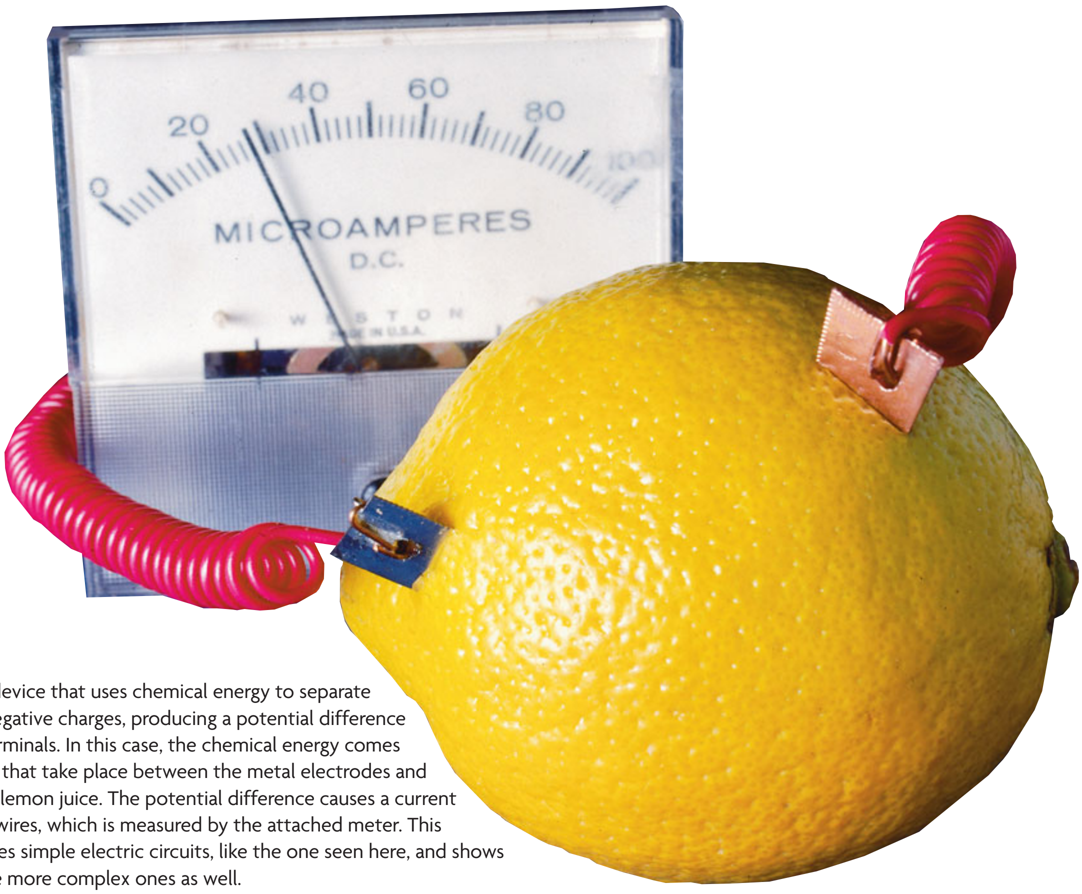
# 21 Electric Current and Direct-Current Circuits




A battery is a device that uses chemical energy to separate positive and negative charges, producing a potential difference between its terminals. In this case, the chemical energy comes from reactions that take place between the metal electrodes and the acid in the lemon juice. The potential difference causes a current to flow in the wires, which is measured by the attached meter. This chapter explores simple electric circuits, like the one seen here, and shows how to analyze more complex ones as well.

As you read this paragraph, your heart is pumping blood through the arteries and veins in your body. In a way, your heart is acting like a battery in an electric circuit: A battery causes electric charge to flow through a closed circuit of wires; your heart causes blood to flow through your body. Just as the flow of blood is important to life, the flow of electric charge is of central importance to modern technology. In this chapter we consider some of the basic properties of moving electric charges, and we apply these results to simple electric circuits.

## 21–1 Electric Current

A flow of electric charge from one place to another is referred to as an **electric current.** Often, the charge is carried by electrons moving through a metal wire. Though the analogy should not be pushed too far, the electrons flowing through a wire are much like water molecules flowing through a garden hose or blood cells flowing through an artery.

To be specific, suppose a charge $\Delta Q$ flows past a given point in a wire in a time $\Delta t$. In such a case, we say that the electric current, $I$, in the wire is:

**Definition of Electric Current, *I***

$$I = \frac{\Delta Q}{\Delta t} \qquad 21\text{–}1$$

SI unit: coulomb per second, C/s = ampere, A

The unit of current, the ampere (A) or *amp* for short, is named for the French physicist André-Marie Ampère (1775–1836) and is defined simply as 1 coulomb per second:

$$1\ \text{A} = 1\ \text{C/s}$$

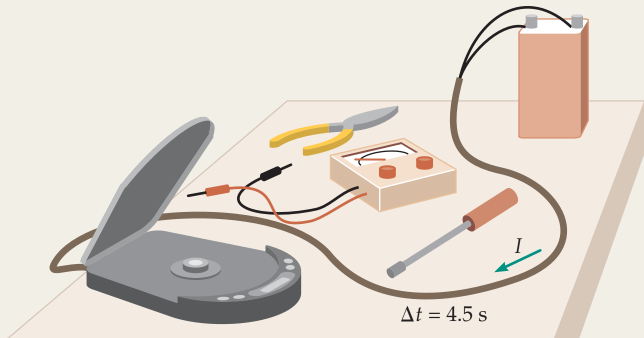
The following Example shows that the number of electrons involved in typical electric circuits, with currents of roughly an amp, is extremely large—not unlike the large number of water molecules flowing through a garden hose.

### EXAMPLE 21–1 MEGA BLASTER

The disk drive in a portable CD player is connected to a battery that supplies it with a current of 0.22 A. How many electrons pass through the drive in 4.5 s?

**PICTURE THE PROBLEM**

Our sketch shows the CD drive with a current $I = 0.22$ A flowing through it. Also indicated is the time $\Delta t = 4.5$ s during which the current flows.




**STRATEGY**

Since we know both the current, $I$, and the length of time, $\Delta t$, we can use the definition of current, $I = \Delta Q/\Delta t$, to find the charge, $\Delta Q$, that flows through the player. Once we know the charge, the number of electrons, $N$, is simply $\Delta Q$ divided by the magnitude of the electron's charge: $N = \Delta Q/e$.

**SOLUTION**

1. Calculate the charge, $\Delta Q$, that flows through the drive: $\Delta Q = I\,\Delta t = (0.22\ \text{A})(4.5\ \text{s}) = 0.99\ \text{C}$

2. Divide by the magnitude of the electron's charge, $e$, to find the number of electrons:
$$N = \frac{\Delta Q}{e} = \frac{0.99\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = 6.2 \times 10^{18}\ \text{electrons}$$

**INSIGHT**

Thus, even a modest current flowing for a brief time corresponds to the transport of an extremely large number of electrons.

**PRACTICE PROBLEM**

How long must this current last if $7.5 \times 10^{18}$ electrons are to flow through the disk drive? [**Answer:** 5.5 s]

*Some related homework problems: Problem 1, Problem 2*

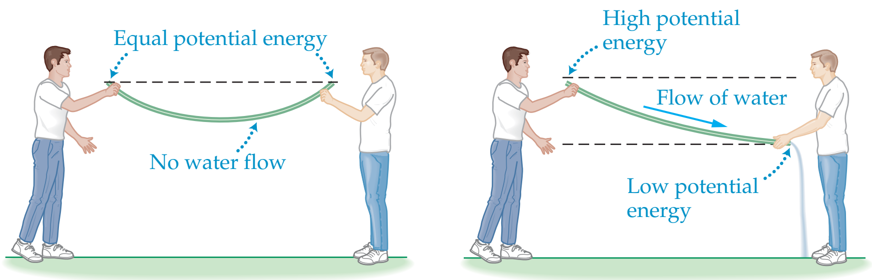
When charge flows through a closed path and returns to its starting point, we refer to the closed path as an *electric circuit*. In this chapter we consider **direct-current circuits,** also known as dc circuits, in which the current always flows in the same direction. Circuits with currents that periodically reverse their direction

▲ Electric currents are not confined to the wires in our houses and machines, but occur in nature as well. A lightning bolt is simply an enormous, brief current. It flows when the difference in electric potential between cloud and ground (or cloud and cloud) becomes so great that it exceeds the breakdown strength of air. An enormous quantity of charge then leaps across the gap in a fraction of a second. Some organisms, such as this electric torpedo ray, have internal organic "batteries" that can produce significant electric potentials. The resulting current is used to stun their prey.

are referred to as **alternating-current circuits.** These AC circuits are considered in detail in Chapter 24.

## Batteries and Electromotive Force

Although electrons move rather freely in metal wires, they do not flow unless the wires are connected to a source of electrical energy. A close analogy is provided by water in a garden hose. Imagine that you and a friend each hold one end of a garden hose filled with water. If the two ends are held at the same level, as in **Figure 21–1 (a)**, the water does not flow. If, however, one end is raised above the other, as in **Figure 21–1 (b)**, water flows from the high end—where the gravitational potential energy is high—to the low end.




**(a)** Equal potential energy → no flow **(b)** Water flows from high potential energy to low

▶ **FIGURE 21–1 Water flow as an analogy for electric current**

Water can flow quite freely through a garden hose, but if both ends are at the same level **(a)**, there is no flow. If the ends are held at different levels **(b)**, the water flows from the region where the gravitational potential energy is high to the region where it is low.

A **battery** performs a similar function in an electric circuit. To put it simply, a battery uses chemical reactions to produce a difference in electric potential between its two ends, or **terminals.** The symbol for a battery is . The terminal corresponding to a high electric potential is denoted by a $+$, and the terminal corresponding to a low electric potential is denoted by a $-$. When the battery is connected to a circuit, electrons move in a closed path from the negative terminal of the battery, through the circuit, and back to the positive terminal.

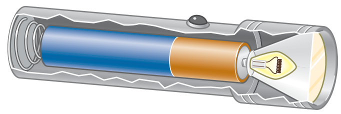
A simple example of an electrical system is shown in **Figure 21–2 (a)**, where we show a battery, a switch, and a lightbulb as they might be connected in a flashlight. In the schematic circuit shown in **Figure 21–2 (b)**, the switch is "open"—creating an **open circuit**—which means there is no closed path through which the electrons can flow. As a result, the light is off. When the switch is closed—which "closes" the circuit—charge flows around the circuit, causing the light to glow.

A mechanical analog to the flashlight circuit is shown in **Figure 21–3**. In this system, the person raising the water from a low to a high level is analogous to the battery, the paddle wheel is analogous to the lightbulb, and the water is analogous to the electric charge. Notice that the person does work in raising the water; later, as the water falls to its original level, it does work on the external world by turning the paddle wheel.

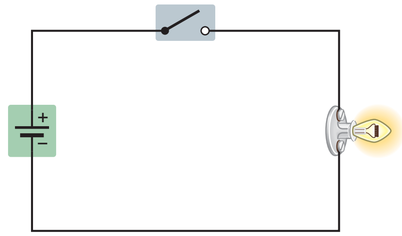
When a battery is disconnected from a circuit and carries no current, the difference in electric potential between its terminals is referred to as its *electromotive force*, or *emf* ($\mathcal{E}$). It follows that the units of emf are the same as those of electric potential, namely, volts. Clearly, then, the electromotive force is not really a force at all. Instead, the emf determines the amount of work a battery does to move a certain amount of charge around a circuit (like the person lifting water in Figure 21–3). To be specific, the magnitude of the work done by a battery of emf $\mathcal{E}$ as a charge $\Delta Q$ moves from one of its terminals to the other is given by Equation 20–2:

$$W = \Delta Q \mathcal{E}$$

We apply this relation to a flashlight circuit in the following Active Example.



**(a)** A simple flashlight



**(b)** Circuit diagram for flashlight

▲ **FIGURE 21–2 The flashlight: A simple electric circuit**

**(a)** A simple flashlight, consisting of a battery, a switch, and a lightbulb. **(b)** When the switch is in the open position, the circuit is "broken," and no charge can flow. When the switch is closed, electrons flow through the circuit and the light glows.

## ACTIVE EXAMPLE 21–1 OPERATING A FLASHLIGHT: FIND THE CHARGE AND THE WORK

A battery with an emf of 1.5 V delivers a current of 0.44 A to a flashlight bulb for 64 s (see Figure 21–2). Find **(a)** the charge that passes through the circuit and **(b)** the work done by the battery.

**SOLUTION** *(Test your understanding by performing the calculations indicated in each step.)*

**Part (a)**

1. Use the definition of current, $I = \Delta Q/\Delta t$, to find the charge that flows through the circuit: $\Delta Q = 28\ \text{C}$

**Part (b)**

2. Once we know $\Delta Q$, we can use $W = \Delta Q\mathcal{E}$ to find the work: $W = 42\ \text{J}$

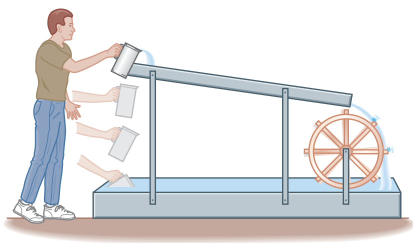
**INSIGHT**

Note that the more charge a battery moves through a circuit, the more work it does. Similarly, the greater the emf, the greater the work. We can see, then, that a car battery that operates at 12 volts and delivers several amps of current does much more work than a flashlight battery—as expected.

**YOUR TURN**

How long must the flashlight battery operate to do 150 J of work?

*(Answers to* **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 21–3 A mechanical analog to the flashlight circuit**

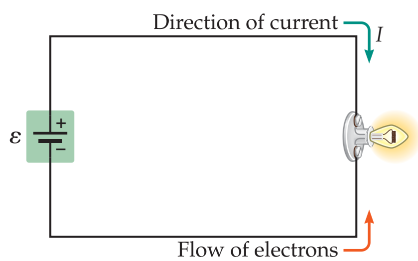
The person lifting the water corresponds to the battery in Figure 21–2, and the paddle wheel corresponds to the lightbulb.

The emf of a battery is the potential difference it can produce between its terminals under ideal conditions. In real batteries, however, there is always some internal loss, leading to a potential difference that is less than the ideal value. In fact, the greater the current flowing through a battery, the greater the reduction in potential difference between its terminals, as we shall see in Section 21–4. Only when the current is zero can a real battery produce its full emf. Because most batteries have relatively small internal losses, we shall treat batteries as ideal—always producing a potential difference precisely equal to $\mathcal{E}$—unless specifically stated otherwise.

When we draw an electric circuit, it will be useful to draw an arrow indicating the flow of current. By convention, the direction of the current arrow is given in terms of a positive test charge, in much the same way that the direction of the electric field is determined:

> The direction of the current in an electric circuit is the direction in which a *positive* test charge would move.

Of course, in typical circuits the charges that flow are actually *negatively* charged electrons. As a result, the flow of electrons and the current arrow point in opposite directions, as indicated in **Figure 21–4**. Notice that a positive charge will flow from

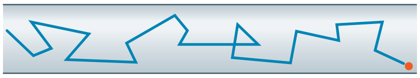



▲ **FIGURE 21–4 Direction of current and electron flow**

In the flashlight circuit, electrons flow from the negative terminal of the battery to the positive terminal. The direction of the current, $I$, is just the opposite: from the positive terminal to the negative terminal.

a region of high electric potential, near the positive terminal of the battery, to a region of low electric potential, near the negative terminal, as one would expect.

Finally, surprising as it may seem, electrons move rather slowly through a typical wire. They suffer numerous collisions with the atoms in the wire, and hence their path is rather tortuous and roundabout, as indicated in **Figure 21–5**. Like a car contending with a series of speed bumps, the electron's average speed, or **drift speed** as it is often called, is limited by the repeated collisions—in fact, their average speed is commonly about $10^{-4}$ m/s. Thus, if you switch on the headlights of a car, for example, an electron leaving the battery will take about an hour to reach the lightbulb, yet the lights seem to shine from the instant the switch is turned on. How is this possible?



▲ **FIGURE 21–5 Path of an electron in a wire**

Typical path of an electron as it bounces off atoms in a metal wire. Because of the tortuous path the electron follows, its average velocity is rather small.

The answer is that as an electron begins to move away from the battery, it exerts a force on its neighbors, causing them to move in the same general direction and, in turn, to exert a force on their neighbors, and so on. This process generates a propagating influence that travels through the wire at nearly the speed of light. The phenomenon is analogous to a bowling ball hitting one end of a line of balls; the effect of the colliding ball travels through the line at roughly the speed of sound, although the individual balls have very little displacement. Similarly, the electrons in a wire move with a rather small average velocity as they collide with and bounce off the atoms making up the wire, whereas the influence they have on one another races ahead and causes the light to shine.

## 21–2 Resistance and Ohm's Law

Electrons flow through metal wires with relative ease. In the ideal case, nothing about the wire would prevent their free motion. Real wires, however, under normal conditions, always affect the electrons to some extent, creating a **resistance** to their motion in much the same way that friction slows a box sliding across the floor.

In order to cause electrons to move against the resistance of a wire, it is necessary to apply a potential difference between its ends. For a wire with constant resistance, $R$, the potential difference, $V$, necessary to create a current, $I$, is given by Ohm's law:

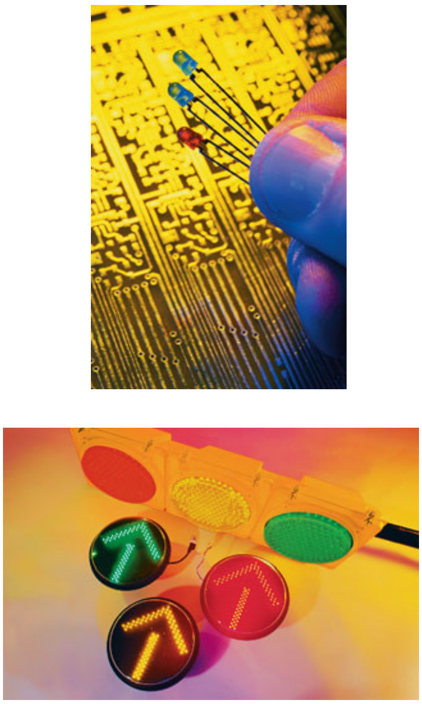
**Ohm's Law**

$$V = IR \tag{21–2}$$

SI unit: volt, V

Ohm's law is named for the German physicist Georg Simon Ohm (1789–1854).

It should be noted at the outset that Ohm's law is not a law of nature but more on the order of a useful rule of thumb—like Hooke's law for springs or the ideal-gas laws that approximate the behavior of real gases. Materials that are well approximated by Ohm's law are said to be "ohmic" in their behavior; they show a simple linear relationship between the voltage applied to them and the current that results. In particular, if one plots current versus voltage for an ohmic material, the result is a straight line, with a constant slope equal to $1/R$. Nonohmic materials, on the other hand, have more complex relationships between voltage and current. A plot of current versus voltage for a nonohmic material is nonlinear; hence, the material does not have a constant resistance. (As an example, see Problem 9.) It is precisely these "nonlinearities," however, that can make such materials so useful in the construction of electronic devices, including the ubiquitous light-emitting diodes (LEDs).



▲ A light-emitting diode (LED) is a relatively small, nonohmic device (top), but groups of LEDs can be used to form displays of practically any size (bottom). Because LEDs are extremely durable, and predicted to last 20 years or more, they are becoming the illumination of choice in high-reliability applications such as traffic lights, emergency exit signs, and brake lights. You'll probably see several on your way home today.

Solving Ohm's law for the resistance, we find

$$R = \frac{V}{I}$$

From this expression it is clear that the units of resistance are volts per amp. In particular, we define 1 volt per amp to be 1 **ohm.** Letting the Greek letter omega, $\Omega$, designate the ohm, we have

$$1\,\Omega = 1\,\mathrm{V/A}$$

A device for measuring resistance is called an ohmmeter. We describe the operation of an ohmmeter in Section 21–8.

### EXERCISE 21–1

A potential difference of 24 V is applied to a 150-Ω resistor. How much current flows through the resistor?

**SOLUTION**

Solving Ohm's law for the current, $I$, we find

$$I = \frac{V}{R} = \frac{24\text{ V}}{150\ \Omega} = \frac{24\text{ V}}{150\text{ V/A}} = 0.16\text{ A}$$

In an electric circuit a resistor is signified by a zigzag line:  . The straight lines in a circuit indicate ideal wires of zero resistance. To indicate the resistance of a real wire or device, we simply include a resistor of the appropriate value in the circuit.

## Resistivity

Suppose you have a piece of wire of length $L$ and cross-sectional area $A$. The resistance of this wire depends on the particular material from which it is constructed. If the wire is made of copper, for instance, its resistance will be less than if it is made from iron. The quantity that characterizes the resistance of a given material is its **resistivity,** $\rho$. For a wire of given dimensions, the greater the resistivity, the greater the resistance.

The resistance of a wire also depends on its length and area. To understand the dependence on $L$ and $A$, consider again the analogy of water flowing through a hose. If the hose is very long, the resistance it presents to the water will be correspondingly large, whereas a wider hose—one with a greater cross-sectional area—will offer less resistance to the water. After all, water flows more easily through a short fire hose than through a long soda straw; hence, the resistance of a hose—and similarly a piece of wire—should be proportional to $L$ and inversely proportional to $A$; that is, proportional to $(L/A)$.

Combining these observations, we can write the resistance of a wire of length $L$, area $A$, and resistivity $\rho$ in the following way:

**Definition of Resistivity, $\rho$**

$$R = \rho\left(\frac{L}{A}\right) \qquad 21\text{–}3$$

Since the units of $L$ are m and the units of $A$ are $m^2$, it follows that the units of resistivity are $(\Omega\cdot\text{m})$. Typical values for $\rho$ are given in Table 21–1. Notice the enormous range in values of $\rho$, with the resistivity of an insulator like rubber about $10^{21}$ times greater than the resistivity of a good conductor like silver.

