UCONN Physics 1201Q : Unit 8 **/ 22** **Name: ____________________**

Worksheet II : Angular Momentum **Date: __________ Period: ____**

Angular momentum and the Principle of Conservation of Angular Momentum

1. (II) (*a*) A grinding wheel 0.35 m in diameter rotates at 2500 rpm. Calculate its angular velocity in $\mathrm{rad/s}$.

(*b*) What are the linear speed and acceleration of a point on the edge of the grinding wheel?

2. (II) A 70-cm-diameter wheel accelerates uniformly about its center from 130 rpm to 280 rpm in 4.0 s. Determine (*a*) its angular acceleration,

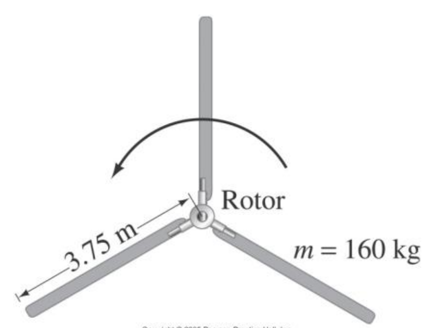
(*b*) the radial and tangential components of the linear acceleration of a point on the edge of the wheel 2.0 s after it has started accelerating.

3. (II) A wheel 33 cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 6.5 s. How far will a point on the edge of the wheel have traveled in this time?

4. (II) A helicopter rotor blade can be considered a long thin rod, as shown in Fig. 8–46.

(*a*) If each of the three rotor helicopter blades is 3.75 m long and has a mass of 160 kg, calculate the moment of inertia of the three rotor blades about the axis of rotation.

(*b*) How much torque must the motor apply to bring the blades up to a speed of $5.0\,\mathrm{rev/s}$ in 8.0 s?



5. (II) A merry-go-round has a mass of 1640 kg and a radius of 7.50 m. How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 8.00 s? Assume it is a solid cylinder.

6. (II) A diver (such as the one shown in Fig. 8–29) can reduce her moment of inertia by a factor of about 3.5 when changing from the straight position to the tuck position. If she makes 2.0 rotations in 1.5 s when in the tuck position, what is her angular speed (rev/s) when in the straight position?

7. (II) A person stands, hands at his side, on a platform that is rotating at a rate of $1.30\ \text{rev/s}$. If he raises his arms to a horizontal position, Fig. 8–48, the speed of rotation decreases to $0.80\ \text{rev/s}$.

(*a*) Why?

(*b*) By what factor has his moment of inertia changed?

1. (*a*) $$\omega = \left(\frac{2500\ \text{rev}}{1\ \text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 261.8\ \text{rad/sec} \approx \boxed{2.6\times10^{2}\ \text{rad/sec}}$$

(*b*) $$v = \omega r = (261.8\ \text{rad/sec})(0.175\ \text{m}) = \boxed{46\ \text{m/s}}$$

$$a_{\text{R}} = \omega^2 r = (261.8\ \text{rad/sec})^2 (0.175\ \text{m}) = \boxed{1.2\times10^{4}\ \text{m/s}^2}$$

2. Convert the rpm values to angular velocities.

$$\omega_0 = \left(130\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = 13.6\ \text{rad/s}$$

$$\omega = \left(280\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = 29.3\ \text{rad/s}$$

(*a*) The angular acceleration is found from Eq. 8-9a.

$$\omega = \omega_0 + \alpha t \ \rightarrow\ \alpha = \frac{\omega - \omega_0}{t} = \frac{29.3\ \text{rad/s} - 13.6\ \text{rad/s}}{4.0\ \text{s}} = 3.93\ \text{rad/s}^2 \approx \boxed{3.9\ \text{rad/s}^2}$$

(*b*) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega = \omega_0 + \alpha t = 13.6\ \text{rad/s} + (3.93\ \text{rad/s}^2)(2.0\ \text{s}) = 21.5\ \text{rad/s}$$

The instantaneous radial acceleration is given by $a_{\text{R}} = \omega^2 r$.

$$a_{\text{R}} = \omega^2 r = (21.5\ \text{rad/s})^2 (0.35\ \text{m}) = \boxed{1.6\times10^{2}\ \text{m/s}^2}$$

The tangential acceleration is given by $a_{\tan} = \alpha r$.

$$a_{\tan} = \alpha r = (3.93\ \text{rad/s}^2)(0.35\ \text{m}) = \boxed{1.4\ \text{m/s}^2}$$

3.

$$\bar{\omega} = \frac{\omega + \omega_0}{2} = \frac{240\ \text{rpm} + 360\ \text{rpm}}{2} = 300\ \text{rpm}$$

$$\theta = \bar{\omega} t = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)(6.5\ \text{s}) = 32.5\ \text{rev}$$

Each revolution corresponds to a circumference of travel distance.

$$32.5\ \text{rev}\left[\frac{\pi(0.33\ \text{m})}{1\ \text{rev}}\right] = \boxed{34\ \text{m}}$$

4. (*a*) The moment of inertia of a thin rod, rotating about its end, is given in Figure 8-21(g). There are three blades to add.

$$I_{\text{total}} = 3\left(\tfrac{1}{3}ML^2\right) = ML^2 = (160\ \text{kg})(3.75\ \text{m})^2 = 2250\ \text{kg}\cdot\text{m}^2 \approx \boxed{2.3\times10^{2}\ \text{kg}\cdot\text{m}^2}$$

(*b*) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{\text{total}}\alpha = I_{\text{total}}\frac{\omega - \omega_0}{t} = (2250\ \text{kg}\cdot\text{m}^2)\frac{(5.0\ \text{rev/sec})(2\pi\ \text{rad/rev})}{8.0\ \text{s}} = \boxed{8.8\times10^{3}\ \text{m}\cdot\text{N}}$$

5. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta KE_{\text{rot}} = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2 = \tfrac{1}{2}\tfrac{1}{2} MR^2\omega_f^2 = \tfrac{1}{4}(1640 \text{ kg})(7.50 \text{ m})^2 \left(\frac{2\pi \text{ rad}}{8.00 \text{ s}}\right)^2 = \boxed{1.42\times 10^4 \text{ J}}$$

6. There is no net torque on the diver because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$L_1 = L_2 \quad \rightarrow \quad I_1\omega_1 = I_2\omega_2 \quad \rightarrow \quad \omega_2 = \omega_1 \frac{I_1}{I_2} = \left(\frac{2 \text{ rev}}{1.5 \text{ sec}}\right)\left(\frac{1}{3.5}\right) = \boxed{0.38 \text{ rev/s}}$$

7. (*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(*b*) $$L_i = L_f \quad \rightarrow \quad I_i\omega_i = I_f\omega_f \quad \rightarrow \quad I_f = I_i \frac{\omega_i}{\omega_f} = I_i \frac{1.30 \text{ rev/s}}{0.80 \text{ rev/s}} = 1.625\, I_i \approx 1.6\, I_i$$

The rotational inertia has increased by a factor of $\boxed{1.6}$.