_2) a \end{aligned}$$

4. Solve for a:

$$a = \left(\frac{m_2}{m_1 + m_2}\right) g$$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

Part (b)

5. Substitute a into the first relation ($T = m_1 a$) to find T:

$$T = m_1 a = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g$$

INSIGHT

We could just as well have determined T using $m_2 g - T = m_2 a$, though the algebra is a bit messier. Also, note that $a = 0$ if $m_2 = 0$, and that $a = g$ if $m_1 = 0$, as expected. Similarly, $T = 0$ if either m_1 or m_2 is zero. This type of check, where you connect equations with physical situations, is one of the best ways to increase your understanding of physics.

PRACTICE PROBLEM

Find the tension for the case $m_1 = 1.50$ kg and $m_2 = 0.750$ kg, and compare the tension to $m_2 g$. [**Answer:** $a = 3.27\ \text{m/s}^2$, $T = 4.91\ \text{N} < m_2 g = 7.36\ \text{N}$]

Some related homework problems: Problem 44, Problem 48

Conceptual Checkpoint 6–4 shows that the tension in the string should be less than $m_2 g$. Let's rewrite our solution for T to show that this is indeed the case. From Example 6–6 we have

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g = \left(\frac{m_1}{m_1 + m_2}\right) m_2 g$$

Since the ratio $m_1/(m_1 + m_2)$ is always less than 1 (as long as m_2 is nonzero), it follows that $T < m_2 g$, as expected.

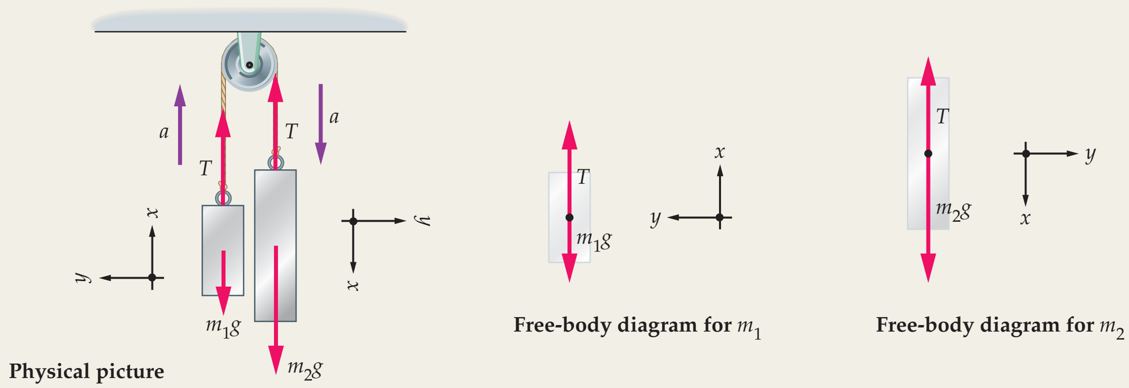
We conclude this section with a classic system that can be used to measure the acceleration of gravity. It is referred to as Atwood's machine, and it is basically two blocks of different mass connected by a string that passes over a pulley. The resulting acceleration of the blocks is related to the acceleration of gravity by a relatively simple expression, which we derive in the following Example.

EXAMPLE 6–7 ATWOOD'S MACHINE

Atwood's machine consists of two masses connected by a string that passes over a pulley, as shown below. Find the acceleration of the masses for general m_1 and m_2, and evaluate for the specific case $m_1 = 3.1$ kg, $m_2 = 4.4$ kg.

PICTURE THE PROBLEM

Our sketch shows Atwood's machine, along with our choice of coordinate directions for the two blocks. Note that both blocks accelerate in the positive x direction with accelerations of equal magnitude, a. From the free-body diagrams we can see that for mass 1 the weight is in the negative x direction and the tension is in the positive x direction. For mass 2, the tension is in the negative x direction and the weight is in the positive x direction. The tension has the same magnitude T for both masses, but their weights are different.



STRATEGY

To find the acceleration of the blocks, we follow the same strategy given in the previous Example. In particular, we start by applying Newton's second law to each block individually, using the fact that $a_{1,x} = a_{2,x} = a$. This gives two equations, both involving the tension T and the acceleration a. Eliminating T allows us to solve for the acceleration.

SOLUTION

1. Begin by writing out the expression $\Sigma F_{1,x} = m_1 a$. Note that two forces act in the x direction; T (positive direction) and $m_1 g$ (negative direction):

$$\sum F_{1,x} = T - m_1 g = m_1 a$$

2. Next, write out $\Sigma F_{2,x} = m_2 a$. The two forces acting in the x direction in this case are $m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$

3. Sum the two relations obtained above to eliminate T:

$$\begin{array}{r} T - m_1 g = m_1 a \\ m_2 g - T = m_2 a \\ \hline (m_2 - m_1)g = (m_1 + m_2)a \end{array}$$

4. Solve for a:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

5. To evaluate the acceleration, substitute numerical values for the masses and for g:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{4.4\text{ kg} - 3.1\text{ kg}}{3.1\text{ kg} + 4.4\text{ kg}}\right)(9.81\text{ m/s}^2) = 1.7\text{ m/s}^2$$

INSIGHT

Since m_2 is greater than m_1, we find that the acceleration is positive, meaning that the masses accelerate in the positive x direction. On the other hand, if m_1 were greater than m_2, we would find that a is negative, indicating that the masses accelerate in the negative x direction. Finally, if $m_1 = m_2$ we have $a = 0$, as expected.

PRACTICE PROBLEM

If m_1 is increased by a small amount, does the acceleration of the blocks increase, decrease, or stay the same? Check your answer by evaluating the acceleration for $m_1 = 3.3$ kg. [**Answer:** If m_1 is increased only slightly, the acceleration will decrease. For $m_1 = 3.3$ kg, we find $a = 1.4\text{ m/s}^2$.]

Some related homework problems: Problem 48, Problem 50

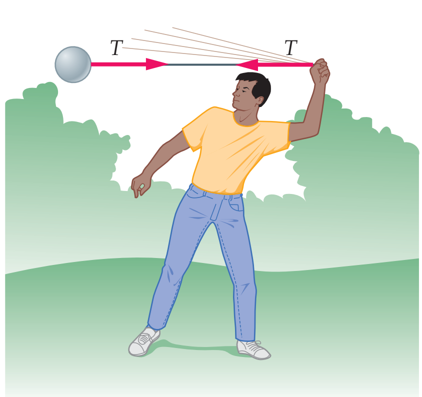
6–5 Circular Motion

According to Newton's second law, if no force acts on an object, it will move with constant speed in a constant direction. A force is required to change the speed, the direction, or both. For example, if you drive a car with constant speed on a circular track, the direction of the car's motion changes continuously. A force must act on the car to cause this change in direction. We would like to know two things about a force that causes circular motion: what is its direction, and what is its magnitude?

First, let's consider the direction of the force. Imagine swinging a ball tied to a string in a circle about your head, as shown in **Figure 6–12**. As you swing the ball, you feel a tension in the string pulling outward. Of course, on the other end of the string, where it attaches to the ball, the tension pulls inward, toward the center of the circle. Thus, the force the ball experiences is a force that is always directed toward the center of the circle. In summary,

> To make an object move in a circle with constant speed, a force must act on it that is directed toward the center of the circle.

Since the ball is acted on by a *force* toward the center of the circle, it follows that it must be *accelerating* toward the center of the circle. This might seem odd at



▲ **FIGURE 6–12 Swinging a ball in a circle** The tension in the string pulls outward on the person's hand and pulls inward on the ball.

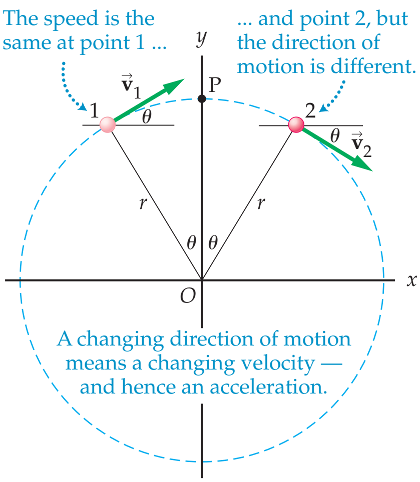


FIGURE 6–13 A particle moving with constant speed in a circular path centered on the origin

The speed of the particle is constant, but its velocity is constantly changing direction. Because the velocity changes, the particle is accelerating.

first: How can a ball that moves with constant speed have an acceleration? The answer is that acceleration is produced whenever the speed or direction of the velocity changes—and in circular motion, the direction changes continuously. The resulting center-directed acceleration is called **centripetal acceleration** (centripetal is from the Latin for "center seeking").

Let's calculate the magnitude of the centripetal acceleration, a_{cp}, for an object moving with a constant speed v in a circle of radius r. **Figure 6–13** shows the circular path of an object, with the center of the circle at the origin. To calculate the acceleration at the top of the circle, at point P, we first calculate the average acceleration from point 1 to point 2:

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} \qquad 6\text{–}10$$

The instantaneous acceleration at P is the limit of $\vec{\mathbf{a}}_{av}$ as points 1 and 2 move closer to P.

Referring to Figure 6–13, we see that $\vec{\mathbf{v}}_1$ is at an angle θ above the horizontal, and $\vec{\mathbf{v}}_2$ is at an angle θ below the horizontal. Both $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ have a magnitude v. Therefore, we can write the two velocities in vector form as follows:

$$\vec{\mathbf{v}}_1 = (v\cos\theta)\hat{\mathbf{x}} + (v\sin\theta)\hat{\mathbf{y}}$$
$$\vec{\mathbf{v}}_2 = (v\cos\theta)\hat{\mathbf{x}} + (-v\sin\theta)\hat{\mathbf{y}}$$

Substituting these results into $\vec{\mathbf{a}}_{av}$ gives

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \frac{-2v\sin\theta}{\Delta t}\hat{\mathbf{y}} \qquad 6\text{–}11$$

Note that $\vec{\mathbf{a}}_{av}$ points in the negative y direction—which, at point P, is toward the center of the circle.

To complete the calculation, we need Δt, the time it takes the object to go from point 1 to point 2. Since the object's speed is v, and the distance from point 1 to point 2 is $d = r(2\theta)$ where θ is measured in radians (see Appendix A, page A-2 for a discussion of radians and degrees), we find

$$\Delta t = \frac{d}{v} = \frac{2r\theta}{v} \qquad 6\text{–}12$$

Combining this result for Δt with the previous result for $\vec{\mathbf{a}}_{av}$ gives

$$\vec{\mathbf{a}}_{av} = \frac{-2v\sin\theta}{(2r\theta/v)}\hat{\mathbf{y}} = -\frac{v^2}{r}\left(\frac{\sin\theta}{\theta}\right)\hat{\mathbf{y}} \qquad 6\text{–}13$$

TABLE 6.2 $\frac{\sin\theta}{\theta}$ **for Values of** θ **Approaching Zero**

θ, radians	$\frac{\sin\theta}{\theta}$
1.00	0.841
0.500	0.959
0.250	0.990
0.125	0.997
0.0625	0.999

To find $\vec{\mathbf{a}}$ at point P, we let points 1 and 2 approach P, which means letting θ go to zero. Table 6–2 shows that as θ goes to zero ($\theta \rightarrow 0$), the ratio $(\sin\theta)/\theta$ goes to 1:

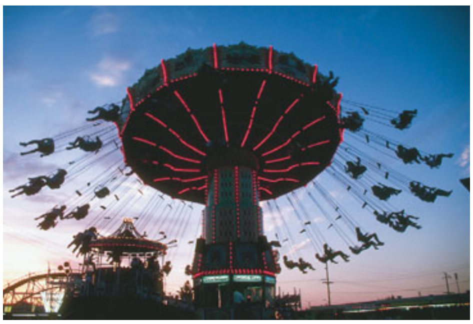
$$\frac{\sin\theta}{\theta} \xrightarrow[\text{as } \theta \rightarrow 0]{} 1$$

Finally, then, the instantaneous acceleration at point P is

$$\vec{\mathbf{a}} = -\frac{v^2}{r}\hat{\mathbf{y}} = -a_{cp}\hat{\mathbf{y}} \qquad 6\text{–}14$$

As mentioned, the direction of the acceleration is toward the center of the circle, and now we see that its magnitude is

$$a_{cp} = \frac{v^2}{r} \qquad 6\text{–}15$$



The people enjoying this carnival ride are experiencing a centripetal acceleration of roughly 10 m/s^2 directed inward, toward the axis of rotation. The force needed to produce this acceleration, which keeps the riders moving in a circular path, is provided by the horizontal component of the tension in the chains.

We can summarize these results as follows:

- When an object moves in a circle of radius r with constant speed v, its centripetal acceleration is $a_{cp} = v^2/r$.
- A force must be applied to an object to give it circular motion. For an object of mass m, the net force acting on it must have a magnitude given by

$$f_{cp} = ma_{cp} = m\frac{v^2}{r} \qquad \text{6–16}$$

and must be directed toward the center of the circle.

Note that the **centripetal force,** f_{cp}, can be produced in any number of ways. For example, f_{cp} might be the tension in a string, as in the example with the ball, or it might be due to friction between tires and the road, as when a car turns a corner. In addition, f_{cp} could be the force of gravity causing a satellite, or the Moon, to orbit the Earth. Thus, f_{cp} is a force that must be present to cause circular motion, but the specific cause of f_{cp} varies from system to system.