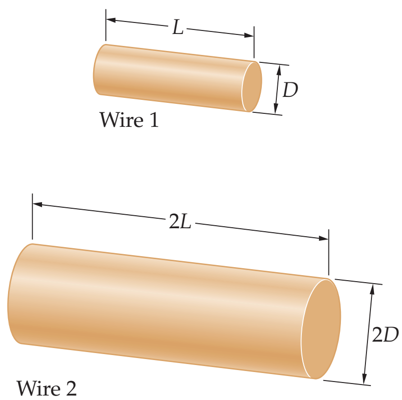
**TABLE 21–1 Resistivities**

| Substance | Resistivity, $\rho$ $(\Omega\cdot\text{m})$ |
|---|---|
| **Insulators** | |
| Quartz (fused) | $7.5 \times 10^{17}$ |
| Rubber | $1$ to $100 \times 10^{13}$ |
| Glass | $1$ to $10{,}000 \times 10^{9}$ |
| **Semiconductors** | |
| Silicon* | 0.10 to 60 |
| Germanium* | 0.001 to 0.5 |
| **Conductors** | |
| Lead | $22 \times 10^{-8}$ |
| Iron | $9.71 \times 10^{-8}$ |
| Tungsten | $5.6 \times 10^{-8}$ |
| Aluminum | $2.65 \times 10^{-8}$ |
| Gold | $2.20 \times 10^{-8}$ |
| Copper | $1.68 \times 10^{-8}$ |
| Silver | $1.59 \times 10^{-8}$ |

*The resistivity of a semiconductor varies greatly with the type and amount of impurities it contains. This property makes them particularly useful in electronic applications.

### CONCEPTUAL CHECKPOINT 21–1 COMPARE THE RESISTANCE

Wire 1 has a length $L$ and a circular cross section of diameter $D$. Wire 2 is constructed from the same material as wire 1 and has the same shape, but its length is $2L$, and its diameter is $2D$. Is the resistance of wire 2 **(a)** the same as that of wire 1, **(b)** twice that of wire 1, or **(c)** half that of wire 1?




**REASONING AND DISCUSSION**

First, the resistance of wire 1 is

$$R_1 = \rho\left(\frac{L}{A}\right) = \rho\frac{L}{(\pi D^2/4)}$$

Note that we have used the fact that the area of a circle of diameter $D$ is $\pi D^2/4$. For wire 2 we replace $L$ with $2L$ and $D$ with $2D$:

$$R_2 = \rho\frac{2L}{[\pi(2D)^2/4]} = \left(\tfrac{1}{2}\right)\rho\frac{L}{(\pi D^2/4)} = \tfrac{1}{2}R_1$$

CONTINUED ON NEXT PAGE

Thus, increasing the length by a factor of 2 increases the resistance by a factor of 2; on the other hand, increasing the diameter by a factor of 2 increases the area, and decreases the resistance, by a factor of 4. Overall, then, the resistance of wire 2 is half that of wire 1.

**ANSWER**

**(c)** The resistance of wire 2 is half that of wire 1; $R_2 = R_1/2$.

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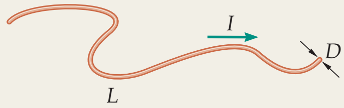
## EXAMPLE 21–2 A CURRENT-CARRYING WIRE

A current of 1.82 A flows through a copper wire 1.75 m long and 1.10 mm in diameter. Find the potential difference between the ends of the wire. (The value of $\rho$ for copper may be found in Table 21–1.)

**PICTURE THE PROBLEM**

The wire carries a current $I = 1.82$ A, and its total length $L$ is 1.75 m. We assume that the wire has a circular cross section, with a diameter $D = 1.10$ mm.




**STRATEGY**

We know from Ohm's law that the potential difference associated with a current $I$ and a resistance $R$ is $V = IR$. We are given the current in the wire, but not the resistance. The resistance is easily determined, however, using $R = \rho(L/A)$ with $A = \pi D^2/4$. Thus, we first calculate $R$ and then substitute the result into $V = IR$ to obtain the potential difference.

**SOLUTION**

**1.** Calculate the resistance of the wire:

$$R = \rho\left(\frac{L}{A}\right) = \rho\left(\frac{L}{\pi D^2/4}\right)$$

$$= (1.72 \times 10^{-8}\ \Omega \cdot \text{m})\left[\frac{1.75\ \text{m}}{\pi(0.00110\ \text{m})^2/4}\right] = 0.0317\ \Omega$$

**2.** Multiply $R$ by the current, $I$, to find the potential difference:

$$V = IR = (1.82\ \text{A})(0.0317\ \Omega) = 0.0577\ \text{V}$$

**INSIGHT**

Copper is an excellent conductor; therefore, both the resistance and the potential difference are quite small.

**PRACTICE PROBLEM**

What diameter of copper wire is needed for there to be a potential difference of 0.100 V? Assume that all other quantities remain the same. [**Answer:** 0.835 mm]

*Some related homework problems: Problem 17, Problem 18*

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## Temperature Dependence and Superconductivity

We know from everyday experience that a wire carrying an electric current can become warm—even quite hot, as in the case of a burner on a stove or the filament in an incandescent lightbulb. This follows from our earlier discussion of the fact that electrons collide with the atoms in a wire as they flow through an electric circuit. These collisions cause the atoms to jiggle with greater kinetic energy about their equilibrium positions. As a result, the temperature of the wire increases (see Section 17–2, and Equation 17–21 in particular). For example, the wire filament in an incandescent lightbulb can reach temperatures of roughly 2800 °C (in comparison, the surface of the Sun has a temperature of about 5500 °C), and the heating coil on a stove has a temperature of about 750 °C.

As a wire is heated, its resistivity tends to increase. This is because atoms that are jiggling more rapidly are more likely to collide with electrons and slow their progress through the wire. In fact, many metals show an approximately linear increase of $\rho$ over a wide range of temperature. Once the dependence of $\rho$ on $T$ is known for a given material, the change in resistivity can be used as a means of measuring temperature.

**REAL-WORLD PHYSICS**
**The bolometer**

The first practical application of this principle was in a device known as the **bolometer.** Invented in 1880, the bolometer is an extremely sensitive thermometer that uses the temperature variation in the resistivity of platinum, nickel, or bismuth as a means of detecting temperature changes as small as 0.0001 C°. Soon after its invention, a bolometer was used to detect infrared radiation from the stars.

Some materials, like semiconductors, actually show a drop in resistivity as temperature is increased. This is because the resistivity of a semiconductor is strongly dependent on the number of electrons that are free to move about and conduct a current. As the temperature is increased in a semiconductor, more electrons are able to break free from their atoms, leading to an increased current and a reduced resistivity. Electronic devices incorporating such temperature-dependent semiconductors are known as **thermistors.** The digital fever thermometer so common in today's hospitals uses a thermistor to provide accurate measurements of a patient's temperature.

**REAL-WORLD PHYSICS**

**Thermistors and fever thermometers**

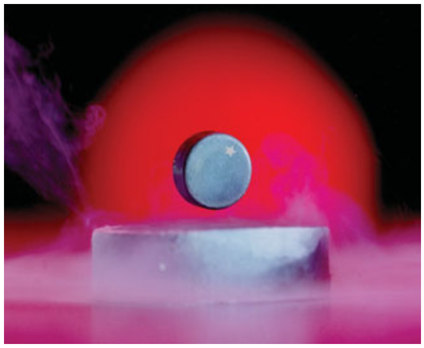
Since resistivity typically increases with temperature, it follows that if a wire is cooled below room temperature, its resistivity will *decrease*. Quite surprising, however, was a discovery made in the laboratory of Heike Kamerlingh-Onnes in 1911. Measuring the resistance of a sample of mercury at temperatures just a few degrees above absolute zero, researchers found that at about 4.2 K the resistance of the mercury suddenly dropped to zero—not just to a very small value, but to *zero*. At this temperature, we say that the mercury becomes **superconducting,** a hitherto unknown phase of matter. Since that time many different superconducting materials have been discovered, with various different **critical temperatures,** $T_c$, at which superconductivity begins. Today we know that superconductivity is a result of quantum effects (Chapter 30).

**REAL-WORLD PHYSICS**

**Superconductors and high-temperature superconductivity**

When a material becomes superconducting, a current can flow through it with absolutely no resistance. In fact, if a current is initiated in a superconducting ring of material, it will flow undiminished for as long as the ring is kept cool enough. In some cases, circulating currents have been maintained for years, with absolutely no sign of diminishing.

In 1986 a new class of superconductors was discovered that has zero resistance at temperatures significantly greater than those of any previously known superconducting materials. At the moment, the highest temperature at which superconductivity has been observed is about 125 K. Since the discovery of these "high-$T_c$" superconductors, hopes have been raised that it may one day be possible to produce room-temperature superconductors. The practical benefits of such a breakthrough, including power transmission with no losses, improved MRI scanners, and magnetically levitated trains, could be immense.



▲ When cooled below their critical temperature, superconductors not only lose their resistance to current flow but also exhibit new magnetic properties, such as repelling an external magnetic field. Here, a superconductor (bottom) levitates a small permanent magnet.

## 21–3 Energy and Power in Electric Circuits

When a charge $\Delta Q$ moves across a potential difference $V$, its electrical potential energy, $U$, changes by the amount

$$\Delta U = (\Delta Q)V$$

Recalling that power is the rate at which energy changes, $P = \Delta U/\Delta t$, we can write the electrical power as follows:

$$P = \frac{\Delta U}{\Delta t} = \frac{(\Delta Q)V}{\Delta t}$$

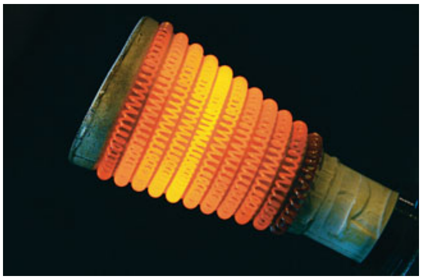
Since the electric current is given by $I = \Delta Q/\Delta t$, we have:

**Electrical Power**

$$P = IV \qquad 21\text{–}4$$

SI unit: watt, W

Thus, a current of 1 amp flowing across a potential difference of 1 V produces a power of 1 W.



▲ The heating element of an electric space heater is nothing more than a length of resistive wire coiled up for compactness. As electric current flows through the wire, the power it dissipates ($P = I^2R$) is converted to heat and light. The coils near the center are the hottest, and hence they glow with a higher-frequency, yellowish light.

## EXERCISE 21–2

A handheld electric fan operates on a 3.00-V battery. If the power generated by the fan is 2.24 W, what is the current supplied by the battery?

**SOLUTION**

Solving $P = IV$ for the current, we obtain

$$I = \frac{P}{V} = \frac{2.24\ \text{W}}{3.00\ \text{V}} = 0.747\ \text{A}$$

The expression $P = IV$ applies to any electrical system. In the special case of a resistor, the electrical power is dissipated in the form of heat. Applying Ohm's law $(V = IR)$ to this case, we can write the power dissipated in a resistor as

$$P = IV = I(IR) = I^2R \quad \text{21–5}$$

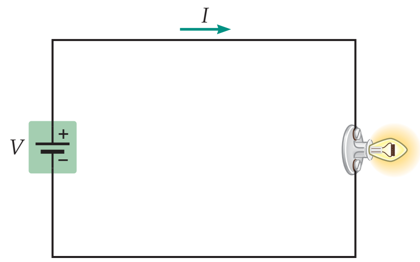
Similarly, using Ohm's law to solve for the current, $I = V/R$, we have

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R} \quad \text{21–6}$$

These relations also apply to incandescent lightbulbs, which are basically resistors that become hot enough to glow.

## CONCEPTUAL CHECKPOINT 21–2 COMPARE LIGHTBULBS

A battery that produces a potential difference $V$ is connected to a 5-W lightbulb. Later, the 5-W lightbulb is replaced with a 10-W lightbulb. **(a)** In which case does the battery supply more current? **(b)** Which lightbulb has the greater resistance?




**REASONING AND DISCUSSION**

**a.** To compare the currents, we need consider only the relation $P = IV$. Solving for the current yields $I = P/V$. When the voltage $V$ is the same, it follows that the greater the power, the greater the current. In this case, then, the current in the 10-W bulb is twice the current in the 5-W bulb.

**b.** We now consider the relation $P = V^2/R$, which gives resistance in terms of voltage and power. In fact, $R = V^2/P$. Again, with $V$ the same, it follows that the smaller the power, the greater the resistance. Thus, the resistance of the 5-W bulb is twice that of the 10-W bulb.

**ANSWER**

**(a)** When the battery is connected to the 10-W bulb, it delivers twice as much current as when it is connected to the 5-W bulb. **(b)** The 5-W bulb has twice as much resistance as the 10-W bulb.

On a microscopic level, the power dissipated by a resistor is the result of incessant collisions between electrons moving through the circuit and atoms making up the resistor. Specifically, the electric potential difference produced by the battery causes electrons to accelerate until they bounce off an atom of the resistor. At this point the electrons transfer energy to the atoms, causing them to jiggle more rapidly. The increased kinetic energy of the atoms is reflected in an increased temperature of the resistor (see Section 17–2). After each collision, the potential difference accelerates the electrons again and the process repeats—like a car bouncing through a series of speed bumps—resulting in a continuous transfer of energy from the electrons to the atoms.

## EXAMPLE 21–3 HEATED RESISTANCE

A battery with an emf of 12 V is connected to a 545-Ω resistor. How much energy is dissipated in the resistor in 65 s?

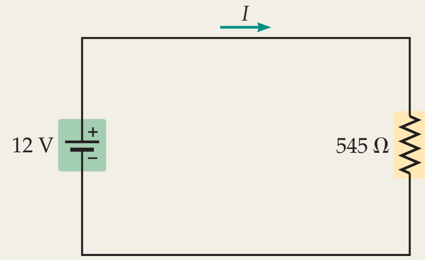
**PICTURE THE PROBLEM**

The circuit, consisting of a battery and a resistor, is shown in our sketch. We show the current flowing from the positive terminal of the 12-V battery, through the 545-Ω resistor, and into the negative terminal of the battery.

**STRATEGY**

We know that a current flowing through a resistor dissipates power (energy per time), which means that the energy it dissipates in a given time is simply the power multiplied by the time: $\Delta U = P\,\Delta t$. The time is given ($\Delta t = 65\text{ s}$), and the power can be found using $P = IV$, $P = I^2R$, or $P = V^2/R$. The last expression is most convenient in this case, because the problem statement gives us the voltage and resistance.

To summarize, we first calculate the power, then multiply by the time.




**SOLUTION**

**1.** Calculate the power dissipated in the resistor: $P = V^2/R = (12\text{ V})^2/(545\ \Omega) = 0.26\text{ W}$

**2.** Multiply the power by the time to find the energy dissipated: $\Delta U = P\,\Delta t = (0.26\text{ W})(65\text{ s}) = 17\text{ J}$

**INSIGHT**

The current in this circuit is $I = V/R = 0.022\text{ A}$. Using this result, we find that the power is $P = I^2R = IV = 0.26\text{ W}$, as expected.

**PRACTICE PROBLEM**

How much energy is dissipated in the resistor if the voltage is doubled to 24 V? [**Answer:** $4(17\text{ J}) = 68\text{ J}$]

*Some related homework problems: Problem 29, Problem 32*



◀ The battery testers now often built into battery packages (left) employ a tapered graphite strip. The narrow end (at bottom in the right-hand photo) has the highest resistance, and thus produces the most heat when a current flows through the strip. The heat is used to produce the display on the front that indicates the strength of the battery—if the current is sufficient to warm even the top of the strip, where the resistance is lowest, the battery is fresh.

**REAL-WORLD PHYSICS**

**"Battery check" meters**

A commonly encountered application of resistance heating is found in the "battery check" meters often included with packs of batteries. To operate one of these meters, you simply press the contacts on either end of the meter against the corresponding terminals of the battery to be checked. This allows a current to flow through the main working element of the meter—a tapered strip of graphite.

The reason the strip is tapered is to provide a variation in resistance. According to the relation given in Equation 21–3, $R = \rho(L/A)$, the smaller the cross-sectional area $A$ of the strip, the larger the resistance $R$. It follows that the narrow end has a higher resistance than the wide end. Because the same current $I$ flows through all parts of the strip, the power dissipated is expressed most conveniently in the form $P = I^2R$. It follows that at the narrow end of the strip, where $R$ is largest, the heating due to the current will be the greatest. Pressing the meter against the terminals of the battery, then, results in an overall warming of the graphite strip, with the narrow end warmer than the wide end.

The final element in the meter is a thin layer of liquid crystal (similar to the material used in LCD displays) that responds to small increases in temperature. In particular, this liquid crystal is black and opaque at room temperature but transparent when heated slightly. The liquid crystal is placed in front of a colored background, which can be seen in those regions where the graphite strip is warm enough to make the liquid crystal transparent. If the battery is weak, only the narrow portion of the strip becomes warm enough, and the meter shows only a small stripe of color. A strong battery, on the other hand, heats the entire strip enough to make the liquid crystal transparent, resulting in a colored stripe the full length of the meter.

### Energy Usage

When you get a bill from the local electric company, you will find the number of kilowatt-hours of electricity that you have used. Notice that a kilowatt-hour (kWh) has the units of energy:

$$1 \text{ kilowatt-hour} = (1000 \text{ W})(3600 \text{ s}) = (1000 \text{ J/s})(3600 \text{ s})$$
$$= 3.6 \times 10^6 \text{ J}$$

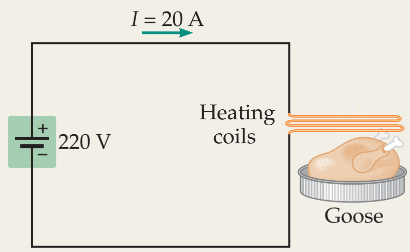
Thus, the electric company is charging for the amount of energy you use—as one would expect—and not for the rate at which you use it. The following Example considers the energy and monetary cost for a typical everyday situation.

### EXAMPLE 21–4 YOUR GOOSE IS COOKED

A holiday goose is cooked in the kitchen oven for 4.00 h. Assume that the stove draws a current of 20.0 A, operates at a voltage of 220.0 V, and uses electrical energy that costs $0.068 per kWh. How much does it cost to cook your goose?

**PICTURE THE PROBLEM**

We show a schematic representation of the stove cooking the goose in our sketch. The current in the circuit is 20.0 A, and the voltage difference across the heating coils is 220 V.




**STRATEGY**

The cost is simply the energy usage (in kWh) times the cost per kilowatt-hour ($0.068). To find the energy used, we note that energy is power multiplied by time. The time is given, and the power associated with a current $I$ and a voltage $V$ is $P = IV$.

Thus, we find the power, multiply by the time, and then multiply by $0.068 to find the cost.

**SOLUTION**

1. Calculate the power delivered to the stove: $P = IV = (20.0 \text{ A})(220.0 \text{ V}) = 4.40 \text{ kW}$
2. Multiply power by time to determine the total energy supplied to the stove during the 4.00 h of cooking: $\Delta U = P\,\Delta t = (4.40 \text{ kW})(4.00 \text{ h}) = 17.6 \text{ kWh}$
3. Multiply by the cost per kilowatt-hour to find the total cost of cooking: $\text{cost} = (17.6 \text{ kWh})(\$0.068/\text{kWh}) = \$1.20$

**INSIGHT**

Thus, your goose can be cooked for just over a dollar.

**PRACTICE PROBLEM**

If the voltage and current are reduced by a factor of 2 each, how long must the goose be cooked to use the same amount of energy? [**Answer:** $4(4.00 \text{ h}) = 16.0 \text{ h}$. *Note*: You should be able to answer a question like this by referring to your previous solution, without repeating the calculation in detail.]

*Some related homework problems: Problem 30, Problem 31*

## 21–4 Resistors in Series and Parallel

Electric circuits often contain a number of resistors connected in various ways. In this section we consider simple circuits containing only resistors and batteries. For each type of circuit considered, we calculate the **equivalent resistance** produced by a group of individual resistors.

### Resistors in Series

When resistors are connected one after the other, end to end, we say that they are in *series*. **Figure 21–6 (a)** shows three resistors, $R_1$, $R_2$, and $R_3$, connected in series. The three resistors acting together have the same effect—that is, they draw the same current—as a single resistor, referred to as the equivalent resistor, $R_{eq}$. This equivalence is illustrated in **Figure 21–6 (b)**. We now calculate the value of the equivalent resistance.

The first thing to notice about the circuit in Figure 21–6 (a) is that the same current $I$ must flow through each of the resistors—there is no other place for the current to go. As a result, the potential differences across the three resistors are

$V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$, respectively. Since the total potential difference from point A to point B must be the emf of the battery, $\mathcal{E}$, it follows that

$$\mathcal{E} = V_1 + V_2 + V_3$$

Writing each of the potentials in terms of the current and resistance, we find

$$\mathcal{E} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

Now, let's compare this expression with the result we obtain for the equivalent circuit in Figure 21–6 (b). In this circuit, the potential difference across the battery is $V = IR_{eq}$. Since this potential must be the same as the emf of the battery, we have

$$\mathcal{E} = IR_{eq}$$

Comparing this expression with $\mathcal{E} = I(R_1 + R_2 + R_3)$, we see that the equivalent resistance is simply the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3$$

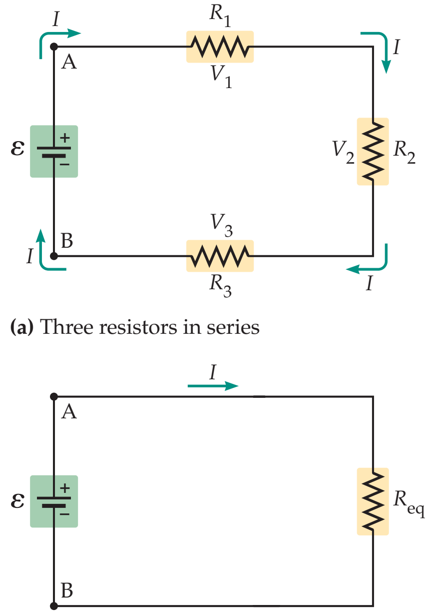
In general, for any number of resistors in series, the equivalent resistance is

**Equivalent Resistance for Resistors in Series**

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R \qquad \text{21–7}$$

SI unit: ohm, $\Omega$

Note that the equivalent resistance is greater than the greatest resistance of any of the individual resistors. Connecting the resistors in series is like making a single resistor increasingly longer; as its length increases so does its resistance.




