

19 Electric Charges, Forces, and Fields



Amber, a form of fossilized tree resin long used to make beautiful beads and other ornaments, has also made contributions to two different sciences. Pieces of amber have preserved prehistoric insects and pollen grains for modern students of evolution. And over 2500 years ago, amber provided Greek scientists with their first opportunity to study electric forces—the subject of this chapter.

We are all made up of electric charges. Every atom in every human body contains both positive and negative charges held together by an attractive force that is similar to gravity—only vastly stronger. Our atoms are bound together by electric forces to form molecules; these molecules, in turn, interact with one another to produce solid bones and liquid blood. In a very real sense, we are walking, talking manifestations of electricity.

In this chapter, we discuss the basic properties of electric charge. Among these are that electric charge comes in discrete units (quantization) and that the total amount of charge in the universe remains constant (conservation). In addition, we present the force law that describes the interactions between electric charges. Finally, we introduce the idea of an electric *field*, and show how it is related to the distribution of charge.

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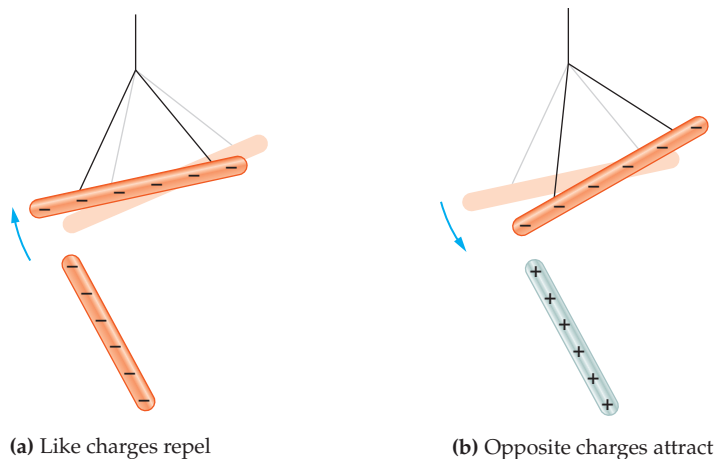
19-1 Electric Charge

The effects of electric charge have been known since at least 600 B.C. About that time, the Greeks noticed that amber—a solid, translucent material formed from the fossilized resin of extinct coniferous trees—has a peculiar property. If a piece of amber is rubbed with animal fur, it attracts small, lightweight objects. This phenomenon is illustrated in **Figure 19-1**.

For some time, it was thought that amber was unique in its ability to become “charged.” Much later, it was discovered that other materials can behave in this way as well. For example, if glass is rubbed with a piece of silk, it too can attract small objects. In this respect, glass and amber seem to be the same. It turns out, however, that these two materials have different types of charge.

To see this, imagine suspending a small, charged rod of amber from a thread, as in **Figure 19-2**. If a second piece of charged amber is brought near the rod, as shown in Figure 19-2 (a), the rod rotates away, indicating a repulsive force between the two pieces of amber. Thus, “like” charges repel. On the other hand, if a piece of charged glass is brought near the amber rod, the amber rotates toward the glass, indicating an attractive force. This is illustrated in Figure 19-2 (b). Clearly, then, the *different* charges on the glass and amber attract one another. We refer to different charges as being the “opposite” of one another, as in the familiar expression “opposites attract.”

We know today that the two types of charge found on amber and glass are, in fact, the only types that exist, and we still use the purely arbitrary names—**positive** (+) charge and **negative** (−) charge—proposed by Benjamin Franklin (1706–1790) in 1747. In accordance with Franklin’s original suggestion, the charge of amber is negative, and the charge of glass is positive (the opposite of negative). Calling the different charges + and − is actually quite useful mathematically; for example, an object that contains an equal amount of positive and negative charge has zero net charge. Objects with zero net charge are said to be electrically **neutral**.



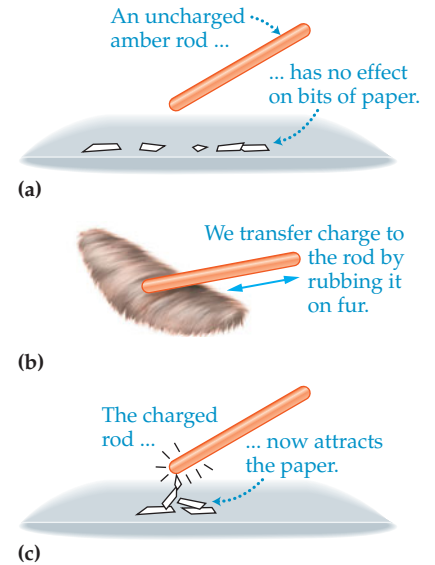
A familiar example of an electrically neutral object is the atom. Atoms have a small, dense nucleus with a positive charge surrounded by a cloud of negatively charged electrons (from the Greek word for amber, *elektron*). A pictorial representation of an atom is shown in **Figure 19-3**.

All electrons have exactly the same electric charge. This charge is very small and is defined to have a magnitude, e , given by the following:

Magnitude of an Electron’s Charge, e

$$e = 1.60 \times 10^{-19} \text{ C}$$

SI unit: coulomb, C



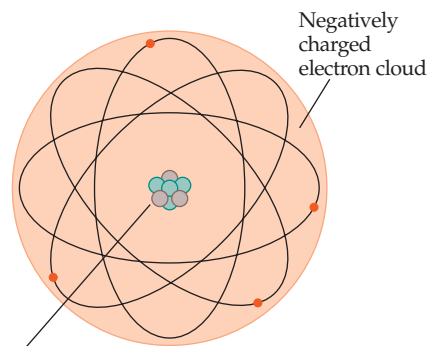
▲ FIGURE 19-1 Charging an amber rod

An uncharged amber rod (a) exerts no force on scraps of paper. When the rod is rubbed against a piece of fur (b), it becomes charged and then attracts the paper (c).

◀ FIGURE 19-2 Likes repel, opposites attract

A charged amber rod is suspended by a string. According to the convention introduced by Benjamin Franklin, the charge on the amber is designated as negative. (a) When another charged amber rod is brought near the suspended rod, it rotates away, indicating a repulsive force between like charges. (b) When a charged glass rod is brought close to the suspended amber rod, the amber rotates toward the glass, indicating an attractive force and the existence of a second type of charge, which we designate as positive.

In this expression, C is a unit of charge referred to as the **coulomb**, named for the French physicist Charles-Augustin de Coulomb (1736–1806). (The precise definition



Positively charged nucleus

▲ **FIGURE 19–3** The structure of an atom

A crude representation of an atom, showing the positively charged nucleus at its center and the negatively charged electrons orbiting about it. More accurately, the electrons should be thought of as forming a “cloud” of negative charge surrounding the nucleus.

of the coulomb is in terms of electric current, which we shall discuss in Chapter 21.) Clearly, the charge on an electron, which is negative, is $-e$. This is one of the defining, or *intrinsic*, properties of the electron. Another intrinsic property of the electron is its mass, m_e :

$$m_e = 9.11 \times 10^{-31} \text{ kg} \quad 19-2$$

In contrast, the charge on a proton—one of the main constituents of nuclei—is *exactly* $+e$. Therefore, the net charge on atoms, which have equal numbers of electrons and protons, is precisely zero. The mass of the proton is

$$m_p = 1.673 \times 10^{-27} \text{ kg} \quad 19-3$$

Note that this is about 2000 times larger than the mass of the electron. The other main constituent of the nucleus is the neutron, which, as its name implies, has zero charge. Its mass is slightly larger than that of the proton:

$$m_n = 1.675 \times 10^{-27} \text{ kg} \quad 19-4$$

Since the magnitude of the charge per electron is $1.60 \times 10^{-19} \text{ C/electron}$, it follows that the number of electrons in 1 C of charge is enormous:

$$\frac{1 \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 6.25 \times 10^{18} \text{ electrons}$$

As we shall see when we consider the force between charges, a coulomb is a significant amount of charge; even a powerful lightning bolt delivers only 20 to 30 C. A more common unit of charge is the microcoulomb, μC , where $1 \mu\text{C} = 10^{-6} \text{ C}$. Still, the amount of charge contained in everyday objects is very large, even in units of the coulomb, as we show in the following Exercise.

EXERCISE 19–1

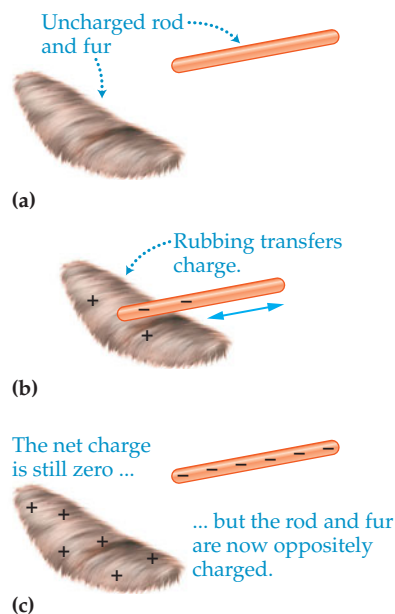
Find the amount of positive electric charge in one mole of helium atoms. (Note that the nucleus of a helium atom consists of two protons and two neutrons.)

SOLUTION

Since each helium atom contains two positive charges of magnitude e , the total positive charge in a mole is

$$N_A(2e) = (6.02 \times 10^{23})(2)(1.60 \times 10^{-19} \text{ C}) = 1.93 \times 10^5 \text{ C}$$

Thus, a mere 4 g of helium contains almost 200,000 C of positive charge, and the same amount of negative charge, as well.



▲ **FIGURE 19–4** Charge transfer

(a) Initially, an amber rod and a piece of fur are electrically neutral; that is, they each contain equal quantities of positive and negative charge. (b) As they are rubbed together, charge is transferred from one to the other. (c) In the end, the fur and the rod have charges of equal magnitude but opposite sign.

Charge Separation

How is it that rubbing a piece of amber with fur gives the amber a charge? Originally, it was thought that the friction of rubbing *created* the observed charge. We now know, however, that rubbing the fur across the amber simply results in a *transfer* of charge from the fur to the amber—with the total amount of charge remaining unchanged. This is indicated in **Figure 19–4**. Before charging, the fur and the amber are both neutral. During the rubbing process some electrons are transferred from the fur to the amber, giving the amber a net negative charge, and leaving the fur with a net positive charge. At no time during this process is charge ever created or destroyed. This, in fact, is an example of one of the fundamental conservation laws of physics:

Conservation of Electric Charge

The total electric charge of the universe is constant. No physical process can result in an increase or decrease in the total amount of electric charge in the universe.