We now show how these results for centripetal force and centripetal acceleration can be applied in practice.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Circular Motion

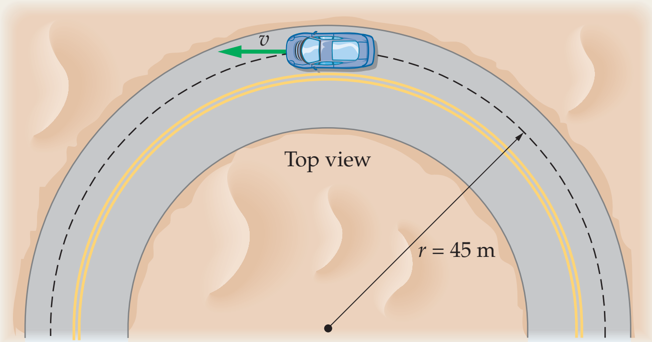
In circular motion, it is convenient to choose the coordinate system so that one axis points toward the center of the circle. Then, we know that the acceleration in that direction must be $a_{cp} = v^2/r$.

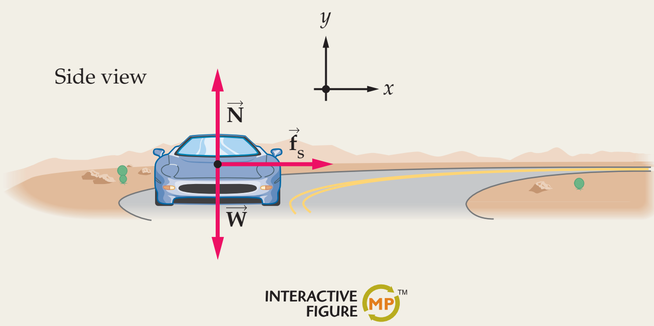
EXAMPLE 6–8 ROUNDING A CORNER

A 1200-kg car rounds a corner of radius $r = 45$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.82$, what is the greatest speed the car can have in the corner without skidding?

PICTURE THE PROBLEM

In the first sketch we show a bird's-eye view of the car as it moves along its circular path. The next sketch shows the car moving directly toward the observer. Note that we have chosen the positive x direction to point toward the center of the circular path, and the positive y axis to point vertically upward. We also indicate the three forces acting on the car: gravity, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$; the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$; and the force of static friction, $\vec{\mathbf{f}}_s = \mu_s N\hat{\mathbf{x}}$.





STRATEGY

In this system, the force of static friction provides the centripetal force required for the car to move in a circular path. That is why the force of friction is at right angles to the car's direction of motion; it is directed toward the center of the circle. In addition, the friction in this case is static because the car's tires are rolling without slipping—always making static contact with the ground. Finally, if the car moves faster, more centripetal force (i.e., more friction) is required. Thus, the greatest speed for the car corresponds to the maximum static friction, $f_s = \mu_s N$. Hence, if we set $\mu_s N$ equal to the centripetal force, $ma_{cp} = mv^2/r$, we can solve for v.

SOLUTION

1. Sum the x components of force to relate the force of static friction to the centripetal acceleration of the car: $\quad \sum F_x = f_s = ma_x$

2. Since the car moves in a circular path, with the center of the circle in the x direction, it follows that $a_x = a_{cp} = v^2/r$. Make this substitution, along with $f_s = \mu_s N$ for the force of static friction: $\quad \mu_s N = ma_{cp} = m\dfrac{v^2}{r}$

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

3. Next, set the sum of the y components of force equal to zero, since $a_y = 0$: $\sum F_y = N - W = ma_y = 0$
4. Solve for the normal force: $N = W = mg$
5. Substitute the result $N = mg$ in Step 2 and solve for v. Notice that the mass of the car cancels: $\mu_s mg = m\frac{v^2}{r}$, $v = \sqrt{\mu_s rg}$
6. Substitute numerical values to determine v: $v = \sqrt{(0.82)(45\ \text{m})(9.81\ \text{m/s}^2)} = 19\ \text{m/s}$

INSIGHT

Note that the maximum speed is less if the radius is smaller (tighter corner) or if μ_s is smaller (slick road). The mass of the vehicle, however, is irrelevant. For example, the maximum speed is precisely the same for a motorcycle rounding this corner as it is for a large, heavily loaded truck.

PRACTICE PROBLEM

Suppose the situation described in this Example takes place on the Moon, where the acceleration of gravity is less than it is on Earth. If a lunar rover goes around this same corner, is its maximum speed greater than, less than, or the same as the result found in Step 4? To check your answer, find the maximum speed for a lunar rover when it rounds a corner with $r = 45$ m and $\mu_s = 0.82$. (On the Moon, $g = 1.62\ \text{m/s}^2$.) [**Answer:** The maximum speed will be less. On the Moon we find $v = 7.7$ m/s.]

Some related homework problems: Problem 55, Problem 57, Problem 61

REAL-WORLD PHYSICS

Skids and banked roadways

If you try to round a corner too rapidly, you may experience a skid; that is, your car may begin to slide sideways across the road. A common bit of road wisdom is that you should turn in the direction of the skid to regain control—which, to most people, sounds counterintuitive. The advice is sound, however. Suppose, for example, that you are turning to the left and begin to skid to the right. If you turn more sharply to the left to try to correct for the skid, you simply reduce the turning radius of your car, r. The result is that the centripetal acceleration, v^2/r, becomes larger, and an even larger force would be required from the road to make the turn. The tendency to skid would therefore be increased. On the other hand, if you turn slightly to the right when you start to skid, you *increase* your turning radius and the centripetal acceleration decreases. In this case your car may stop skidding, and you can then regain control of your vehicle.

You may also have noticed that many roads are tilted, or banked, when they round a corner. The same type of banking is observed on many automobile racetracks as well. Next time you drive around a banked curve, notice that the banking tilts you in toward the center of the circular path you are following. This is by

▲ The steeply banked track at the Talladega Speedway in Alabama (left) helps to keep the rapidly moving cars from skidding off along a tangential path. Even when there is no solid roadway, however, banking can still help—airplanes bank when making turns (center) to keep from "skidding" sideways. Banking is beneficial in another way as well. Occupants of cars on a banked roadway or of a banking airplane feel no sideways force when the banking angle is just right, so turns become a safer and more comfortable experience. For this reason, some trains use hydraulic suspension systems to bank when rounding corners (right), even though the tracks themselves are level.

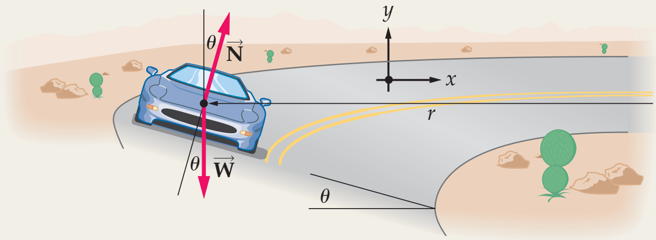
design. On a banked curve, the normal force exerted by the road contributes to the required centripetal force. If the tilt angle is just right, the normal force provides all of the centripetal force so that the car can negotiate the curve even if there is no friction between its tires and the road. The next Example determines the optimum banking angle for a given speed and given radius of turn.

EXAMPLE 6–9 BANK ON IT

If a roadway is banked at the proper angle, a car can round a corner without any assistance from friction between the tires and the road. Find the appropriate banking angle for a 900-kg car traveling at 20.5 m/s in a turn of radius 85.0 m.

PICTURE THE PROBLEM

Note that we choose the positive y axis to point vertically upward and the positive x direction to point toward the center of the circular path. Since $\vec{\mathbf{N}}$ is perpendicular to the banked roadway, it is at an angle θ to the y axis. Therefore, $\vec{\mathbf{N}} = (N \sin\theta)\hat{\mathbf{x}} + (N\cos\theta)\hat{\mathbf{y}}$ and $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$.



STRATEGY

In order for the car to move in a circular path, there must be a force acting on it in the positive x direction. Since the weight $\vec{\mathbf{W}}$ has no x component, it follows that the normal force $\vec{\mathbf{N}}$ must supply the needed centripetal force. Thus, we find N by setting $\Sigma F_y = ma_y = 0$, since there is no motion in the y direction. Then we use N in $\Sigma F_x = ma_x = mv^2/r$ to find the angle θ.

SOLUTION

1. Start by determining N from the condition $\Sigma F_y = 0$:

$$\sum F_y = N\cos\theta - W = 0$$

$$N = \frac{W}{\cos\theta} = \frac{mg}{\cos\theta}$$

2. Next, set $\Sigma F_x = mv^2/r$:

$$\sum F_x = N\sin\theta$$

$$= ma_x = ma_{cp} = m\frac{v^2}{r}$$

3. Substitute $N = mg/\cos\theta$ (from $\Sigma F_y = 0$, Step 1) and solve for θ, using the fact that $\sin\theta/\cos\theta = \tan\theta$. Notice that, once again, the mass of the car cancels:

$$N\sin\theta = \frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

$$\tan\theta = \frac{v^2}{gr} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$$

4. Substitute numerical values to determine θ:

$$\theta = \tan^{-1}\left[\frac{(20.5\text{ m/s})^2}{(9.81\text{ m/s}^2)(85.0\text{ m})}\right] = 26.7°$$

INSIGHT

The symbolic result in Step 3 shows that the banking angle increases with increasing speed and decreasing radius of turn, as one would expect.

From the point of view of a passenger, the experience of rounding a properly banked corner is basically the same as riding on a level road—there are no "sideways forces" to make the turn uncomfortable. There is one small difference, however—the passenger feels heavier due to the increased normal force.

PRACTICE PROBLEM

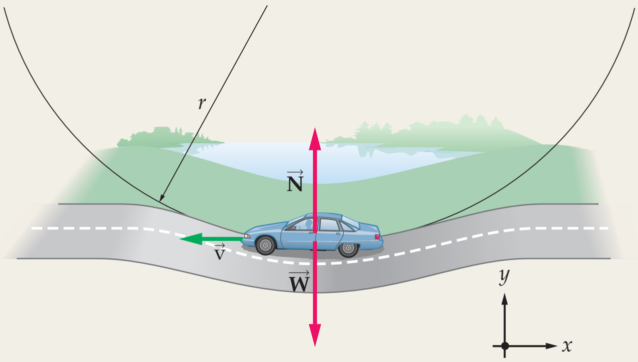
A turn of radius 65 m is banked at 30.0°. What speed should a car have in order to make the turn with no assistance from friction? [**Answer:** $v = 19$ m/s]

Some related homework problems: Problem 58, Problem 107

If you've ever driven through a dip in the road, you know that you feel momentarily heavier near the bottom of the dip, just like a passenger in Example 6–9. This change in apparent weight is due to the approximately circular motion of the car, as we show next.

ACTIVE EXAMPLE 6–4 FIND THE NORMAL FORCE

While driving along a country lane with a constant speed of 17.0 m/s, you encounter a dip in the road. The dip can be approximated as a circular arc, with a radius of 65.0 m. What is the normal force exerted by a car seat on an 80.0-kg passenger when the car is at the bottom of the dip?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write $\Sigma F_y = ma_y$ for the passenger: $N - mg = ma_y$
2. Replace a_y with the centripetal acceleration: $a_y = v^2/r$
3. Solve for N: $N = mg + mv^2/r$
4. Substitute numerical values: $N = 1140$ N

INSIGHT

At the bottom of the dip the normal force is greater than the weight of the passenger, since it must also supply the centripetal force. As a result, the passenger feels heavier than usual. In this case, the 80.0-kg passenger feels as if his mass has increased by 45%, to 116 kg!

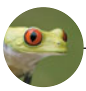
The same physics applies to a jet pilot who pulls a plane out of a high-speed dive. In that case, the magnitude of the effect can be much larger, resulting in a decrease of blood flow to the brain and eventually to loss of consciousness. Here's a case where basic physics really can be a matter of life and death.

YOUR TURN

At what speed will the magnitude of the normal force be equal to 1250 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

▲ A laboratory centrifuge of the kind commonly used to separate blood components.

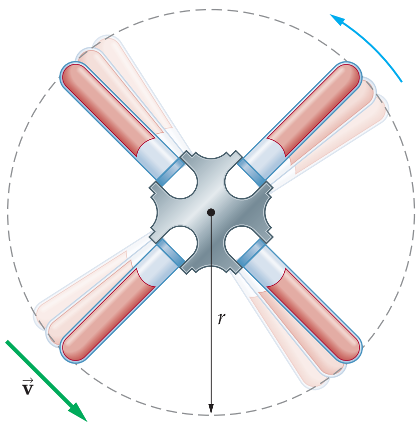


REAL-WORLD PHYSICS: BIO

Centrifuges and ultracentrifuges

A similar calculation can be applied to a car going over the top of a bump. In that case, circular motion results in a reduced apparent weight.

Finally, we determine the acceleration produced in a **centrifuge,** a common device in biological and medical laboratories that uses large centripetal accelerations to perform such tasks as separating red and white blood cells from serum. A simplified top view of a centrifuge is shown in **Figure 6–14**.



▲ **FIGURE 6–14 Simplified top view of a centrifuge in operation**

EXERCISE 6–1

The centrifuge in Figure 6–14 rotates at a rate that gives the bottom of the test tube a linear speed of 89.3 m/s. If the bottom of the test tube is 8.50 cm from the axis of rotation, what is the centripetal acceleration experienced there?

SOLUTION

Applying the relation $a_{cp} = v^2/r$ yields

$$a_{cp} = \frac{v^2}{r} = \frac{(89.3\ \text{m/s})^2}{0.0850\ \text{m}} = 93{,}800\ \text{m/s}^2 = 9560g$$

In this expression, g is the acceleration of gravity, 9.81 m/s^2.