**(a)** Three resistors in series

**(b)** Equivalent resistance has the same current

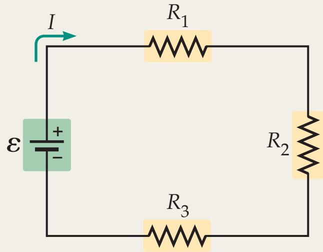
▲ **FIGURE 21–6 Resistors in series**
**(a)** Three resistors, $R_1$, $R_2$, and $R_3$, connected in series. Note that the same current $I$ flows through each resistor. **(b)** The equivalent resistance, $R_{eq} = R_1 + R_2 + R_3$, has the same current $I$ flowing through it as the current $I$ in the original circuit.

## EXAMPLE 21–5 THREE RESISTORS IN SERIES

A circuit consists of three resistors connected in series to a 24.0-V battery. The current in the circuit is 0.0320 A. Given that $R_1 = 250.0\ \Omega$ and $R_2 = 150.0\ \Omega$, find **(a)** the value of $R_3$ and **(b)** the potential difference across each resistor.

**PICTURE THE PROBLEM**

The circuit is shown in our sketch. Note that the 24.0-V battery delivers the same current, $I = 0.0320$ A, to each of the three resistors. This is the key characteristic of a series circuit.




**STRATEGY**

**a.** First, we can obtain the equivalent resistance of the circuit using Ohm's law (as in Equation 21–2); $R_{eq} = \mathcal{E}/I$. Since the resistors are in series, we also know that $R_{eq} = R_1 + R_2 + R_3$. We can solve this relation for the only unknown, $R_3$.

**b.** We can then calculate the potential difference across each resistor using Ohm's law, $V = IR$.

**SOLUTION**

**Part (a)**

1. Use Ohm's law to find the equivalent resistance of the circuit: $R_{eq} = \dfrac{\mathcal{E}}{I} = \dfrac{24.0\text{ V}}{0.0320\text{ A}} = 7.50 \times 10^2\ \Omega$

2. Set $R_{eq}$ equal to the sum of the individual resistances, and solve for $R_3$:

$$R_{eq} = R_1 + R_2 + R_3$$
$$R_3 = R_{eq} - R_1 - R_2$$
$$= 7.50 \times 10^2\ \Omega - 250.0\ \Omega - 150.0\ \Omega = 3.50 \times 10^2\ \Omega$$

**Part (b)**

3. Use Ohm's law to determine the potential difference across $R_1$: $V_1 = IR_1 = (0.0320\text{ A})(250.0\ \Omega) = 8.00\text{ V}$

4. Find the potential difference across $R_2$: $V_2 = IR_2 = (0.0320\text{ A})(150.0\ \Omega) = 4.80\text{ V}$

5. Find the potential difference across $R_3$: $V_3 = IR_3 = (0.0320\text{ A})(3.50 \times 10^2\ \Omega) = 11.2\text{ V}$

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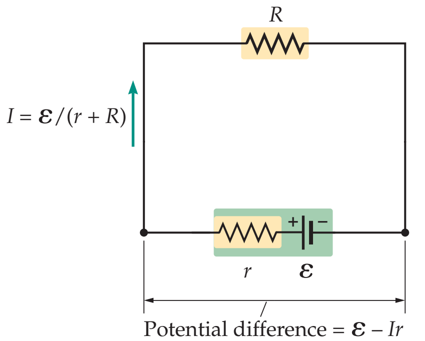
**INSIGHT**

Note that the greater the resistance, the greater the potential difference. In addition, the sum of the individual potential differences is 8.00 V + 4.80 V + 11.2 V = 24.0 V, as expected.

**PRACTICE PROBLEM**

Find the power dissipated in each resistor. [**Answer:** $P_1 = 0.256$ W, $P_2 = 0.154$ W, $P_3 = 0.358$ W]

*Some related homework problems: Problem 43, Problem 44*




▲ **FIGURE 21–7 The internal resistance of a battery**

Real batteries always dissipate some energy in the form of heat. These losses can be modeled by a small "internal" resistance, $r$, within the battery. As a result, the potential difference between the terminals of a real battery is less than its ideal emf, $\mathcal{E}$. For example, if a battery produces a current $I$, the potential difference between its terminals is $\mathcal{E} - Ir$. In the case shown here, a battery is connected in series with the resistor $R$. Instead of producing the current $I = \mathcal{E}/R$, as in the ideal case, it produces the current $I = \mathcal{E}/(r + R)$.

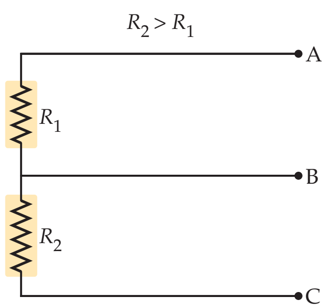
An everyday example of resistors in series is the **internal resistance,** $r$, of a battery. As was mentioned in Section 21–1, real batteries have internal losses that cause the potential difference between their terminals to be less than $\mathcal{E}$ and to depend on the current in the battery. The simplest way to model a real battery is to imagine it to consist of an ideal battery of emf $\mathcal{E}$ in series with an internal resistance $r$, as shown in **Figure 21–7**. If this battery is then connected to an external resistance, $R$, the equivalent resistance of the circuit is $r + R$. As a result, the current flowing through the circuit is $I = \mathcal{E}/(r + R)$, and the potential difference between the terminals of the battery is $\mathcal{E} - Ir$. Thus, we see that the potential difference produced by the battery is less than $\mathcal{E}$ by an amount that is proportional to the current $I$. Only in the limit of zero current, or zero internal resistance, will the battery produce its full emf. (See Problems 51, 54, 116, and 121 for examples of batteries with internal resistance.)

Another application of resistors in series is the three-way lightbulb circuit shown in **Figure 21–8**. In this circuit, the two resistors represent two different filaments within a single bulb that are connected to a constant potential difference $V$. At the "high" setting, the lower-resistance filament, $R_1$, is connected to the electrical outlet via terminals A and B, and the brightest light is obtained $(P = V^2/R)$. At the "middle" setting, the higher-resistance filament, $R_2$, is connected to the outlet via terminals B and C, resulting in a dimmer light. Finally, at the "low" setting, both filaments are connected in series via terminals A and C. This setting gives the greatest equivalent resistance, and thus the lowest light output.

An alternative method of producing a three-way lightbulb is to connect the resistors in parallel. This will be discussed in the next subsection.

**REAL-WORLD PHYSICS**

**Three-way lightbulbs**




▲ **FIGURE 21–8 A three-way bulb**

The circuit diagram for a three-way lightbulb. For the brightest light, terminals A and B are connected to the household electrical line, so the current passes through the low-resistance filament $R_1$. For intermediate brightness, terminals B and C are used, so the current passes through the higher-resistance filament $R_2$. For the lowest light output, terminals A and C are used, so the current passes through both $R_1$ and $R_2$ in series.

## Resistors in Parallel

Resistors are in *parallel* when they are connected across the same potential difference, as in **Figure 21–9 (a)**. In a case like this, the current has parallel paths through which it can flow. As a result, the total current in the circuit, $I$, is equal to the sum of the currents through each of the three resistors:

$$I = I_1 + I_2 + I_3$$

Since the potential difference is the same for each of the resistors, it follows that the currents flowing through them are as follows:

$$I_1 = \frac{\mathcal{E}}{R_1}, \quad I_2 = \frac{\mathcal{E}}{R_2}, \quad I_3 = \frac{\mathcal{E}}{R_3}$$

Summing these three currents, we find

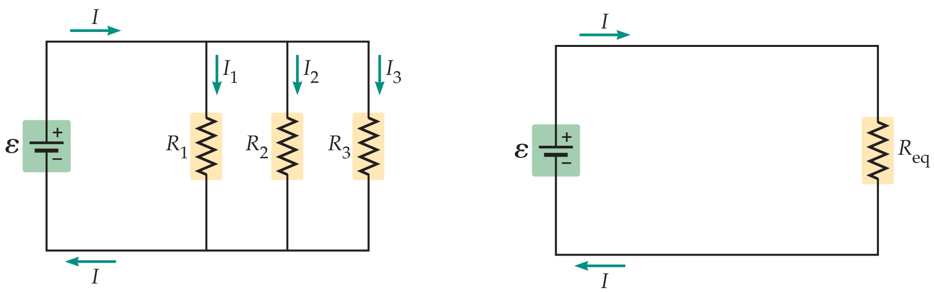
$$I = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad \text{21–8}$$

Now, in the equivalent circuit shown in **Figure 21–9 (b)**, Ohm's law gives $\mathcal{E} = IR_{eq}$, or

$$I = \mathcal{E}\left(\frac{1}{R_{eq}}\right) \qquad \text{21–9}$$

Comparing Equations 21–8 and 21–9, we find that the equivalent resistance for three resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$




**(a)** Three resistors in parallel

**(b)** Equivalent resistance has the same current

**FIGURE 21–9 Resistors in parallel**
**(a)** Three resistors, $R_1$, $R_2$, and $R_3$, connected in parallel. Note that each resistor is connected across the same potential difference $\mathcal{E}$. **(b)** The equivalent resistance, $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$, has the same current flowing through it as the total current $I$ in the original circuit.

In general, for any number of resistors in parallel, we have:

**Equivalent Resistance for Resistors in Parallel**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R} \quad \text{21–10}$$

SI unit: ohm, Ω

**PROBLEM-SOLVING NOTE**

**The Equivalent Resistance of Resistors in Parallel**

After summing the inverse of resistors in parallel, remember to take one more inverse at the end of your calculation to find the equivalent resistance.

As a simple example, consider a circuit with two identical resistors $R$ connected in parallel. The equivalent resistance in this case is given by

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

Solving for $R_{eq}$, we find $R_{eq} = \frac{1}{2}R$. If we connect three such resistors in parallel, the corresponding result is

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

In this case, $R_{eq} = \frac{1}{3}R$. Thus, the more resistors we connect in parallel, the smaller the equivalent resistance. Each time we add a new resistor in parallel with the others, we give the battery a new path through which current can flow—analogous to opening an additional lane of traffic on a busy highway. Stated another way, giving the current multiple paths through which it can flow is equivalent to using a wire with a greater cross-sectional area. From either point of view, the fact that more current flows with the same potential difference means that the equivalent resistance has been reduced.

Finally, if any one of the resistors in a parallel connection is equal to zero, the equivalent resistance is also zero. This situation is referred to as a **short circuit,** and is illustrated in **Figure 21–10**. In this case, all of the current flows through the path of zero resistance.

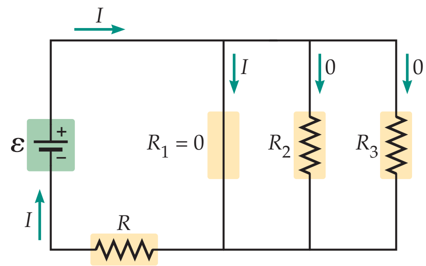


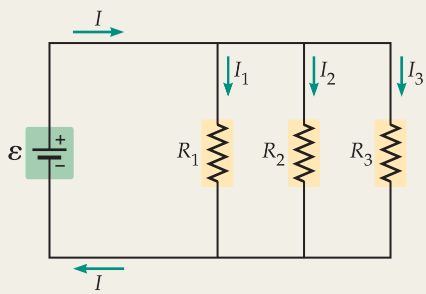

**FIGURE 21–10 A short circuit**
If one of the resistors in parallel with others is equal to zero, all the current flows through that portion of the circuit, giving rise to a short circuit. In this case, resistors $R_2$ and $R_3$ are "shorted out," and the current in the circuit is $I = \mathcal{E}/R$.

## EXAMPLE 21–6 THREE RESISTORS IN PARALLEL

Consider a circuit with three resistors, $R_1 = 250.0\ \Omega$, $R_2 = 150.0\ \Omega$, and $R_3 = 350.0\ \Omega$, connected in parallel with a 24.0-V battery. Find **(a)** the total current supplied by the battery and **(b)** the current through each resistor.

**PICTURE THE PROBLEM**

The accompanying sketch indicates the parallel connection of the resistors with the battery. Notice that each of the resistors experiences precisely the same potential difference; namely, the 24.0 V produced by the battery. This is the feature that characterizes parallel connections.




**STRATEGY**

**a.** We can find the total current from $I = \mathcal{E}/R_{eq}$, where $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$.

**b.** For each resistor, the current is given by Ohm's law, $I = \mathcal{E}/R$.

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**SOLUTION**

**Part (a)**

1. Find the equivalent resistance of the circuit:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{250.0\ \Omega} + \frac{1}{150.0\ \Omega} + \frac{1}{350.0\ \Omega} = 0.01352\ \Omega^{-1}$$

$$R_{eq} = (0.01352\ \Omega^{-1})^{-1} = 73.96\ \Omega$$

2. Use Ohm's law to find the total current:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{24.0\ \text{V}}{73.96\ \Omega} = 0.325\ \text{A}$$

**Part (b)**

3. Calculate $I_1$ using $I_1 = \mathcal{E}/R_1$ with $\mathcal{E} = 24.0$ V:

$$I_1 = \frac{\mathcal{E}}{R_1} = \frac{24.0\ \text{V}}{250.0\ \Omega} = 0.0960\ \text{A}$$

4. Repeat the preceding calculation for resistors 2 and 3:

$$I_2 = \frac{\mathcal{E}}{R_2} = \frac{24.0\ \text{V}}{150.0\ \Omega} = 0.160\ \text{A}$$

$$I_3 = \frac{\mathcal{E}}{R_3} = \frac{24.0\ \text{V}}{350.0\ \Omega} = 0.0686\ \text{A}$$

**INSIGHT**

As expected, the smallest resistor, $R_2$, carries the greatest current. The three currents combined yield the total current, as they must; that is, $I_1 + I_2 + I_3 = 0.0960\ \text{A} + 0.160\ \text{A} + 0.0686\ \text{A} = 0.325\ \text{A} = I$.

**PRACTICE PROBLEM**

Find the power dissipated in each resistor. [**Answer:** $P_1 = 2.30$ W, $P_2 = 3.84$ W, $P_3 = 1.65$ W]

*Some related homework problems: Problem 45, Problem 46*

In comparing Examples 21–5 and 21–6 note the differences in the power dissipated in each circuit. First, the total power dissipated in the parallel circuit is much greater than that dissipated in the series circuit. This is due to the fact that the equivalent resistance of the parallel circuit is smaller than the equivalent resistance of the series circuit, and the power delivered by a voltage $V$ to a resistance $R$ is inversely proportional to the resistance $(P = V^2/R)$. In addition, note that the smallest resistor, $R_2$, has the smallest power in the series circuit but the largest power in the parallel circuit. These issues are explored further in the following Conceptual Checkpoint.

## CONCEPTUAL CHECKPOINT 21–3 SERIES VERSUS PARALLEL

Two identical lightbulbs are connected to a battery, either in series or in parallel. Are the bulbs in series **(a)** brighter than, **(b)** dimmer than, or **(c)** the same brightness as the bulbs in parallel?

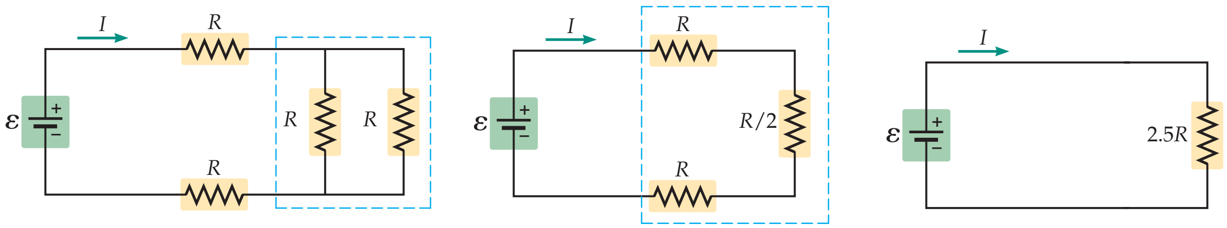
**REASONING AND DISCUSSION**

Both sets of lightbulbs are connected to the same potential difference, $V$; hence, the power delivered to the bulbs is $V^2/R_{eq}$, where $R_{eq}$ is twice the resistance of a bulb in the series circuit and half the resistance of a bulb in the parallel circuit. As a result, more power is converted to light in the parallel circuit.

**ANSWER**

**(b)** The bulbs connected in series are dimmer than the bulbs connected in parallel.

Finally, note that a three-way lightbulb can also be produced by simply wiring two filaments in parallel. For example, one filament might have a power of 50 W and the second filament a power of 100 W. One setting of the switch sends current through the 50-W filament, the next setting sends current through the 100-W filament, and the third setting connects the two filaments in parallel. With the third connection, each filament produces the same power as before—since each is connected to the same potential difference—giving a total power of 50 W + 100 W = 150 W.




**(a)** Replace parallel resistors **(b)** Replace series resistors **(c)** Final equivalent resistance

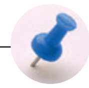
▲ **FIGURE 21–11 Analyzing a complex circuit of resistors**

**(a)** The two vertical resistors are in parallel with one another; hence, they can be replaced with their equivalent resistance, $R/2$. **(b)** Now the circuit consists of three resistors in series. The equivalent resistance of these three resistors is 2.5 $R$. **(c)** The original circuit reduced to a single equivalent resistance.

## Combination Circuits

The rules we have developed for series and parallel resistors can be applied to more complex circuits as well. For example, consider the circuit shown in **Figure 21–11 (a)**, where four resistors, each equal to $R$, are connected in a way that combines series and parallel features. To analyze this circuit, we first note that the two vertically oriented resistors are connected in parallel with one another. Therefore, the equivalent resistance of this unit is given by $1/R_{eq} = 1/R + 1/R$, or $R_{eq} = R/2$. Replacing these two resistors with $R/2$ yields the circuit shown in **Figure 21–11 (b)**, which consists of three resistors in series. As a result, the equivalent resistance of the entire circuit is $R + R/2 + R = 2.5R$, as indicated in **Figure 21–11 (c)**. Similar methods can be applied to a wide variety of circuits.

**PROBLEM-SOLVING NOTE**



**Analyzing a Complex Circuit**

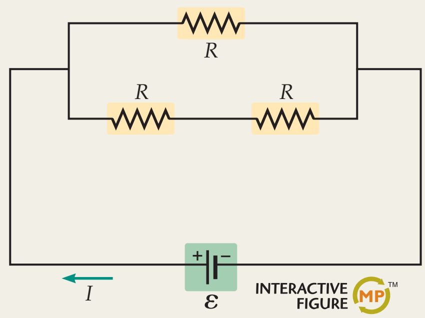
When considering an electric circuit with resistors in series and parallel, work from the smallest units of the circuit outward to ever larger units.

## EXAMPLE 21–7 COMBINATION SPECIAL

In the circuit shown in the diagram, the emf of the battery is 12.0 V, and each resistor has a resistance of 200.0 Ω. Find **(a)** the current supplied by the battery to this circuit and **(b)** the current through the lower two resistors.

**PICTURE THE PROBLEM**

The circuit for this problem has three resistors connected to a battery. Note that the lower two resistors are in series with one another, and in parallel with the upper resistor. The battery has an emf of 12.0 V.




**STRATEGY**

**a.** The current supplied by the battery, $I$, is given by Ohm's law, $I = \mathcal{E}/R_{eq}$, where $R_{eq}$ is the equivalent resistance of the three resistors. To find $R_{eq}$, we first note that the lower two resistors are in series, giving a net resistance of $2R$. Next, the upper resistor, $R$, is in parallel with $2R$. Calculating this equivalent resistance yields the desired $R_{eq}$.

**b.** Because the voltage across the lower two resistors is $\mathcal{E}$, the current through them is $I_{lower} = \mathcal{E}/R_{eq,lower} = \mathcal{E}/2R$.

**SOLUTION**

**Part (a)**

1. Calculate the equivalent resistance of the lower two resistors:

$$R_{eq,lower} = R + R = 2R$$

2. Calculate the equivalent resistance of $R$ in parallel with $2R$:

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$

$$R_{eq} = \tfrac{2}{3}R = \tfrac{2}{3}(200.0\ \Omega) = 133.3\ \Omega$$

3. Find the current supplied by the battery, $I$:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{12.0\ \text{V}}{133.3\ \Omega} = 0.0900\ \text{A}$$

**Part (b)**

4. Use $\mathcal{E}$ and $R_{eq,lower}$ to find the current in the lower two resistors:

$$I_{lower} = \frac{\mathcal{E}}{R_{eq,lower}} = \frac{12.0\ \text{V}}{2(200.0\ \Omega)} = 0.0300\ \text{A}$$

**INSIGHT**

Note that the total resistance of the three 200.0-Ω resistors is less than 200.0 Ω—in fact, it is only 133.3 Ω. We also see that 0.0300 A flows through the lower two resistors, and therefore twice that much—0.0600 A—flows through the upper resistor.