When charge is transferred from one object to another, it is generally due to the movement of electrons. In a typical solid, the nuclei of the atoms are fixed in

position. The outer electrons of these atoms, however, are often weakly bound and fairly easily separated. As a piece of fur rubs across amber, for example, some of the electrons that were originally part of the fur are separated from their atoms and deposited onto the amber. The atom that loses an electron is now a **positive ion**, and the atom that receives an extra electron becomes a **negative ion**. This is charging by separation.

In general, when two materials are rubbed together, the magnitude *and* sign of the charge that each material acquires depend on how strongly it holds onto its electrons. For example, if silk is rubbed against glass, the glass acquires a positive charge, as was mentioned earlier in this section. It follows that electrons have moved from the glass to the silk, giving the silk a *negative* charge. If silk is rubbed against amber, however, the silk becomes *positively* charged, as electrons in this case pass from the silk to the amber.

These results can be understood by referring to Table 19-1, which presents the relative charging due to rubbing—also known as **triboelectric charging**—for a variety of materials. The more plus signs associated with a material, the more readily it gives up electrons and becomes positively charged. Similarly, the more minus signs for a material, the more readily it acquires electrons. For example, we know that amber becomes negatively charged when rubbed against fur, but a greater negative charge is obtained if rubber, PVC, or Teflon is rubbed with fur instead. In general, when two materials in Table 19-1 are rubbed together, the one higher in the list becomes positively charged, and the one lower in the list becomes negatively charged. The greater the separation on the list, the greater the magnitude of the charge.

Charge separation occurs not only when one object is rubbed against another, but also when objects collide. For example, colliding crystals of ice in a rain cloud can cause charge separation that may ultimately result in bolts of lightning to bring the charges together again. Similarly, particles in the rings of Saturn are constantly undergoing collisions and becoming charged as a result. In fact, when the *Voyager* spacecraft examined the rings of Saturn, it observed electrostatic discharges, similar to lightning bolts on Earth. In addition, ghostly radial “spokes” that extend across the rings of Saturn—which cannot be explained by gravitational forces alone—are also the result of electrostatic interactions.

TABLE 19-1 Triboelectric Charging

Material	Relative charging with rubbing
Rabbit fur	++++++
Glass	+++++
Human hair	++++
Nylon	+++
Silk	++
Paper	+
Cotton	-
Wood	--
Amber	---
Rubber	----
PVC	-----
Teflon	-----



▲ The Van de Graaff generator (left) that these children are touching can produce very large charges of static electricity. Since they are clearly not frightened, why is their hair standing on end? On a smaller scale, if you rub a balloon against a cloth surface, the balloon acquires a negative electric charge. The balloon can then attract a stream of water (right), even though water molecules themselves are electrically neutral. This phenomenon occurs because the water molecules, though they have no net charge, are polar: one end of the molecule has a slight positive charge and the other a slight negative charge. Under the influence of the balloon's negative charge, the water molecules orient themselves so that their positive ends point toward the balloon. This alignment ensures that the electrical attraction between the balloon and the positive part of each molecule exceeds the repulsion between the balloon and the negative part of each molecule.

CONCEPTUAL CHECKPOINT 19–1 COMPARE THE MASS

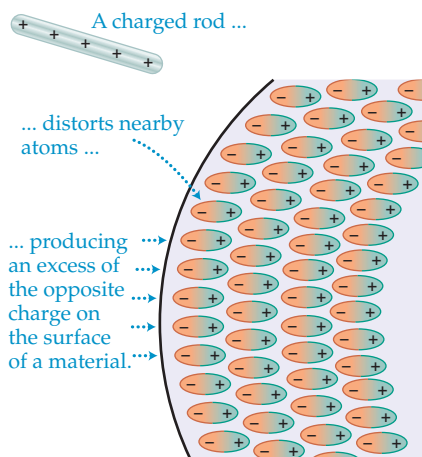
Is the mass of an amber rod after charging with fur **(a)** greater than, **(b)** less than, or **(c)** the same as its mass before charging?

REASONING AND DISCUSSION

Since an amber rod becomes negatively charged, it has acquired electrons from the fur. Each electron has a small, but nonzero, mass. Therefore, the mass of the rod increases ever so slightly as it is charged.

ANSWER

(a) The mass of the amber rod is greater after charging.



▲ **FIGURE 19–5** Electrical polarization

When a charged rod is far from a neutral object, the atoms in the object are undistorted, as in Figure 19–3. As the rod is brought closer, however, the atoms distort, producing an excess of one type of charge on the surface of the object (in this case a negative charge). This induced charge is referred to as a polarization charge. Because the sign of the polarization charge is the opposite of the sign of the charge on the rod, there is an attractive force between the rod and the object.

Since electrons always have the charge $-e$, and protons always have the charge $+e$, it follows that all objects must have a net charge that is an integral multiple of e . This conclusion was confirmed early in the twentieth century by the American physicist Robert A. Millikan (1868–1953) in a classic series of experiments. He found that the charge on an object can be $\pm e$, $\pm 2e$, $\pm 3e$, and so on, but never $1.5e$ or $-9.3847e$, for example. We describe this restriction by saying that electric charge is **quantized**.

Polarization

We know that charges of opposite sign attract, but it is also possible for a charged rod to attract small objects that have zero net charge. The mechanism responsible for this attraction is called **polarization**.

To see how polarization works, consider **Figure 19–5**. Here we show a positively charged rod held close to an enlarged view of a neutral object. An atom near the surface of the neutral object will become elongated because the negative electrons in it are attracted to the rod while the positive protons are repelled. As a result, a net negative charge develops on the surface near the rod—the so-called polarization charge. The attractive force between the rod and this *induced* polarization charge leads to a net attraction between the rod and the entire neutral object.

Of course, the same conclusion is reached if we consider a negative rod held near a neutral object—except in this case the polarization charge is positive. Thus, the effect of polarization is to give rise to an attractive force regardless of the sign of the charged object. It is for this reason that both charged amber and charged glass attract neutral objects—even though their charges are opposite.

A potentially dangerous, and initially unsuspected, medical application of polarization occurs in endoscopic surgery. In these procedures, a tube carrying a small video camera is inserted into the body. The resulting video image is produced by electrons striking the inside surface of a computer monitor’s screen, which is kept positively charged to attract the electrons. Minute airborne particles in the operating room—including dust, lint, and skin cells—are polarized by the positive charge on the screen, and are attracted to its exterior surface.

The problem comes when a surgeon touches the screen to point out an important feature to others in the medical staff. Even the slightest touch can transfer particles—many of which carry bacteria—from the screen to the surgeon’s finger and from there to the patient. In fact, the surgeon’s finger doesn’t even have to touch the screen—as the finger approaches the screen, it too becomes polarized, and hence, it can attract particles from the screen, or directly from the air. Situations like these have resulted in infections, and surgeons are now cautioned not to bring their fingers near the video monitor.



REAL-WORLD PHYSICS: BIO

Bacterial infection from endoscopic surgery

19–2 Insulators and Conductors

Suppose you rub one end of an amber rod with fur, being careful not to touch the other end. The result is that the rubbed portion becomes charged, whereas the other end remains neutral. In particular, the negative charge transferred to the rubbed end stays put; it does not move about from one end of the rod to the other. Materials like

amber, in which charges are not free to move, are referred to as **insulators**. Most insulators are nonmetallic substances, and most are also good thermal insulators.

In contrast, most metals are good **conductors** of electricity, in the sense that they allow charges to move about more or less freely. For example, suppose an uncharged metal sphere is placed on an insulating base. If a charged rod is brought into contact with the sphere, as in **Figure 19-6 (a)**, some charge will be transferred to the sphere at the point of contact. The charge does not stay put, however. Since the metal is a good conductor of electricity, the charges are free to move about the sphere, which they do because of their mutual repulsion. The result is a uniform distribution of charge over the surface of the sphere, as shown in **Figure 19-6 (b)**. Note that the insulating base prevents charge from flowing away from the sphere into the ground.

On a microscopic level, the difference between conductors and insulators is that the atoms in conductors allow one or more of their outermost electrons to become detached. These detached electrons, often referred to as “conduction electrons,” can move freely throughout the conductor. In a sense, the conduction electrons behave almost like gas molecules moving about within a container. Insulators, in contrast, have very few, if any, free electrons; the electrons are bound to their atoms and cannot move from place to place within the material.

Some materials have properties that are intermediate between those of a good conductor and a good insulator. These materials, referred to as **semiconductors**, can be fine-tuned to display almost any desired degree of conductivity by controlling the concentration of the various components from which they are made. The great versatility of semiconductors is one reason they have found such wide areas of application in electronics and computers.

Exposure to light can sometimes determine whether a given material is an insulator or a conductor. An example of such a **photoconductive** material is selenium, which conducts electricity when light shines on it but is an insulator when in the dark. Because of this special property, selenium plays a key role in the production of photocopiers. To see how, we first note that at the heart of every photocopier is a selenium-coated aluminum drum. Initially, the selenium is given a positive charge and kept in the dark—which causes it to retain its charge. When flash lamps illuminate a document to be copied, an image of the document falls on the drum. Where the document is light, the selenium is illuminated and becomes a conductor, and the positive charge flows away into the aluminum drum, leaving the selenium uncharged. Where the document is dark, the selenium is not illuminated, meaning that it is an insulator, and its charge remains in place. At this point, a negatively charged “toner” powder is wiped across the drum, where it sticks to those positively charged portions of the drum that were not illuminated. Next, the drum is brought into contact with paper, transferring the toner to it. Finally, the toner is fused into the paper fibers with heat, the drum is cleaned of excess toner, and the cycle repeats. Thus, a slight variation in electrical properties due to illumination is the basis of an entire technology.

