CHAPTER 6: Work and Energy

Answers to Questions

1. Some types of physical labor, particularly if it involves lifting objects, such as shoveling dirt or carrying shingles up to a roof, are "work" in the physics sense of the word. Or, pushing a lawn mower would be work corresponding to the physics definition. When we use the word "work" for employment, such as "go to work" or "school work", there is often no sense of physical labor or of moving something through a distance by a force.

2. The normal force can do work on an object if the normal force has a component in the direction of displacement of an object. If someone were to jump up in the air, then the floor pushing upward on the person (the normal force) would do positive work and increase the person's kinetic energy. Likewise when they hit the floor coming back down, the force of the floor pushing upwards (the normal force) would do negative work and decrease the person's kinetic energy.

3. The kinetic force of friction opposes the relative motion between two objects. As in the example suggested, as the tablecloth is pulled from under the dishes, the relative motion is for the dishes to be left behind as the tablecloth is pulled, and so the kinetic friction opposes that and moves the dishes in the same direction as the tablecloth. This is a force that is in the direction of displacement, and so positive work is done. Also note that the cloth is moving faster than the dishes in this case, so that the friction is kinetic, not static.

4. (*a*) In this case, the same force is applied to both springs. Spring 1 will stretch less, and so more work is done on spring 2.
   (*b*) In this case, both springs are stretched the same distance. It takes more force to stretch spring 1, and so more work is done on spring 1.

5. Your gravitational PE will change according to $\Delta PE = mg\Delta y$. If we choose some typical values of $m = 80\text{ kg}$ and $\Delta y = 0.75\text{ m}$, then $\Delta PE = (80\text{ kg})(9.8\,\text{m/s}^2)(0.75\text{ m}) = 590\text{ J}$

6. The two launches will result in the same largest angle. Applying conservation of energy between the launching point and the highest point, we have $E_1 = E_2 \quad \rightarrow \quad \frac{1}{2}mv^2 + mgh = mgh_{\text{max}}$. The direction of the launching velocity does not matter, and so the same maximum height (and hence maximum angle) will results from both launches. Also, for the first launch, the ball will rise to some maximum height and then come back to the launch point with the same speed as when launched. That then exactly duplicates the second launch.

7. If the instructor releases the ball without pushing it, the ball should return to exactly the same height (barring any dissipative forces) and just touch the instructor's nose as it stops. But if the instructor pushes the ball, giving it extra kinetic energy and hence a larger total energy, the ball will then swing to a higher point before stopping, and hit the instructor in the face when it returns.

8. Start the description with the child suspended in mid-air, at the top of a hop. All of the energy is gravitational PE at that point. Then, the child falls, and gains kinetic energy. When the child reaches the ground, most of the energy is kinetic. As the spring begins to compress, the kinetic energy is changed into elastic PE. The child also goes down a little bit further as the spring compresses, and so more gravitational PE is also changed into elastic PE. At the very bottom of a hop, the energy is all elastic PE. Then as the child rebounds, the elastic PE is turned into kinetic energy and gravitational PE. When the child reaches the top of the bounce, all of the elastic PE has been changed into gravitational PE, because the child has a speed of 0 at the top. Then the cycle starts over again. Due to friction, the child must also add energy to the system by pushing down on the pogo stick while it is on the ground, getting a more forceful reaction from the ground.

9. (*a*) If there is no friction to dissipate any of the energy, then the gravitational PE that the child has at the top of the hill all turns into kinetic energy at the bottom of the hill. The same kinetic energy will be present regardless of the slope – the final speed is completely determined by the height. The time it takes to reach the bottom of the hill will be longer for a smaller slope.
(*b*) If there is friction, then the longer the path is, the more work that friction will do, and so the slower the speed will be at the bottom. So for a steep hill, the sled will have a greater speed at the bottom than for a shallow hill.