Thus, a centrifuge can produce centripetal accelerations that are many thousand times greater than the acceleration of gravity. In fact, devices referred to as **ultracentrifuges** can produce accelerations as great as 1 million g. Even in the relatively modest case considered in Exercise 6–1, the forces involved in a centrifuge can be quite significant. For example, if the contents of the test tube have a mass of 12.0 g, the centripetal force that must be exerted by the bottom of the tube is $(0.0120 \text{ kg})(9560\, g) = 1130$ N, or about 250 lb!

Finally, an object moving in a circular path may increase or decrease its speed. In such a case, the object has both an acceleration tangential to its path that changes its speed, $\vec{\mathbf{a}}_t$, and a centripetal acceleration perpendicular to its path, $\vec{\mathbf{a}}_{cp}$, that changes its direction of motion. Such a situation is illustrated in **Figure 6–15**. The total acceleration of the object is the vector sum of $\vec{\mathbf{a}}_t$ and $\vec{\mathbf{a}}_{cp}$. We will explore this case more fully in Chapter 10.

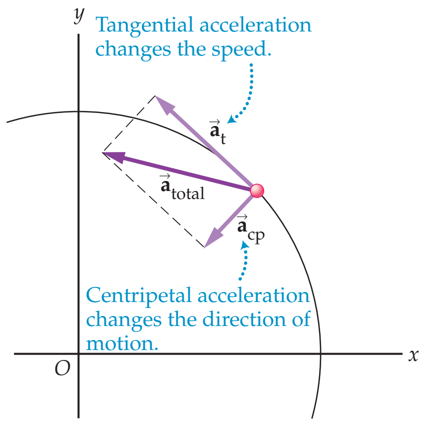


FIGURE 6–15 A particle moving in a circular path with tangential acceleration In this case, the particle's speed is increasing at the rate given by a_t.

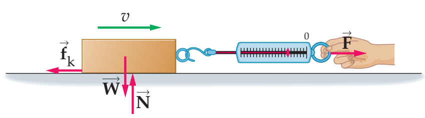
THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The equations of kinematics from Chapters 2 and 4 proved useful again in this chapter. See, in particular, Examples 6–1 and 6–2.	Our discussion of springs, and Hooke's law in particular, will be of importance when we consider oscillations in Chapter 13.
The discussion related to Figure 5–15 about angles on an inclined surface came into play when identifying the angles in Examples 6–2 and 6–9.	The basic ideas of translational equilibrium (Section 6–3) will be extended to more general objects in Chapter 11.
Our derivation of the direction and magnitude of centripetal acceleration (Section 6–5) made extensive use of our knowledge of vectors and how to resolve them into components.	Circular motion will come up again in a number of situations, but especially when we consider orbital motion in Chapter 12 and the Bohr model of the hydrogen atom in Chapter 31.

CHAPTER SUMMARY

6–1 FRICTIONAL FORCES

Frictional forces are due to the microscopic roughness of surfaces in contact. As a rule of thumb, friction is independent of the area of contact and independent of the relative speed of the surfaces.



Kinetic Friction

Friction experienced by surfaces that are in contact and moving relative to one another. The force of kinetic friction is given by

$$f_k = \mu_k N \qquad 6\text{–}1$$

In this expression, μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force.

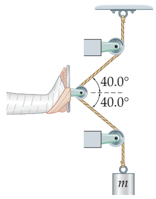
Static Friction

Friction experienced by surfaces that are in static contact. The maximum force of static friction is given by

$$f_{s,max} = \mu_s N \qquad 6\text{–}3$$

In this expression, μ_s is the coefficient of static friction and N is the magnitude of the normal force. The force of static friction can have any magnitude between zero and its maximum value.

6–2 STRINGS AND SPRINGS



Strings and springs provide a common way of exerting forces on objects. Ideal strings and springs are massless.

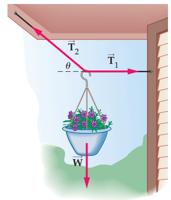
Tension
The force transmitted through a string. The tension is the same throughout the length of an ideal string.

Hooke's Law
The force exerted by an ideal spring stretched by the amount x is

$$F_x = -kx \tag{6–4}$$

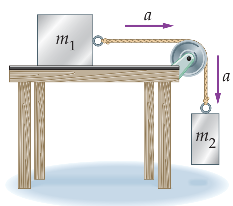
In words, the force exerted by a spring is proportional to the amount of stretch or compression, and is in the opposite direction.

6–3 TRANSLATIONAL EQUILIBRIUM



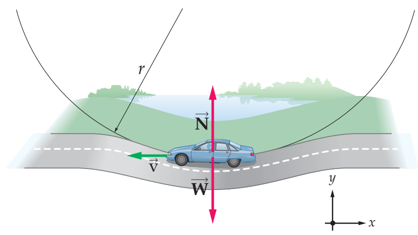
An object is in translational equilibrium if the net force acting on it is zero. Equivalently, an object is in equilibrium if it has zero acceleration.

6–4 CONNECTED OBJECTS



Connected objects are linked physically, and hence they are linked mathematically as well. For example, objects connected by strings have the same magnitude of acceleration.

6–5 CIRCULAR MOTION



An object moving with speed v in a circle of radius r has an acceleration of magnitude v^2/r directed toward the center of the circle: This is referred to as the centripetal acceleration, a_{cp}. If the object has a mass m, the force required for the circular motion is

$$f_{cp} = ma_{cp} = mv^2/r \tag{6–16}$$

PROBLEM-SOLVING SUMMARY

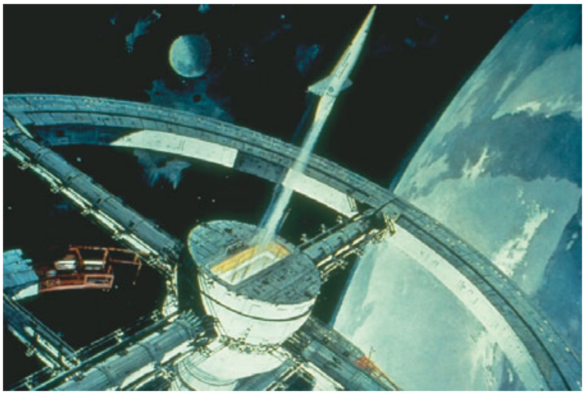
Type of Calculation	Relevant Physical Concepts	Related Examples
Find the acceleration when kinetic friction is present.	First, find the magnitude of the normal force, N. The corresponding kinetic friction has a magnitude of $f_k = \mu_k N$ and points opposite to the direction of motion. Include this force with the others when applying Newton's second law.	Examples 6–1, 6–2
Solve problems involving static friction.	Start by finding the magnitude of the normal force, N. The corresponding static friction has a magnitude between zero and $\mu_s N$. Its direction opposes motion.	Example 6–3 Active Example 6–1
Find the acceleration and the tension for masses connected by a string.	Apply Newton's second law to each mass separately. This generates two equations, which can be solved for the two unknowns, a and T.	Examples 6–6, 6–7
Solve problems involving circular motion.	Set up the coordinate system so that one axis points to the center of the circle. When applying Newton's second law to that direction, set the acceleration equal to $a_{cp} = v^2/r$.	Examples 6–8, 6–9 Active Example 6–4

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A clothesline always sags a little, even if nothing hangs from it. Explain.
2. In the *Jurassic Park* sequel, *The Lost World*, a man tries to keep a large vehicle from going over a cliff by connecting a cable from his Jeep to the vehicle. The man then puts the Jeep in gear and spins the rear wheels. Do you expect that spinning the tires will increase the force exerted by the Jeep on the vehicle? Why or why not?
3. When a traffic accident is investigated, it is common for the length of the skid marks to be measured. How could this information be used to estimate the initial speed of the vehicle that left the skid marks?
4. In a car with rear-wheel drive, the maximum acceleration is often less than the maximum deceleration. Why?
5. A train typically requires a much greater distance to come to rest, for a given initial speed, than does a car. Why?
6. Give some everyday examples of situations in which friction is beneficial.
7. At the local farm, you buy a flat of strawberries and place them on the backseat of the car. On the way home, you begin to brake as you approach a stop sign. At first the strawberries stay put, but as you brake a bit harder, they begin to slide off the seat. Explain.
8. It is possible to spin a bucket of water in a vertical circle and have none of the water spill when the bucket is upside down. How would you explain this to members of your family?
9. Water sprays off a rapidly turning bicycle wheel. Why?
10. Can an object be in equilibrium if it is moving? Explain.
11. In a dramatic circus act, a motorcyclist drives his bike around the inside of a vertical circle. How is this possible, considering that the motorcycle is upside down at the top of the circle?
12. The gravitational attraction of the Earth is only slightly less at the altitude of an orbiting spacecraft than it is on the Earth's surface. Why is it, then, that astronauts feel weightless?
13. A popular carnival ride has passengers stand with their backs against the inside wall of a cylinder. As the cylinder begins to spin, the passengers feel as if they are being pushed against the wall. Explain.
14. Referring to Question 13, after the cylinder reaches operating speed, the floor is lowered away, leaving the passengers "stuck" to the wall. Explain.
15. Your car is stuck on an icy side street. Some students on their way to class see your predicament and help out by sitting on the trunk of your car to increase its traction. Why does this help?
16. The parking brake on a car causes the rear wheels to lock up. What would be the likely consequence of applying the parking brake in a car that is in rapid motion? (*Note:* Do *not* try this at home.)
17. **BIO** The foot of your average gecko is covered with billions of tiny hair tips—called spatulae—that are made of keratin, the protein found in human hair. A subtle shift of the electron distribution in both the spatulae and the wall to which a gecko clings produces an adhesive force by means of the van der Waals interaction between molecules. Suppose a gecko uses its spatulae to cling to a vertical windowpane. If you were to describe this situation in terms of a coefficient of static friction, μ_s, what value would you assign to μ_s? Is this a sensible way to model the gecko's feat? Explain.
18. Discuss the physics involved in the spin cycle of a washing machine. In particular, how is circular motion related to the removal of water from the clothes?
19. The gas pedal and the brake pedal are capable of causing a car to accelerate. Can the steering wheel also produce an acceleration? Explain.
20. In the movie *2001: A Space Odyssey*, a rotating space station provides "artificial gravity" for its inhabitants. How does this work?



The rotating space station from the movie *2001: A Space Odyssey* (Conceptual Question 20)

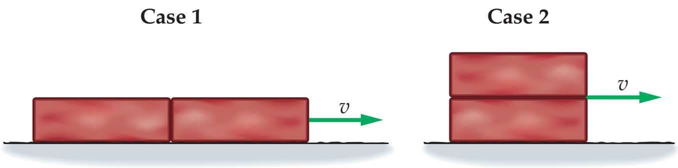
21. When rounding a corner on a bicycle or a motorcycle, the driver leans inward, toward the center of the circle. Why?
22. In *Robin Hood: Prince of Thieves,* starring Kevin Costner, Robin swings between trees on a vine that is on fire. At the lowest point of his swing, the vine burns through and Robin begins to fall. The next shot, from high up in the trees, shows Robin falling straight downward. Would you rate the physics of this scene "Good," "Bad," or "Ugly"? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 6–1 FRICTIONAL FORCES

1. • **CE Predict/Explain** You push two identical bricks across a tabletop with constant speed, v, as shown in **Figure 6–16**. In case 1, you place the bricks end to end; in case 2, you stack the bricks one on top of the other. **(a)** Is the force of kinetic friction in case 1 greater than, less than, or equal to the force of kinetic friction in case 2? **(b)** Choose the *best explanation* from among the following:



▲ **FIGURE 6–16** Problem 1

I. The normal force in case 2 is larger, and hence the bricks press down more firmly against the tabletop.
II. The normal force is the same in the two cases, and friction is independent of surface area.
III. Case 1 has more surface area in contact with the tabletop, and this leads to more friction.

2. • **CE Predict/Explain** Two drivers traveling side-by-side at the same speed suddenly see a deer in the road ahead of them and begin braking. Driver 1 stops by locking up his brakes and screeching to a halt; driver 2 stops by applying her brakes just to the verge of locking, so that the wheels continue to turn until her car comes to a complete stop. **(a)** All other factors being equal, is the stopping distance of driver 1 greater than, less than, or equal to the stopping distance of driver 2? **(b)** Choose the *best explanation* from among the following:
I. Locking up the brakes gives the greatest possible braking force.
II. The same tires on the same road result in the same force of friction.
III. Locked-up brakes lead to sliding (kinetic) friction, which is less than rolling (static) friction.

3. • A baseball player slides into third base with an initial speed of 4.0 m/s. If the coefficient of kinetic friction between the player and the ground is 0.46, how far does the player slide before coming to rest?

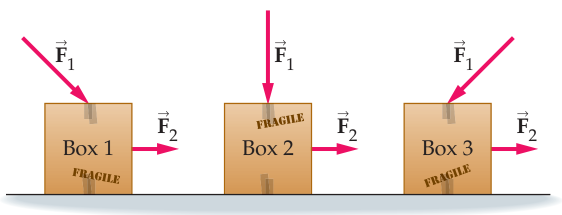
4. • A child goes down a playground slide with an acceleration of 1.26 m/s^2. Find the coefficient of kinetic friction between the child and the slide if the slide is inclined at an angle of 33.0° below the horizontal.

5. • Hopping into your Porsche, you floor it and accelerate at 12 m/s^2 without spinning the tires. Determine the minimum coefficient of static friction between the tires and the road needed to make this possible.