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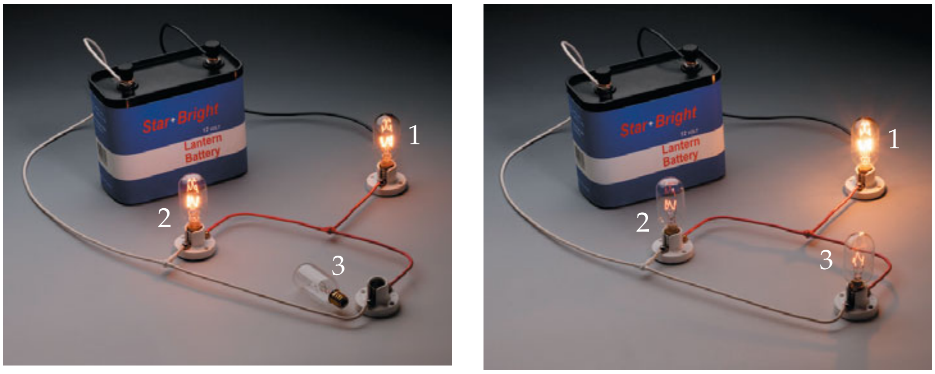
**PRACTICE PROBLEM**

Suppose the upper resistor is changed from $R$ to $2R$, and the lower two resistors remain the same. **(a)** Will the current supplied by the battery increase, decrease, or stay the same? **(b)** Find the new current. [**Answer:** (a) The current will decrease because there is greater resistance to its flow; (b) 0.0600 A.]

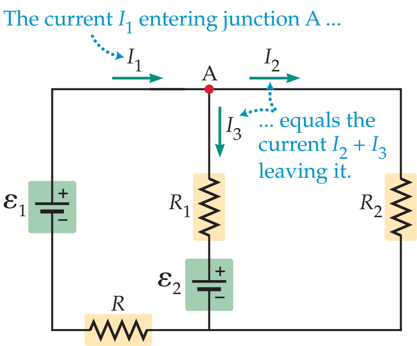
*Some related homework problems: Problem 48, Problem 49, Problem 51*




▶ The electric circuit in these photos starts with two identical lightbulbs (1 and 2) in series with a battery, as we see on the left. The bulbs are equally bright. Now, *before you examine the photo to the right*, consider the effect of adding a third identical bulb (3) to the circuit by placing it in the empty socket. What happens to the brightness of bulbs 1 and 2? As you can see, adding bulb 3 creates a new path for the current and increases the total current in the circuit by a factor of 4/3 (check this yourself). The current passing through bulb 1 is equally split between bulbs 2 and 3, however, and the new current in bulb 2 is now only $\frac{1}{2}(4/3) = 2/3$ of its original value. Thus, bulb 1 brightens and bulb 2 becomes dimmer.

## 21–5 Kirchhoff's Rules

To find the currents and voltages in a general electric circuit, we use two rules first introduced by the German physicist Gustav Kirchhoff (1824–1887). The *Kirchhoff rules* are simply ways of expressing charge conservation (the junction rule) and energy conservation (the loop rule) in a closed circuit. Since these conservation laws are always obeyed in nature, the Kirchhoff rules are completely general.




▲ **FIGURE 21–12 Kirchhoff's junction rule**
Kirchhoff's junction rule states that the sum of the currents entering a junction must equal the sum of the currents leaving the junction. In this case, for the junction labeled A, $I_1 = I_2 + I_3$, or $I_1 - I_2 - I_3 = 0$.

### The Junction Rule

The junction rule follows from the observation that the current entering any point in a circuit must equal the current leaving that point. If this were not the case, charge would either build up or disappear from a circuit.

As an example, consider the circuit shown in **Figure 21–12**. At point A, three wires join to form a **junction.** (In general, a *junction* is any point in a circuit where three or more wires meet.) The current carried by each of the three wires is indicated in the figure. Notice that the current entering the junction is $I_1$; the current leaving the junction is $I_2 + I_3$. Setting the incoming and outgoing currents equal, we have $I_1 = I_2 + I_3$, or equivalently

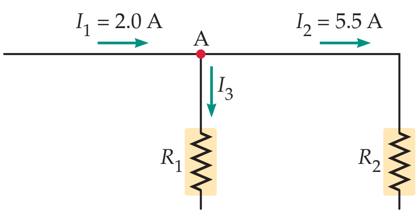
$$I_1 - I_2 - I_3 = 0$$

This is Kirchhoff's junction rule applied to the junction at point A.

In general, if we associate a + sign with currents entering a junction and a − sign with currents leaving a junction, Kirchhoff's junction rule can be stated as follows:

> The algebraic sum of all currents meeting at any junction in a circuit must equal zero.

In the example just discussed, $I_1$ enters the junction (+), $I_2$ and $I_3$ leave the junction (−); hence, the algebraic sum of currents at the junction is $I_1 - I_2 - I_3$. Setting this sum equal to zero recovers our previous result.




▲ **FIGURE 21–13 A specific application of Kirchhoff's junction rule**
Applying Kirchhoff's junction rule to the junction A, $I_1 - I_2 - I_3 = 0$, yields the result $I_3 = -3.5$ A. The minus sign indicates that $I_3$ flows opposite to the direction shown; that is, $I_3$ is actually upward.

In some cases we may not know the direction of all the currents meeting at a junction in advance. When this happens, we simply choose a direction for the unknown currents, apply the junction rule, and continue as usual. If the value we obtain for a given current is negative, it simply means that the direction we chose was wrong; the current actually flows in the opposite direction.

For example, suppose we know both the direction and magnitude of the currents $I_1$ and $I_2$ in **Figure 21–13**. To find the third current, we apply the junction rule—but first we must choose a direction for $I_3$. If we choose $I_3$ to point downward, as shown in the figure, the junction rule gives

$$I_1 - I_2 - I_3 = 0$$

Solving for $I_3$, we have

$$I_3 = I_1 - I_2 = 2.0\text{ A} - 5.5\text{ A} = -3.5\text{ A}$$

Since $I_3$ is negative, we conclude that the actual direction of this current is upward; that is, the 2.0-A current and the 3.5-A current enter the junction and combine to yield the 5.5-A current that leaves the junction.

## The Loop Rule

Imagine taking a day hike on a mountain path. First, you gain altitude to reach a scenic viewpoint; later you descend below your starting point into a valley; finally, you gain altitude again and return to the trailhead. During the hike you sometimes increase your gravitational potential energy, and sometimes you decrease it, but the net change at the end of the hike is zero—after all, you return to the same altitude from which you started. Kirchhoff's loop rule is an application of the same idea to an electric circuit.

For example, consider the simple circuit shown in **Figure 21–14**. The electric potential increases by the amount $\mathcal{E}$ in going from point A to point B, since we move from the low-potential $(-)$ terminal of the battery to the high-potential $(+)$ terminal. This is like gaining altitude in the hiking analogy. Next, there is no potential change as we go from point B to point C, since these points are connected by an ideal wire. As we move from point C to point D, however, the potential does change—recall that a potential difference is required to force a current through a resistor. We label the potential difference across the resistor $\Delta V_{CD}$. Finally, there is no change in potential between points D and A, since they too are connected by an ideal wire.

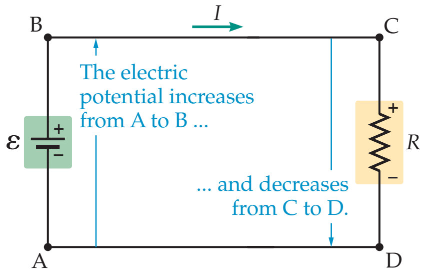



**FIGURE 21–14 Kirchhoff's loop rule** Kirchhoff's loop rule states that as one moves around a closed loop in a circuit, the algebraic sum of all potential differences must be zero. The electric potential increases as one moves from the $-$ to the $+$ plate of a battery; it decreases as one moves through a resistor in the direction of the current.

We can now apply the idea that the net change in electric potential (the analog to gravitational potential energy in the hike) must be zero around any closed loop. In this case, we have

$$\mathcal{E} + \Delta V_{CD} = 0$$

Thus, we find that $\Delta V_{CD} = -\mathcal{E}$; that is, the electric potential *decreases* as one moves across the resistor *in the direction of the current*. To indicate this drop in potential, we label the side where the current enters the resistor with a $+$ (indicating high potential) and the side where the current leaves the resistor with a $-$ (indicating low potential). Finally, we can use Ohm's law to set the magnitude of the potential drop equal to $IR$ and find the current in the circuit:

$$|\Delta V_{CD}| = \mathcal{E} = IR$$

$$I = \frac{\mathcal{E}}{R}$$

This, of course, is the expected result.

In general, Kirchhoff's loop rule can be stated as follows:

> The algebraic sum of all potential differences around any closed loop in a circuit is zero.

We now consider a variety of applications in which both the junction rule and the loop rule are used to find the various currents and potentials in a circuit.

**PROBLEM-SOLVING NOTE**

**Applying Kirchhoff's Rules**

When applying Kirchhoff's rules, be sure to use the appropriate sign for currents and potential differences.

## Applications

We begin by considering the relatively simple circuit shown in **Figure 21–15**. The currents and voltages in this circuit can be found by considering various parallel and series combinations of the resistors, as we did in the previous section. Thus, Kirchhoff's rules are not strictly needed in this case. Still, applying the rules to this circuit illustrates many of the techniques that can be used when studying more complex circuits.

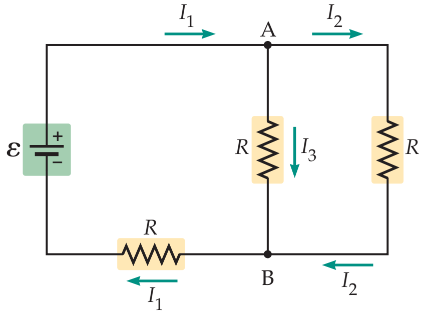
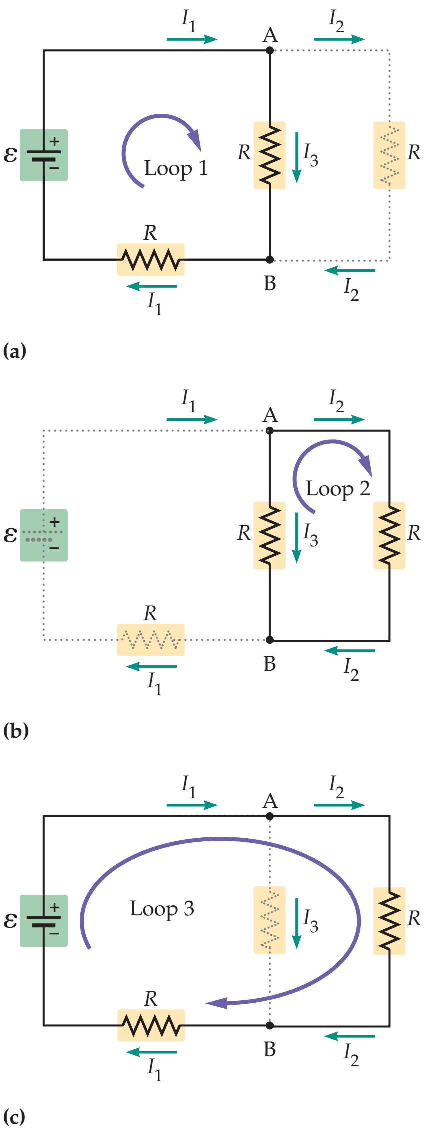



**FIGURE 21–15 Analyzing a simple circuit** A simple circuit that can be studied using either equivalent resistance or Kirchhoff's rules.




▲ **FIGURE 21–16 Using loops to analyze a circuit**

Three loops associated with the circuit in Figure 21–15.

Let's suppose that all the resistors have the value $R = 100.0\ \Omega$, and that the emf of the battery is $\mathcal{E} = 15.0$ V. The equivalent resistance of the resistors can be obtained by noting that the vertical resistors are connected in parallel with one another and in series with the horizontal resistor. The vertical resistors combine to give a resistance of $R/2$, which, when added to the horizontal resistor, gives an equivalent resistance of $R_{eq} = 3R/2 = 150.0\ \Omega$. The current in the circuit, then, is $I = \mathcal{E}/R_{eq} = 15.0\ \text{V}/150.0\ \Omega = 0.100$ A.

Now we approach the same problem from the point of view of Kirchhoff's rules. First, we apply the junction rule to point A:

$$I_1 - I_2 - I_3 = 0 \qquad \text{(junction A)} \qquad 21\text{–}11$$

Note that current $I_1$ splits at point A into currents $I_2$ and $I_3$, which combine again at point B to give $I_1$ flowing through the horizontal resistor. We can apply the junction rule to point B, which gives $-I_1 + I_2 + I_3 = 0$, but since this differs from Equation 21–11 by only a minus sign, no new information is gained.

Next, we apply the loop rule. Since there are three unknowns, $I_1$, $I_2$, and $I_3$, we need three independent equations for a full solution. One has already been given by the junction rule; thus, we expect that two loop equations will be required to complete the solution. To begin, we consider loop 1, which is shown in **Figure 21–16 (a)**. We choose to move around this loop in the clockwise direction. (If we were to choose the counterclockwise direction instead, the same information would be obtained.) For loop 1, then, we have an increase in potential as we move across the battery, a drop in potential across the vertical resistor of $I_3R$, and another drop in potential across the horizontal resistor, this time of magnitude $I_1R$. Applying the loop rule, we find the following:

$$\mathcal{E} - I_3R - I_1R = 0 \qquad \text{(loop 1)} \qquad 21\text{–}12$$

Similarly, we can apply the loop rule to loop 2, shown in **Figure 21–16 (b)**. In this case we cross the right-hand vertical resistor in the direction of the current, implying a drop in potential, and we cross the left-hand vertical resistor against the current, implying an increase in potential. Therefore, the loop rule gives

$$I_3R - I_2R = 0 \qquad \text{(loop 2)} \qquad 21\text{–}13$$

There is a third possible loop, shown in **Figure 21–16 (c)**, but the information it gives is not different from that already obtained. In fact, *any two of the three loops* complete our solution.

Note that $R$ cancels in Equation 21–13; hence, we see that $I_3 - I_2 = 0$, or $I_3 = I_2$. Substituting this result into the junction rule (Equation 21–11), we obtain

$$\begin{aligned} I_1 - I_2 - I_3 &= I_1 - I_2 - I_2 \\ &= I_1 - 2I_2 = 0 \end{aligned}$$

Solving this equation for $I_2$ gives us $I_2 = I_1/2 = I_3$. Finally, using the first loop equation (Equation 21–12), we find

$$\mathcal{E} - (I_1/2)R - I_1R = \mathcal{E} - \tfrac{3}{2}I_1R = 0$$

Note that the only unknown in this equation is current $I_1$. Solving for this current, we find

$$I_1 = \frac{\mathcal{E}}{\frac{3}{2}R} = \frac{15.0\ \text{V}}{\frac{3}{2}(100.0\ \Omega)} = 0.100\ \text{A}$$

As expected, our result using Kirchhoff's rules agrees with the result obtained previously. Finally, the other two currents in the circuit are $I_2 = I_3 = I_1/2 = 0.0500$ A.

## EXERCISE 21–3

Write the loop equation for loop 3 in Figure 21–16 (c).

**SOLUTION**

Proceeding in a clockwise direction, as indicated in the figure, we find

$$\mathcal{E} - I_2R - I_1R = 0$$

Since $I_2$ and $I_3$ are equal (loop 2), it follows that loop 1 ($\mathcal{E} - I_3R - I_1R = 0$) and loop 3 ($\mathcal{E} - I_2R - I_1R = 0$) give the same information. If we proceed in a counterclockwise direction around loop 3, we find

$$-\mathcal{E} + I_2R + I_1R = 0$$

Notice that this result is the same as the clockwise result except for an overall minus sign, and, therefore, it contains no new information. In general, it does not matter in which direction we choose to go around a loop.

---

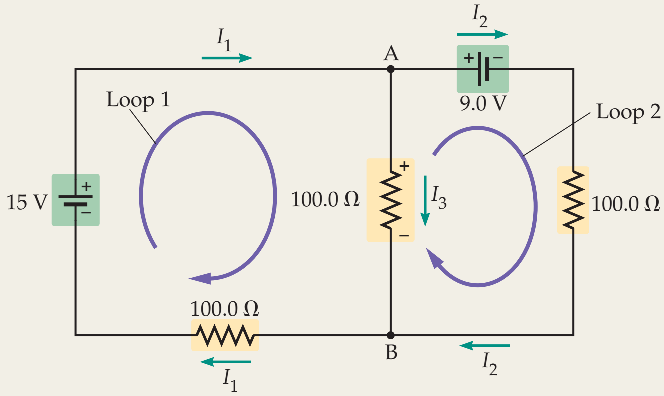
Clearly, the Kirchhoff approach is more involved than the equivalent-resistance method. However, it is not possible to analyze all circuits in terms of equivalent resistances. In such cases, Kirchhoff's rules are the only option, as illustrated in the next Active Example.

**ACTIVE EXAMPLE 21–2 TWO LOOPS, TWO BATTERIES: FIND THE CURRENTS**

Find the currents in the circuit shown.

**SOLUTION** *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply the junction rule to point A: $I_1 - I_2 - I_3 = 0$
2. Apply the loop rule to loop 1 (let $R = 100.0\ \Omega$): $15\ \text{V} - I_3R - I_1R = 0$
3. Apply the loop rule to loop 2 (let $R = 100.0\ \Omega$): $-9.0\ \text{V} - I_2R + I_3R = 0$
4. Solve for $I_1$, $I_2$, and $I_3$: $I_1 = 0.070\ \text{A}$, $I_2 = -0.010\ \text{A}$, $I_3 = 0.080\ \text{A}$




**INSIGHT**

Note that $I_2$ is negative. This means that its direction is opposite to that shown in the circuit diagram.

**YOUR TURN**

Suppose the polarity of the 9.0-V battery is reversed. What are the currents in this case?

*(Answers to* **Your Turn** *problems are given in the back of the book.)*

## 21–6 Circuits Containing Capacitors

To this point we have considered only resistors and batteries in electric circuits. Capacitors, which can also play an important role, are represented by a set of parallel lines (reminiscent of a parallel-plate capacitor): ⊣⊢ . We now investigate simple circuits involving batteries and capacitors, leaving for the next section circuits that combine all three circuit elements.

### Capacitors in Parallel

The simplest way to combine capacitors, as we shall see, is by connecting them in parallel. For example, **Figure 21–17 (a)** shows three capacitors connected in parallel with a battery of emf $\varepsilon$. As a result, each capacitor has the same potential difference, $\varepsilon$, between its plates. The magnitudes of the charges on each capacitor are as follows:

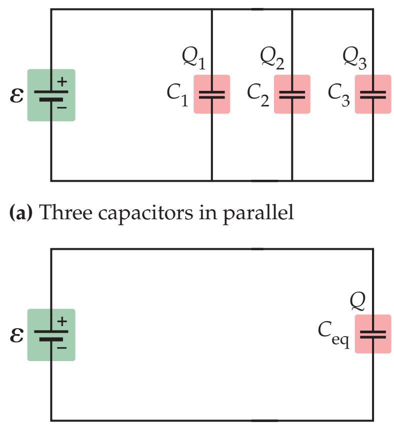
$$Q_1 = C_1\mathcal{E}, \qquad Q_2 = C_2\mathcal{E}, \qquad Q_3 = C_3\mathcal{E}$$

As a result, the total charge on the three capacitors is

$$Q = Q_1 + Q_2 + Q_3 = \mathcal{E}C_1 + \mathcal{E}C_2 + \mathcal{E}C_3 = (C_1 + C_2 + C_3)\mathcal{E}$$

If an equivalent capacitor is used to replace the three in parallel, as in **Figure 21–17 (b)**, the charge on its plates must be the same as the total charge on the individual capacitors:

$$Q = C_{eq}\mathcal{E}$$




**(a)** Three capacitors in parallel

**(b)** Equivalent capacitance with same total charge

▲ **FIGURE 21–17 Capacitors in parallel**
**(a)** Three capacitors, $C_1$, $C_2$, and $C_3$, connected in parallel. Note that each capacitor is connected across the same potential difference, $\mathcal{E}$. **(b)** The equivalent capacitance, $C_{eq} = C_1 + C_2 + C_3$, has the same charge on its plates as the total charge on the three original capacitors.

Comparing $Q = C_{eq}\mathcal{E}$ with $Q = (C_1 + C_2 + C_3)\mathcal{E}$, we see that the equivalent capacitance is simply

$$C_{eq} = C_1 + C_2 + C_3$$

In general, the equivalent capacitance of capacitors connected in parallel is the sum of the individual capacitances:

**Equivalent Capacitance for Capacitors in Parallel**

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C \qquad \text{21–14}$$

SI unit: farad, F

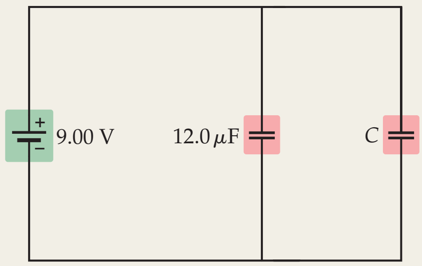
Thus, connecting capacitors in parallel produces an equivalent capacitance greater than the greatest individual capacitance. It is as if the plates of the individual capacitors are connected together to give one large set of plates, with a correspondingly large capacitance.

## EXAMPLE 21–8 ENERGY IN PARALLEL

Two capacitors, one 12.0 $\mu$F and the other of unknown capacitance $C$, are connected in parallel across a battery with an emf of 9.00 V. The total energy stored in the two capacitors is 0.0115 J. What is the value of the capacitance $C$?

**PICTURE THE PROBLEM**

The circuit, consisting of one 9.00-V battery and two capacitors, is illustrated in the diagram. The total energy of 0.0115 J stored in the two capacitors is the same as the energy stored in the equivalent capacitance for this circuit.




**STRATEGY**

Recall from Chapter 20 that the energy stored in a capacitor can be written as $U = \frac{1}{2}CV^2$. It follows, then, that for an equivalent capacitance, $C_{eq}$, the energy is $U = \frac{1}{2}C_{eq}V^2$. Since we know the energy and voltage, we can solve this relation for the equivalent capacitance. Finally, the equivalent capacitance is the sum of the individual capacitances, $C_{eq} = 12.0\ \mu\text{F} + C$. We use this relation to solve for $C$.