10. If we assume that all of the arrow's kinetic energy is converted into work done against friction, then the following relationship exists:

$$W = \Delta KE = KE_f - KE_i \rightarrow F_{\mathrm{fr}} d \cos 180^\circ = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2 \rightarrow -F_{\mathrm{fr}} d = -\tfrac{1}{2} m v_0^2 \rightarrow$$

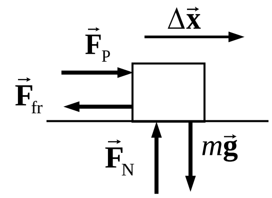
$$d = \frac{m v_0^2}{2F_{\mathrm{fr}}}$$

Thus the distance is proportional to the square of the initial velocity. So if the initial velocity is doubled, the distance will be multiplied by a factor of 4. Thus the faster arrow penetrates 4 times further than the slower arrow.

11. The superball cannot rebound to a height greater than its original height when dropped. If it did, it would violate conservation of energy. When a ball collides with the floor, the KE of the ball is converted into elastic PE by deforming the ball, much like compressing a spring. Then as the ball springs back to its original shape, that elastic PE is converted to back to KE. But that process is "lossy" – not all of the elastic PE gets converted back to KE. Some of the PE is lost, primarily to friction. The superball rebounds higher than many other balls because it is less "lossy" in its rebound than many other materials.

Solutions to Problems

1. Draw a free-body diagram for the crate as it is being pushed across the floor. Since it is not accelerating vertically, $F_N = mg$. Since it is not accelerating horizontally, $F_P = F_{fr} = \mu_k F_N = \mu_k mg$. The work done to move it across the floor is the work done by the pushing force. The angle between the pushing force and the direction of motion is 0°.



$$W_{push} = F_{push} d \cos 0^\circ = \mu_k mgd\,(1) = (0.50)(160\text{ kg})\left(9.80\,\text{m/s}^2\right)(10.3\text{ m})$$
$$= \boxed{8.1\times10^3\,\text{J}}$$

2. Since the acceleration of the box is constant, use Eq. 2-11b to find the distance moved. Assume that the box starts from rest.

$$\Delta x = x - x_0 = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}\left(2.0\,\text{m/s}^2\right)(7\text{ s})^2 = 49\text{ m}$$

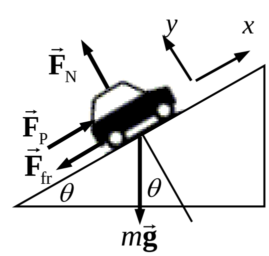
Then the work done in moving the crate is

$$W = F\Delta x \cos 0^\circ = ma\Delta x = (5\text{ kg})\left(2.0\,\text{m/s}^2\right)(49\text{ m}) = \boxed{4.9\times10^2\,\text{J}}$$

3. The first book is already in position, so no work is required to position it. The second book must be moved upwards by a distance *d*, by a force equal to its weight, *mg*. The force and the displacement are in the same direction, so the work is *mgd*. The third book will need to be moved a distance of 2*d* by the same size force, so the work is 2*mgd*, This continues through all seven books, with each needing to be raised by an additional amount of *d* by a force of *mg*. The total work done is

$$W = mgd + 2mgd + 3mgd + 4mgd + 5mgd + 6mgd + 7mgd$$
$$= 28mgd = 28(1.7\text{ kg})\left(9.8\,\text{m/s}^2\right)(0.043\text{ m}) = \boxed{2.0\times10^1\,\text{J}}.$$

4. Draw a free-body diagram of the car on the incline. Include a frictional force, but ignore it in part (*a*) of the problem. The minimum work will occur when the car is moved at a constant velocity.



(*a*) Write Newton's 2nd law in both the *x* and *y* directions, noting that the car is unaccelerated.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad\rightarrow\quad F_N = mg\cos\theta$$
$$\sum F_x = F_P - mg\sin\theta = 0 \quad\rightarrow\quad F_P = mg\sin\theta$$

The work done by $\vec{\mathbf{F}}_P$ in moving the car a distance *d* along the plane (parallel to $\vec{\mathbf{F}}_P$) is given by

$$W_P = F_P d\cos 0^\circ = mgd\sin\theta = (950\text{ kg})\left(9.80\,\text{m/s}^2\right)(810\text{ m})\sin 9.0^\circ = \boxed{1.2\times10^6\,\text{J}}$$