6. • When you push a 1.80-kg book resting on a tabletop, it takes 2.25 N to start the book sliding. Once it is sliding, however, it takes only 1.50 N to keep the book moving with constant speed. What are the coefficients of static and kinetic friction between the book and the tabletop?

7. • In Problem 6, what is the frictional force exerted on the book when you push on it with a force of 0.75 N?

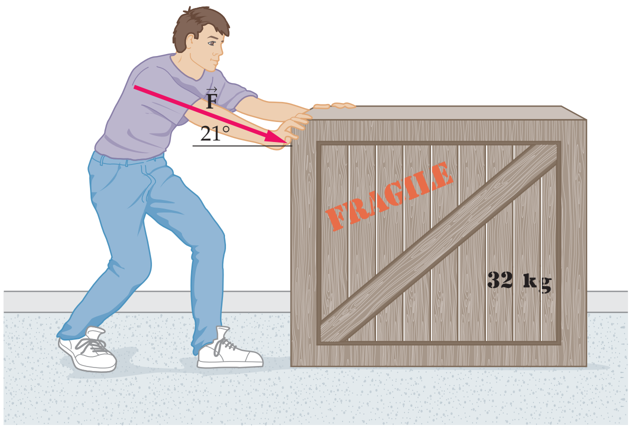
8. •• **CE** The three identical boxes shown in **Figure 6–17** remain at rest on a rough, horizontal surface, even though they are acted on by two different forces, $\vec{F}_1$ and $\vec{F}_2$. All of the forces labeled $\vec{F}_1$ have the same magnitude; all of the forces labeled $\vec{F}_2$ are identical to one another. Rank the boxes in order of increasing magnitude of the force static friction between them and the surface. Indicate ties where appropriate.



▲ **FIGURE 6–17** Problem 8

9. •• **IP** A tie of uniform width is laid out on a table, with a fraction of its length hanging over the edge. Initially, the tie is at rest. **(a)** If the fraction hanging from the table is increased, the tie eventually slides to the ground. Explain. **(b)** What is the coefficient of static friction between the tie and the table if the tie begins to slide when one-fourth of its length hangs over the edge?

10. •• To move a large crate across a rough floor, you push on it with a force F at an angle of 21° below the horizontal, as shown in **Figure 6–18**. Find the force necessary to start the crate moving, given that the mass of the crate is 32 kg and the coefficient of static friction between the crate and the floor is 0.57.



▲ **FIGURE 6–18** Problems 10, 11, and 106

11. •• In Problem 10, find the acceleration of the crate if the applied force is 330 N and the coefficient of kinetic friction is 0.45.

12. •• **IP** A 48-kg crate is placed on an inclined ramp. When the angle the ramp makes with the horizontal is increased to 26°, the crate begins to slide downward. **(a)** What is the coefficient of static friction between the crate and the ramp? **(b)** At what angle does the crate begin to slide if its mass is doubled?

13. •• **IP** A 97-kg sprinter wishes to accelerate from rest to a speed of 13 m/s in a distance of 22 m. **(a)** What coefficient of static friction is required between the sprinter's shoes and the track? **(b)** Explain the strategy used to find the answer to part (a).

14. •• **Coffee To Go** A person places a cup of coffee on the roof of her car while she dashes back into the house for a forgotten item. When she returns to the car, she hops in and takes off with the coffee cup still on the roof. **(a)** If the coefficient of static friction between the coffee cup and the roof of the car is 0.24, what is the maximum acceleration the car can have without causing the cup to slide? Ignore the effects of air resistance. **(b)** What is the smallest amount of time in which the person can accelerate the car from rest to 15 m/s and still keep the coffee cup on the roof?

15. •• **IP Force Times Distance I** At the local hockey rink, a puck with a mass of 0.12 kg is given an initial speed of $v = 5.3$ m/s. **(a)** If the coefficient of kinetic friction between the ice and the puck is 0.11, what distance d does the puck slide before coming to rest? **(b)** If the mass of the puck is doubled, does the frictional force F exerted on the puck increase, decrease, or stay the same? Explain. **(c)** Does the stopping distance of the puck increase, decrease, or stay the same when its mass is doubled? Explain. **(d)** For the situation considered in part (a), show that $Fd = \frac{1}{2}mv^2$.

(The significance of this result will be discussed in Chapter 7, where we will see that $\frac{1}{2}mv^2$ is the kinetic energy of an object.)

16. •• **IP** **Force Times Time** At the local hockey rink, a puck with a mass of 0.12 kg is given an initial speed of $v_0 = 6.7$ m/s. **(a)** If the coefficient of kinetic friction between the ice and the puck is 0.13, how much time t does it take for the puck to come to rest? **(b)** If the mass of the puck is doubled, does the frictional force F exerted on the puck increase, decrease, or stay the same? Explain. **(c)** Does the stopping time of the puck increase, decrease, or stay the same when its mass is doubled? Explain. **(d)** For the situation considered in part (a), show that $Ft = mv_0$. (The significance of this result will be discussed in Chapter 9, where we will see that mv is the momentum of an object.)

17. •• **Force Times Distance II** A block of mass $m = 1.95$ kg slides with an initial speed $v_i = 4.33$ m/s on a smooth, horizontal surface. The block now encounters a rough patch with a coefficient of kinetic friction given by $\mu_k = 0.260$. The rough patch extends for a distance $d = 0.125$ m, after which the surface is again frictionless. **(a)** What is the acceleration of the block when it is in the rough patch? **(b)** What is the final speed, v_f, of the block when it exits the rough patch? **(c)** Show that $-Fd = -(\mu_k mg)d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. (The significance of this result will be discussed in Chapter 7, where we will see that $\frac{1}{2}mv^2$ is the kinetic energy of an object.)

18. ••• **IP** The coefficient of kinetic friction between the tires of your car and the roadway is μ. **(a)** If your initial speed is v and you lock your tires during braking, how far do you skid? Give your answer in terms of v, μ, and m, the mass of your car. **(b)** If you double your speed, what happens to the stopping distance? **(c)** What is the stopping distance for a truck with twice the mass of your car, assuming the same initial speed and coefficient of kinetic friction?

SECTION 6–2 STRINGS AND SPRINGS

19. • **CE** A certain spring has a force constant k. **(a)** If this spring is cut in half, does the resulting half spring have a force constant that is greater than, less than, or equal to k? **(b)** If two of the original full-length springs are connected end to end, does the resulting double spring have a force constant that is greater than, less than, or equal to k?

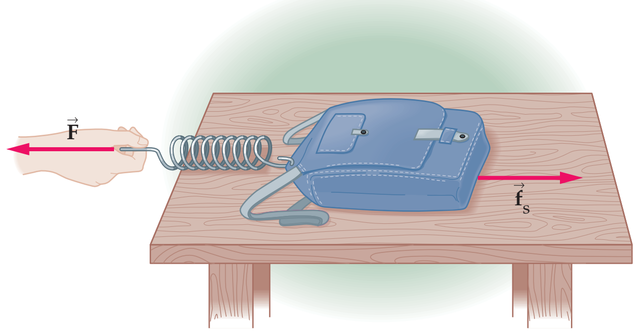
20. • Pulling up on a rope, you lift a 4.35-kg bucket of water from a well with an acceleration of 1.78 m/s^2. What is the tension in the rope?

21. • When a 9.09-kg mass is placed on top of a vertical spring, the spring compresses 4.18 cm. Find the force constant of the spring.

22. • A 110-kg box is loaded into the trunk of a car. If the height of the car's bumper decreases by 13 cm, what is the force constant of its rear suspension?

23. • A 50.0-kg person takes a nap in a backyard hammock. Both ropes supporting the hammock are at an angle of 15.0° above the horizontal. Find the tension in the ropes.

24. • **IP** A backpack full of books weighing 52.0 N rests on a table in a physics laboratory classroom. A spring with a force constant of 150 N/m is attached to the backpack and pulled horizontally, as indicated in **Figure 6–19**. **(a)** If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table? **(b)** Does your answer to part (a) change if the mass of the backpack is doubled? Explain.



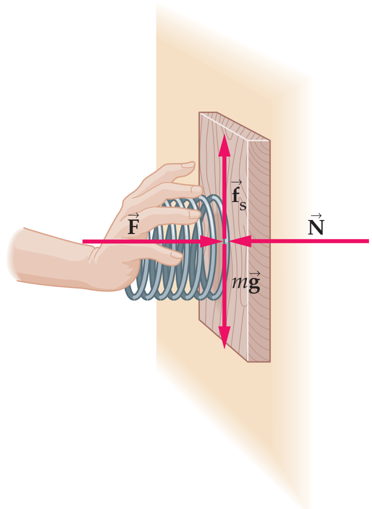
▲ **FIGURE 6–19** Problems 24 and 25

25. • If the 52.0-N backpack in Problem 24 begins to slide when the spring ($k = 150$ N/m) stretches by 2.50 cm, what is the coefficient of static friction between the backpack and the table?

26. •• **IP** The equilibrium length of a certain spring with a force constant of $k = 250$ N/m is 0.18 m. **(a)** What is the magnitude of the force that is required to hold this spring at twice its equilibrium length? **(b)** Is the magnitude of the force required to keep the spring compressed to half its equilibrium length greater than, less than, or equal to the force found in part (a)? Explain.

27. •• **IP** Illinois Jones is being pulled from a snake pit with a rope that breaks if the tension in it exceeds 755 N. **(a)** If Illinois Jones has a mass of 70.0 kg and the snake pit is 3.40 m deep, what is the minimum time that is required to pull our intrepid explorer from the pit? **(b)** Explain why the rope breaks if Jones is pulled from the pit in less time than that calculated in part (a).

28. •• **IP** A spring with a force constant of 120 N/m is used to push a 0.27-kg block of wood against a wall, as shown in **Figure 6–20**. **(a)** Find the minimum compression of the spring needed to keep the block from falling, given that the coefficient of static friction between the block and the wall is 0.46. **(b)** Does your answer to part (a) change if the mass of the block of wood is doubled? Explain.

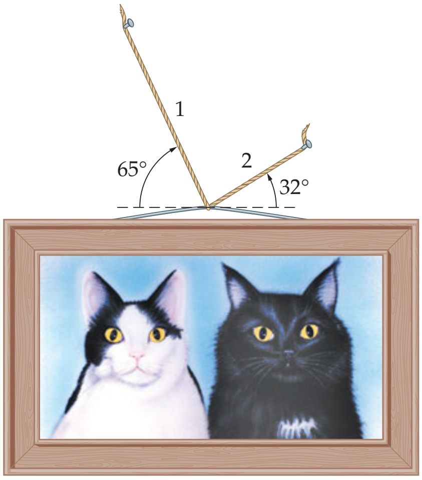


▲ **FIGURE 6–20** Problem 28

29. •• **IP** Your friend's 13.6-g graduation tassel hangs on a string from his rearview mirror. **(a)** When he accelerates from a stoplight, the tassel deflects backward toward the rear of the car. Explain. **(b)** If the tassel hangs at an angle of 6.44° relative to the vertical, what is the acceleration of the car?

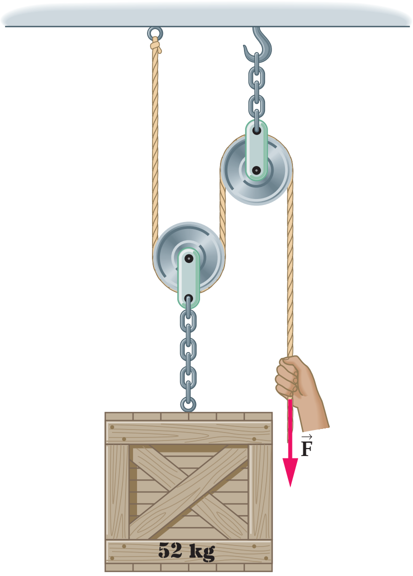
30. •• In Problem 29, **(a)** find the tension in the string holding the tassel. **(b)** At what angle to the vertical will the tension in the string be twice the weight of the tassel?

31. •• **IP** A picture hangs on the wall suspended by two strings, as shown in **Figure 6–21**. The tension in string 1 is 1.7 N. **(a)** Is the tension in string 2 greater than, less than, or equal to 1.7 N? Explain. **(b)** Verify your answer to part (a) by calculating the tension in string 2. **(c)** What is the weight of the picture?



▲ **FIGURE 6–21** Problems 31 and 83

32. •• **Mechanical Advantage** The pulley system shown in **Figure 6–22** is used to lift a 52-kg crate. Note that one chain connects the upper pulley to the ceiling and a second chain connects the lower pulley to the crate. Assuming the masses of the chains, pulleys, and ropes are negligible, determine **(a)** the force $\vec{F}$ required to lift the crate with constant speed, **(b)** the tension in the upper chain, and **(c)** the tension in the lower chain.



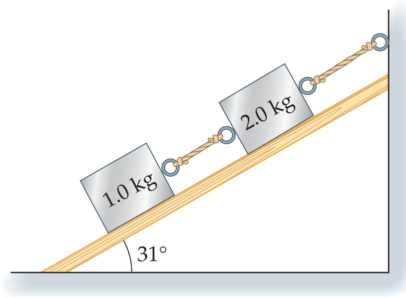
▲ **FIGURE 6–22** Problems 32 and 33

33. •• In Problem 32, determine **(a)** the force $\vec{F}$, **(b)** the tension in the upper chain, and **(c)** the tension in the lower chain, given that the crate is rising with an acceleration of 2.3 m/s^2.