**SOLUTION**

1. Solve $U = \frac{1}{2}C_{eq}V^2$ for the equivalent capacitance:

$$U = \tfrac{1}{2}C_{eq}V^2$$

$$C_{eq} = \frac{2U}{V^2}$$

2. Substitute numerical values to find $C_{eq}$:

$$C_{eq} = \frac{2U}{V^2} = \frac{2(0.0115\ \text{J})}{(9.00\ \text{V})^2} = 284\ \mu\text{F}$$

3. Solve for $C$ in terms of the equivalent capacitance:

$$C_{eq} = 12.0\ \mu\text{F} + C$$

$$C = C_{eq} - 12.0\ \mu\text{F} = 284\ \mu\text{F} - 12.0\ \mu\text{F} = 272\ \mu\text{F}$$

**INSIGHT**

The energy stored in the 12.0-$\mu$F capacitor is $U = \frac{1}{2}CV^2 = 0.000486$ J. In comparison, the 272-$\mu$F capacitor stores an energy equal to 0.0110 J. Thus, the larger capacitor stores the greater amount of energy. Though this may seem only natural, one needs to be careful. When we examine capacitors in *series* later in this section, we shall find exactly the opposite result.

**PRACTICE PROBLEM**

What is the total charge stored on the two capacitors? [**Answer:** $Q = C_{eq}\mathcal{E} = 2.56 \times 10^{-3}$ C]

*Some related homework problems: Problem 72, Problem 73*

**REAL-WORLD PHYSICS**

**"Touch-sensitive" lamps**

Although you probably haven't realized it, when you turn on a "touch sensitive" lamp, you are part of a circuit with capacitors in parallel. In fact, you are one of the capacitors! When you touch such a lamp, a small amount of charge moves onto your body—your body is like the plate of a capacitor. Because you have effectively increased the plate area—as always happens when capacitors are connected in parallel—the capacitance of the circuit increases. The electronic circuitry in the lamp senses this increase in capacitance and triggers the switch to turn the light on or off.

## Capacitors in Series

You have probably noticed from Equation 21–14 that capacitors connected in *parallel* combine in the same way as resistors connected in *series*. Similarly, capacitors connected in *series* obey the same rules as resistors connected in *parallel*, as we now show.

Consider three capacitors—initially uncharged—connected in series with a battery, as in **Figure 21–18 (a)**. The battery causes the left plate of $C_1$ to acquire a positive charge, $+Q$. This charge, in turn, attracts a negative charge $-Q$ onto the right plate of the capacitor. Because the capacitors start out uncharged, there is zero net charge between $C_1$ and $C_2$. As a result, the negative charge $-Q$ on the right plate of $C_1$ leaves a corresponding positive charge $+Q$ on the upper plate of $C_2$. The charge $+Q$ on the upper plate of $C_2$ attracts a negative charge $-Q$ onto its lower plate, leaving a corresponding positive charge $+Q$ on the right plate of $C_3$. Finally, the positive charge on the right plate of $C_3$ attracts a negative charge $-Q$ onto its left plate. The result is that all three capacitors have charge of the same magnitude on their plates.

With the same charge $Q$ on all the capacitors, the potential difference for each is as follows:

$$V_1 = \frac{Q}{C_1}, \qquad V_2 = \frac{Q}{C_2}, \qquad V_3 = \frac{Q}{C_3}$$

Since the total potential difference across the three capacitors must equal the emf of the battery, we have

$$\mathcal{E} = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad \text{21–15}$$

An equivalent capacitor connected to the same battery, as in **Figure 21–18 (b)**, will satisfy the relation $Q = C_{eq}\mathcal{E}$, or

$$\mathcal{E} = Q\left(\frac{1}{C_{eq}}\right) \qquad \text{21–16}$$

A comparison of Equations 21–15 and 21–16 yields the result

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus, in general, we have the following rule for combining capacitors in series:

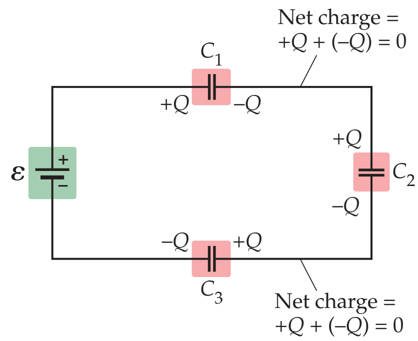
**Equivalent Capacitance for Capacitors in Series**

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C} \qquad \text{21–17}$$

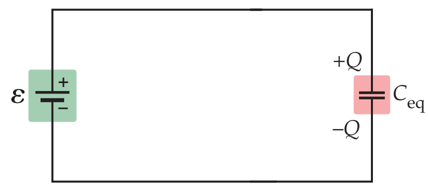
SI unit: farad, F

It follows, then, that the equivalent capacitance of a group of capacitors connected in series is less than the smallest individual capacitance. In this case, it is as if the plate separations of the individual capacitors add to give a larger effective separation, and a correspondingly smaller capacitance.

More complex circuits, with some capacitors in series and others in parallel, can be handled in the same way as was done earlier with resistors. This is illustrated in the following Active Example.




**(a)** Three capacitors in series




**(b)** Equivalent capacitance with same total charge

▲ **FIGURE 21–18 Capacitors in series**
**(a)** Three capacitors, $C_1$, $C_2$, and $C_3$, connected in series. Note that each capacitor has the same magnitude charge on its plates. **(b)** The equivalent capacitance, $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$, has the same charge as the original capacitors.

**PROBLEM-SOLVING NOTE**

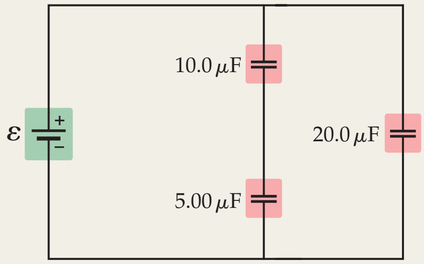
**Finding the Equivalent Capacitance of a Circuit**

When calculating the equivalent capacitance of capacitors in series, be sure to take one final inverse at the end of your calculation to find $C_{eq}$. Also, when considering circuits with capacitors in both series and parallel, start with the smallest units of the circuit and work your way out to the larger units.

## ACTIVE EXAMPLE 21–3 FIND THE EQUIVALENT CAPACITANCE AND THE STORED ENERGY

Consider the electric circuit shown here, consisting of a 12.0-V battery and three capacitors connected partly in series and partly in parallel. Find **(a)** the equivalent capacitance of this circuit and **(b)** the total energy stored in the capacitors.

**SOLUTION** *(Test your understanding by performing the calculations indicated in each step.)*




**Part (a)**

1. Find the equivalent capacitance of a 10.0-$\mu$F capacitor in series with a 5.00-$\mu$F capacitor: 3.33 $\mu$F
2. Find the equivalent capacitance of a 3.33-$\mu$F capacitor in parallel with a 20.0-$\mu$F capacitor: $C_{eq} = 23.3\ \mu F$

**Part (b)**

3. Calculate the stored energy using $U = \frac{1}{2}C_{eq}V^2$: $U = 1.68 \times 10^{-3}$ J

**INSIGHT**

Notice that the 10.0-$\mu$F capacitor and the 5.00-$\mu$F capacitor are connected in series. As you might expect, one of these capacitors stores twice as much energy as the other. Which is it? Check the Your Turn question for the answer.

**YOUR TURN**

Is the energy stored in the 10.0-$\mu$F capacitor greater than or less than the energy stored in the 5.0-$\mu$F capacitor? Explain. Check your answer by calculating the energy stored in each of the capacitors.

*(Answers to* **Your Turn** *problems are given in the back of the book.)*

## 21–7 *RC* Circuits

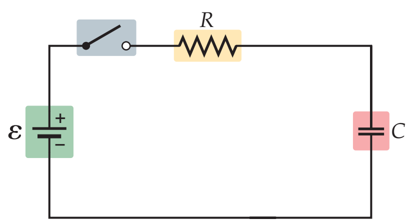
When the switch is closed on a circuit containing only batteries and capacitors, the charge on the capacitor plates appears almost instantaneously—essentially at the speed of light. This is not the case, however, in circuits that also contain resistors. In these situations, the resistors limit the rate at which charge can flow, and an appreciable amount of time may be required before the capacitors acquire a significant charge. A useful analogy is the amount of time needed to fill a bucket with water. If you use a fire hose, which has little resistance to the flow of water, the bucket fills almost instantly. If you use a garden hose, which presents a much greater resistance to the water, filling the bucket may take a minute or more.

The simplest example of such a circuit, a so-called ***RC* circuit,** is shown in **Figure 21–19**. Initially (before $t = 0$) the switch is open, and there is no current in the resistor or charge on the capacitor. At $t = 0$ the switch is closed and current begins to flow. If the resistor was not present, the capacitor would immediately take on the charge $Q = C\mathcal{E}$. The effect of the resistor, however, is to slow the charging process—in fact, the larger the resistance, the longer it takes for the capacitor to charge. One way to think of this is to note that as long as a current flows in the circuit, as in Figure 21–19 (b), there is a potential drop across the resistor; hence, the potential difference between the plates of the capacitor is less than the emf of the battery. With less voltage across the capacitor there will be less charge on its plates compared with the charge that would result if the plates were connected directly to the battery.

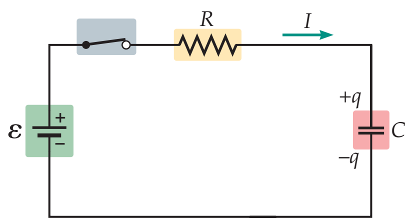
The methods of calculus can be used to show that the charge on the capacitor in Figure 21–19 varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$

In this expression, $e$ is Euler's number ($e = 2.718\ldots$) or, more precisely, the base of natural logarithms (see Appendix A). The quantity $\tau$ is referred to as the **time constant** of the circuit. The time constant is related to the resistance and capacitance of a circuit by the following simple relation: $\tau = RC$. As we shall see, $\tau$ can be thought of as a characteristic time for the behavior of an $RC$ circuit.




**(a)** $t < 0$




**(b)** $t > 0$

▲ **FIGURE 21–19 A typical *RC* circuit**
**(a)** Before the switch is closed ($t < 0$) there is no current in the circuit and no charge on the capacitor. **(b)** After the switch is closed ($t > 0$), current flows and the charge on the capacitor builds up over a finite time. As $t \to \infty$ the charge on the capacitor approaches $Q = C\mathcal{E}$.

For example, at time $t = 0$ the exponential term is $e^{-0/\tau} = e^0 = 1$; therefore, the charge on the capacitor is zero at $t = 0$, as expected:

$$q(0) = C\mathcal{E}(1 - 1) = 0$$

In the opposite limit, $t \to \infty$, the exponential vanishes: $e^{-\infty/\tau} = 0$. Thus the charge in this limit is $C\mathcal{E}$:

$$q(t \to \infty) = C\mathcal{E}(1 - 0) = C\mathcal{E}$$

This is just the charge $Q$ the capacitor would have had from $t = 0$ on if there had been no resistor in the circuit. Finally, at time $t = \tau$ the charge on the capacitor is $q = C\mathcal{E}(1 - e^{-1}) = C\mathcal{E}(1 - 0.368) = 0.632C\mathcal{E}$, which is 63.2% of its final charge. The charge on the capacitor as a function of time is plotted in **Figure 21–20**.

Before we continue, let's check to see that the quantity $\tau = RC$ is in fact a time. Suppose, for example, that the resistor and capacitor in an $RC$ circuit have the values $R = 120\ \Omega$ and $C = 3.5\ \mu\text{F}$, respectively. Multiplying $R$ and $C$ we find

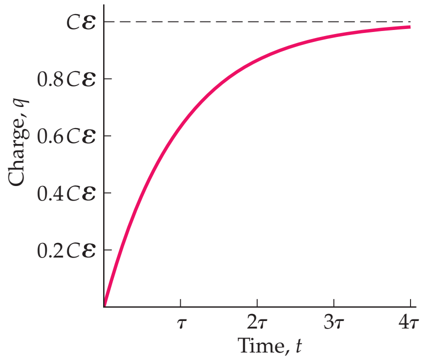
$$\tau = RC = (120\ \text{ohm})(3.5 \times 10^{-6}\ \text{farad})$$
$$= \left(\frac{120\ \text{volt}}{\text{coulomb/second}}\right)\left(\frac{3.5 \times 10^{-6}\ \text{coulomb}}{\text{volt}}\right) = 4.2 \times 10^{-4}\ \text{second}$$

The tick marks on the horizontal axis in Figure 21–20 indicate the times $\tau$, $2\tau$, $3\tau$, and $4\tau$. Notice that the capacitor is almost completely charged by the time $t = 4\tau$.

Figure 21–20 also shows that the charge on the capacitor increases rapidly initially, indicating a large current in the circuit. Eventually, the charging slows down, because the greater the charge on the capacitor, the harder it is to transfer additional charge against the electrical repulsive force. Later, the charge barely changes with time, which means that the current is essentially zero. In fact, the mathematical expression for the current—again derived from calculus—is the following:

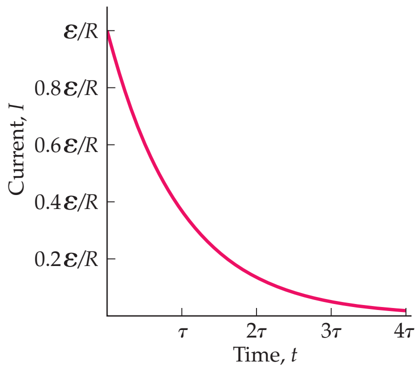
$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad \text{21–19}$$

This expression is plotted in **Figure 21–21**, where we see that significant variation in the current occurs over times ranging from $t = 0$ to $t \sim 4\tau$. At time $t = 0$ the current is $I(0) = \mathcal{E}/R$, which is the value it would have if the capacitor were replaced by an ideal wire. As $t \to \infty$, the current approaches zero, as expected: $I(t \to \infty) \to 0$. In this limit, the capacitor is essentially fully charged, so that no more charge can flow onto its plates. Thus, in this limit, the capacitor behaves like an open switch.




▲ **FIGURE 21–20 Charge versus time for the *RC* circuit in Figure 21–19**

The horizontal axis shows time in units of the characteristic time, $\tau = RC$. The vertical axis shows the magnitude of the charge on the capacitor in units of $C\mathcal{E}$.




▲ **FIGURE 21–21 Current versus time for the *RC* circuit in Figure 21–19**

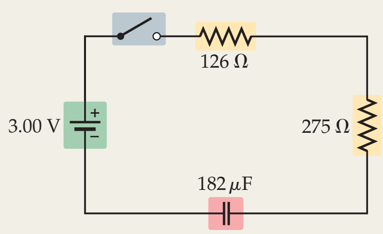
Initially the current is $\mathcal{E}/R$, the same as if the capacitor were not present. The current approaches zero after a period equal to several time constants, $\tau = RC$.

## EXAMPLE 21–9 CHARGING A CAPACITOR

A circuit consists of a 126-Ω resistor, a 275-Ω resistor, a 182-μF capacitor, a switch, and a 3.00-V battery all connected in series. Initially the capacitor is uncharged and the switch is open. At time $t = 0$ the switch is closed. **(a)** What charge will the capacitor have a long time after the switch is closed? **(b)** At what time will the charge on the capacitor be 80.0% of the value found in part (a)?

### PICTURE THE PROBLEM

The circuit described in the problem statement is shown with the switch in the open position. Once the switch is closed at $t = 0$, current will flow in the circuit and charge will begin to accumulate on the capacitor plates.




INTERACTIVE FIGURE

### STRATEGY

**a.** A long time after the switch is closed, the current stops and the capacitor is fully charged. At this point, the voltage across the capacitor is equal to the emf of the battery. Therefore, the charge on the capacitor is $Q = C\mathcal{E}$.

**b.** To find the time when the charge will be 80.0% of the full charge, $Q = C\mathcal{E}$, we can set $q(t) = C\mathcal{E}(1 - e^{-t/\tau}) = 0.800C\mathcal{E}$ and solve for the desired time, $t$.

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**SOLUTION**

**Part (a)**

1. Evaluate $Q = C\mathcal{E}$ for this circuit: $Q = C\mathcal{E} = (182\ \mu F)(3.00\ V) = 546\ \mu C$

**Part (b)**

2. Set $q(t) = 0.800C\mathcal{E}$ in $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and cancel $C\mathcal{E}$:
$q(t) = 0.800C\mathcal{E} = C\mathcal{E}(1 - e^{-t/\tau})$
$0.800 = 1 - e^{-t/\tau}$

3. Solve for $t$ in terms of the time constant $\tau$:
$e^{-t/\tau} = 1 - 0.800 = 0.200$
$t = -\tau \ln(0.200)$

4. Calculate $\tau$ and use the result to find the time $t$:
$\tau = RC = (126\ \Omega + 275\ \Omega)(182\ \mu F) = 73.0\ ms$
$t = -(73.0\ ms)\ln(0.200)$
$= -(73.0\ ms)(-1.61) = 118\ ms$

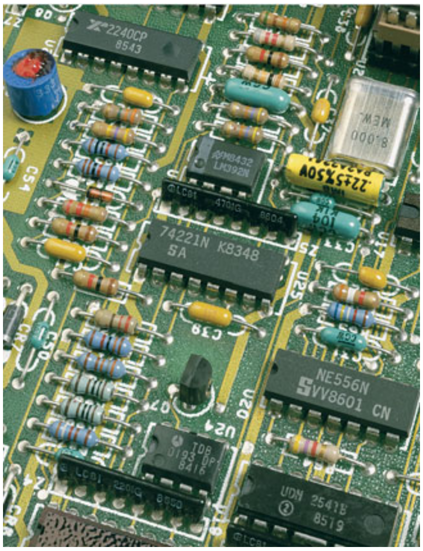
**INSIGHT**

Note that the time required for the charge on a capacitor to reach 80.0% of its final value is 1.61 time constants. This result is independent of the values of $R$ and $C$ in an $RC$ circuit.

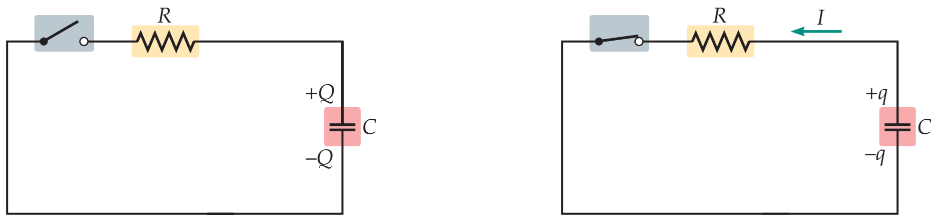
**PRACTICE PROBLEM**

What is the current in this circuit at the time found in part (b)? [**Answer:** $I(t) = (\mathcal{E}/R)e^{-t/\tau} = [(3.00\ V)/(126\ \Omega + 275\ \Omega)](0.200) = (7.48\ mA)(0.200) = 1.50\ mA$]

*Some related homework problems: Problem 79, Problem 82*



▲ A modern-day circuit board incorporates numerous resistors (cylinders with colored bands) and capacitors (yellow cylinders and metal container).




**(a)** $t < 0$ **(b)** $t > 0$

▲ **FIGURE 21–22 Discharging a capacitor**

**(a)** A charged capacitor is connected to a resistor. Initially the circuit is open, and no current can flow. **(b)** When the switch is closed, current flows from the + plate of the capacitor to the − plate. The charge remaining on the capacitor approaches zero after several time units, $RC$.

Similar behavior occurs when a charged capacitor is allowed to discharge, as in **Figure 21–22**. In this case, the initial charge on the capacitor is $Q$. If the switch is closed at $t = 0$, the charge for later times is

$$q(t) = Qe^{-t/\tau} \qquad 21\text{–}20$$

Like charging, the discharging of a capacitor occurs with a characteristic time $\tau = RC$.

To summarize, circuits with resistors and capacitors have the following general characteristics:

- Charging and discharging occur over a finite, characteristic time given by the time constant, $\tau = RC$.
- At $t = 0$ current flows freely through a capacitor being charged; it behaves like a short circuit.
- As $t \to \infty$ the current flowing into a capacitor approaches zero. In this limit, a capacitor behaves like an open switch.

We explore these features further in the following Conceptual Checkpoint.

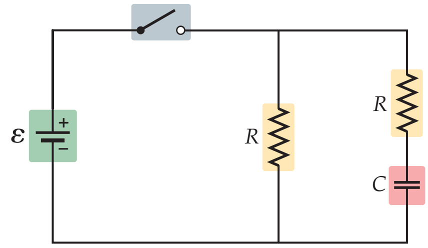
**PROBLEM-SOLVING NOTE**

**The Limiting Behavior of Capacitors**

Capacitors in dc circuits act like short circuits at $t = 0$ and open circuits as $t \to \infty$.

## CONCEPTUAL CHECKPOINT 21–4 CURRENT IN AN *RC* CIRCUIT

What current flows through the battery in this circuit **(a)** immediately after the switch is closed and **(b)** a long time after the switch is closed?




**REASONING AND DISCUSSION**

**a.** Immediately after the switch is closed, the capacitor acts like a short circuit; that is, as if the battery were connected to two resistors $R$ in parallel. The equivalent resistance in this case is $R/2$; therefore, the current is $I = \mathcal{E}/(R/2) = 2\mathcal{E}/R$.

**b.** After current has been flowing in the circuit for a long time, the capacitor acts like an open switch. Now current can flow only through the one resistor, $R$; hence, the current is $I = \mathcal{E}/R$, half of its initial value.