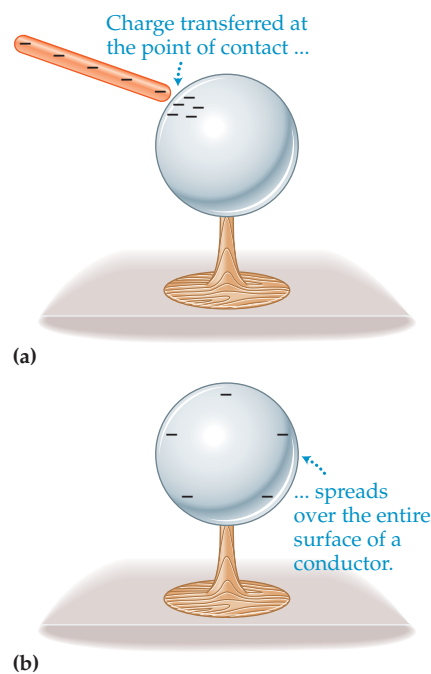
The operation of a laser printer is basically the same as that of a photocopier, with the difference that in the laser printer the selenium-coated drum is illuminated with a computer-controlled laser beam. As the laser sweeps across the selenium, the computer turns the beam on and off to produce areas that will print light or dark, respectively.

19-3 Coulomb's Law

We have already discussed the fact that electric charges exert forces on one another. The precise law describing these forces was first determined by Coulomb in the late 1780s. His result is remarkably simple. Suppose, for example, that an idealized point charge q_1 is separated by a distance r from another point charge q_2 . Both charges are at rest; that is, the system is **electrostatic**. According to Coulomb's law, the magnitude of the electrostatic force between these charges is proportional to the product of the magnitude of the charges, $|q_1||q_2|$, and inversely proportional to the square of the distance, r^2 , between them:



▲ People who work with electricity must be careful to use gloves made of nonconducting materials. Rubber, an excellent insulator, is often used for this purpose.



▲ **FIGURE 19-6** Charging a conductor

(a) When an uncharged metal sphere is touched by a charged rod, some charge is transferred at the point of contact.

(b) Because like charges repel, and charges move freely on a conductor, the transferred charge quickly spreads out and covers the entire surface of the sphere.

REAL-WORLD PHYSICS

Photocopiers and laser printers



Coulomb's Law for the Magnitude of the Electrostatic Force Between Point Charges

$$F = k \frac{|q_1||q_2|}{r^2}$$

19-5

SI unit: newton, N

In this expression, the proportionality constant k has the value

$$k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

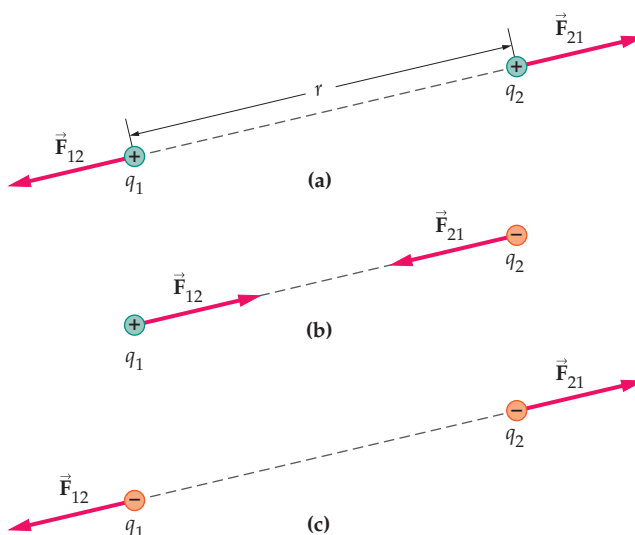
19-6

Note that the units of k are simply those required for the force F to have the units of newtons.

The direction of the force in Coulomb's law is along the line connecting the two charges. In addition, we know from the observations described in Section 19-1 that like charges repel and opposite charges attract. These properties are illustrated in **Figure 19-7**, where force vectors are shown for charges of various signs. Thus, when applying Coulomb's law, we first calculate the magnitude of the force using Equation 19-5, and then determine its direction with the "likes repel, opposites attract" rule.

▶ FIGURE 19-7 Forces between point charges

The forces exerted by two point charges on one another are always along the line connecting the charges. If the charges have the same sign, as in (a) and (c), the forces are repulsive; that is, each charge experiences a force that points away from the other charge. Charges of opposite sign, as in (b), experience attractive forces. Notice that in all cases the forces exerted on the two charges form an action-reaction pair. That is, $\vec{F}_{21} = -\vec{F}_{12}$.



Finally, note how Newton's third law applies to each of the cases shown in Figure 19-7. For example, the force exerted on charge 1 by charge 2, \vec{F}_{12} , is always equal in magnitude and opposite in direction to the force exerted on charge 2 by charge 1, \vec{F}_{21} ; that is, $\vec{F}_{21} = -\vec{F}_{12}$.

CONCEPTUAL CHECKPOINT 19-2 WHERE DO THEY COLLIDE?

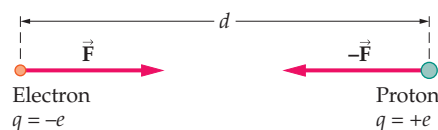
An electron and a proton, initially separated by a distance d , are released from rest simultaneously. The two particles are free to move. When they collide, are they (a) at the midpoint of their initial separation, (b) closer to the initial position of the proton, or (c) closer to the initial position of the electron?

REASONING AND DISCUSSION

Because of Newton's third law, the forces exerted on the electron and proton are equal in magnitude and opposite in direction. For this reason, it might seem that the particles meet at the midpoint. The masses of the particles, however, are quite different. In fact, as mentioned in Section 19-1, the mass of the proton is about 2000 times greater than the mass of the electron; therefore, the proton's acceleration ($a = F/m$) is about 2000 times less than the electron's acceleration. As a result, the particles collide near the initial position of the proton. More specifically, they collide at the location of the center of mass of the system, which remains at rest throughout the process.

ANSWER

(b) The particles collide near the initial position of the proton.



It is interesting to note the similarities and differences between Coulomb's law, $F = k|q_1||q_2|/r^2$, and Newton's law of gravity, $F = Gm_1m_2/r^2$. In each case, the force decreases as the square of the distance between the two objects. In addition, both forces depend on a product of intrinsic quantities: in the case of the electric force the intrinsic quantity is the charge; in the case of gravity it is the mass.

Equally significant, however, are the differences. In particular, the force of gravity is always attractive, whereas the electric force can be attractive or repulsive. As a result, the net electric force between neutral objects, such as the Earth and the Moon, is essentially zero because attractive and repulsive forces cancel one another. Since gravity is always attractive, however, the net gravitational force between the Earth and the Moon is nonzero. Thus, in astronomy, gravity rules, and electric forces play hardly any role.

Just the opposite is true in atomic systems. To see this, let's compare the electric and gravitational forces between a proton and an electron in a hydrogen atom. Taking the distance between the two particles to be the radius of hydrogen, $r = 5.29 \times 10^{-11}$ m, we find that the gravitational force has a magnitude

$$\begin{aligned} F_g &= G \frac{m_e m_p}{r^2} \\ &= (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(9.11 \times 10^{-31} \text{ kg})(1.673 \times 10^{-27} \text{ kg})}{(5.29 \times 10^{-11} \text{ m})^2} \\ &= 3.63 \times 10^{-47} \text{ N} \end{aligned}$$

Similarly, the magnitude of the electric force between the electron and the proton is

$$\begin{aligned} F_e &= k \frac{|q_1||q_2|}{r^2} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{|-1.60 \times 10^{-19} \text{ C}||1.60 \times 10^{-19} \text{ C}|}{(5.29 \times 10^{-11} \text{ m})^2} \\ &= 8.22 \times 10^{-8} \text{ N} \end{aligned}$$

Taking the ratio, we find that the electric force is greater than the gravitational force by a factor of

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{8.22 \times 10^{-8} \text{ N}}{3.63 \times 10^{-47} \text{ N}} = 2.26 \times 10^{39} \\ &= 2,260,000,000,000,000,000,000,000,000,000,000,000,000,000 \end{aligned}$$

This huge factor explains why a small piece of charged amber can lift bits of paper off the ground, even though the entire mass of the Earth is pulling downward on the paper.

Clearly, then, the force of gravity plays essentially no role in atomic systems. The reason gravity dominates in astronomy is that, even though the force is incredibly weak, it always attracts, giving a larger net force the larger the astronomical body. The electric force, on the other hand, is very strong but cancels for neutral objects.

Next, we use the electric force to get an idea of the speed of an electron in a hydrogen atom and the frequency of its orbital motion.

EXAMPLE 19-1 THE BOHR ORBIT

In an effort to better understand the behavior of atomic systems, the Danish physicist Niels Bohr (1885–1962) introduced a simple model for the hydrogen atom. In the Bohr model, as it is known today, the electron is imagined to move in a circular orbit about a stationary proton. The force responsible for the electron's circular motion is the electric force of attraction between the electron and the proton. **(a)** Given that the radius of the electron's orbit is 5.29×10^{-11} m, and its mass is $m_e = 9.11 \times 10^{-31}$ kg, find the electron's speed. **(b)** What is the frequency of the electron's orbital motion?

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PROBLEM-SOLVING NOTE

Distance Dependence of the Coulomb Force

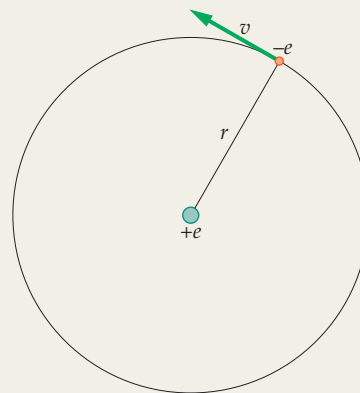


The Coulomb force has an inverse-square dependence on distance. Be sure to divide the product of the charges, $k|q_1||q_2|$, by r^2 when calculating the force.

CONTINUED FROM PREVIOUS PAGE

PICTURE THE PROBLEM

Our sketch shows the electron moving with a speed v in its orbit of radius r . Because the proton is so much more massive than the electron, it is essentially stationary at the center of the orbit. Note that the electron has a charge $-e$ and the proton has a charge $+e$.