(*b*) Now include the frictional force, given by $F_{fr} = \mu_k F_N$. We still assume that the car is not accelerated. We again write Newton's 2nd law for each direction. The *y*-forces are unchanged by the addition of friction, and so we still have $F_N = mg\cos\theta$.

$$\sum F_x = F_P - F_{fr} - mg\sin\theta = 0 \quad\rightarrow\quad F_P = F_{fr} + mg\sin\theta = \mu_k mg\cos\theta + mg\sin\theta.$$

The work done by $\vec{\mathbf{F}}_P$ in moving the car a distance *d* along the plane (parallel to $\vec{\mathbf{F}}_P$) is given by

$$W_P = F_P d\cos 0^\circ = mgd\left(\sin\theta + \mu_k\cos\theta\right)$$
$$= (950\text{ kg})\left(9.80\,\text{m/s}^2\right)(810\text{ m})\left(\sin 9.0^\circ + 0.25\cos 9.0^\circ\right) = \boxed{3.0\times10^6\,\text{J}}$$

5. The work done will be the area under the F_x vs. x graph.

(*a*) From $x = 0.0$ to $x = 10.0$ m, the shape under the graph is trapezoidal. The area is

$$W_a = (400 \text{ N})\frac{10 \text{ m} + 4 \text{ m}}{2} = \boxed{2.8\times10^3 \text{ J}}$$

(*b*) From $x = 10.0$ m to $x = 15.0$ m, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

$$W_a = (-200 \text{ N})\frac{5 \text{ m} + 2 \text{ m}}{2} = -700 \text{ J}.$$

Thus the total work from $x = 0.0$ to $x = 15.0$ m is $2800 \text{ J} - 700 \text{ J} = \boxed{2.1\times10^3 \text{ J}}$

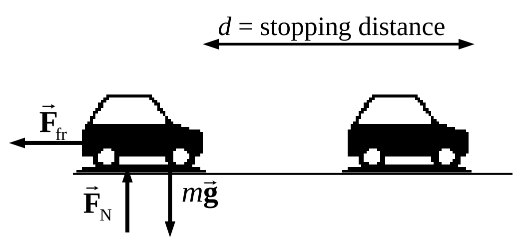
6. Find the velocity from the kinetic energy, using Eq. 6-3.

$$KE = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2(6.21\times10^{-21}\text{ J})}{5.31\times10^{-26}}} = \boxed{484\text{ m/s}}$$

7. The force exerted by the bow on the arrow is in the same direction as the displacement of the arrow. Thus $W = Fd\cos0^\circ = Fd = (110 \text{ N})(0.78 \text{ m}) = 85.8 \text{ J}$. But that work changes the KE of the arrow, by the work-energy theorem. Thus

$$Fd = W = KE_2 - KE_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad\rightarrow\quad v_2 = \sqrt{\frac{2Fd}{m} + v_1^2} = \sqrt{\frac{2(85.8 \text{ J})}{0.088 \text{ kg}} + 0} = \boxed{44\text{ m/s}}$$

8. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be static friction since the driver locked the brakes. Thus $F_{\text{fr}} = \mu_k F_{\text{N}}$. Since the car is on a level surface, the normal force is equal to the car's weight, and so $F_{\text{fr}} = \mu_k mg$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.



$$W = \Delta KE \quad\rightarrow\quad F_{\text{fr}}d\cos180^\circ = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad\rightarrow\quad -\mu_k mgd = 0 - \tfrac{1}{2}mv_1^2 \quad\rightarrow$$

$$v_1 = \sqrt{2\mu_k gd} = \sqrt{2(0.42)(9.8\text{ m/s}^2)(88 \text{ m})} = \boxed{27\text{ m/s}}$$

The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.