**ANSWER**

**(a)** The current is $2\mathcal{E}/R$; **(b)** the current is $\mathcal{E}/R$.

---

**REAL-WORLD PHYSICS**

**Delay circuits in windshield wipers and turn signals**

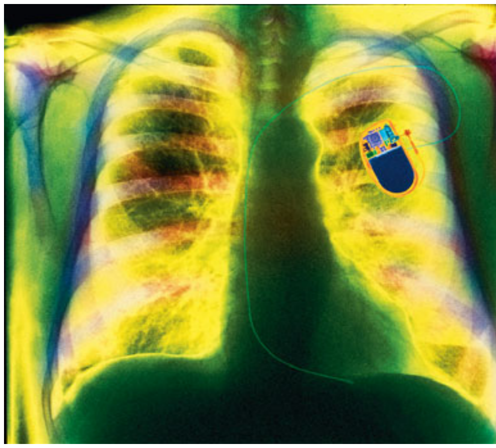
The fact that *RC* circuits have a characteristic time makes them useful in a variety of different applications. On a rather mundane level, *RC* circuits are used to determine the time delay on windshield wipers. When you adjust the delay knob in your car, you change a resistance or a capacitance, which in turn changes the time constant of the circuit. This results in a greater or a smaller delay. The blinking rate of turn signals is also determined by the time constant of an *RC* circuit.

**REAL-WORLD PHYSICS: BIO**

**Pacemakers**

A more critical application of *RC* circuits is the heart pacemaker. In the simplest case, these devices use an *RC* circuit to deliver precisely timed pulses directly to the heart. The more sophisticated pacemakers available today can even "sense" when a patient's heart rate falls below a predetermined value. The pacemaker then begins sending appropriate pulses to the heart to increase its rate. Many pacemakers can even be reprogrammed after they are surgically implanted to respond to changes in a patient's condition.

Normally, the heart's rate of beating is determined by its own natural pacemaker, the sinoatrial or SA node, located in the upper right chamber of the heart. If the SA node is not functioning properly, it may cause the heart to beat slowly or irregularly. To correct the problem, a pacemaker is implanted just under the collarbone, and an electrode is introduced intravenously via the cephalic vein. The distal end of the electrode is positioned, with the aid of fluoroscopic guidance, in the right ventricular apex. From that point on, the operation of the pacemaker follows the basic principles of electric circuits, as described in this chapter.



▲ An X-ray showing a pacemaker installed in a person's chest. The timing of the electrical pulses that keep the heart beating regularly is determined by an *RC* circuit powered by a small, long-lived battery.

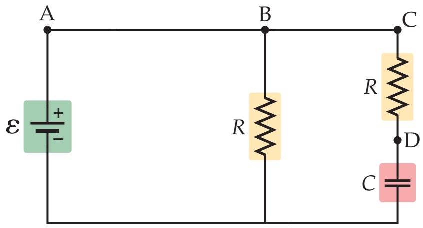
## *21–8 Ammeters and Voltmeters

Devices for measuring currents and voltages in a circuit are referred to as **ammeters** and **voltmeters,** respectively. In each case, the ideal situation is for the meter to measure the desired quantity without altering the characteristics of the circuit being studied. This is accomplished in different ways for these two types of meters, as we shall see.

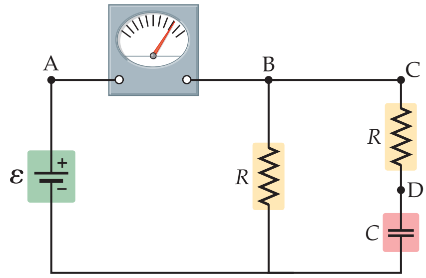
First, the ammeter is designed to measure the flow of current through a particular portion of a circuit. For example, we may want to know the current flowing between points A and B in the circuit shown in **Figure 21–23 (a)**. To measure this current, we insert the ammeter into the circuit in such a way that all the current flowing from A to B must also flow through the meter. This is done by connecting the meter "in series" with the other circuit elements between A and B, as indicated in **Figure 21–23 (b)**.

If the ammeter has a finite resistance—which must be the case for real meters—the presence of the meter in the circuit will alter the current it is intended to measure. Thus, an *ideal* ammeter would be one with zero resistance. In practice, if the resistance of the ammeter is much less than the other resistances in the circuit, its reading will be reasonably accurate.


▲ A typical digital multimeter, which can measure resistance (teal settings), current (yellow settings), or voltage (red settings). This meter is measuring the voltage of a "9 volt" battery.




**(a)** Typical electric circuit



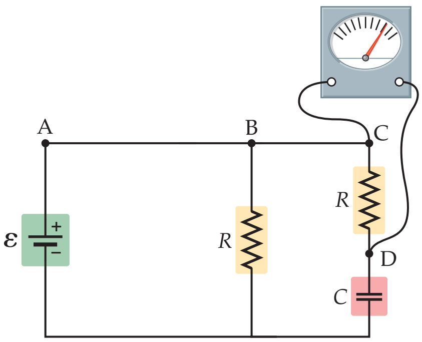

**(b)** Measuring the current between A and B

▲ **FIGURE 21–23 Measuring the current in a circuit**
To measure the current flowing between points A and B in **(a)**, an ammeter is inserted into the circuit, as shown in **(b)**. An ideal ammeter would have zero resistance.

Second, a voltmeter measures the potential drop between any two points in a circuit. Referring again to the circuit in Figure 21–23 (a), we may wish to know the difference in potential between points C and D. To measure this voltage, we connect the voltmeter "in parallel" to the circuit at the appropriate points, as in **Figure 21–24**.

A real voltmeter always allows some current to flow through it, which means that the current flowing through the circuit is less than before the meter was connected. As a result, the measured voltage is altered from its ideal value. An *ideal* voltmeter, then, would be one in which the resistance is infinite, so that the current it draws from the circuit is negligible. In practical situations it is sufficient that the resistance of the meter be much greater than the resistances in the circuit.

Sometimes the functions of an ammeter, voltmeter, and ohmmeter are combined in a single device called a **multimeter.** Adjusting the settings on a multimeter allows a variety of circuit properties to be measured.




**Measuring the voltage between C and D**

◀ **FIGURE 21–24 Measuring the voltage in a circuit**
The voltage difference between points C and D can be measured by connecting a voltmeter in parallel to the original circuit. An ideal voltmeter would have infinite resistance.

## THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

**LOOKING BACK**

The concept of electric potential energy (Chapter 20) is used in Section 21–3, where we talk about the energy associated with an electric circuit.

We also discuss the power of an electric circuit in Section 21–3. For this we refer back to mechanics, where power was originally introduced in Chapter 7.

Capacitors, first introduced in Chapter 20, are used in dc circuits in Section 21–6.

**LOOKING AHEAD**

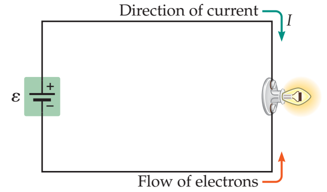
A dc circuit with a current flowing through it will play an important role in our discussion of magnetism in Chapter 22. We will also consider the magnetic force exerted on a current-carrying wire in Chapter 22.

In Chapter 24 we extend our discussion of electric circuits from those in which the current flows in only one direction (dc) to circuits in which the current alternates in direction (ac, or alternating current). We will again use resistors and capacitors in the ac circuits.

A simple dc circuit appears in Chapter 30, where we discuss the photoelectric effect and its importance in the development of quantum mechanics.

# CHAPTER SUMMARY

## 21–1 ELECTRIC CURRENT




Electric current is the flow of electric charge.

**Definition**
If a charge $\Delta Q$ passes a given point in the time $\Delta t$, the corresponding electric current is

$$I = \frac{\Delta Q}{\Delta t} \qquad 21\text{–}1$$

**Ampere**
The unit of current is the ampere, or amp for short. By definition, 1 amp is one coulomb per second; 1 A = 1 C/s.

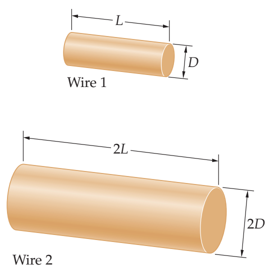
**Battery**
A battery is a device that uses chemical reactions to produce a potential difference between its two terminals.

**Electromotive Force**
The electromotive force, or emf, $\mathcal{E}$, is the potential difference between the terminals of a battery under ideal conditions.

**Work Done by a Battery**
As a battery moves a charge $\Delta Q$ around a circuit, it does the work $W = (\Delta Q)\mathcal{E}$.

**Direction of Current**
By definition, the direction of the current $I$ in a circuit is the direction in which *positive* charges would move. The actual charge carriers, however, are generally electrons; hence, they move in the opposite direction to $I$.

## 21–2 RESISTANCE AND OHM'S LAW




When electrons move through a wire, they encounter resistance to their motion. In order to move electrons against this resistance, it is necessary to apply a potential difference between the ends of the wire.

**Ohm's Law**
To produce a current $I$ through a wire with resistance $R$ the following potential difference, $V$, is required:

$$V = IR \qquad 21\text{–}2$$

**Resistivity**
The resistivity $\rho$ of a material determines how much resistance it gives to the flow of electric current.

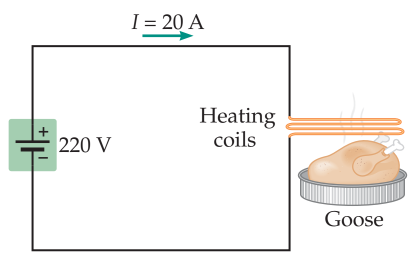
**Resistance of a Wire**
The resistance of a wire of length $L$, cross-sectional area $A$, and resistivity $\rho$ is

$$R = \rho\left(\frac{L}{A}\right) \qquad 21\text{–}3$$

**Temperature Dependence**
The resistivity of most metals increases approximately linearly with temperature.

**Superconductivity**
Below a certain critical temperature, $T_c$, certain materials lose all electrical resistance. A current flowing in a superconductor can continue undiminished as long as its temperature is maintained below $T_c$.

## 21–3 ENERGY AND POWER IN ELECTRIC CIRCUITS




In general, energy is required to cause an electric current to flow through a circuit. The rate at which the energy must be supplied is the power.

**Electrical Power**
If a current $I$ flows across a potential difference $V$, the corresponding electrical power is

$$P = IV \qquad 21\text{–}4$$

**Power Dissipation in a Resistor**
If a potential difference $V$ produces a current $I$ in a resistor $R$, the electrical power converted to heat is

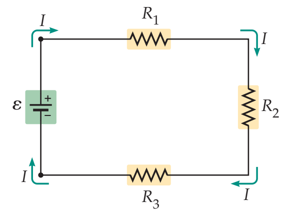
$$P = I^2R = V^2/R \qquad \text{21–5, 21–6}$$

**Energy Usage and the Kilowatt-Hour**
The energy equivalent of one kilowatt-hour (kWh) is

$$1\text{ kWh} = 3.6 \times 10^6\text{ J}$$

## 21–4 RESISTORS IN SERIES AND PARALLEL

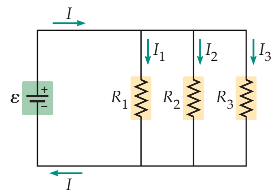
Resistors connected end to end—so that the same current flows through each one—are said to be in series. Resistors connected across the same potential difference—allowing parallel paths for the current to flow—are said to be connected in parallel.




**Series**
The equivalent resistance, $R_{eq}$, of resistors connected in series is equal to the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R \qquad \text{21–7}$$

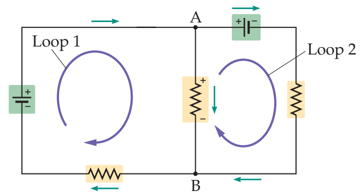
**Parallel**
The equivalent resistance, $R_{eq}$, of resistors connected in parallel is given by the following:




$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R} \qquad \text{21–10}$$

## 21–5 KIRCHHOFF'S RULES

Kirchhoff's rules are statements of charge conservation and energy conservation as applied to closed electric circuits.

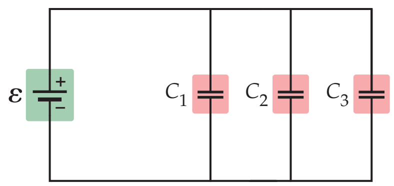



**Junction Rule (Charge Conservation)**
The algebraic sum of all currents meeting at a junction must equal zero. Currents entering the junction are taken to be positive; currents leaving are taken to be negative.

**Loop Rule (Energy Conservation)**
The algebraic sum of all potential differences around a closed loop is zero. The potential increases in going from the − to the + terminal of a battery and decreases when crossing a resistor in the direction of the current.

## 21–6 CIRCUITS CONTAINING CAPACITORS

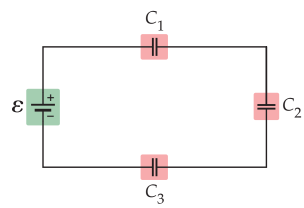
Capacitors connected end to end—so that the same charge is on each one—are said to be in series. Capacitors connected across the same potential difference are said to be connected in parallel.




**Parallel**
The equivalent capacitance, $C_{eq}$, of capacitors connected in parallel is equal to the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C \qquad \text{21–14}$$

**Series**
The equivalent capacitance, $C_{eq}$, of capacitors connected in series is given by




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C} \qquad \text{21–17}$$

## 21–7 *RC* CIRCUITS

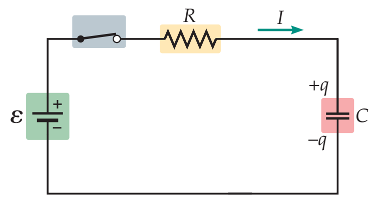
In circuits containing both resistors and capacitors, there is a characteristic time, $\tau = RC$, during which significant changes occur. This time is referred to as the time constant. The simplest such circuit, known as an *RC* circuit, consists of one resistor and one capacitor connected in series.

**Charging a Capacitor**
The charge on a capacitor in an *RC* circuit varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$

The corresponding current is given by

$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad 21\text{–}19$$




**Discharging a Capacitor**
If a capacitor in an *RC* circuit starts with a charge $Q$ at time $t = 0$, its charge at all later times is

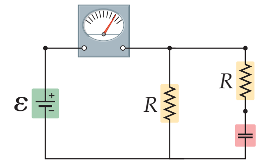
$$q(t) = Qe^{-t/\tau} \qquad 21\text{–}20$$

**Behavior near $t = 0$**
Just after the switch is closed in an *RC* circuit, capacitors behave like ideal wires—that is, they offer no resistance to the flow of current.

**Behavior as $t \to \infty$**
Long after the switch is closed in an *RC* circuit, capacitors behave like open circuits.

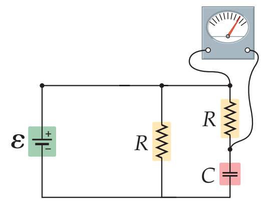
## *21–8 AMMETERS AND VOLTMETERS

Ammeters and voltmeters are devices for measuring currents and voltages, respectively, in electric circuits.




**Ammeter**
An ammeter is connected in series with the section of the circuit in which the current is to be measured. In the ideal case, an ammeter's resistance is zero.

**Voltmeter**
A voltmeter is connected in parallel with the portion of the circuit to be measured. In the ideal case, a voltmeter's resistance is infinite.




## PROBLEM-SOLVING SUMMARY

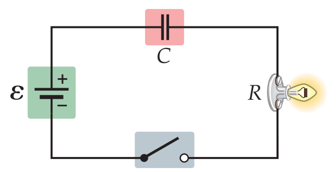
| Type of Problem | Relevant Physical Concepts | Related Examples |
|---|---|---|
| Find the work done by a battery. | The work done by a battery is the charge that passes through the battery times the emf of the battery: $W = \Delta Q\mathcal{E}$. | Active Example 21–1 |
| Relate resistance to resistivity. | The resistance of a wire is its resistivity, $\rho$, times its length, divided by its cross-sectional area: $R = \rho(L/A)$. | Example 21–2 |
| Relate the power in an electric circuit to the current, voltage, and resistance. | The basic definition of electrical power is current times voltage: $P = IV$. Using Ohm's law when appropriate, the power can also be expressed as $P = I^2R$ and $P = V^2/R$. | Examples 21–3, 21–4 |
| Determine the equivalent resistance of resistors in series and parallel. | Resistors in series simply add: $R_{eq} = R_1 + R_2 + \cdots$; resistors in parallel add in terms of inverses: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$. | Examples 21–5, 21–6, 21–7 |
| Find the current in a circuit containing resistors that are not simply in series or parallel. | Apply Kirchhoff's junction rule (the algebraic sum of currents at a junction must be zero) and loop rule (the algebraic sum of potential difference around a loop is zero). | Active Example 21–2 |
| Determine the equivalent capacitance of capacitors in series and parallel. | Capacitors in parallel simply add: $C_{eq} = C_1 + C_2 + \cdots$; capacitors in series add in terms of inverses: $1/C_{eq} = 1/C_1 + 1/C_2 + \cdots$. | Example 21–8<br>Active Example 21–3 |
| Find the charge and the current in an *RC* circuit as a function of time. | The charge and current in an *RC* circuit during charging vary exponentially with time as follows: $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$; $I(t) = (\mathcal{E}/R)e^{-t/\tau}$. The characteristic time is $\tau = RC$. | Example 21–9 |

## CONCEPTUAL QUESTIONS



(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. What is the direction of the electric current produced by an electron that falls toward the ground?
2. Your body is composed of electric charges. Does it follow, then, that you produce an electric current when you walk?
3. Suppose you charge a comb by rubbing it through your hair. Do you produce a current when you walk across the room carrying the comb?
4. Suppose you charge a comb by rubbing it through the fur on your dog's back. Do you produce a current when you walk across the room carrying the comb?
5. An electron moving through a wire has an average drift speed that is very small. Does this mean that its instantaneous velocity is also very small?
6. Are car headlights connected in series or parallel? Give an everyday observation that supports your answer.
7. Give an example of how four resistors of resistance $R$ can be combined to produce an equivalent resistance of $R$.
8. Is it possible to connect a group of resistors of value $R$ in such a way that the equivalent resistance is less than $R$? If so, give a specific example.
9. What physical quantity do resistors connected in series have in common?
10. What physical quantity do resistors connected in parallel have in common?
11. Explain how electrical devices can begin operating almost immediately after you throw a switch, even though individual electrons in the wire may take hours to reach the device.
12. Explain the difference between resistivity and resistance.
13. Explain why birds can roost on high-voltage wire without being electrocuted.
14. List two electrical applications that would benefit from room-temperature superconductors. List two applications for which room-temperature superconductivity would not be beneficial.
15. On what basic conservation laws are Kirchhoff's rules based?
16. What physical quantity do capacitors connected in series have in common?
17. What physical quantity do capacitors connected in parallel have in common?
18. Consider the circuit shown in Figure 21–25, in which a light of resistance $R$ and a capacitor of capacitance $C$ are connected in series. The capacitor has a large capacitance, and is initially uncharged. The battery provides enough power to light the bulb when connected to the battery directly. Describe the behavior of the light after the switch is closed.




▲ **FIGURE 21–25** Conceptual Question 18

## PROBLEMS AND CONCEPTUAL EXERCISES

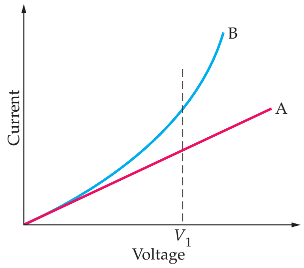
*Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book.* **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

### SECTION 21–1 ELECTRIC CURRENT

1. • How many coulombs of charge are in one ampere-hour?
2. • A flashlight bulb carries a current of 0.18 A for 78 s. How much charge flows through the bulb in this time? How many electrons?
3. • The picture tube in a particular television draws a current of 15 A. How many electrons strike the viewing screen every second?
4. • **IP** A car battery does 260 J of work on the charge passing through it as it starts an engine. **(a)** If the emf of the battery is 12 V, how much charge passes through the battery during the start? **(b)** If the emf is doubled to 24 V, does the amount of charge passing through the battery increase or decrease? By what factor?
5. • Highly sensitive ammeters can measure currents as small as 10.0 fA. How many electrons per second flow through a wire with a 10.0-fA current?
6. •• A television set connected to a 120-V outlet consumes 78 W of power. **(a)** How much current flows through the television? **(b)** How long does it take for 10 million electrons to pass through the TV?
7. •• **BIO** **Pacemaker Batteries** Pacemakers designed for long-term use commonly employ a lithium–iodine battery capable of supplying 0.42 A·h of charge. **(a)** How many coulombs of charge can such a battery supply? **(b)** If the average current produced by the pacemaker is 5.6 $\mu$A, what is the expected lifetime of the device?

### SECTION 21–2 RESISTANCE AND OHM'S LAW

8. • **CE** A conducting wire is quadrupled in length and tripled in diameter. **(a)** Does its resistance increase, decrease, or stay the same? Explain. **(b)** By what factor does its resistance change?
9. • **CE** Figure 21–26 shows a plot of current versus voltage for two different materials, A and B. Which of these materials satisfies Ohm's law? Explain.




▲ **FIGURE 21–26** Problems 9 and 10

**10.** • **CE Predict/Explain** Current-versus-voltage plots for two materials, A and B, are shown in Figure 21–26. **(a)** Is the resistance of material A greater than, less than, or equal to the resistance of material B at the voltage $V_1$? **(b)** Choose the *best explanation* from among the following:

**I.** Curve B is higher in value than curve A.
**II.** A larger slope means a larger value of $I/V$, and hence a smaller value of $R$.
**III.** Curve B has the larger slope at the voltage $V_1$ and hence the larger resistance.

**11.** • **CE** Two cylindrical wires are made of the same material and have the same length. If wire B is to have nine times the resistance of wire A, what must be the ratio of their radii, $r_B/r_A$?