**STRATEGY**

- The idea behind this model is that a force is required to make the electron move in a circular path, and this force is provided by the electric force of attraction between the electron and the proton. Thus, as with any circular motion, we set the force acting on the electron equal to its mass times its centripetal acceleration. This allows us to solve for the centripetal acceleration, $a_{cp} = v^2/r$ (Equation 6-14), which in turn gives us the speed v .
- The frequency of the electron's orbital motion is $f = 1/T$, where T is the period of the motion; that is, the time for one complete orbit. The time for an orbit, in turn, is the circumference divided by the speed, or $T = C/v = 2\pi r/v$. Taking the inverse immediately yields the frequency.

SOLUTION**Part (a)**

- Set the Coulomb force between the electron and proton equal to the centripetal force required for the electron's circular orbit:

$$k \frac{|q_1||q_2|}{r^2} = m_e a_{cp}$$

$$k \frac{e^2}{r^2} = m_e \frac{v^2}{r}$$

- Solve for the speed of the electron, v :

$$v = e \sqrt{\frac{k}{m_e r}}$$

- Substitute numerical values:

$$v = (1.60 \times 10^{-19} \text{ C}) \sqrt{\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{(9.11 \times 10^{-31} \text{ kg})(5.29 \times 10^{-11} \text{ m})}}$$

$$= 2.19 \times 10^6 \text{ m/s}$$

Part (b)

- Calculate the time for one orbit, T , which is the distance ($C = 2\pi r$) divided by the speed (v):

$$T = \frac{C}{v} = \frac{2\pi r}{v} = \frac{2\pi(5.29 \times 10^{-11} \text{ m})}{2.19 \times 10^6 \text{ m/s}} = 1.52 \times 10^{-16} \text{ s}$$

- Take the inverse of T to find the frequency:

$$f = \frac{1}{T} = \frac{1}{1.52 \times 10^{-16} \text{ s}} = 6.58 \times 10^{15} \text{ Hz}$$

INSIGHT

If you could travel around the world at this speed, your trip would take only about 18 s, but your centripetal acceleration would be a more-than-lethal 75,000 times the acceleration of gravity. As it is, the centripetal acceleration of the electron in this "Bohr" orbit around the proton is about 10^{22} times greater than the acceleration of gravity on the surface of the Earth.

The frequency of the orbit is also incredibly large. We won't encounter frequencies this high again until we study light waves in Chapter 25.

PRACTICE PROBLEM

The second Bohr orbit has a radius that is four times the radius of the first orbit. What is the speed of an electron in this orbit? [Answer: $v = 1.09 \times 10^6 \text{ m/s}$]

Some related homework problems: Problem 19, Problem 28, Problem 37

Another indication of the strength of the electric force is given in the following Exercise.

EXERCISE 19-2

Find the electric force between two 1.00-C charges separated by 1.00 m.

SOLUTION

Substituting $q_1 = q_2 = 1.00 \text{ C}$ and $r = 1.00 \text{ m}$ in Coulomb's law, we find

$$F = k \frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.00 \text{ C})(1.00 \text{ C})}{(1.00 \text{ m})^2} = 8.99 \times 10^9 \text{ N}$$

Exercise 19-2 shows that charges of one coulomb exert a force of about a million tons on one another when separated by a distance of a meter. If the charge in your body could be separated into a pile of positive charge on one side of the room and a pile of negative charge on the other side, the force needed to hold them apart would be roughly 10^{10} tons! Thus, everyday objects are never far from electrical neutrality, since disturbing neutrality requires such tremendous forces.

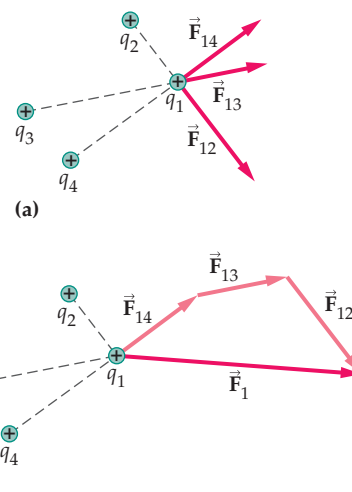
Superposition of Forces

The electric force, like all forces, is a vector quantity. Hence, when a charge experiences forces due to two or more other charges, the net force on it is simply the *vector* sum of the forces taken individually. For example, in Figure 19-8, the total force on charge 1, \vec{F}_1 , is the vector sum of the forces due to charges 2, 3, and 4:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

This is referred to as the **superposition** of forces.

Notice that the total force acting on a given charge is the sum of interactions involving just *two* charges at a time, with the force between each pair of charges given by Coulomb's law. For example, the total force acting on charge 1 in Figure 19-8 is the sum of the forces between q_1 and q_2 , q_1 and q_3 , and q_1 and q_4 . Therefore, superposition of forces can be thought of as the generalization of Coulomb's law to systems containing more than two charges. In our first numerical Example of superposition, we consider three charges in a line.



▲ FIGURE 19-8 Superposition of forces
(a) Forces are exerted on q_1 by the charges q_2 , q_3 , and q_4 . These forces are \vec{F}_{12} , \vec{F}_{13} , and \vec{F}_{14} , respectively. (b) The net force acting on q_1 , which we label \vec{F}_1 , is the vector sum of \vec{F}_{12} , \vec{F}_{13} , and \vec{F}_{14} .

EXAMPLE 19-2 NET FORCE

A charge $q_1 = -5.4 \mu\text{C}$ is at the origin, and a charge $q_2 = -2.2 \mu\text{C}$ is on the x axis at $x = 1.00$ m. Find the net force acting on a charge $q_3 = +1.6 \mu\text{C}$ located at $x = 0.75$ m.

PICTURE THE PROBLEM

The physical situation is shown in our sketch, with each charge at its appropriate location. Notice that the forces exerted on charge q_3 by the charges q_1 and q_2 are in opposite directions. We give the force on q_3 due to q_1 the label \vec{F}_{31} , and the force on q_3 due to q_2 the label \vec{F}_{32} .

STRATEGY

The net force on q_3 is the vector sum of the forces due to q_1 and q_2 . In particular, note that \vec{F}_{31} points in the negative x direction ($-\hat{x}$), whereas \vec{F}_{32} points in the positive x direction (\hat{x}). The magnitude of \vec{F}_{31} is $k|q_1||q_3|/r^2$, with $r = 0.75$ m. Similarly, the magnitude of \vec{F}_{32} is $k|q_2||q_3|/r^2$, with $r = 0.25$ m.

SOLUTION

1. Find the force acting on q_3 due to q_1 . Since this force is in the negative x direction, as indicated in the sketch, we give it a negative sign:

$$\begin{aligned}\vec{F}_{31} &= -k \frac{|q_1||q_3|}{r^2} \hat{x} \\ &= -(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\ &\quad \times \frac{(5.4 \times 10^{-6} \text{ C})(1.6 \times 10^{-6} \text{ C})}{(0.75 \text{ m})^2} \hat{x} \\ &= -0.14 \text{ N} \hat{x}\end{aligned}$$

2. Find the force acting on q_3 due to q_2 . Since this force is in the positive x direction, as indicated in the sketch, we give it a positive sign:

$$\begin{aligned}\vec{F}_{32} &= k \frac{|q_2||q_3|}{r^2} \hat{x} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\ &\quad \times \frac{(2.2 \times 10^{-6} \text{ C})(1.6 \times 10^{-6} \text{ C})}{(0.25 \text{ m})^2} \hat{x} \\ &= 0.51 \text{ N} \hat{x}\end{aligned}$$

3. Superpose these forces to find the total force, \vec{F}_3 , acting on q_3 :

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{31} + \vec{F}_{32} = -0.14 \text{ N} \hat{x} + 0.51 \text{ N} \hat{x} \\ &= 0.37 \text{ N} \hat{x}\end{aligned}$$

INSIGHT

The net force acting on q_3 has a magnitude of 0.37 N, and it points in the positive x direction. As usual, notice that we use only magnitudes for the charges in the numerator of Coulomb's law.

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PRACTICE PROBLEMFind the net force on q_3 if it is at the location $x = 0.25$ m. [Answer: $\vec{F}_3 = -1.2 \text{ N}\hat{x}$]

Some related homework problems: Problem 23, Problem 26, Problem 27

ACTIVE EXAMPLE 19-1 FIND THE LOCATION OF ZERO NET FORCEIn Example 19-2, the net force acting on the charge q_3 is to the right. To what value of x should q_3 be moved for the net force on it to be zero?**SOLUTION** (Test your understanding by performing the calculations indicated in each step.)

1. Write the magnitude of the force due to q_1 : $F_{31} = k|q_1||q_3|/x^2$
2. Write the magnitude of the force due to q_2 : $F_{32} = k|q_2||q_3|/(1.00 \text{ m} - x)^2$
3. Set these forces equal to one another, and cancel common terms: $|q_1|/x^2 = |q_2|/(1.00 \text{ m} - x)^2$
4. Take the square root of both sides and solve for x : $x = 0.61 \text{ m}$

INSIGHTTherefore, if q_3 is placed between $x = 0.61 \text{ m}$ and $x = 1.00 \text{ m}$, the net force acting on it is to the right, in agreement with Example 19-2. On the other hand, if q_3 is placed between $x = 0$ and $x = 0.61 \text{ m}$, the net force acting on it is to the left. This agrees with the result in the Practice Problem of Example 19-2.**YOUR TURN**

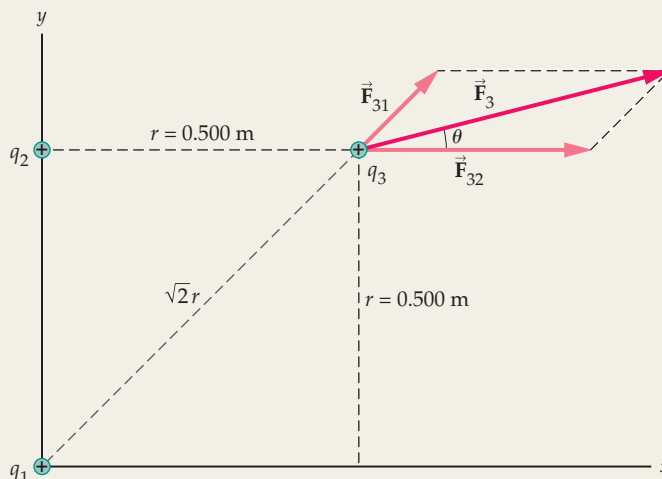
If the magnitude of each charge in this system is doubled, does the point of zero net force move to the right, move to the left, or remain in the same place? Explain.