SECTION 6–3 TRANSLATIONAL EQUILIBRIUM

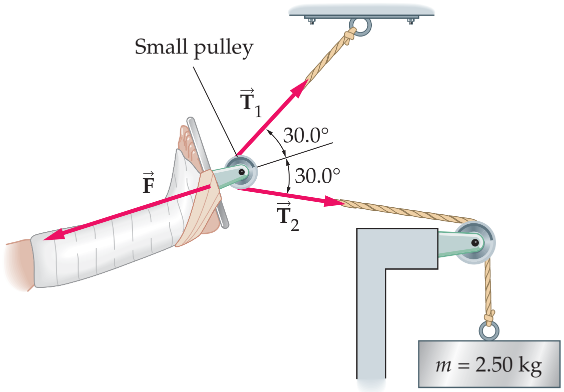
34. • Pulling the string on a bow back with a force of 28.7 lb, an archer prepares to shoot an arrow. If the archer pulls in the center of the string, and the angle between the two halves is 138°, what is the tension in the string?

35. • In **Figure 6–23** we see two blocks connected by a string and tied to a wall. The mass of the lower block is 1.0 kg; the mass of the upper block is 2.0 kg. Given that the angle of the incline is 31°, find the tensions in **(a)** the string connecting the two blocks and **(b)** the string that is tied to the wall.



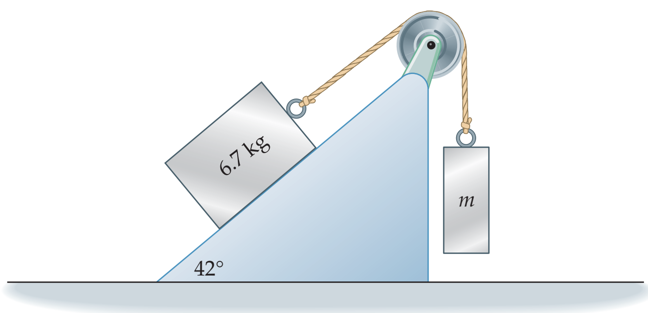
▲ **FIGURE 6–23** Problem 35

36. • **BIO Traction** After a skiing accident, your leg is in a cast and supported in a traction device, as shown in **Figure 6–24**. Find the magnitude of the force $\vec{F}$ exerted by the leg on the small pulley. (By Newton's third law, the small pulley exerts an equal and opposite force on the leg.) Let the mass m be 2.50 kg.



▲ **FIGURE 6–24** Problems 36 and 69

37. • Two blocks are connected by a string, as shown in **Figure 6–25**. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of 6.7 kg. Find the mass of the hanging block that will cause the system to be in equilibrium. (The pulley is assumed to be ideal.)



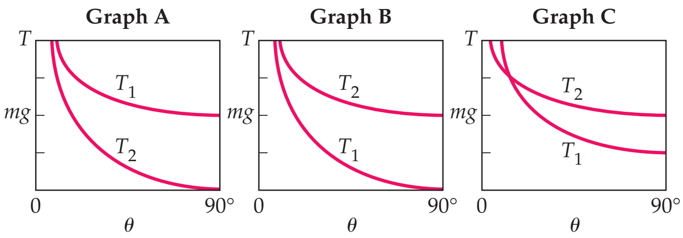
▲ **FIGURE 6–25** Problem 37

38. •• **CE Predict/Explain (a)** Referring to the hanging planter in Example 6–5, which of the three graphs (A, B, or C) in **Figure 6–26** shows an accurate plot of the tensions T_1 and T_2 as a function of the angle θ? **(b)** Choose the *best explanation* from among the following:

I. The two tensions must be equal at some angle between $\theta = 0$ and $\theta = 90°$.

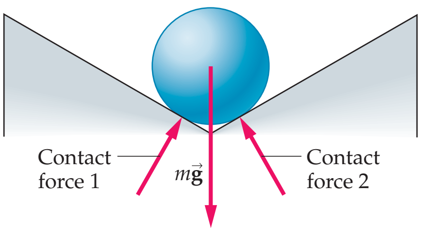
II. T_2 is greater than T_1 at all angles, and is equal to mg at $\theta = 90°$.

III. T_2 is less than T_1 at all angles, and is equal to 0 at $\theta = 90°$.



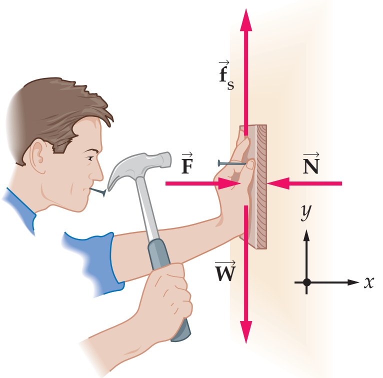
▲ **FIGURE 6–26** Problem 38

39. •• A 0.15-kg ball is placed in a shallow wedge with an opening angle of 120°, as shown in **Figure 6–27**. For each contact point between the wedge and the ball, determine the force exerted on the ball. Assume the system is frictionless.



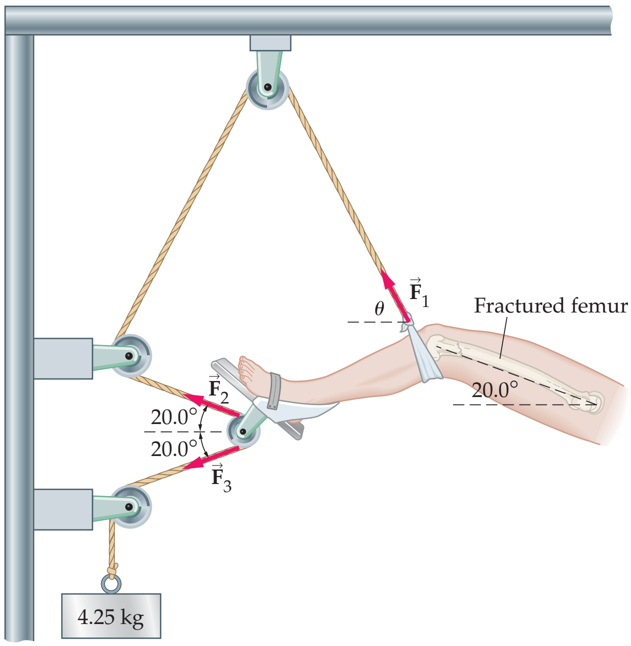
▲ **FIGURE 6–27** Problem 39

40. •• **IP** You want to nail a 1.6-kg board onto the wall of a barn. To position the board before nailing, you push it against the wall with a horizontal force $\vec{F}$ to keep it from sliding to the ground (**Figure 6–28**). **(a)** If the coefficient of static friction between the board and the wall is 0.79, what is the least force you can apply and still hold the board in place? **(b)** What happens to the force of static friction if you push against the wall with a force greater than that found in part (a)?



▲ **FIGURE 6–28** Problem 40

41. ••• **BIO The Russell Traction System** To immobilize a fractured femur (the thigh bone), doctors often utilize the Russell traction system illustrated in **Figure 6–29**. Notice that one force is applied directly to the knee, $\vec{F}_1$, while two other forces, $\vec{F}_2$ and $\vec{F}_3$, are applied to the foot. The latter two forces combine to give a force $\vec{F}_2 + \vec{F}_3$ that is transmitted through the lower leg to the knee. The result is that the knee experiences the total force $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. The goal of this traction system is to have $\vec{F}_{total}$ directly in line with the fractured femur, at an angle of 20.0° above the horizontal. Find **(a)** the angle θ required to produce this alignment of $\vec{F}_{total}$ and **(b)** the magnitude of the force, $\vec{F}_{total}$ that is applied to the femur in this case. (Assume the pulleys are ideal.)



▲ **FIGURE 6–29** Problem 41

SECTION 6–4 CONNECTED OBJECTS

42. • **CE** In Example 6–6 (Connected Blocks), suppose m_1 and m_2 are both increased by a factor of 2. **(a)** Does the acceleration of the blocks increase, decrease, or stay the same? **(b)** Does the tension in the string increase, decrease, or stay the same?

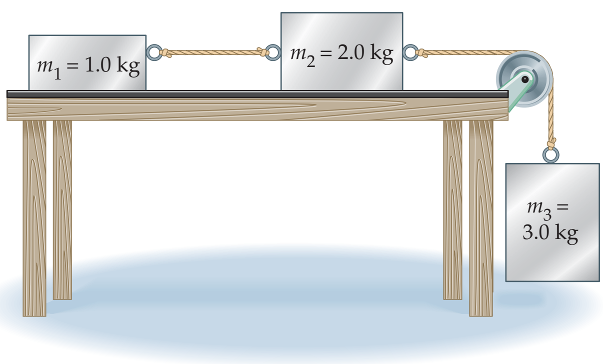
43. • **CE Predict/Explain** Suppose m_1 and m_2 in Example 6–7 (Atwood's Machine) are both increased by 1 kg. Does the acceleration of the blocks increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The net force acting on the blocks is the same, but the total mass that must be accelerated is greater.

II. The difference in the masses is the same, and this is what determines the net force on the system.

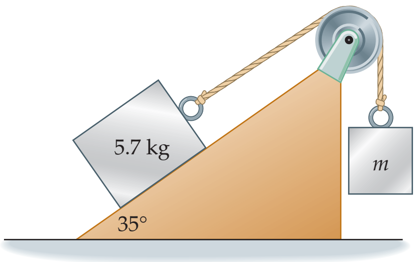
III. The force exerted on each block is greater, leading to an increased acceleration.

44. • Find the acceleration of the masses shown in **Figure 6–30**, given that $m_1 = 1.0$ kg, $m_2 = 2.0$ kg, and $m_3 = 3.0$ kg. Assume the table is frictionless and the masses move freely.



▲ **FIGURE 6–30** Problems 44, 47, and 103

45. • Two blocks are connected by a string, as shown in **Figure 6–31**. The smooth inclined surface makes an angle of 35° with the horizontal, and the block on the incline has a mass of 5.7 kg. The mass of the hanging block is $m = 3.2$ kg. Find **(a)** the direction and **(b)** the magnitude of the hanging block's acceleration.

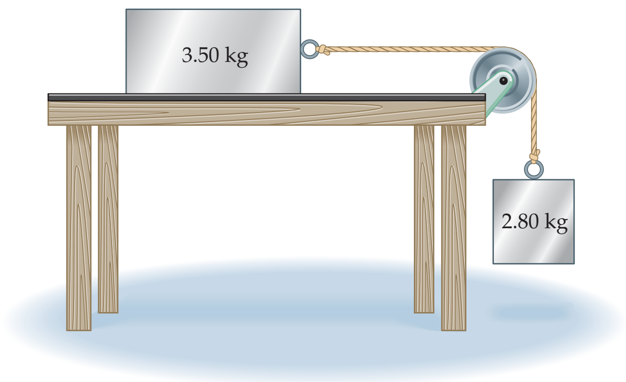


▲ **FIGURE 6–31** Problems 45 and 46

46. • Referring to Problem 45, find **(a)** the direction and **(b)** the magnitude of the hanging block's acceleration if its mass is $m = 4.2$ kg.

47. •• Referring to Figure 6–30, find the tension in the string connecting **(a)** m_1 and m_2 and **(b)** m_2 and m_3. Assume the table is frictionless and the masses move freely.

48. •• **IP** A 3.50-kg block on a smooth tabletop is attached by a string to a hanging block of mass 2.80 kg, as shown in **Figure 6–32**. The blocks are released from rest and allowed to move freely. **(a)** Is the tension in the string greater than, less than, or equal to the weight of the hanging mass? Find **(b)** the acceleration of the blocks and **(c)** the tension in the string.



▲ **FIGURE 6–32** Problem 48

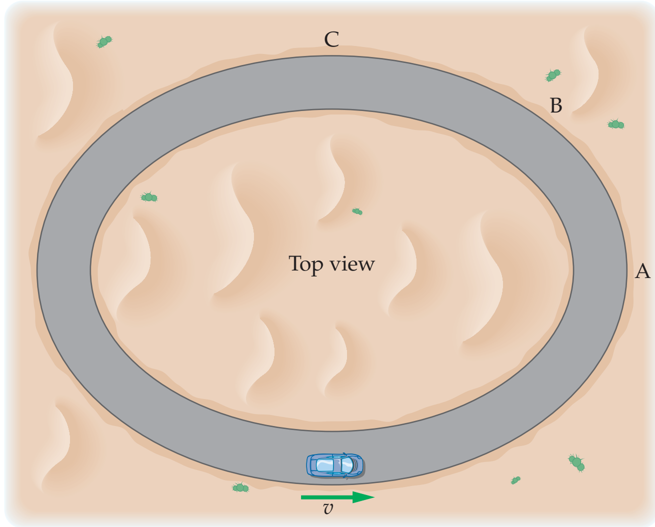
49. •• **IP** A 7.7-N force pulls horizontally on a 1.6-kg block that slides on a smooth horizontal surface. This block is connected by a horizontal string to a second block of mass $m_2 = 0.83$ kg on the same surface. **(a)** What is the acceleration of the blocks? **(b)** What is the tension in the string? **(c)** If the mass of block 1 is increased, does the tension in the string increase, decrease, or stay the same?

50. ••• **Buckets and a Pulley** Two buckets of sand hang from opposite ends of a rope that passes over an ideal pulley. One bucket is full and weighs 120 N; the other bucket is only partly filled and weighs 63 N. **(a)** Initially, you hold onto the lighter bucket to keep it from moving. What is the tension in the rope? **(b)** You release the lighter bucket and the heavier one descends. What is the tension in the rope now? **(c)** Eventually the heavier bucket lands and the two buckets come to rest. What is the tension in the rope now?