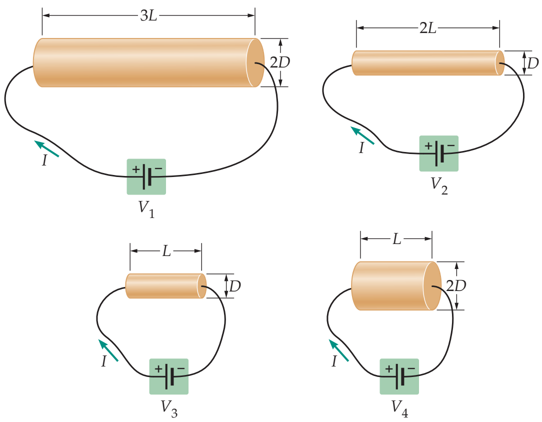
**12.** • A silver wire is 5.9 m long and 0.49 mm in diameter. What is its resistance?

**13.** • When a potential difference of 18 V is applied to a given wire, it conducts 0.35 A of current. What is the resistance of the wire?

**14.** • The tungsten filament of a lightbulb has a resistance of 0.07 Ω. If the filament is 27 cm long, what is its diameter?

**15.** • What is the resistance of 6.0 mi of copper wire with a diameter of 0.55 mm?

**16.** •• **CE** The four conducting cylinders shown in **Figure 21–27** are all made of the same material, though they differ in length and/or diameter. They are connected to four different batteries, which supply the necessary voltages to give the circuits the same current, $I$. Rank the four voltages, $V_1$, $V_2$, $V_3$, and $V_4$, in order of increasing value. Indicate ties where appropriate.




▲ **FIGURE 21–27** Problem 16

**17.** •• **IP** A bird lands on a bare copper wire carrying a current of 32 A. The wire is 8 gauge, which means that its cross-sectional area is 0.13 cm$^2$. **(a)** Find the difference in potential between the bird's feet, assuming they are separated by a distance of 6.0 cm. **(b)** Will your answer to part (a) increase or decrease if the separation between the bird's feet increases? Explain.

**18.** •• A current of 0.96 A flows through a copper wire 0.44 mm in diameter when it is connected to a potential difference of 15 V. How long is the wire?

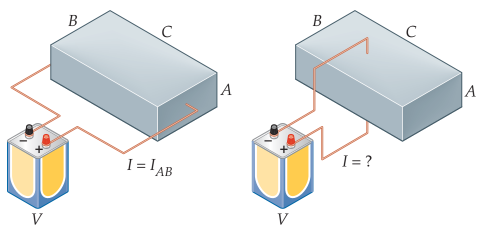
**19.** •• **IP BIO Current Through a Cell Membrane** A typical cell membrane is 8.0 nm thick and has an electrical resistivity of $1.3 \times 10^7\ \Omega \cdot \text{m}$. **(a)** If the potential difference between the inner and outer surfaces of a cell membrane is 75 mV, how much current flows through a square area of membrane 1.0 $\mu$m on a side? **(b)** Suppose the thickness of the membrane is doubled, but the resistivity and potential difference remain the same. Does the current increase or decrease? By what factor?

**20.** •• When a potential difference of 12 V is applied to a wire 6.9 m long and 0.33 mm in diameter, the result is an electric current of 2.1 A. What is the resistivity of the wire?

**21.** •• **IP (a)** What is the resistance per meter of an aluminum wire with a cross-sectional area of $2.4 \times 10^{-7}\ \text{m}^2$. **(b)** Would your answer to part (a) increase, decrease, or stay the same if the diameter of the wire were increased? Explain. **(c)** Repeat part (a) for a wire with a cross-sectional area of $3.6 \times 10^{-7}\ \text{m}^2$.

**22.** •• **BIO Resistance and Current in the Human Finger** The interior of the human body has an electrical resistivity of $0.15\ \Omega \cdot \text{m}$. **(a)** Estimate the resistance for current flowing the length of your index finger. (For this calculation, ignore the much higher resistivity of your skin.) **(b)** Your muscles will contract when they carry a current greater than 15 mA. What voltage is required to produce this current through your finger?

**23.** ••• Consider a rectangular block of metal of height $A$, width $B$, and length $C$, as shown in **Figure 21–28**. If a potential difference $V$ is maintained between the two $A \times B$ faces of the block, a current $I_{AB}$ is observed to flow. Find the current that flows if the same potential difference $V$ is applied between the two $B \times C$ faces of the block. Give your answer in terms of $I_{AB}$.




▲ **FIGURE 21–28** Problem 23

## SECTION 21–3 ENERGY AND POWER IN ELECTRIC CIRCUITS

**24.** • **CE** Light A has four times the power rating of light B when operated at the same voltage. **(a)** Is the resistance of light A greater than, less than, or equal to the resistance of light B? Explain. **(b)** What is the ratio of the resistance of light A to the resistance of light B?

**25.** • **CE** Two lightbulbs operate on the same potential difference. Bulb A has four times the power output of bulb B. **(a)** Which bulb has the greater current passing through it? Explain. **(b)** What is the ratio of the current in bulb A to the current in bulb B?

**26.** • **CE** Two lightbulbs operate on the same current. Bulb A has four times the power output of bulb B. **(a)** Is the potential difference across bulb A greater than or less than the potential difference across bulb B? Explain. **(b)** What is the ratio of the potential difference across bulb A to that across bulb B?

**27.** • A 75-V generator supplies 3.8 kW of power. How much current does the generator produce?

**28.** • A portable CD player operates with a current of 22 mA at a potential difference of 4.1 V. What is the power usage of the player?

**29.** • Find the power dissipated in a 25-Ω electric heater connected to a 120-V outlet.

**30.** • The current in a 120-V reading lamp is 2.6 A. If the cost of electrical energy is \$0.075 per kilowatt-hour, how much does it cost to operate the light for an hour?

**31.** • It costs 2.6 cents to charge a car battery at a voltage of 12 V and a current of 15 A for 120 minutes. What is the cost of electrical energy per kilowatt-hour at this location?

**32.** •• **IP** A 75-W lightbulb operates on a potential difference of 95 V. Find **(a)** the current in the bulb and **(b)** the resistance of the bulb. **(c)** If this bulb is replaced with one whose resistance is half the value found in part (b), is its power rating greater than or less than 75 W? By what factor?

**33.** •• **Rating Car Batteries** Car batteries are rated by the following two numbers: **(1)** cranking amps = current the battery can produce for 30.0 seconds while maintaining a terminal voltage of at least 7.2 V and **(2)** reserve capacity = number of minutes the battery can produce a 25-A current while maintaining a terminal voltage of at least 10.5 V. One particular battery is advertised as having 905 cranking amps and a 155-minute reserve capacity. Which of these two ratings represents the greater amount of energy delivered by the battery?

## SECTION 21–4 RESISTORS IN SERIES AND PARALLEL

**34.** • **CE Predict/Explain** A dozen identical lightbulbs are connected to a given emf. **(a)** Will the lights be brighter if they are connected in series or in parallel? **(b)** Choose the *best explanation* from among the following:

- **I.** When connected in parallel each bulb experiences the maximum emf and dissipates the maximum power.
- **II.** Resistors in series have a larger equivalent resistance and dissipate more power.
- **III.** Resistors in parallel have a smaller equivalent resistance and dissipate less power.

**35.** • **CE Predict/Explain** A *fuse* is a device to protect a circuit from the effects of a large current. The fuse is a small strip of metal that burns through when the current in it exceeds a certain value, thus producing an open circuit. **(a)** Should a fuse be connected in series or in parallel with the circuit it is intended to protect? **(b)** Choose the *best explanation* from among the following:

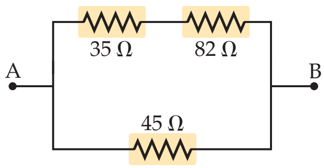
- **I.** Either connection is acceptable; the main thing is to have a fuse in the circuit.
- **II.** The fuse should be connected in parallel, otherwise it will interrupt the current in the circuit.
- **III.** With the fuse connected in series, the current in the circuit drops to zero as soon as the fuse burns through.

**36.** • **CE** A circuit consists of three resistors, $R_1 < R_2 < R_3$, connected in series to a battery. Rank these resistors in order of increasing **(a)** current through them and **(b)** potential difference across them. Indicate ties where appropriate.

**37.** • **CE Predict/Explain** Two resistors are connected in parallel. **(a)** If a third resistor is now connected in parallel with the original two, does the equivalent resistance of the circuit increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:

- **I.** Adding a resistor generally tends to increase the resistance, but putting it in parallel tends to decrease the resistance; therefore the effects offset and the resistance stays the same.
- **II.** Adding more resistance to the circuit will increase the equivalent resistance.
- **III.** The third resistor gives yet another path for current to flow in the circuit, which means that the equivalent resistance is less.

**38.** • Find the equivalent resistance between points A and B for the group of resistors shown in **Figure 21–29**.




▲ **FIGURE 21–29** Problems 38 and 115

**39.** • What is the minimum number of 65-Ω resistors that must be connected in parallel to produce an equivalent resistance of 11 Ω or less?

**40.** •• Four lightbulbs (A, B, C, D) are connected together in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed:

| | A | B | C | D |
|---|---|---|---|---|
| A removed | * | on | on | on |
| B removed | on | * | on | off |
| C removed | off | off | * | off |
| D removed | on | off | on | * |

Draw a circuit diagram for these bulbs.

**41.** •• Your toaster has a power cord with a resistance of 0.020 Ω connected in series with a 9.6-Ω nichrome heating element. If the potential difference between the terminals of the toaster is 120 V, how much power is dissipated in **(a)** the power cord and **(b)** the heating element?

**42.** •• A hobbyist building a radio needs a 150-Ω resistor in her circuit, but has only a 220-Ω, a 79-Ω, and a 92-Ω resistor available. How can she connect these resistors to produce the desired resistance?

**43.** •• A circuit consists of a 12.0-V battery connected to three resistors (42 Ω, 17 Ω, and 110 Ω) in series. Find **(a)** the current that flows through the battery and **(b)** the potential difference across each resistor.

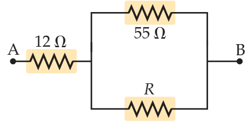
**44.** •• **IP** Three resistors, 11 Ω, 53 Ω, and $R$, are connected in series with a 24.0-V battery. The total current flowing through the battery is 0.16 A. **(a)** Find the value of resistance $R$. **(b)** Find the potential difference across each resistor. **(c)** If the voltage of the battery had been greater than 24.0 V, would your answer to part (a) have been larger or smaller? Explain.

**45.** •• A circuit consists of a battery connected to three resistors (65 Ω, 25 Ω, and 170 Ω) in parallel. The total current through the resistors is 1.8 A. Find **(a)** the emf of the battery and **(b)** the current through each resistor.

**46.** •• **IP** Three resistors, 22 Ω, 67 Ω, and $R$, are connected in parallel with a 12.0-V battery. The total current flowing through the battery is 0.88 A. **(a)** Find the value of resistance $R$. **(b)** Find the current through each resistor. **(c)** If the total current in the battery had been greater than 0.88 A, would your answer to part (a) have been larger or smaller? Explain.

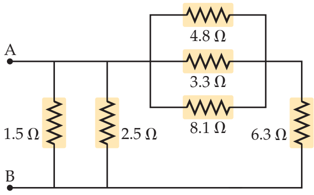
**47.** •• An 89-Ω resistor has a current of 0.72 A and is connected in series with a 130-Ω resistor. What is the emf of the battery to which the resistors are connected?

**48.** •• The equivalent resistance between points A and B of the resistors shown in **Figure 21–30** is 26 Ω. Find the value of resistance $R$.




▲ **FIGURE 21–30** Problems 48, 52, and 98

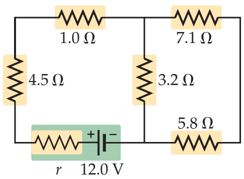
49\. •• Find the equivalent resistance between points A and B shown in **Figure 21–31**.




▲ **FIGURE 21–31** Problems 49 and 53

50\. •• How many 65-W lightbulbs can be connected in parallel across a potential difference of 85 V before the total current in the circuit exceeds 2.1 A?

51\. •• The circuit in **Figure 21–32** includes a battery with a finite internal resistance, $r = 0.50\ \Omega$. **(a)** Find the current flowing through the 7.1-Ω and the 3.2-Ω resistors. **(b)** How much current flows through the battery? **(c)** What is the potential difference between the terminals of the battery?



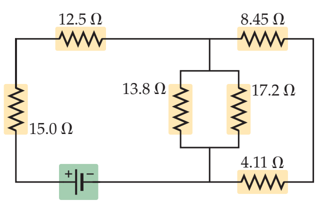

▲ **FIGURE 21–32** Problems 51 and 54

52\. •• **IP** A 12-V battery is connected to terminals A and B in Figure 21–30. **(a)** Given that $R = 85\ \Omega$, find the current in each resistor. **(b)** Suppose the value of $R$ is increased. For each resistor in turn, state whether the current flowing through it increases or decreases. Explain.

53\. •• **IP** The terminals A and B in Figure 21–31 are connected to a 9.0-V battery. **(a)** Find the current flowing through each resistor. **(b)** Is the potential difference across the 6.3-Ω resistor greater than, less than, or the same as the potential difference across the 1.5-Ω resistor? Explain.

54\. •• **IP** Suppose the battery in Figure 21–32 has an internal resistance $r = 0.25\ \Omega$. **(a)** How much current flows through the battery? **(b)** What is the potential difference between the terminals of the battery? **(c)** If the 3.2-Ω resistor is increased in value, will the current in the battery increase or decrease? Explain.

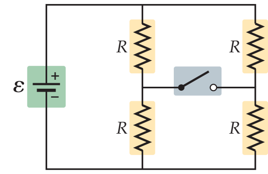
55\. ••• **IP** The current flowing through the 8.45-Ω resistor in **Figure 21–33** is 1.52 A. **(a)** What is the voltage of the battery? **(b)** If the 17.2-Ω resistor is increased in value, will the current provided by the battery increase, decrease, or stay the same? Explain.




▲ **FIGURE 21–33** Problems 55 and 56

56\. ••• The current in the 13.8-Ω resistor in Figure 21–33 is 0.795 A. Find the current in the other resistors in the circuit.

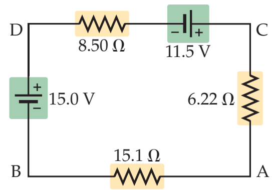
57\. ••• **IP** Four identical resistors are connected to a battery as shown in **Figure 21–34**. When the switch is open, the current through the battery is $I_0$. **(a)** When the switch is closed, will the current through the battery increase, decrease, or stay the same? Explain. **(b)** Calculate the current that flows through the battery when the switch is closed. Give your answer in terms of $I_0$.




▲ **FIGURE 21–34** Problem 57

## SECTION 21–5 KIRCHHOFF'S RULES

58\. • Find the magnitude and direction (clockwise or counterclockwise) of the current in **Figure 21–35**.

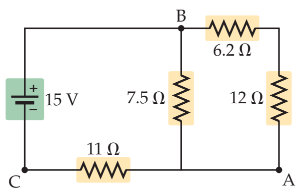



▲ **FIGURE 21–35** Problems 58, 59, and 60

59\. • **IP** Suppose the polarity of the 11.5-V battery in Figure 21–35 is reversed. **(a)** Do you expect this to increase or decrease the amount of current flowing in the circuit? Explain. **(b)** Calculate the magnitude and direction (clockwise or counterclockwise) of the current in this case.

60\. •• **IP** It is given that point A in Figure 21–35 is grounded ($V = 0$). **(a)** Is the potential at point B greater than or less than zero? Explain. **(b)** Is the potential at point C greater than or less than zero? Explain. **(c)** Calculate the potential at point D.

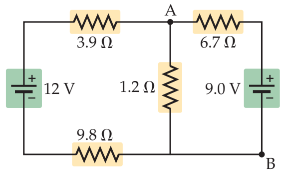
61\. •• Consider the circuit shown in **Figure 21–36**. Find the current through each resistor using **(a)** the rules for series and parallel resistors and **(b)** Kirchhoff's rules.




▲ **FIGURE 21–36** Problems 61 and 62

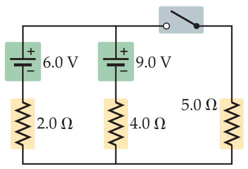
62\. •• Suppose point A is grounded ($V = 0$) in Figure 21–36. Find the potential at points B and C.

63\. •• **IP** **(a)** Find the current in each resistor in **Figure 21–37**. **(b)** Is the potential at point A greater than, less than, or equal to the potential at point B? Explain. **(c)** Determine the potential difference between the points A and B.




▲ **FIGURE 21–37** Problem 63

64. ••• Two batteries and three resistors are connected as shown in **Figure 21–38**. How much current flows through each battery when the switch is **(a)** closed and **(b)** open?




▲ **FIGURE 21–38** Problem 64

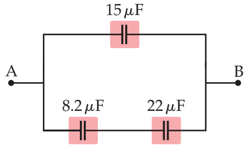
## SECTION 21–6 CIRCUITS CONTAINING CAPACITORS

65. • **CE** Two capacitors, $C_1 = C$ and $C_2 = 2C$, are connected to a battery. **(a)** Which capacitor stores more energy when they are connected to the battery in series? Explain. **(b)** Which capacitor stores more energy when they are connected in parallel? Explain.

66. • **CE Predict/Explain** Two capacitors are connected in series. **(a)** If a third capacitor is now connected in series with the original two, does the equivalent capacitance increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:
    **I.** Adding a capacitor generally tends to increase the capacitance, but putting it in series tends to decrease the capacitance; therefore, the net result is no change.
    **II.** Adding a capacitor in series will increase the total amount of charge stored, and hence increase the equivalent capacitance.
    **III.** Adding a capacitor in series decreases the equivalent capacitance since each capacitor now has less voltage across it, and hence stores less charge.

67. • **CE Predict/Explain** Two capacitors are connected in parallel. **(a)** If a third capacitor is now connected in parallel with the original two, does the equivalent capacitance increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:
    **I.** Adding a capacitor tends to increase the capacitance, but putting it in parallel tends to decrease the capacitance; therefore, the net result is no change.
    **II.** Adding a capacitor in parallel will increase the total amount of charge stored, and hence increase the equivalent capacitance.
    **III.** Adding a capacitor in parallel decreases the equivalent capacitance since each capacitor now has less voltage across it, and hence stores less charge.

68. • Find the equivalent capacitance between points A and B for the group of capacitors shown in **Figure 21–39**.

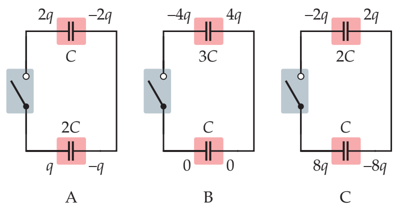



▲ **FIGURE 21–39** Problems 68 and 72

69. • A 12-V battery is connected to three capacitors in series. The capacitors have the following capacitances: 4.5 μF, 12 μF, and 32 μF. Find the voltage across the 32-μF capacitor.

70. •• **CE** You conduct a series of experiments in which you connect the capacitors $C_1$ and $C_2 > C_1$ to a battery in various ways. The experiments are as follows: **A**, $C_1$ alone connected to the battery; **B**, $C_2$ alone connected to the battery; **C**, $C_1$ and $C_2$ connected to the battery in series; **D**, $C_1$ and $C_2$ connected to the battery in parallel. Rank these four experiments in order of increasing equivalent capacitance. Indicate ties where appropriate.

71. •• **CE** Three different circuits, each containing a switch and two capacitors, are shown in **Figure 21–40**. Initially, the plates of the capacitors are charged as shown. The switches are then closed, allowing charge to move freely between the capacitors. Rank the circuits in order of increasing final charge on the left plate of **(a)** the upper capacitor and **(b)** the lower capacitor. Indicate ties where appropriate.



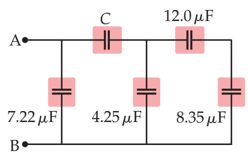

▲ **FIGURE 21–40** Problem 71

72. •• Terminals A and B in Figure 21–39 are connected to a 9.0-V battery. Find the energy stored in each capacitor.

73. •• **IP** Two capacitors, one 7.5 μF and the other 15 μF, are connected in parallel across a 15-V battery. **(a)** Find the equivalent capacitance of the two capacitors. **(b)** Which capacitor stores more charge? Explain. **(c)** Find the charge stored on each capacitor.

74. •• **IP** Two capacitors, one 7.5 μF and the other 15 μF, are connected in series across a 15-V battery. **(a)** Find the equivalent capacitance of the two capacitors. **(b)** Which capacitor stores more charge? Explain. **(c)** Find the charge stored on each capacitor.

75. •• The equivalent capacitance of the capacitors shown in **Figure 21–41** is 9.22 μF. Find the value of capacitance $C$.

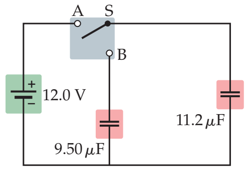



▲ **FIGURE 21–41** Problems 75 and 118

76. ••• Two capacitors, $C_1$ and $C_2$, are connected in series and charged by a battery. Show that the energy stored in $C_1$ plus the energy stored in $C_2$ is equal to the energy stored in the equivalent capacitor, $C_{eq}$, when it is connected to the same battery.

77. ••• With the switch in position A, the 11.2-μF capacitor in **Figure 21–42** is fully charged by the 12.0-V battery, and the

9.50-$\mu$F capacitor is uncharged. The switch is now moved to position B. As a result, charge flows between the capacitors until they have the same voltage across their plates. Find this voltage.




▲ **FIGURE 21–42** Problem 77

## SECTION 21–7 *RC* CIRCUITS

78. • The switch on an *RC* circuit is closed at $t = 0$. Given that $\mathcal{E} = 9.0$ V, $R = 150\ \Omega$, and $C = 23\ \mu$F, how much charge is on the capacitor at time $t = 4.2$ ms?

79. • The capacitor in an *RC* circuit ($R = 120\ \Omega, C = 45\ \mu$F) is initially uncharged. Find **(a)** the charge on the capacitor and **(b)** the current in the circuit one time constant ($\tau = RC$) after the circuit is connected to a 9.0-V battery.