(Answers to Your Turn problems are given in the back of the book.)

**PROBLEM-SOLVING NOTE****Determining the Direction of the Electric Force**

When determining the total force acting on a charge, begin by calculating the magnitude of each of the individual forces acting on it. Next, assign appropriate directions to the forces based on the principle that “opposites attract, likes repel” and perform a vector sum.

Next we consider systems in which the individual forces are not along the same line. In such cases, it is often useful to resolve the individual force vectors into components and then perform the required vector sum component by component. This technique is illustrated in the following Example and Conceptual Checkpoint.

EXAMPLE 19-3 SUPERPOSITIONThree charges, each equal to $+2.90 \mu\text{C}$, are placed at three corners of a square 0.500 m on a side, as shown in the diagram. Find the magnitude and direction of the net force on charge 3.**PICTURE THE PROBLEM**The positions of the three charges are shown in the sketch. We also show the force produced by charge 1, \vec{F}_{31} , and the force produced by charge 2, \vec{F}_{32} . Note that \vec{F}_{31} is 45° above the x axis and that \vec{F}_{32} is in the positive x direction. Also, the distance from charge 2 to charge 3 is $r = 0.500 \text{ m}$, and the distance from charge 1 to charge 3 is $\sqrt{2}r$.**STRATEGY**To find the net force, we first calculate the magnitudes of \vec{F}_{31} and \vec{F}_{32} and then their components. Summing these components yields the components of the net force, \vec{F}_3 . Once we know the components of \vec{F}_3 , we can calculate its magnitude and direction in the same way as for any other vector.

SOLUTION

1. Find the magnitude of
- \vec{F}_{31}
- :

$$F_{31} = k \frac{|q_1||q_3|}{(\sqrt{2}r)^2}$$

$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.90 \times 10^{-6} \text{ C})^2}{2(0.500 \text{ m})^2}$$

$$= 0.151 \text{ N}$$

2. Find the magnitude of
- \vec{F}_{32}
- :

$$F_{32} = k \frac{|q_2||q_3|}{r^2}$$

$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.90 \times 10^{-6} \text{ C})^2}{(0.500 \text{ m})^2}$$

$$= 0.302 \text{ N}$$

3. Calculate the components of
- \vec{F}_{31}
- and
- \vec{F}_{32}
- :

$$F_{31,x} = F_{31} \cos 45.0^\circ = (0.151 \text{ N})(0.707) = 0.107 \text{ N}$$

$$F_{31,y} = F_{31} \sin 45.0^\circ = (0.151 \text{ N})(0.707) = 0.107 \text{ N}$$

$$F_{32,x} = F_{32} \cos 0^\circ = (0.302 \text{ N})(1) = 0.302 \text{ N}$$

$$F_{32,y} = F_{32} \sin 0^\circ = (0.302 \text{ N})(0) = 0$$

4. Find the components of
- \vec{F}_3
- :

$$F_{3,x} = F_{31,x} + F_{32,x} = 0.107 \text{ N} + 0.302 \text{ N} = 0.409 \text{ N}$$

$$F_{3,y} = F_{31,y} + F_{32,y} = 0.107 \text{ N} + 0 = 0.107 \text{ N}$$

5. Find the magnitude of
- \vec{F}_3
- :

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2}$$

$$= \sqrt{(0.409 \text{ N})^2 + (0.107 \text{ N})^2} = 0.423 \text{ N}$$

6. Find the direction of
- \vec{F}_3
- :

$$\theta = \tan^{-1}\left(\frac{F_{3,y}}{F_{3,x}}\right) = \tan^{-1}\left(\frac{0.107 \text{ N}}{0.409 \text{ N}}\right) = 14.7^\circ$$

INSIGHT

Thus, the net force on charge 3 has a magnitude of 0.423 N and points in a direction 14.7° above the x axis. Note that charge 1, which is $\sqrt{2}$ times farther away from charge 3 than is charge 2, produces only half as much force as charge 2.

PRACTICE PROBLEM

Find the magnitude and direction of the net force on charge 3 if its magnitude is doubled to $5.80 \mu\text{C}$. Assume that charge 1 and charge 2 are unchanged. [Answer: $F_3 = 2(0.423 \text{ N}) = 0.846 \text{ N}$, $\theta = 14.7^\circ$. Note that the angle is unchanged.]

Some related homework problems: Problem 31, Problem 32

CONCEPTUAL CHECKPOINT 19-3 COMPARE THE FORCE

A charge $-q$ is to be placed at either point A or point B in the accompanying figure. Assume points A and B lie on a line that is midway between the two positive charges. Is the net force experienced at point A (a) greater than, (b) equal to, or (c) less than the net force experienced at point B?

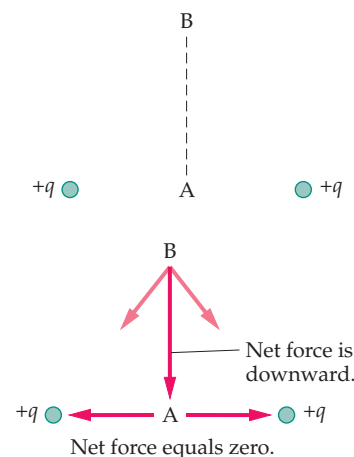
REASONING AND DISCUSSION

Point A is closer to the two positive charges than is point B. As a result, the force exerted by each positive charge will be greater when the charge $-q$ is placed at A. The net force, however, is zero at point A, since the equal attractive forces due to the two positive charges cancel, as shown in the diagram.

At point B, on the other hand, the attractive forces combine to give a net downward force. Hence, the charge $-q$ will experience a greater net force at point B.

ANSWER

(c) The net force at point A is less than the net force at point B.



Spherical Charge Distributions

Although Coulomb's law is stated in terms of point charges, it can be applied to any type of charge distribution by using the appropriate mathematics. For example, suppose a sphere has a charge Q distributed uniformly over its surface. If a

point charge q is outside the sphere, a distance r from its center, the methods of calculus show that the magnitude of the force between the point charge and the sphere is simply

$$F = k \frac{|q||Q|}{r^2}$$

In situations like this, the spherical charge distribution behaves the same as if all its charge were concentrated in a point at its center. For point charges inside a charged spherical shell, the net force exerted by the shell is zero. In general, the electrical behavior of spherical *charge* distributions is analogous to the gravitational behavior of spherical *mass* distributions.

In the next Active Example, we consider a system in which a charge Q is distributed uniformly over the surface of a sphere. In such a case it is often convenient to specify the amount of *charge per area* on the sphere. This is referred to as the **surface charge density**, σ . If a sphere has an area A and a surface charge density σ , its total charge is

$$Q = \sigma A \quad 19-7$$

Note that the SI unit of σ is C/m^2 . If the radius of the sphere is R , then $A = 4\pi R^2$, and $Q = \sigma(4\pi R^2)$.



PROBLEM-SOLVING NOTE

Spherical Charge Distributions

Remember that a uniform spherical charge distribution can be replaced with a point charge only when considering points outside the charge distribution.

ACTIVE EXAMPLE 19-2 FIND THE FORCE EXERTED BY A SPHERE

An insulating sphere of radius $R = 0.10$ m has a uniform surface charge density equal to $5.9 \mu C/m^2$. A point charge of magnitude $0.71 \mu C$ is 0.45 m from the center of the sphere. Find the magnitude of the force exerted by the sphere on the point charge.

SOLUTION (Test your understanding by performing the calculations indicated in each step.)

- | | |
|---|------------------------|
| 1. Find the area of the sphere: | $A = 0.13 \text{ m}^2$ |
| 2. Calculate the total charge on the sphere: | $Q = 0.77 \mu C$ |
| 3. Use Coulomb's law to calculate the magnitude of the force between the sphere and the point charge: | $F = 0.024 \text{ N}$ |

INSIGHT

As long as the point charge is outside the sphere, and the charge distribution remains spherically uniform, the sphere may be treated as a point charge.

YOUR TURN

Suppose the sphere in this problem is replaced by one with half the radius, but with the same surface charge density. Is the force exerted by this sphere greater than, less than, or the same as the force exerted by the original sphere? Explain.

(Answers to **Your Turn** problems are given in the back of the book.)

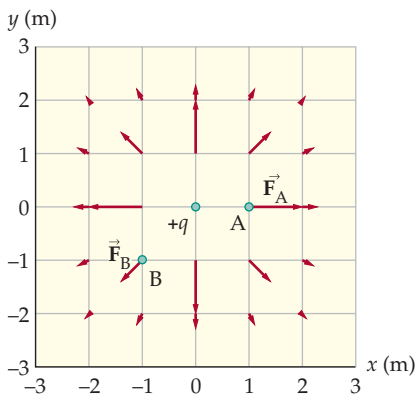


FIGURE 19-9 An electrostatic force field

The positive charge $+q$ at the origin of this coordinate system exerts a different force on a given charge at every point in space. Here we show the force vectors associated with q for a grid of points.

19-4 The Electric Field

You have probably encountered the notion of a “force field” in various science fiction novels and movies. A concrete example of a force field is provided by the force between electric charges. Consider, for example, a positive point charge q at the origin of a coordinate system, as in **Figure 19-9**. If a positive “test charge,” q_0 , is placed at point A, the force exerted on it by q is indicated by the vector \vec{F}_A . On the other hand, if the test charge is placed at point B, the force it experiences there is \vec{F}_B . At every point in space there is a corresponding force. In this sense, **Figure 19-9** allows us to visualize the “force field” associated with the charge q .

Since the magnitude of the force at every point in **Figure 19-9** is proportional to q_0 (due to Coulomb's law), it is convenient to divide by q_0 and define a *force per*

charge at every point in space that is independent of q_0 . We refer to the force per charge as the electric field, \vec{E} . Its precise definition is as follows:

Definition of the Electric Field, \vec{E}

If a test charge q_0 experiences a force \vec{F} at a given location, the electric field \vec{E} at that location is

$$\vec{E} = \frac{\vec{F}}{q_0} \quad 19-8$$

SI unit: N/C

It should be noted that this definition applies whether the force \vec{F} is due to a single charge, as in Figure 19-9, or to a group of charges. In addition, it is assumed that the test charge is small enough that it does not disturb the position of any other charges in the system.