SECTION 6–5 CIRCULAR MOTION

51. • **CE** Suppose you stand on a bathroom scale and get a reading of 700 N. In principle, would the scale read more, less, or the same if the Earth did not rotate?

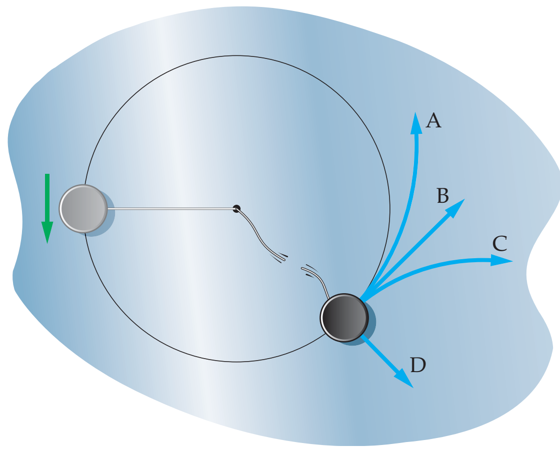
52. • **CE** A car drives with constant speed on an elliptical track, as shown in **Figure 6–33**. Rank the points A, B, and C in order of increasing likelihood that the car might skid. Indicate ties where appropriate.



▲ **FIGURE 6–33** Problem 52

53. • **CE** A car is driven with constant speed around a circular track. Answer each of the following questions with "Yes" or "No." **(a)** Is the car's velocity constant? **(b)** Is its speed constant? **(c)** Is the magnitude of its acceleration constant? **(d)** Is the direction of its acceleration constant?

54. • **CE** A puck attached to a string undergoes circular motion on an air table. If the string breaks at the point indicated in **Figure 6–34**, is the subsequent motion of the puck best described by path A, B, C, or D?



▲ **FIGURE 6–34** Problem 54

55. • When you take your 1300-kg car out for a spin, you go around a corner of radius 59 m with a speed of 16 m/s. The coefficient of static friction between the car and the road is 0.88. Assuming your car doesn't skid, what is the force exerted on it by static friction?

56. • Find the linear speed of the bottom of a test tube in a centrifuge if the centripetal acceleration there is 52,000 times the acceleration of gravity. The distance from the axis of rotation to the bottom of the test tube is 7.5 cm.

57. • **BIO A Human Centrifuge** To test the effects of high acceleration on the human body, the National Aeronautics and Space Administration (NASA) has constructed a large centrifuge at the Manned Spacecraft Center in Houston. In this device, astronauts are placed in a capsule that moves in a circular path with a radius of 15 m. If the astronauts in this centrifuge experience a centripetal acceleration 9.0 times that of gravity, what is the linear speed of the capsule?

58. • A car goes around a curve on a road that is banked at an angle of 33.5°. Even though the road is slick, the car will stay on the road without any friction between its tires and the road when its speed is 22.7 m/s. What is the radius of the curve?

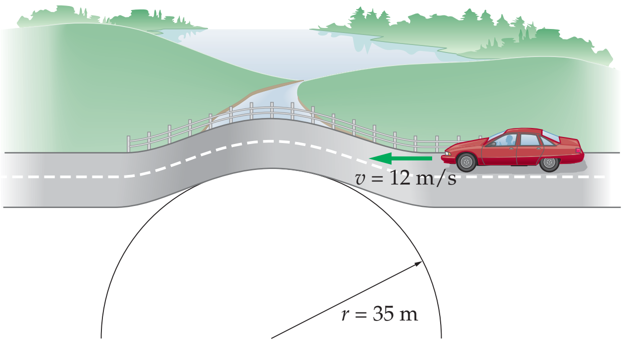
59. •• Jill of the Jungle swings on a vine 6.9 m long. What is the tension in the vine if Jill, whose mass is 63 kg, is moving at 2.4 m/s when the vine is vertical?

60. •• **IP** In Problem 59, **(a)** how does the tension in the vine change if Jill's speed is doubled? Explain. **(b)** How does the tension change if her mass is doubled instead? Explain.

61. •• **IP** **(a)** As you ride on a Ferris wheel, your apparent weight is different at the top than at the bottom. Explain. **(b)** Calculate your apparent weight at the top and bottom of a Ferris wheel, given that the radius of the wheel is 7.2 m, it completes one revolution every 28 s, and your mass is 55 kg.

A Ferris Wheel (Problems 61 and 84)

62. •• Driving in your car with a constant speed of 12 m/s, you encounter a bump in the road that has a circular cross section, as indicated in **Figure 6–35**. If the radius of curvature of the bump is 35 m, find the apparent weight of a 67-kg person in your car as you pass over the top of the bump.



▲ **FIGURE 6–35** Problems 62 and 63

63. •• Referring to Problem 62, at what speed must you go over the bump if people in your car are to feel "weightless"?

64. •• **IP** You swing a 4.6-kg bucket of water in a vertical circle of radius 1.3 m. **(a)** What speed must the bucket have if it is to complete the circle without spilling any water? **(b)** How does your answer depend on the mass of the bucket?

GENERAL PROBLEMS

65. • **CE** If you weigh yourself on a bathroom scale at the equator, is the reading you get greater than, less than, or equal to the reading you get if you weigh yourself at the North Pole?

66. • **CE** An object moves on a flat surface with an acceleration of constant magnitude. If the acceleration is always perpendicular to the object's direction of motion, **(a)** is the shape of the object's path circular, linear, or parabolic? **(b)** During its motion, does the object's velocity change in direction but not magnitude, change in magnitude but not direction, or change in both magnitude and direction? **(c)** Does its speed increase, decrease, or stay the same?

67. • **CE BIO** **Maneuvering a Jet** Humans lose consciousness if exposed to prolonged accelerations of more than about $7g$. This is of concern to jet fighter pilots, who may experience centripetal accelerations of this magnitude when making high-speed turns. Suppose we would like to decrease the centripetal acceleration of a jet. Rank the following changes in flight path in order of how effective they are in decreasing the centripetal acceleration, starting with the least effective: **A**, decrease the turning radius by a factor of two; **B**, decrease the speed by a factor of three; or **C**, increase the turning radius by a factor of four.

68. • **CE BIO** **Gravitropism** As plants grow, they tend to align their stems and roots along the direction of the gravitational field. This tendency, which is related to differential concentrations of plant hormones known as auxins, is referred to as *gravitropism*. As an illustration of gravitropism, experiments show that seedlings placed in pots on the rim of a rotating turntable do not grow in the vertical direction. Do you expect their stems to tilt inward—toward the axis of rotation—or outward—away from the axis of rotation?

69. • **BIO** A skateboard accident leaves your leg in a cast and supported by a traction device, as in Figure 6–24. Find the mass m that must be attached to the rope if the net force exerted by the small pulley on the foot is to have a magnitude of 37 N.

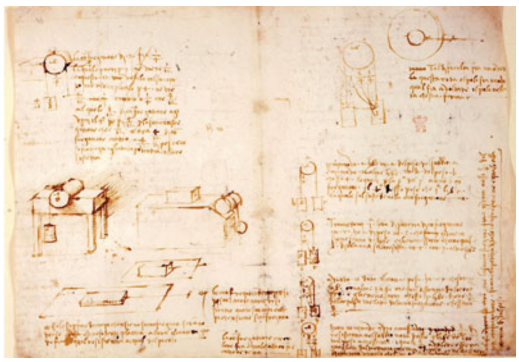
70. • Find the centripetal acceleration at the top of a test tube in a centrifuge, given that the top is 4.2 cm from the axis of rotation and that its linear speed is 77 m/s.

71. • Find the coefficient of kinetic friction between a 3.85-kg block and the horizontal surface on which it rests if an 850-N/m spring must be stretched by 6.20 cm to pull it with constant speed. Assume that the spring pulls in the horizontal direction.

72. • A child goes down a playground slide that is inclined at an angle of 26.5° below the horizontal. Find the acceleration of the child given that the coefficient of kinetic friction between the child and the slide is 0.315.

73. • When a block is placed on top of a vertical spring, the spring compresses 3.15 cm. Find the mass of the block, given that the force constant of the spring is 1750 N/m.

74. •• **The da Vinci Code** Leonardo da Vinci (1452–1519) is credited with being the first to perform quantitative experiments on friction, though his results weren't known until centuries later, due in part to the secret code (mirror writing) he used in his notebooks. Leonardo would place a block of wood on an inclined



Sketches from the notebooks of Leonardo da Vinci showing experiments he performed on friction (Problem 74) plane and measure the angle at which the block begins to slide. He reports that the coefficient of static friction was 0.25 in his experiments. At what angle did Leonardo's blocks begin to slide?

75. •• A force of 9.4 N pulls horizontally on a 1.1-kg block that slides on a rough, horizontal surface. This block is connected by a horizontal string to a second block of mass $m_2 = 1.92$ kg on the same surface. The coefficient of kinetic friction is $\mu_k = 0.24$ for both blocks. **(a)** What is the acceleration of the blocks? **(b)** What is the tension in the string?

76. •• You swing a 3.25-kg bucket of water in a vertical circle of radius 0.950 m. At the top of the circle the speed of the bucket is 3.23 m/s; at the bottom of the circle its speed is 6.91 m/s. Find the tension in the rope tied to the bucket at **(a)** the top and **(b)** the bottom of the circle.

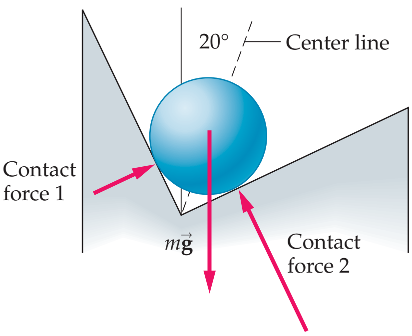
77. •• A 14-g coin slides upward on a surface that is inclined at an angle of 18° above the horizontal. The coefficient of kinetic friction between the coin and the surface is 0.23; the coefficient of static friction is 0.35. Find the magnitude and direction of the force of friction **(a)** when the coin is sliding and **(b)** after it comes to rest.

78. •• In Problem 77, the angle of the incline is increased to 25°. Find the magnitude and direction of the force of friction when the coin is **(a)** sliding upward initially and **(b)** sliding back downward later.

79. •• A physics textbook weighing 22 N rests on a table. The coefficient of static friction between the book and the table is $\mu_s = 0.60$; the coefficient of kinetic friction is $\mu_k = 0.40$. You push horizontally on the book with a force that gradually increases from 0 to 15 N, and then slowly decreases to 5.0 N, as indicated in the following table. For each value of the applied force given in the table, give the magnitude of the force of friction and state whether the book is accelerating, decelerating, at rest, or moving with constant speed.

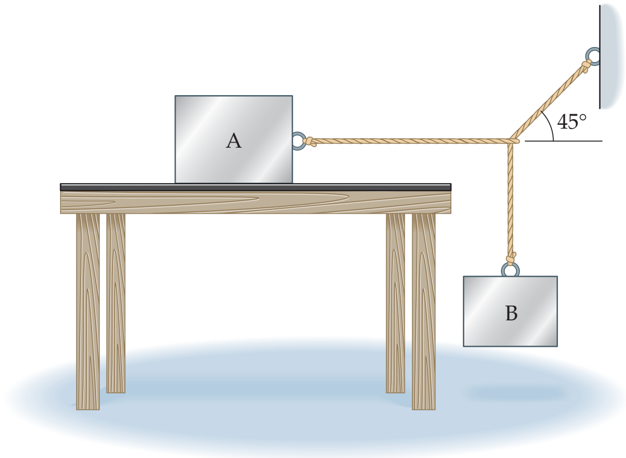
Applied force	Friction force	Motion
0		
5.0 N		
11 N		
15 N		
11 N		
8.0 N		
5.0 N		

80. •• A ball of mass m is placed in a wedge, as shown in **Figure 6–36**, in which the two walls meet at a right angle. Assuming the walls of the wedge are frictionless, determine the magnitude of **(a)** contact force 1 and **(b)** contact force 2.



▲ **FIGURE 6–36** Problem 80

81. •• **IP** The blocks shown in **Figure 6–37** are at rest. **(a)** Find the frictional force exerted on block A given that the mass of block A is 8.82 kg, the mass of block B is 2.33 kg, and the coefficient of static friction between block A and the surface on which it rests is 0.320. **(b)** If the mass of block A is doubled, does the frictional force exerted on it increase, decrease, or stay the same? Explain.



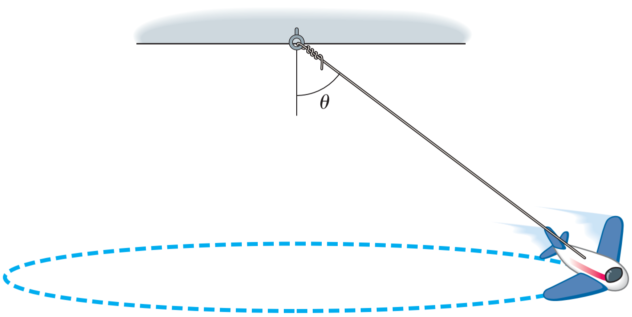
▲ **FIGURE 6–37** Problems 81 and 82

82. •• In part (a) of Problem 81, what is the maximum mass block B can have and the system still be in equilibrium?

83. •• **IP** A picture hangs on the wall suspended by two strings, as shown in Figure 6–21. The tension in string 2 is 1.7 N. **(a)** Is the tension in string 1 greater than, less than, or equal to 1.7 N? Explain. **(b)** Verify your answer to part (a) by calculating the tension in string 1. **(c)** What is the mass of the picture?