80. •• **CE** Three *RC* circuits have the emf, resistance, and capacitance given in the accompanying table. Initially, the switch on the circuit is open and the capacitor is uncharged. Rank these circuits in order of increasing **(a)** initial current (immediately after the switch is closed) and **(b)** time for the capacitor to acquire half its final charge. Indicate ties where appropriate.

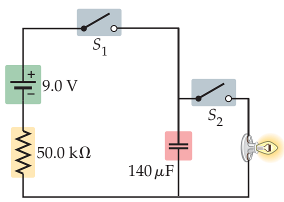
| | $\mathcal{E}$ (V) | $R$ ($\Omega$) | $C$ ($\mu$F) |
|---|---|---|---|
| Circuit A | 12 | 4 | 3 |
| Circuit B | 9 | 3 | 1 |
| Circuit C | 9 | 9 | 2 |

81. •• Consider an *RC* circuit with $\mathcal{E} = 12.0$ V, $R = 175\ \Omega$, and $C = 55.7\ \mu$F. Find **(a)** the time constant for the circuit, **(b)** the maximum charge on the capacitor, and **(c)** the initial current in the circuit.

82. •• The resistor in an *RC* circuit has a resistance of 145 $\Omega$. **(a)** What capacitance must be used in this circuit if the time constant is to be 3.5 ms? **(b)** Using the capacitance determined in part (a), calculate the current in the circuit 7.0 ms after the switch is closed. Assume that the capacitor is uncharged initially and that the emf of the battery is 9.0 V.

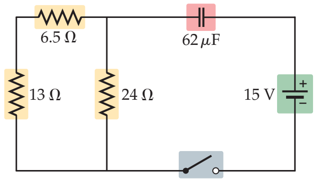
83. •• A flash unit for a camera has a capacitance of 1500 $\mu$F. What resistance is needed in this *RC* circuit if the flash is to charge to 90% of its full charge in 21 s?

84. •• **Figure 21–43** shows a simplified circuit for a photographic flash unit. This circuit consists of a 9.0-V battery, a 50.0-k$\Omega$ resistor, a 140-$\mu$F capacitor, a flashbulb, and two switches. Initially, the capacitor is uncharged and the two switches are open. To charge the unit, switch $S_1$ is closed; to fire the flash, switch $S_2$ (which is connected to the camera's shutter) is closed. How long does it take to charge the capacitor to 5.0 V?




▲ **FIGURE 21–43** Problem 84

85. •• **IP** Consider the *RC* circuit shown in **Figure 21–44**. Find **(a)** the time constant and **(b)** the initial current for this circuit. **(c)** It is desired to increase the time constant of this circuit by adjusting the value of the 6.5-$\Omega$ resistor. Should the resistance of this resistor be increased or decreased to have the desired effect? Explain.




▲ **FIGURE 21–44** Problems 85 and 119

86. ••• The capacitor in an *RC* circuit is initially uncharged. In terms of $R$ and $C$, determine **(a)** the time required for the charge on the capacitor to rise to 50% of its final value and **(b)** the time required for the initial current to drop to 10% of its initial value.

## GENERAL PROBLEMS

87. • **CE** A given car battery is rated as 250 amp-hours. Is this rating a measure of energy, power, charge, voltage, or current? Explain.

88. • **CE Predict/Explain** The resistivity of tungsten increases with temperature. **(a)** When a light containing a tungsten filament heats up, does its power consumption increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
    **I.** The voltage is unchanged, and therefore an increase in resistance implies a reduced power, as we can see from $P = V^2/R$.
    **II.** Increasing the resistance increases the power, as is clear from $P = I^2R$.
    **III.** The power consumption is independent of resistance, as we can see from $P = IV$.

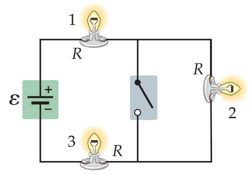
89. • **CE** A cylindrical wire is to be doubled in length, but it is desired that its resistance remain the same. **(a)** Must its radius be increased or decreased? Explain. **(b)** By what factor must the radius be changed?

90. • **CE Predict/Explain** An electric space heater has a power rating of 500 W when connected to a given voltage $V$. **(a)** If two of these heaters are connected in series to the same voltage, is the power consumed by the two heaters greater than, less than, or equal to 1000 W? **(b)** Choose the *best explanation* from among the following:
    **I.** Each heater consumes 500 W; therefore two of them will consume $500\ W + 500\ W = 1000\ W$.
    **II.** The voltage is the same, but the resistance is doubled by connecting the heaters in series. Therefore, the power consumed ($P = V^2/R$) is less than 1000 W.
    **III.** Connecting two heaters in series doubles the resistance. Since power depends on the resistance squared, it follows that the power consumed is greater than 1000 W.

91. • **CE** Two resistors, $R_1 = R$ and $R_2 = 2R$, are connected to a battery. **(a)** Which resistor dissipates more power when they are connected to the battery in series? Explain. **(b)** Which resistor dissipates more power when they are connected in parallel? Explain.

92. • **CE** Consider the circuit shown in **Figure 21–45**, in which three lights, each with a resistance $R$, are connected in series. The circuit also contains an open switch. **(a)** When the switch is closed, does the intensity of light 2 increase, decrease, or stay

the same? Explain. **(b)** Do the intensities of lights 1 and 3 increase, decrease, or stay the same when the switch is closed? Explain.

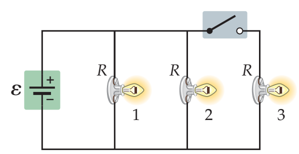



▲ **FIGURE 21–45** Problems 92, 93, and 94

93. • **CE Predict/Explain (a)** Referring to Problem 92 and the circuit in Figure 21–45, does the current supplied by the battery increase, decrease, or remain the same when the switch is closed? **(b)** Choose the *best explanation* from among the following:
   **I.** The current decreases because only two resistors can draw current from the battery when the switch is closed.
   **II.** Closing the switch makes no difference to the current since the second resistor is still connected to the battery as before.
   **III.** Closing the switch shorts out the second resistor, decreases the total resistance of the circuit, and increases the current.

94. • **CE Predict/Explain (a)** Referring to Problem 92 and the circuit in Figure 21–45, does the total power dissipated in the circuit increase, decrease, or remain the same when the switch is closed? **(b)** Choose the *best explanation* from among the following:
   **I.** Closing the switch shorts out one of the resistors, which means that the power dissipated decreases.
   **II.** The equivalent resistance of the circuit is reduced by closing the switch, but the voltage remains the same. Therefore, from $P = V^2/R$ we see that the power dissipated increases.
   **III.** The power dissipated remains the same because power, $P = IV$, is independent of resistance.

95. • **CE** Consider the circuit shown in **Figure 21–46**, in which three lights, each with a resistance *R*, are connected in parallel. The circuit also contains an open switch. **(a)** When the switch is closed, does the intensity of light 3 increase, decrease, or stay the same? Explain. **(b)** Do the intensities of lights 1 and 2 increase, decrease, or stay the same when the switch is closed? Explain.




▲ **FIGURE 21–46** Problems 95, 96, and 97

96. • **CE Predict/Explain (a)** When the switch is closed in the circuit shown in Figure 21–46, does the current supplied by the battery increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   **I.** The current increases because three resistors are drawing current from the battery when the switch is closed, rather than just two.
   **II.** Closing the switch makes no difference to the current because the voltage is the same as before.
   **III.** Closing the switch decreases the current because an additional resistor is added to the circuit.

97. • **CE Predict/Explain (a)** When the switch is closed in the circuit shown in Figure 21–46, does the total power dissipated in the circuit increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   **I.** Closing the switch adds one more resistor to the circuit. This makes it harder for the battery to supply current, which decreases the power dissipated.
   **II.** The equivalent resistance of the circuit is reduced by closing the switch, but the voltage remains the same. Therefore, from $P = V^2/R$ we see that the power dissipated increases.
   **III.** The power dissipated remains the same because power, $P = IV$, is independent of resistance.

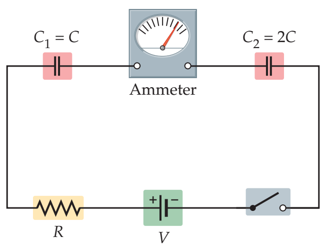
98. • Suppose that points A and B in Figure 21–30 are connected to a 12-V battery. Find the power dissipated in each of the resistors assuming that $R = 65\ \Omega$.

99. • You are given resistors of 413 Ω, 521 Ω, and 146 Ω. Describe how these resistors must be connected to produce an equivalent resistance of 255 Ω.

100. • You are given capacitors of 18 μF, 7.2 μF, and 9.0 μF. Describe how these capacitors must be connected to produce an equivalent capacitance of 22 μF.

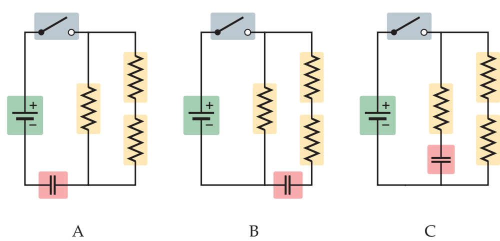
101. • Suppose your car carries a charge of 85 μC. What current does it produce as it travels from Dallas to Fort Worth (35 mi) in 0.75 h?

102. •• **CE** The circuit shown in **Figure 21–47** shows a resistor and two capacitors connected in series with a battery of voltage *V*. The circuit also has an ammeter and a switch. Initially, the switch is open and both capacitors are uncharged. The following questions refer to a time long after the switch is closed and current has ceased to flow. **(a)** In terms of *V*, what is the voltage across the capacitor $C_1$? **(b)** In terms of *CV*, what is the charge on the right plate of $C_2$? **(c)** What is the net charge that flowed through the ammeter during charging? Give your answer in terms of *CV*.




▲ **FIGURE 21–47** Problem 102

103. •• **CE** The three circuits shown in **Figure 21–48** have identical batteries, resistors, and capacitors. Initially, the switches are open and the capacitors are uncharged. Rank the circuits in order of increasing **(a)** final charge on the capacitor and **(b)** time for the current to drop to 90% of its initial value. Indicate ties where appropriate.

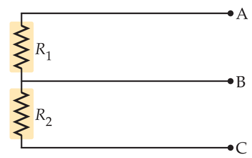



▲ **FIGURE 21–48** Problem 103

104. •• It is desired to construct a 5.0-Ω resistor from a 1.2-m length of tungsten wire. What diameter is needed for this wire?

**105.** •• **Electrical Safety Codes** For safety reasons, electrical codes have been established that limit the amount of current a wire of a given size can carry. For example, an 18-gauge (cross-sectional area = 1.17 mm$^2$), rubber-insulated extension cord with copper wires can carry a maximum current of 5.0 A. Find the voltage drop in a 12-ft, 18-gauge extension cord carrying a current of 5.0 A. (*Note:* In an extension cord, the current must flow through two lengths—down and back.)

**106.** •• **A Three-Way Lightbulb** A three-way lightbulb has two filaments with resistances $R_1$ and $R_2$ connected in series. The resistors are connected to three terminals, as indicated in **Figure 21–49**, and the light switch determines which two of the three terminals are connected to a potential difference of 120 V at any given time. When terminals A and B are connected to 120 V the bulb uses 75.0 W of power. When terminals A and C are connected to 120 V the bulb uses 50.0 W of power. **(a)** What is the resistance $R_1$? **(b)** What is the resistance $R_2$? **(c)** How much power does the bulb use when 120 V is connected to terminals B and C?

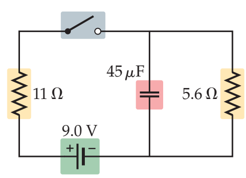



▲ **FIGURE 21–49** Problem 106

**107.** •• A portable CD player uses a current of 7.5 mA at a potential difference of 3.5 V. **(a)** How much energy does the player use in 35 s? **(b)** Suppose the player has a mass of 0.65 kg. For what length of time could the player operate on the energy required to lift it through a height of 1.0 m?

**108.** •• An electrical heating coil is immersed in 4.6 kg of water at 22 °C. The coil, which has a resistance of 250 Ω, warms the water to 32 °C in 15 min. What is the potential difference at which the coil operates?

**109.** •• **IP** Consider the circuit shown in **Figure 21–50**. **(a)** Is the current flowing through the battery immediately after the switch is closed greater than, less than, or the same as the current flowing through the battery long after the switch is closed? Explain. **(b)** Find the current flowing through the battery immediately after the switch is closed. **(c)** Find the current in the battery long after the switch is closed.




▲ **FIGURE 21–50** Problems 109 and 116

**110.** •• A silver wire and a copper wire have the same volume and the same resistance. Find the ratio of their radii, $r_{silver}/r_{copper}$.

**111.** •• Two resistors are connected in series to a battery with an emf of 12 V. The voltage across the first resistor is 2.7 V and the current through the second resistor is 0.15 A. Find the resistance of the two resistors.

**112.** •• **BIO** **Pacemaker Pulses** A pacemaker sends a pulse to a patient's heart every time the capacitor in the pacemaker charges to a voltage of 0.25 V. It is desired that the patient receive 75 pulses per minute. Given that the capacitance of the pacemaker is 110 μF and that the battery has a voltage of 9.0 V, what value should the resistance have?

**113.** •• A long, thin wire has a resistance $R$. The wire is now cut into three segments of equal length, which are connected in parallel. In terms of $R$, what is the equivalent resistance of the three wire segments?

**114.** •• Three resistors ($R$, $\frac{1}{2}R$, $2R$) are connected to a battery. **(a)** If the resistors are connected in series, which one has the greatest rate of energy dissipation? **(b)** Repeat part (a), this time assuming that the resistors are connected in parallel.

**115.** •• **IP** Suppose we connect a 12.0-V battery to terminals A and B in Figure 21–29. **(a)** Is the current in the 45-Ω resistor greater than, less than, or the same as the current in the 35-Ω resistor? Explain. **(b)** Calculate the current flowing through each of the three resistors in this circuit.

**116.** •• **IP** Suppose the battery in Figure 21–50 has an internal resistance of 0.73 Ω. **(a)** What is the potential difference across the terminals of the battery when the switch is open? **(b)** When the switch is closed, does the potential difference of the battery increase or decrease? Explain. **(c)** Find the potential difference across the battery after the switch has been closed a long time.

**117.** •• **National Electric Code** In the United States, the National Electric Code sets standards for maximum safe currents in insulated copper wires of various diameters. The accompanying table gives a portion of the code. Notice that wire diameters are identified by the *gauge* of the wire, and that 1 mil = $10^{-3}$ in. Find the maximum power dissipated per length in **(a)** an 8-gauge wire and **(b)** a 10-gauge wire.

| Gauge | Diameter (mils) | Safe current (A) |
|---|---|---|
| 8 | 129 | 35 |
| 10 | 102 | 25 |

**118.** ••• **IP** A 15.0-V battery is connected to terminals A and B in Figure 21–41. **(a)** Given that $C$ = 15.0 μF, find the charge on each of the capacitors. **(b)** Find the total energy stored in this system. **(c)** If the 7.22-μF capacitor is increased in value, will the total energy stored in the circuit increase or decrease? Explain.

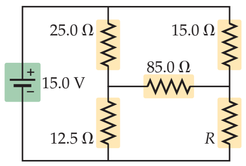
**119.** ••• **IP** The switch in the *RC* circuit shown in Figure 21–44 is closed at $t = 0$. **(a)** How much power is dissipated in each resistor just after $t = 0$ and in the limit $t \to \infty$? **(b)** What is the charge on the capacitor at the time $t$ = 0.35 ms? **(c)** How much energy is stored in the capacitor in the limit $t \to \infty$? **(d)** If the voltage of the battery is doubled, by what factor does your answer to part (c) change? Explain.

**120.** ••• Two resistors, $R_1$ and $R_2$, are connected in parallel and connected to a battery. Show that the power dissipated in $R_1$ plus the power dissipated in $R_2$ is equal to the power dissipated in the equivalent resistor, $R_{eq}$, when it is connected to the same battery.

**121.** ••• A battery has an emf $\mathcal{E}$ and an internal resistance $r$. When the battery is connected to a 25-Ω resistor, the current through the battery is 0.65 A. When the battery is connected to a 55-Ω resistor, the current is 0.45 A. Find the battery's emf and internal resistance.

**122.** ••• When two resistors, $R_1$ and $R_2$, are connected in series across a 6.0-V battery, the potential difference across $R_1$ is 4.0 V. When $R_1$ and $R_2$ are connected in parallel to the same battery, the current through $R_2$ is 0.45 A. Find the values of $R_1$ and $R_2$.

**123.** ••• The circuit shown in **Figure 21–51** is known as a Wheatstone bridge. Find the value of the resistor $R$ such that the current through the 85.0-Ω resistor is zero.



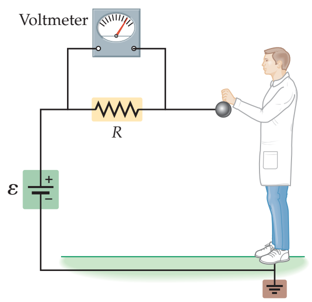

▲ **FIGURE 21–51** Problem 123

## PASSAGE PROBLEMS

### BIO Footwear Safety

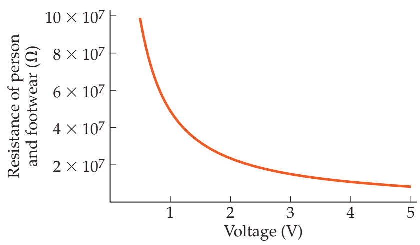
The American National Standards Institute (ANSI) specifies safety standards for a number of potential workplace hazards. For example, ANSI requires that footwear provide protection against the effects of compression from a static weight, impact from a dropped object, puncture from a sharp tool, and cuts from saws. In addition, to protect against the potentially lethal effects of an electrical shock, ANSI provides standards for the electrical resistance that a person and footwear must offer to the flow of electric current.

Specifically, regulation ANSI Z41-1999 states that the resistance of a person and his or her footwear must be tested with the circuit shown in **Figure 21–52**. In this circuit, the voltage supplied by the battery is $\mathcal{E} = 50.0$ V and the resistance in the circuit is $R = 1.00$ MΩ. Initially the circuit is open and no current flows. When a person touches the metal sphere attached to the battery, however, the circuit is closed and a small current flows through the person, the shoes, and back to the battery. The amount of current flowing through the person can be determined by using a voltmeter to measure the voltage drop $V$ across the resistor $R$. To be safe, the current should not exceed 150 $\mu$A.




▲ **FIGURE 21–52** Problems 124, 125, 126, and 127

Notice that the experimental setup in Figure 21–52 is a dc circuit with two resistors in series—the resistance $R$ and the resistance of the person and footwear, $R_{pf}$. It follows that the current in the circuit is $I = \mathcal{E}/(R + R_{pf})$. We also know that the current is $I = V/R$, where $V$ is the reading of the voltmeter. These relations can be combined to relate the voltage $V$ to the resistance $R_{pf}$, with the result shown in **Figure 21–53**. According to ANSI regulations, Type II footwear must give a resistance $R_{pf}$ in the range of $0.1 \times 10^7$ Ω to $100 \times 10^7$ Ω.




▲ **FIGURE 21–53** Problems 124, 125, 126, and 127

**124.** • Suppose the voltmeter measures a potential difference of 3.70 V across the resistor. What is the current that flows through the person's body?

**A.** $3.70 \times 10^{-6}$ A **B.** $5.00 \times 10^{-5}$ A

**C.** 0.0740 A **D.** 3.70 A

**125.** • What is the resistance of the person and footwear when the voltmeter reads 3.70 V?

**A.** $1.25 \times 10^7$ Ω **B.** $1.35 \times 10^7$ Ω

**C.** $4.63 \times 10^7$ Ω **D.** $1.71 \times 10^8$ Ω

**126.** • The resistance of a given person and footwear is $4.00 \times 10^7$ Ω. What is the reading on the voltmeter when this person is tested?

**A.** 0.976 V **B.** 1.22 V

**C.** 1.25 V **D.** 50.0 V

**127.** • Suppose that during one test a person's shoes become wet when water spills onto the floor. When this happens, do you expect the reading on the voltmeter to increase, decrease, or stay the same?

## INTERACTIVE PROBLEMS

**128.** •• **Referring to Example 21–7** Suppose the three resistors in this circuit have the values $R_1 = 100.0$ Ω, $R_2 = 200.0$ Ω, and $R_3 = 300.0$ Ω, and that the emf of the battery is 12.0 V. (The resistor numbers are given in the Interactive Figure.) **(a)** Find the potential difference across each resistor. **(b)** Find the current that flows through each resistor.

**129.** •• **Referring to Example 21–7** Suppose $R_1 = R_2 = 225$ Ω and $R_3 = R$. The emf of the battery is 12.0 V. (The resistor numbers are given in the Interactive Figure.) **(a)** Find the value of $R$ such that the current supplied by the battery is 0.0750 A. **(b)** Find the value of $R$ that gives a potential difference of 2.65 V across resistor 2.

**130.** •• **IP Referring to Example 21–9** Suppose the resistance of the 126-Ω resistor is reduced by a factor of 2. The other resistor is 275 Ω, the capacitor is 182 $\mu$F, and the battery has an emf of 3.00 V. **(a)** Does the final value of the charge on the capacitor increase, decrease, or stay the same? Explain. **(b)** Does the time for the capacitor to charge to 80.0% of its final value increase, decrease, or stay the same? Explain. **(c)** Find the time referred to in part (b).

**131.** •• **IP Referring to Example 21–9** Suppose the capacitance of the 182-$\mu$F capacitor is reduced by a factor of 2. The two resistors are 126 Ω and 275 Ω, and the battery has an emf of 3.00 V. **(a)** Find the final value of the charge on the capacitor. **(b)** Does the time for the capacitor to charge to 80.0% of its final value increase, decrease, or stay the same? Explain. **(c)** Find the time referred to in part (b).