To summarize, *the electric field is the force per charge at a given location*. Therefore, if we know the electric field vector \vec{E} at a given point, the force that a charge q experiences at that point is

$$\vec{F} = q\vec{E} \quad 19-9$$

Notice that the direction of the force depends on the sign of the charge. In particular,

- A positive charge experiences a force *in the direction* of \vec{E} .
- A negative charge experiences a force *in the opposite direction* of \vec{E} .

Finally, the magnitude of the force is the product of the magnitudes of q and \vec{E} :

- The magnitude of the force acting on a charge q is $F = |q|E$.

As we continue in this chapter, we will determine the electric field for a variety of different charge distributions. In some cases, \vec{E} will decrease with distance as $1/r^2$ (a point charge), in other cases as $1/r$ (a line of charge), and in others \vec{E} will be a constant (a charged plane). Before we calculate the electric field itself, however, we first consider the force exerted on charges by a constant electric field.

PROBLEM-SOLVING NOTE

The Force Exerted by an Electric Field



The force exerted on a charge by an electric field can point in only one of two directions—parallel or antiparallel to the direction of the field.

EXAMPLE 19-4 FORCE FIELD

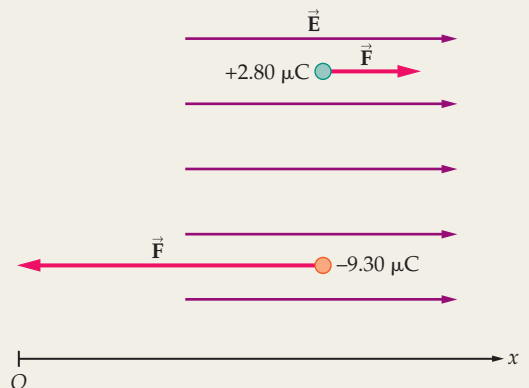
In a certain region of space, a uniform electric field has a magnitude of 4.60×10^4 N/C and points in the positive x direction. Find the magnitude and direction of the force this field exerts on a charge of (a) $+2.80 \mu\text{C}$ and (b) $-9.30 \mu\text{C}$.

PICTURE THE PROBLEM

In our sketch we indicate the uniform electric field and the two charges mentioned in the problem. Note that the positive charge experiences a force in the positive x direction (the direction of \vec{E}), and the negative charge experiences a force in the negative x direction (opposite to \vec{E}).

STRATEGY

To find the magnitude of each force, we use $F = |q|E$. The direction has already been indicated in our sketch.



SOLUTION

Part (a)

1. Find the magnitude of the force on the $+2.80\text{-}\mu\text{C}$ charge:

$$F = |q|E = (2.80 \times 10^{-6} \text{ C})(4.60 \times 10^4 \text{ N/C}) = 0.129 \text{ N}$$

Part (b)

2. Find the magnitude of the force on the $-9.30\text{-}\mu\text{C}$ charge:

$$F = |q|E = (9.30 \times 10^{-6} \text{ C})(4.60 \times 10^4 \text{ N/C}) = 0.428 \text{ N}$$

INSIGHT

To summarize, the force on the $+2.80\text{-}\mu\text{C}$ charge is of magnitude 0.129 N in the positive x direction; the force on the $-9.30\text{-}\mu\text{C}$ charge is of magnitude 0.428 N in the negative x direction.

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PRACTICE PROBLEMIf the $+2.80\text{-}\mu\text{C}$ charge experiences a force of 0.25 N , what is the magnitude of the electric field? [Answer: $E = 8.9 \times 10^4\text{ N/C}$]

Some related homework problems: Problem 44, Problem 47

**REAL-WORLD PHYSICS****Electrodialysis for water purification**

The fact that charges of opposite sign experience forces in opposite directions in an electric field is used to purify water in the process known as **electrodialysis**. This process depends on the fact that most minerals that dissolve in water dissociate into positive and negative ions. Probably the most common example is table salt (NaCl), which dissociates into positive sodium ions (Na^+) and negative chlorine ions (Cl^-). When brackish water is passed through a strong electric field in an electrodialysis machine, the mineral ions move in opposite directions and pass through two different types of semipermeable membrane—one that allows only positive ions to pass through, the other only negative ions. This process leaves water that is purified of dissolved minerals and suitable for drinking.

The Electric Field of a Point Charge

Perhaps the simplest example of an electric field is the field produced by an idealized point charge. To be specific, suppose a positive point charge q is at the origin in **Figure 19–10**. If a positive test charge q_0 is placed a distance r from the origin, the force it experiences is directed away from the origin and is of magnitude

$$F = k \frac{|q||q_0|}{r^2}$$

Applying our definition of the electric field in Equation 19–8, we find that the magnitude of the field is

$$E = \frac{F}{q_0} = \frac{\left(k \frac{|q||q_0|}{r^2}\right)}{q_0} = k \frac{|q|}{r^2}$$

Since a positive charge experiences a force that is radially outward, that too is the direction of \vec{E} .

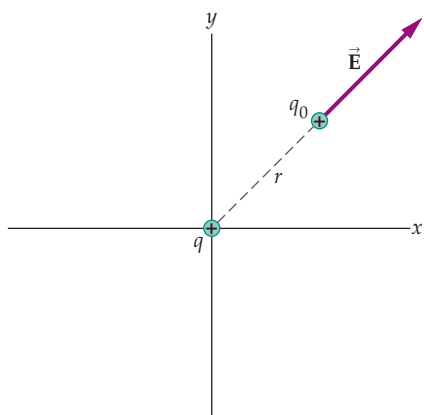
In general, then, we can say that the electric field a distance r from a point charge q has the following magnitude:

Magnitude of the Electric Field Due to a Point Charge

$$E = k \frac{|q|}{r^2}$$

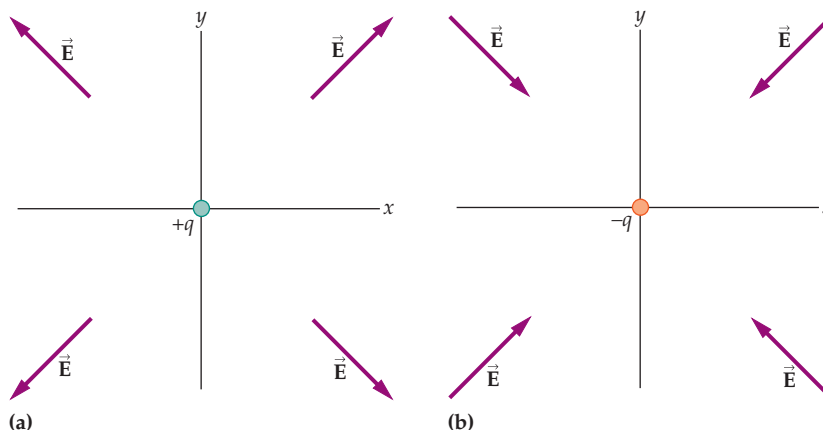
19–10

If the charge q is positive, the field points radially outward from the charge; if it is negative, the field is radially inward. This is illustrated in **Figure 19–11**. Thus, to



▲ FIGURE 19–10 The electric field of a point charge

The electric field \vec{E} due to a positive charge q at the origin is radially outward. Its magnitude is $E = k|q|/r^2$.



► FIGURE 19–11 The direction of the electric field

(a) The electric field due to a positive charge at the origin points radially outward. (b) If the charge at the origin is negative, the electric field is radially inward.

determine the electric field due to a point charge, we first use Equation 19-10 to find its magnitude, and then use the rule illustrated in Figure 19-11 to find its direction.

EXERCISE 19-3

Find the electric field produced by a $1.0\text{-}\mu\text{C}$ point charge at a distance of (a) 0.75 m and (b) 1.5 m .

SOLUTION

- a. Applying Equation 19-10 with $q = 1.0\ \mu\text{C}$ and $r = 0.75\text{ m}$ yields

$$E = k \frac{|q|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.0 \times 10^{-6} \text{ C})}{(0.75 \text{ m})^2} = 1.6 \times 10^4 \text{ N/C}$$

- b. Noting that E depends on $1/r^2$, we see that doubling the distance from 0.75 m to 1.5 m results in a reduction in the electric field by a factor of 4:

$$E = \frac{1}{4}(1.6 \times 10^4 \text{ N/C}) = 0.40 \times 10^4 \text{ N/C}$$

Superposition of Fields

Many electrical systems consist of more than two charges. In such cases, the total electric field can be found by using superposition—just as when we find the total force due to a system of charges. In particular, the total electric field is found by calculating the vector sum—often using components—of the electric fields due to each charge separately.

For example, let's calculate the total electric field at point P in **Figure 19-12**. First we sketch the directions of the fields \vec{E}_1 and \vec{E}_2 due to the charges $q_1 = +q$ and $q_2 = +q$, respectively. In particular, if a positive test charge is at point P, the force due to q_1 is down and to the right, whereas the force due to q_2 is up and to the right. From the geometry of the figure we see that \vec{E}_1 is at an angle θ below the x axis, and—by symmetry— \vec{E}_2 is at the same angle θ above the axis. Since the two charges have the same magnitude, and the distances from P to the charges are the same, it follows that \vec{E}_1 and \vec{E}_2 have the same magnitude:

$$E_1 = E_2 = E = k \frac{|q|}{d^2}$$

To find the net electric field \vec{E}_{net} , we use components. First, consider the y direction. In this case, we have $E_{1,y} = -E \sin \theta$ and $E_{2,y} = +E \sin \theta$. Hence, the y component of the net electric field is zero:

$$E_{\text{net},y} = E_{1,y} + E_{2,y} = -E \sin \theta + E \sin \theta = 0$$

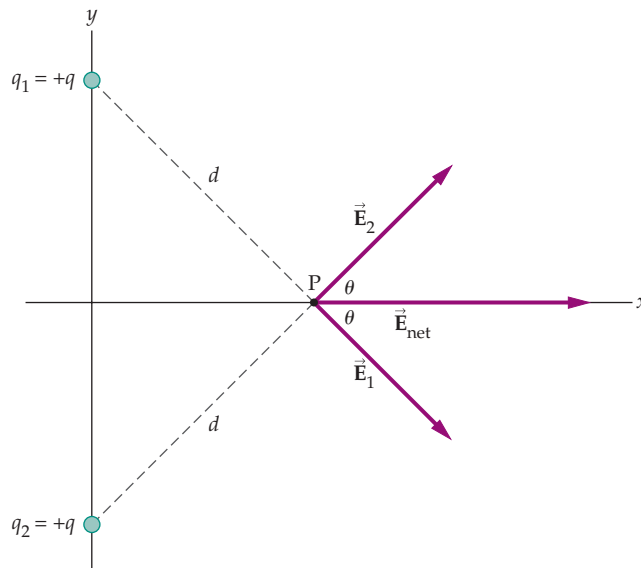


FIGURE 19-12 Superposition of the electric field

The net electric field at the point P is the vector sum of the fields due to the charges q_1 and q_2 . Note that \vec{E}_1 and \vec{E}_2 point away from the charges q_1 and q_2 , respectively. This is as expected, since both of these charges are positive.