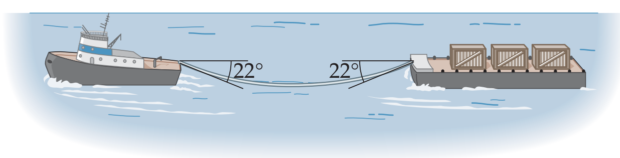
84. •• **IP** Referring to Problem 61, suppose the Ferris wheel rotates fast enough to make you feel "weightless" at the top. **(a)** How many seconds does it take to complete one revolution in this case? **(b)** How does your answer to part (a) depend on your mass? Explain. **(c)** What are the direction and magnitude of your acceleration when you are at the bottom of the wheel? Assume that its rotational speed has remained constant.

85. •• **A Conical Pendulum** A 0.075-kg toy airplane is tied to the ceiling with a string. When the airplane's motor is started, it moves with a constant speed of 1.21 m/s in a horizontal circle of radius 0.44 m, as illustrated in **Figure 6–38**. Find **(a)** the angle the string makes with the vertical and **(b)** the tension in the string.



▲ **FIGURE 6–38** Problem 85

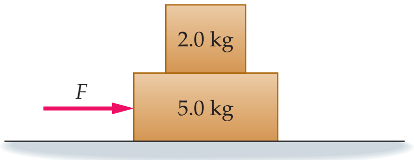
86. •• A tugboat tows a barge at constant speed with a 3500-kg cable, as shown in **Figure 6–39**. If the angle the cable makes with the hor-



▲ **FIGURE 6–39** Problem 86

izontal where it attaches to the barge and the tugboat is 22°, find the force the cable exerts on the barge in the forward direction.

87. •• **IP** Two blocks, stacked one on top of the other, can move without friction on the horizontal surface shown in **Figure 6–40**. The surface between the two blocks is rough, however, with a coefficient of static friction equal to 0.47. **(a)** If a horizontal force F is applied to the 5.0-kg bottom block, what is the maximum value F can have before the 2.0-kg top block begins to slip? **(b)** If the mass of the top block is increased, does the maximum value of F increase, decrease, or stay the same? Explain.



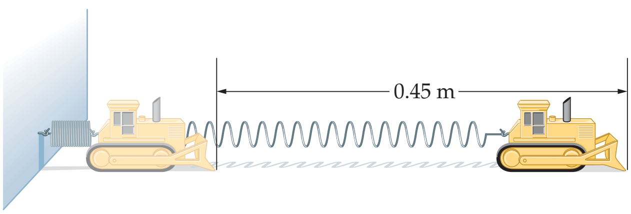
▲ **FIGURE 6–40** Problem 87

88. •• Find the coefficient of kinetic friction between a 4.7-kg block and the horizontal surface on which it rests if an 89-N/m spring must be stretched by 2.2 cm to pull the block with constant speed. Assume the spring pulls in a direction 13° above the horizontal.

89. •• **IP** In a daring rescue by helicopter, two men with a combined mass of 172 kg are lifted to safety. **(a)** If the helicopter lifts the men straight up with constant acceleration, is the tension in the rescue cable greater than, less than, or equal to the combined weight of the men? Explain. **(b)** Determine the tension in the cable if the men are lifted with a constant acceleration of 1.10 m/s^2.

90. •• At the airport, you pull a 18-kg suitcase across the floor with a strap that is at an angle of 45° above the horizontal. Find **(a)** the normal force and **(b)** the tension in the strap, given that the suitcase moves with constant speed and that the coefficient of kinetic friction between the suitcase and the floor is 0.38.

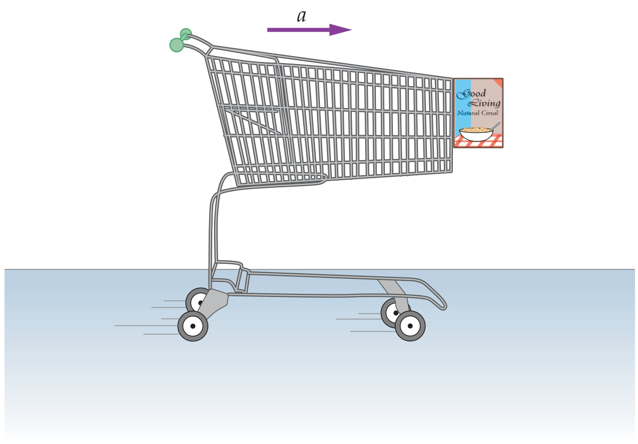
91. •• **IP** A light spring with a force constant of 13 N/m is connected to a wall and to a 1.2-kg toy bulldozer, as shown in **Figure 6–41**. When the electric motor in the bulldozer is turned on, it stretches the spring for a distance of 0.45 m before its tread begins to slip on the floor. **(a)** Which coefficient of friction (static or kinetic) can be determined from this information? Explain. **(b)** What is the numerical value of this coefficient of friction?



▲ **FIGURE 6–41** Problem 91

92. •• **IP** A 0.16-g spider hangs from the middle of the first thread of its future web. The thread makes an angle of 7.2° with the horizontal on both sides of the spider. **(a)** What is the tension in the thread? **(b)** If the angle made by the thread had been less than 7.2°, would its tension have been greater than, less than, or the same as in part (a)? Explain.

93. •• Find the acceleration the cart in **Figure 6–42** must have in order for the cereal box at the front of the cart not to fall. Assume that the coefficient of static friction between the cart and the box is 0.38.



▲ **FIGURE 6–42** Problem 93

94. •• **IP** **Playing a Violin** The tension in a violin string is 2.7 N. When pushed down against the neck of the violin, the string makes an angle of 4.1° with the horizontal. **(a)** With what force must you push down on the string to bring it into contact with the neck? **(b)** If the angle were less than 4.1°, would the required force be greater than, less than, or the same as in part (a)? Explain.

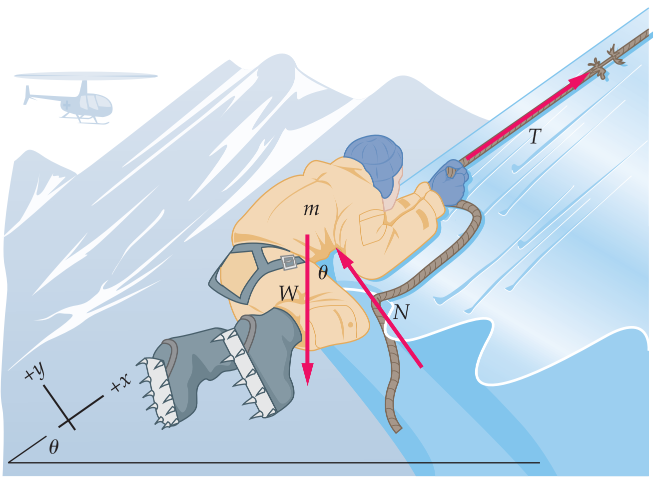
95. •• **IP** A pair of fuzzy dice hangs from a string attached to your rearview mirror. As you turn a corner with a radius of 98 m and a constant speed of 27 mi/h, what angle will the dice make with the vertical? Why is it unnecessary to give the mass of the dice?

96. •• Find the tension in each of the two ropes supporting a hammock if one is at an angle of 18° above the horizontal and the other is at an angle of 35° above the horizontal. The person sleeping in the hammock (unconcerned about tensions and ropes) has a mass of 68 kg.

97. •• As your plane circles an airport, it moves in a horizontal circle of radius 2300 m with a speed of 390 km/h. If the lift of the airplane's wings is perpendicular to the wings, at what angle should the plane be banked so that it doesn't tend to slip sideways?

98. •• **IP** A block with a mass of 3.1 kg is placed at rest on a surface inclined at an angle of 45° above the horizontal. The coefficient of static friction between the block and the surface is 0.50, and a force of magnitude F pushes upward on the block, parallel to the inclined surface. **(a)** The block will remain at rest only if F is greater than a minimum value, F_{min}, and less than a maximum value, F_{max}. Explain the reasons for this behavior. **(b)** Calculate F_{min}. **(c)** Calculate F_{max}.

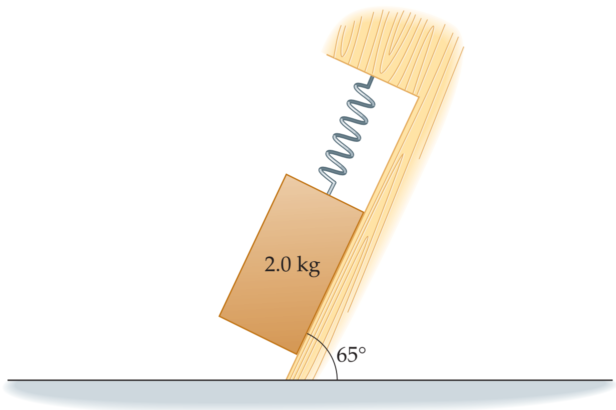
99. •• A mountain climber of mass m hangs onto a rope to keep from sliding down a smooth, ice-covered slope (**Figure 6–43**). Find a formula for the tension in the rope when the slope is inclined at an angle θ above the horizontal. Check your results in the limits $\theta = 0$ and $\theta = 90°$.



▲ **FIGURE 6–43** Problem 99

100. •• A child sits on a rotating merry-go-round, 2.3 m from its center. If the speed of the child is 2.2 m/s, what is the minimum coefficient of static friction between the child and the merry-go-round that will prevent the child from slipping?

101. ••• A 2.0-kg box rests on a plank that is inclined at an angle of 65° above the horizontal. The upper end of the box is attached to a spring with a force constant of 360 N/m, as shown in **Figure 6–44**. If the coefficient of static friction between the box and the plank is 0.22, what is the maximum amount the spring can be stretched and the box remain at rest?



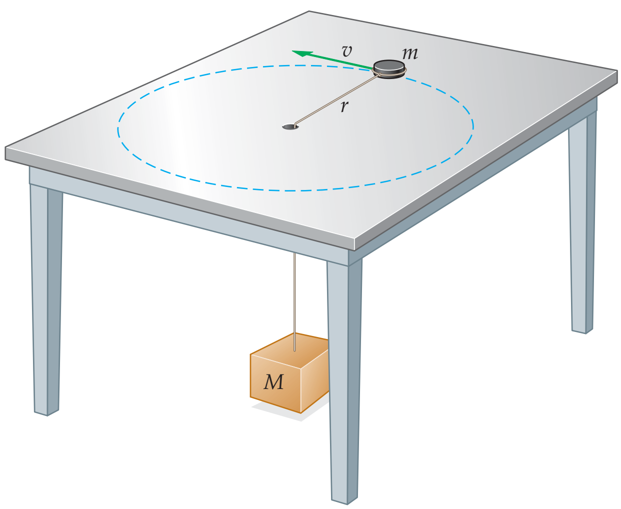
▲ **FIGURE 6–44** Problem 101

102. ••• A wood block of mass m rests on a larger wood block of mass M that rests on a wooden table. The coefficients of static and kinetic friction between all surfaces are μ_s and μ_k, respectively. What is the minimum horizontal force, F, applied to the lower block that will cause it to slide out from under the upper block?

103. ••• Find the tension in each of the two strings shown in Figure 6–30 for general values of the masses. Your answer should be in terms of m_1, m_2, m_3, and g.

104. ••• The coefficient of static friction between a rope and the table on which it rests is μ_s. Find the fraction of the rope that can hang over the edge of the table before it begins to slip.

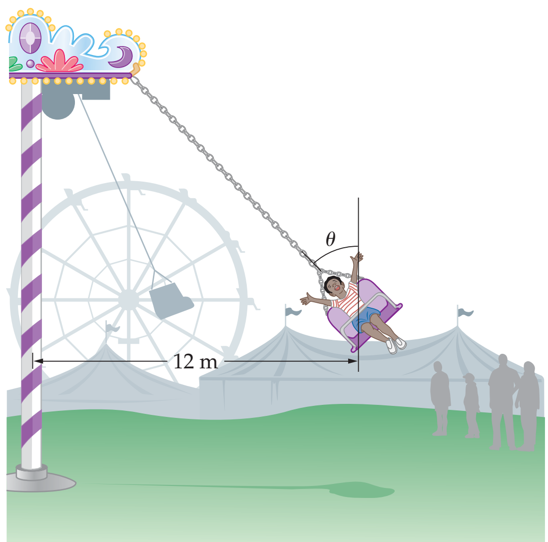
105. ••• A hockey puck of mass m is attached to a string that passes through a hole in the center of a table, as shown in **Figure 6–45**. The hockey puck moves in a circle of radius r. Tied to the other end of the string, and hanging vertically beneath the table, is a mass M. Assuming the tabletop is perfectly smooth, what speed must the hockey puck have if the mass M is to remain at rest?



▲ **FIGURE 6–45** Problem 105

106. ••• **The Force Needed to Move a Crate** To move a crate of mass m across a rough floor, you push down on it at an angle θ, as shown in Figure 6–18 for the special case of $\theta = 21°$. **(a)** Find the force necessary to start the crate moving as a function of θ, given that the coefficient of static friction between the crate and the floor is μ_s. **(b)** Show that it is impossible to move the crate, no matter how great the force, if the coefficient of static friction is greater than or equal to $1/\tan\theta$.

107. ••• **IP** A popular ride at amusement parks is illustrated in **Figure 6–46**. In this ride, people sit in a swing that is suspended from a rotating arm. Riders are at a distance of 12 m from the axis of rotation and move with a speed of 25 mi/h. **(a)** Find the centripetal acceleration of the riders. **(b)** Find the angle θ the supporting wires make with the vertical. **(c)** If you observe a ride like that in Figure 6–46, or as shown in the photo on page 170, you will notice that all the swings are at the same angle θ to the vertical, regardless of the weight of the rider. Explain.