9. If the rock has 80.0 J of work done to it, and it loses all 80.0 J by stopping, then the force of gravity must have done –80.0 J of work on the rock. The force is straight down, and the displacement is straight up, so the angle between the force and the displacement is 180°. The work done by the gravity force can be used to find the distance the rock rises.

$$W_{\text{G}} = F_{\text{G}}d\cos\theta = mgd\cos180^\circ = -80.0 \text{ J}$$

$$d = \frac{W_{\text{G}}}{-mg} = \frac{-80.0 \text{ J}}{-(1.85 \text{ kg})(9.80\text{ m/s}^2)} = \boxed{4.41 \text{ m}}$$

10. Assume that all of the kinetic energy of the car becomes PE of the compressed spring.

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2 \quad \rightarrow \quad k = \frac{mv^2}{x^2} = \frac{(1200\text{ kg})\left[(65\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(2.2\text{ m})^2} = \boxed{8.1\times10^4\text{ N/m}}$$

11. (*a*) The change in PE is given by

$$\Delta PE_G = mg(y_2 - y_1) = (55\text{ kg})(9.80\,\text{m/s}^2)(3300\text{ m} - 1600\text{ m}) = \boxed{9.2\times10^5\,\text{J}}$$

(*b*) The minimum work required by the hiker would equal the change in PE, which is $\boxed{9.2\times10^5\,\text{J}}$.

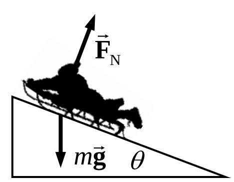
(*c*) $\boxed{\text{Yes}}$. The actual work may be more than this, because the climber almost certainly had to overcome some dissipative forces such as air friction. Also, as the person steps up and down, they do not get the full amount of work back from each up-down event. For example, there will be friction in their joints and muscles.

12. The spring will stretch enough to hold up the mass. The force exerted by the spring will be equal to the weight of the mass.

$$mg = k(\Delta x) \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(2.5\text{ kg})(9.80\,\text{m/s}^2)}{53\,\text{N/m}} = 0.46\text{ m}$$

Thus the ruler reading will be $\boxed{46\,\text{cm} + 15\,\text{cm} = 61\,\text{cm}}$.

13. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for PE $(y = 0)$. We have $y_1 = 0$, $v_2 = 0$, and $y_2 = 1.35$ m. Solve for v_1, the speed at the bottom.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \quad \rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80\,\text{m/s}^2)(1.35\text{ m})} = \boxed{5.14\,\text{m/s}}$$

Notice that the angle is not used in the calculation.

14. We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for PE $(y = 0)$. We have $y_1 = 0$, $v_2 = 0.70\,\text{m/s}$, and $y_2 = 2.10$ m. Solve for v_1, the speed at the bottom.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + 0 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

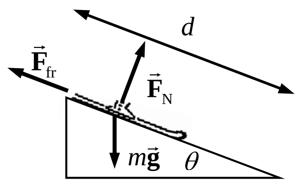
$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(0.70\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(2.10\text{ m})} = \boxed{6.45\,\text{m/s}}$$

15. Apply the conservation of energy to the child, considering work done by gravity and work changed into thermal energy. Subscript 1 represents the child at the top of the slide, and subscript 2 represents the child at the bottom of the slide. The ground is the zero location for PE $(y = 0)$. We have $v_1 = 0$, $y_1 = 3.5 \text{ m}$, $v_2 = 2.2 \text{ m/s}$, and $y_2 = 0$. Solve for the work changed into thermal energy.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + W_{\text{thermal}} \rightarrow$$

$$W_{\text{thermal}} = mgy_1 - \tfrac{1}{2}mv_2^2 = (21.7 \text{ kg})(9.8 \text{ m/s}^2)(3.5 \text{ m}) - \tfrac{1}{2}(21.7 \text{ kg})(2.2 \text{ m/s})^2 = \boxed{6.9\times10^2 \text{ J}}$$

16. (*a*) See the free-body diagram for the ski. Write Newton's 2nd law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.



$$\sum F_\perp = F_N - mg\cos\theta \rightarrow F_N = mg\cos\theta \rightarrow$$

$$F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$$

Now use conservation of energy, including the non-conservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational PE $(y = 0)$. We have $v_1 = 0$, $y_1 = d\sin\theta$, and $y_2 = 0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$

$$W_{NC} = \Delta KE + \Delta PE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgy_2 - mgy_1 \rightarrow W_{NC} + E_1 = E_2$$

$$F_{fr}d\cos180^\circ + \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow -\mu_k mgd\cos\theta + mgd\sin\theta = \tfrac{1}{2}mv_2^2 \rightarrow$$