Referring again to Figure 19–12, it is apparent that this result could have been anticipated by symmetry considerations. Finally, we determine the x component of E_{net} :

$$E_{\text{net},x} = E_{1,x} + E_{2,x} = E \cos \theta + E \cos \theta = 2E \cos \theta$$

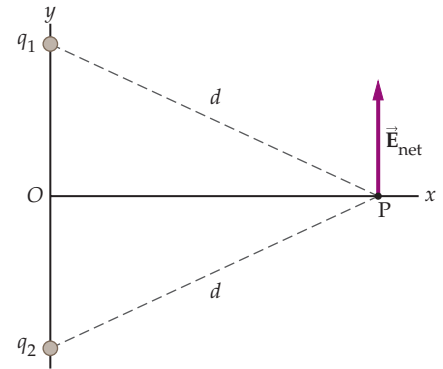
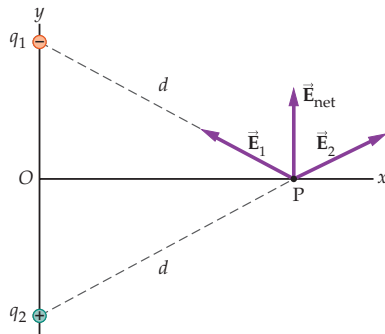
Thus, the net electric field at P is in the positive x direction, as shown in Figure 19–12, and has a magnitude equal to $2E \cos \theta$.

CONCEPTUAL CHECKPOINT 19–4 THE SIGN OF THE CHARGES

Two charges, q_1 and q_2 , have equal magnitudes q and are placed as shown in the figure to the right. The net electric field at point P is vertically upward. Do we conclude that (a) q_1 is positive, q_2 is negative; (b) q_1 is negative, q_2 is positive; or (c) q_1 and q_2 have the same sign?

REASONING AND DISCUSSION

If the net electric field at P is vertically upward, the x components of \vec{E}_1 and \vec{E}_2 must cancel, and the y components must both be in the positive y direction. The only way for this to happen is to have q_1 negative and q_2 positive, as shown in the following diagram.



With this choice, a positive test charge at P is attracted to q_1 (so that \vec{E}_1 is up and to the left) and repelled from q_2 (so that \vec{E}_2 is up and to the right).

ANSWER

(b) q_1 is negative, q_2 is positive.

We conclude this section by considering the same physical system presented in Example 19–3, this time from the point of view of the electric field.

EXAMPLE 19–5 SUPERPOSITION IN THE FIELD

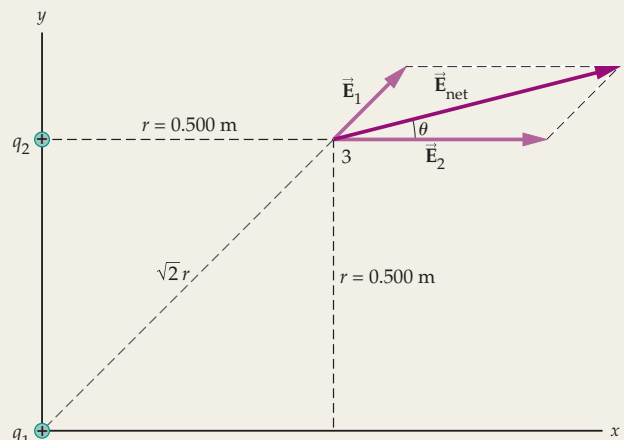
Two charges, each equal to $+2.90 \mu\text{C}$, are placed at two corners of a square 0.500 m on a side, as shown in the sketch. Find the magnitude and direction of the net electric field at a third corner of the square, the point labeled 3 in the sketch.

PICTURE THE PROBLEM

The positions of the two charges are shown in the sketch. We also show the electric field produced by each charge. The key difference between this sketch and the one in Example 19–3 is that in this case there is no charge at point 3; the electric field still exists there, even though it has no charge on which to exert a force.

STRATEGY

In analogy with Example 19–3, we first calculate the magnitudes of \vec{E}_1 and \vec{E}_2 and then their components. Summing these components yields the components of the net electric field, \vec{E}_{net} . Once we know the components of \vec{E}_{net} , we find its magnitude and direction in the same way as for \vec{F}_{net} in Example 19–3.



SOLUTION1. Find the magnitude of \vec{E}_1 :

$$\begin{aligned} E_1 &= k \frac{|q_1|}{(\sqrt{2}r)^2} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.90 \times 10^{-6} \text{ C})}{2(0.500 \text{ m})^2} \\ &= 5.21 \times 10^4 \text{ N/C} \end{aligned}$$

2. Find the magnitude of \vec{E}_2 :

$$\begin{aligned} E_2 &= k \frac{|q_2|}{r^2} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2.90 \times 10^{-6} \text{ C})}{(0.500 \text{ m})^2} \\ &= 1.04 \times 10^5 \text{ N/C} \end{aligned}$$

3. Calculate the components of \vec{E}_1 and \vec{E}_2 :

$$\begin{aligned} E_{1,x} &= E_1 \cos 45.0^\circ \\ &= (5.21 \times 10^4 \text{ N/C})(0.707) = 3.68 \times 10^4 \text{ N/C} \\ E_{1,y} &= E_1 \sin 45.0^\circ \\ &= (5.21 \times 10^4 \text{ N/C})(0.707) = 3.68 \times 10^4 \text{ N/C} \\ E_{2,x} &= E_2 \cos 0^\circ \\ &= (1.04 \times 10^5 \text{ N/C})(1) = 1.04 \times 10^5 \text{ N/C} \\ E_{2,y} &= E_2 \sin 0^\circ = (1.04 \times 10^5 \text{ N/C})(0) = 0 \end{aligned}$$

4. Find the components of \vec{E}_{net} :

$$\begin{aligned} E_{\text{net},x} &= E_{1,x} + E_{2,x} \\ &= 3.68 \times 10^4 \text{ N/C} + 1.04 \times 10^5 \text{ N/C} \\ &= 1.41 \times 10^5 \text{ N/C} \\ E_{\text{net},y} &= E_{1,y} + E_{2,y} \\ &= 3.68 \times 10^4 \text{ N/C} + 0 = 3.68 \times 10^4 \text{ N/C} \end{aligned}$$

5. Find the magnitude of \vec{E}_{net} :

$$\begin{aligned} E_{\text{net}} &= \sqrt{E_{\text{net},x}^2 + E_{\text{net},y}^2} \\ &= \sqrt{(1.41 \times 10^5 \text{ N/C})^2 + (3.68 \times 10^4 \text{ N/C})^2} \\ &= 1.46 \times 10^5 \text{ N/C} \end{aligned}$$

6. Find the direction of \vec{E}_{net} :

$$\begin{aligned} \theta &= \tan^{-1}(E_{\text{net},y}/E_{\text{net},x}) \\ &= \tan^{-1}\left(\frac{3.68 \times 10^4 \text{ N/C}}{1.41 \times 10^5 \text{ N/C}}\right) = 14.6^\circ \end{aligned}$$

INSIGHT

Note that, as one would expect, the direction of the net electric field is the same as the direction of the net force in Example 19-3 (except for a small discrepancy in the last decimal place due to rounding off in the calculations). In addition, the magnitude of the force exerted by the electric field on a charge of $2.90 \mu\text{C}$ is $F = qE_{\text{net}} = (2.90 \mu\text{C})(1.46 \times 10^5 \text{ N/C}) = 0.423 \text{ N}$, the same as was found in Example 19-3.

PRACTICE PROBLEM

Find the magnitude and direction of the net electric field at the bottom right corner of the square. [Answer: $E_{\text{net}} = 1.46 \times 10^5 \text{ N/C}$, $\theta = -14.6^\circ$]

Some related homework problems: Problem 50, Problem 51

Many aquatic creatures are capable of producing electric fields. For example, African freshwater fishes in the family Mormyridae can generate weak electric fields from modified tail muscles and are able to detect variations in this field as they move through their environment. With this capability, these nocturnal feeders have an electrical guidance system that assists them in locating obstacles, enemies, and food. Much stronger electric fields are produced by electric eels and electric skates. In particular, the electric eel *Electrophorus electricus* generates

REAL-WORLD PHYSICS: BIO**Electric fish**



REAL-WORLD PHYSICS: BIO

Electrical shark repellent

electric fields great enough to kill small animals and to stun larger animals, including humans.

Sharks are well known for their sensitivity to weak electric fields in their surroundings. In fact, they possess specialized organs for this purpose, known as the ampullae of Lorenzini, which assist in the detection of prey. Recently, this sensitivity has been put to use as a method of repelling sharks in order to protect swimmers and divers. A device called the SharkPOD (Protective Oceanic Device) consists of two metal electrodes, one attached to a diver's air tank, the other to one of the diver's fins. These electrodes produce a strong electric field that completely surrounds the diver and causes sharks to turn away out to a distance of up to 7 m.

The SharkPOD was used in the 2000 Summer Olympic Games in Sydney to protect swimmers competing in the triathlon. The swimming part of the event was held in Sydney harbor, where great white sharks are a common sight. To protect the swimmers, divers wearing the SharkPOD swam along the course, a couple meters below the athletes. The race was completed without incident.