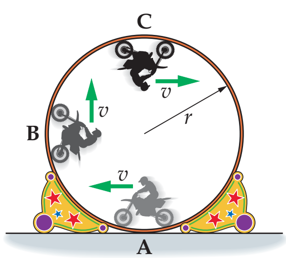


▲ **FIGURE 6–46** Problem 107

108. ••• **A Conveyor Belt** A box is placed on a conveyor belt that moves with a constant speed of 1.25 m/s. The coefficient of kinetic friction between the box and the belt is 0.780. **(a)** How much time does it take for the box to stop sliding relative to the belt? **(b)** How far does the box move in this time?

109. ••• You push a box along the floor against a constant force of friction. When you push with a horizontal force of 75 N, the acceleration of the box is 0.50 m/s^2; when you increase the force to 81 N, the acceleration is 0.75 m/s^2. Find **(a)** the mass of the box and **(b)** the coefficient of kinetic friction between the box and the floor.

110. ••• As part of a circus act, a person drives a motorcycle with constant speed v around the inside of a vertical track of radius r, as indicated in **Figure 6–47**. If the combined mass of the motorcycle and rider is m, find the normal force exerted on the motorcycle by the track at the points **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 6–47** Problem 110

PASSAGE PROBLEMS

BIO Nasal Strips

People in all walks of life use nasal strips, or external nasal dilator strips (ENDS), to alleviate a number of respiratory problems. First introduced to eliminate snoring, they are now finding use in a number of other areas. For example, dentists have found that nasal strips help patients breathe better during dental procedures, making the experience considerably more pleasant for both doctor and patient. Surprisingly, horse owners have also discovered the advantage of nasal strips, and have begun to apply large "horse-sized" strips to saddle horses—as well as racing thoroughbreds—to reduce fatigue and lung stress.

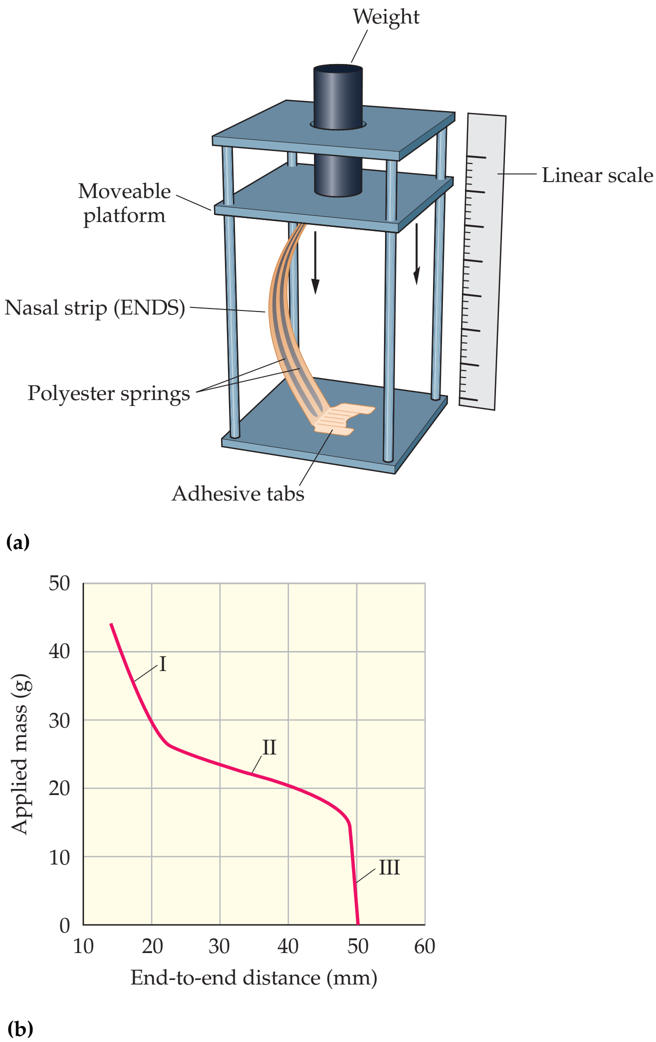
One of the great advantages of ENDS is that no drugs are involved; the strips are a purely mechanical device, consisting of two flat, polyester springs enclosed by an adhesive tape covering. When applied to the nose, they exert an outward force that enlarges the nasal passages and reduces the resistance to air flow (see the illustration in Active Example 6–2). The mechanism shown in **Figure 6–48 (a)** is used to measure the behavior of these strips. For example, if a 30-g weight is placed on the moveable platform (of negligible mass), the strip is found to compress from an initial length of 50 mm to a reduced length of 19 mm, as can be seen in **Figure 6–48 (b)**.

A thoroughbred racehorse with a nasal strip.
Did it win by a nose?

111. • On the straight-line segment I in Figure 6–48 (b) we see that increasing the applied mass from 26 g to 44 g results in a reduction of the end-to-end distance from 21 mm to 14 mm. What is the force constant in N/m on segment I?

A. 2.6 N/m **B.** 3.8 N/m

C. 9.8 N/m **D.** 25 N/m



▲ **FIGURE 6–48** Problems 111, 112, 113, and 114

112. • Is the force constant on segment II greater than, less than, or equal to the force constant on segment I?

113. • Which of the following is the best estimate for the force constant on segment II?

A. 0.83 N/m **B.** 1.3 N/m

C. 2.5 N/m **D.** 25 N/m

114. • Rank the straight segments I, II, and III in order of increasing "stiffness" of the nasal strip.

INTERACTIVE PROBLEMS

115. •• **IP Referring to Example 6–3** Suppose the coefficients of static and kinetic friction between the crate and the truck bed are 0.415 and 0.382, respectively. **(a)** Does the crate begin to slide at a tilt angle that is greater than, less than, or equal to 23.2°? **(b)** Verify your answer to part (a) by determining the angle at which the crate begins to slide. **(c)** Find the length of time it takes for the crate to slide a distance of 2.75 m when the tilt angle has the value found in part (b).

116. •• **IP Referring to Example 6–3** The crate begins to slide when the tilt angle is 17.5°. When the crate reaches the bottom of the flatbed, after sliding a distance of 2.75 m, its speed is 3.11 m/s. Find **(a)** the coefficient of static friction and **(b)** the coefficient of kinetic friction between the crate and the flatbed.

117. •• **Referring to Example 6–6** Suppose that the mass on the frictionless tabletop has the value $m_1 = 2.45$ kg. **(a)** Find the value of m_2 that gives an acceleration of 2.85 m/s^2. **(b)** What is the corresponding tension, T, in the string? **(c)** Calculate the ratio $T/m_2 g$ and show that it is less than 1, as expected.

118. •• **Referring to Example 6–8** **(a)** At what speed will the force of static friction exerted on the car by the road be equal to half the weight of the car? The mass of the car is $m = 1200$ kg, the radius of the corner is $r = 45$ m, and the coefficient of static friction between the tires and the road is $\mu_s = 0.82$. **(b)** Suppose that the mass of the car is now doubled, and that it moves with a speed that again makes the force of static friction equal to half the car's weight. Is this new speed greater than, less than, or equal to the speed in part (a)?

PHYSICS IN

PERSPECTIVE

Force, Acceleration, and Motion

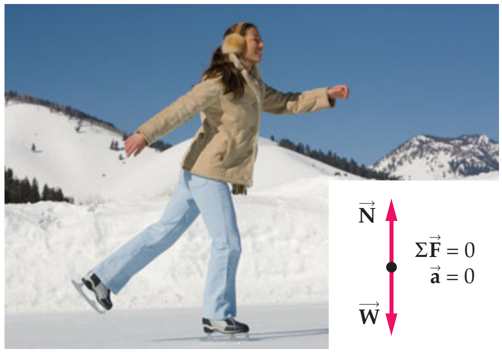
Motion does not require a force—but a *change* in motion does. On these pages we explore the connections between forces, as described in Newton's laws, and the types of motion we've studied in the first six chapters.

1 Objects that experience zero net force obey Newton's first law

If the net force $\vec{\mathbf{F}}_{\text{net}} = \Sigma\vec{\mathbf{F}}$ acting on an object is zero, the object's motion doesn't change—the object either remains at rest or continues to move with constant velocity, as Newton's first law states.

At rest

Motion at constant velocity



This behavior is consistent with Newton's second law for a net force of zero:

If the net force acting on an object is zero … $\Sigma\vec{\mathbf{F}} = 0$, then $\vec{\mathbf{a}} = \dfrac{\Sigma\vec{\mathbf{F}}}{m} = 0$ … the object has zero acceleration.

2 All objects experience forces—the question is whether the object experiences a *net* force

All objects—moving or at rest—are acted on by forces. Even in outer space, objects experience gravitational and other forces. Therefore, the *net* force on the object is the quantity that matters.

Newton's first law seems at odds with our experience: If we stop exerting a force on a moving object, the object usually stops. But that is because we must counter friction and drag forces. In the photo, the net force *on the couch* is zero even though the person exerts a steady push.

3 Moving at constant velocity is equivalent to being at rest

When you sit in a jet flying in a straight line, you feel the same as when you are sitting at home, and objects around you behave the same.

From the point of view of physics, *there is no difference* between these situations; Newton's laws hold in both. We say that both represent *inertial frames of reference*.

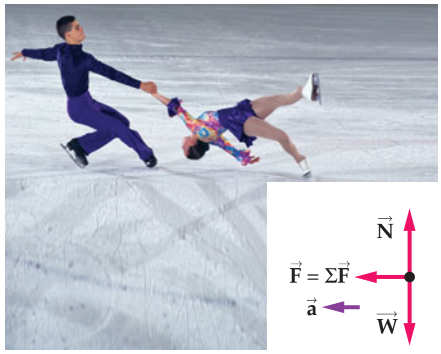
4 Objects that experience a nonzero net force obey Newton's second law

A nonzero net force accelerates an object—that is, causes its velocity to change in magnitude, direction, or both. We have studied the following three special types of accelerated motion:

Linear accelerated motion

Projectile motion

Circular motion



Accelerated motion obeys Newton's second law:

If a nonzero net force acts on an object … $\Sigma\vec{F} \neq 0$, then $\vec{a} = \dfrac{\Sigma\vec{F}}{m} \neq 0$ … the object has an acceleration in the direction of the net force that is proportional to $\Sigma\vec{F}$ and inversely proportional to m.

5 The acceleration points in the direction of the net force

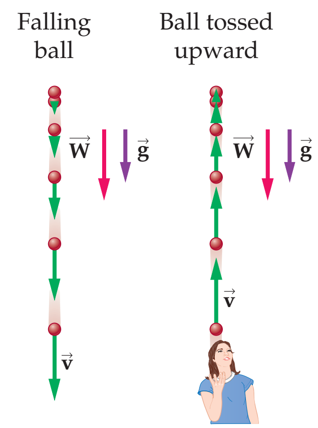
Linear accelerated motion
- Net force is parallel to motion.
- Velocity changes in magnitude but not in direction.

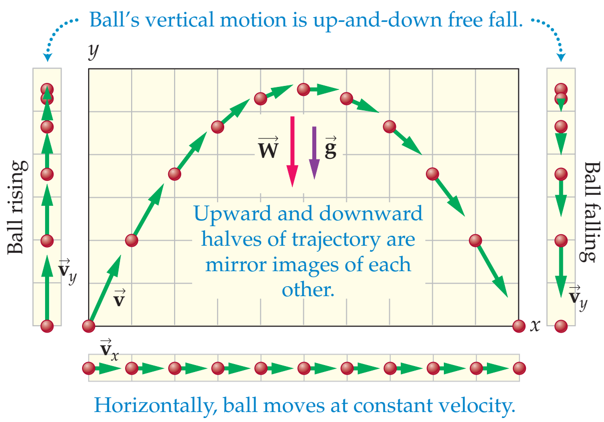
Special case: free fall
Constant downward acceleration $\vec{g}$

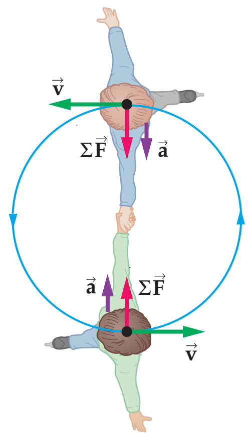
Parabolic motion
- Constant net force acts at angle to motion.
- Velocity changes in both magnitude and direction.

Special case: projectile motion
Constant downward acceleration $\vec{g}$

Circular motion (constant speed)
- Net force is constant in magnitude but always points toward the center of the circle. Thus, the net force is always at a right angle to the object's velocity.
- Velocity changes in direction but not in magnitude.

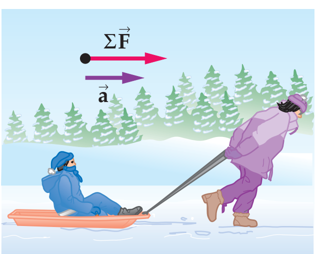


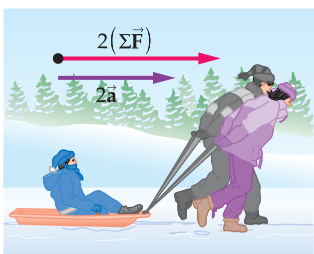




6 The acceleration magnitude is proportional to *F* and inversely proportional to *m*

Doubling the net force acting on an object doubles the object's acceleration ($\vec{a} \propto \vec{F}$).





Doubling the object's mass m *halves* its acceleration ($\vec{a} \propto 1/m$).