$$v_2 = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80 \text{ m/s}^2)(75 \text{ m})(\sin22^\circ - 0.090\cos22^\circ)}$$

$$= 20.69 \text{ m/s} \approx \boxed{21 \text{ m/s}}$$

(*b*) Now, on the level ground, $F_f = \mu_k mg$, and there is no change in PE. Let us again use conservation of energy, including the non-conservative friction force, to relate position 2 with position 3. Subscript 3 represents the ski at the end of the travel on the level, having traveled a distance d_3 on the level. We have $v_2 = 20.69 \text{ m/s}$, $y_2 = 0$, $v_3 = 0$, and $y_3 = 0$.

$$W_{NC} + E_2 = E_3 \rightarrow F_f d_3\cos180^\circ + \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \rightarrow$$

$$-\mu_k mgd_3 + \tfrac{1}{2}mv_2^2 = 0 \rightarrow d_3 = \frac{v_2^2}{2g\mu_k} = \frac{(20.69 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(0.090)} = 242.7 \text{ m} \approx \boxed{2.4\times10^2 \text{ m}}$$

17. (*a*) $1\ \text{kW}\cdot\text{h} = 1\ \text{kW}\cdot\text{h}\left(\dfrac{1000\ \text{W}}{1\ \text{kW}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right)\left(\dfrac{1\ \text{J/s}}{1\ \text{W}}\right) = \boxed{3.6\times10^{6}\ \text{J}}$

(*b*) $(520\ \text{W})(1\ \text{month}) = (520\ \text{W})(1\ \text{month})\left(\dfrac{1\ \text{kW}}{1000\ \text{W}}\right)\left(\dfrac{30\ \text{d}}{1\ \text{month}}\right)\left(\dfrac{24\ \text{h}}{1\ \text{d}}\right) = 374\ \text{kW}\cdot\text{h}$

$\approx \boxed{370\ \text{kW}\cdot\text{h}}$

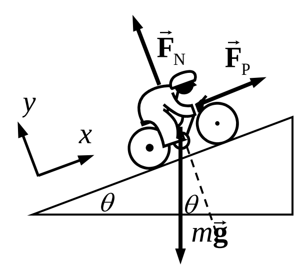
(*c*) $374\ \text{kW}\cdot\text{h} = 374\ \text{kW}\cdot\text{h}\left(\dfrac{3.6\times10^{6}\ \text{J}}{1\ \text{kW}\cdot\text{h}}\right) = \boxed{1.3\times10^{9}\ \text{J}}$

(*d*) $(374\ \text{kW}\cdot\text{h})\left(\dfrac{\$0.12}{1\ \text{kW}\cdot\text{h}}\right) = \$44.88 \approx \boxed{\$45}$

Kilowatt-hours is a measure of energy, not power, and so $\boxed{\text{no}}$, the actual rate at which the energy is used does not figure into the bill. They could use the energy at a constant rate, or at a widely varying rate, and as long as the total used is 370 kilowatt-hours, the price would be $45.

18. Since $P = \dfrac{W}{t}$, we have $W = Pt = 3.0\ \text{hp}\left(\dfrac{746\ \text{W}}{1\ \text{hp}}\right)(1\ \text{hr})\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right) = \boxed{8.1\times10^{6}\ \text{J}}$

19. See the free-body diagram for the bicycle on the hill. Write Newton's 2nd law for the *x* direction, noting that the acceleration is 0. Solve for the magnitude of $\vec{\mathbf{F}}_{\text{P}}$. The power output related to that force is given by Eq. 6-17, $P = F_{\text{P}}v$. Use that relationship to find the velocity.



$$\sum F_x = F_{\text{P}} - mg\sin\theta = 0 \quad\rightarrow\quad F_{\text{P}} = mg\sin\theta$$

$$P = vF_{\text{P}} \quad\rightarrow\quad v = \frac{P}{F_{\text{P}}} = \frac{P}{mg\sin\theta} = \frac{(0.25\ \text{hp})(746\ \text{W/hp})}{(68\ \text{kg})(9.8\ \text{m/s}^2)\sin 6.0^\circ}$$

$$= \boxed{2.7\ \text{m/s}}$$