19-5 Electric Field Lines

When looking at plots like those in Figures 19-09 and 19-11, it is tempting to imagine a pictorial representation of the electric field. This thought is reinforced when one considers a photograph like **Figure 19-13**, which shows grass seeds suspended in oil. Because of polarization effects, the grass seeds tend to align in the direction of the electric field, much like the elongated atoms shown in Figure 19-5. In this case, the seeds are aligned radially, due to the electric field of the charged rod seen "end on" in the middle of the photograph. Clearly, a set of radial lines would seem to represent the electric field in this case.

In fact, an entirely consistent method of drawing electric field lines is obtained by using the following set of rules:

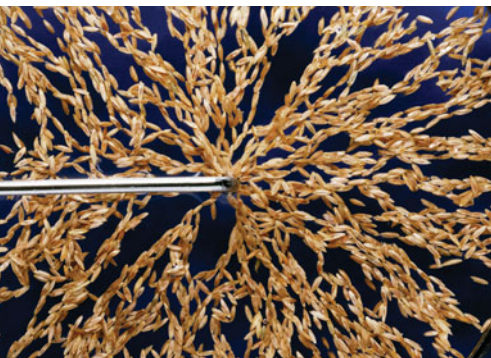
Rules for Drawing Electric Field Lines

Electric field lines:

1. Point in the *direction* of the electric field vector \vec{E} at every point;
2. *Start* at positive (+) charges or at infinity;
3. *End* at negative (-) charges or at infinity;
4. Are more *dense* where \vec{E} has a greater magnitude. In particular, the number of lines entering or leaving a charge is proportional to the magnitude of the charge.

We now show how these rules are applied.

For example, the electric field lines for two different point charges are presented in **Figure 19-14**. First, we know that the electric field points directly away from the charge (+ q) in Figure 19-14 (a); hence from rule 1 the field lines are radial. In agreement with rule 2 the field lines start on a + charge, and in agreement

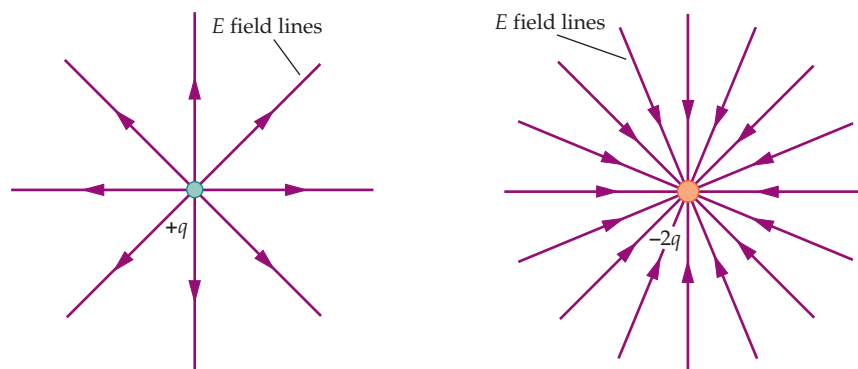


▲ **FIGURE 19-13** Grass seeds in an electric field

Grass seeds aligning with electric field lines.

▶ **FIGURE 19-14** Electric field lines for a point charge

(a) Near a positive charge the field lines point radially away from the charge. The lines start on the positive charge and end at infinity. (b) Near a negative charge the field lines point radially inward. They start at infinity and end on a negative charge and are more dense where the field is more intense. Notice that the number of lines drawn for part (b) is twice the number drawn for part (a), a reflection of the relative magnitudes of the charges.



(a) E field lines point away from positive charges (b) E field lines point toward negative charges

with rule 3 they end at infinity. Finally, as anticipated from rule 4, the field lines are closer together near the charge, where the field is more intense. Similar considerations apply to Figure 19-14 (b), where the charge is $-2q$. In this case, however, the direction of the field lines is reversed and the number of lines is doubled.

CONCEPTUAL CHECKPOINT 19-5 INTERSECT OR NOT?

Which of the following statements is correct: Electric field lines **(a)** can or **(b)** cannot intersect?

REASONING AND DISCUSSION

By definition, electric field lines are always tangent to the electric field. Since the electric force, and hence the electric field, can point in only one direction at any given location, it follows that field lines cannot intersect. If they did, the field at the intersection point would have two conflicting directions.

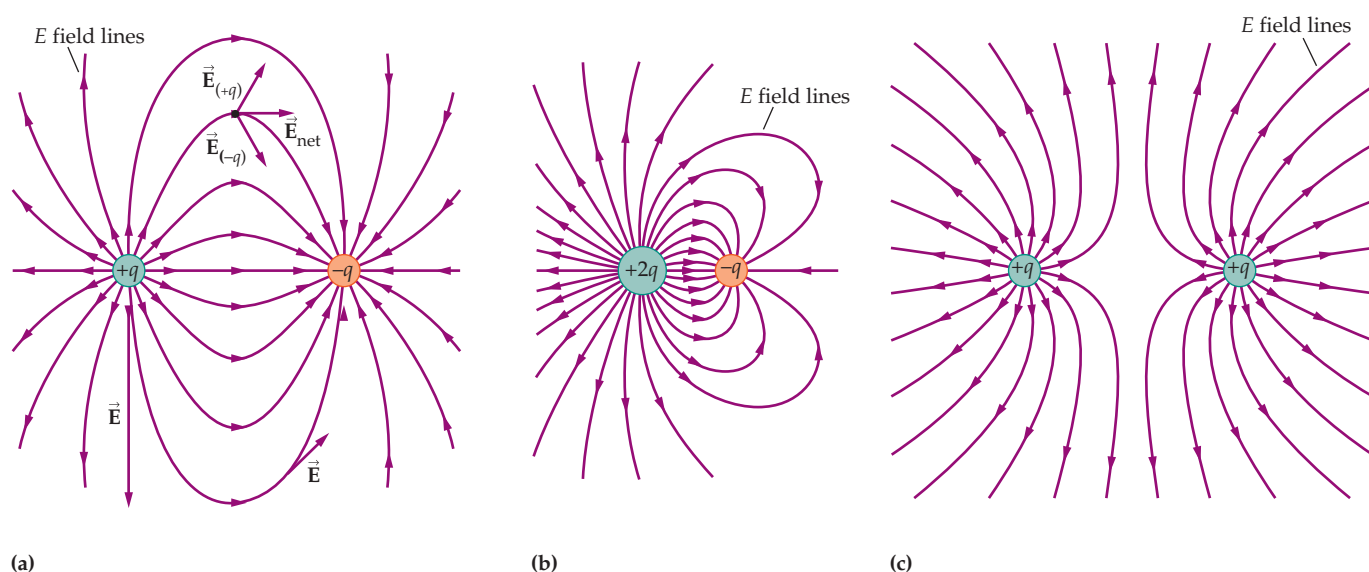
ANSWER

(b) Electric field lines cannot intersect.

Figure 19-15 shows examples of electric field lines for various combinations of charges. In systems like these, we draw a set of curved field lines that are tangent to the electric field vector, \vec{E} , at every point. This is illustrated for a variety of points in Figure 19-15 (a), and similar considerations apply to all such field diagrams. In addition, note that the magnitude of \vec{E} is greater in those regions of Figure 19-15 where the field lines are more closely packed together. Clearly, then, we expect an intense electric field between the charges in Figure 19-15 (b) and a vanishing field between the charges in Figure 19-15 (c).

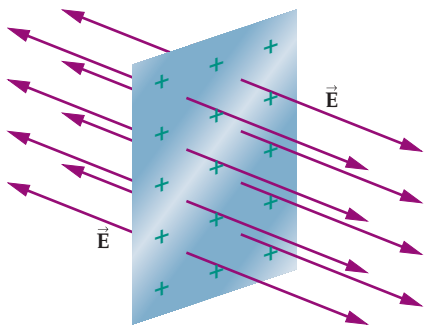
Of particular interest is the $+q$ and $-q$ charge combination in Figure 19-15 (a). In general, a system of equal and opposite charges separated by a nonzero distance is known as an **electric dipole**. The total charge of the dipole is zero, but because the charges are separated, the electric field does not vanish. Instead, the field lines form “loops” that are characteristic of dipoles.

Many molecules are polar—water is a common example—which means they have an excess of positive charge near one end and a corresponding excess of



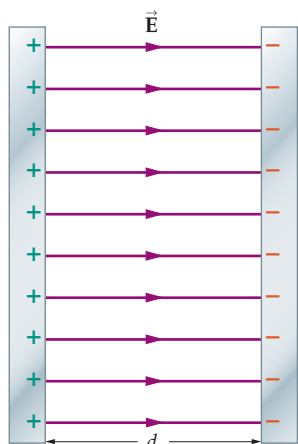
▲ FIGURE 19-15 Electric field lines for systems of charges

(a) The electric field lines for a dipole form closed loops that become more widely spaced with distance from the charges. Note that at each point in space, the electric field vector \vec{E} is tangent to the field lines. (b) In a system with a net charge, some field lines extend to infinity. If the charges have opposite signs, some field lines start on one charge and terminate on the other charge. (c) All of the field lines in a system with charges of the same sign extend to infinity.



◀ **FIGURE 19-16** The electric field of a charged plate

The electric field near a large charged plate is uniform in direction and magnitude.



▲ **FIGURE 19-17** A parallel-plate capacitor

In the ideal case, the electric field is uniform between the plates and zero outside.

negative charge near the other end. As a result, they produce an electric dipole field. Similarly, a typical bar magnet produces a *magnetic* dipole field, as we shall see in Chapter 22.

Finally, the electric field representations in Figures 19-14 and 19-15 are two-dimensional “slices” through the full field, which is three-dimensional. Therefore, one should imagine a similar set of field lines in these figures coming out of the page and going into the page.

Parallel-Plate Capacitor

A particularly simple and important field picture results when charge is spread uniformly over a large plate, as illustrated in **Figure 19-16**. At points that are not near the edge of the plate, the electric field is uniform in both direction and magnitude. That is, the field points in a single direction—perpendicular to the plate—and its magnitude is independent of the distance from the plate. This result can be proved using Gauss’s law, as we show in Section 19-7.

If two such conducting plates with opposite charge are placed parallel to one another and separated by a distance d , as in **Figure 19-17**, the result is referred to as a **parallel-plate capacitor**. The field for such a system is uniform between the plates, and zero outside the plates. This is the ideal case, which is exactly true for an infinite plate and a good approximation for real plates. The field lines are illustrated in Figure 19-17. Parallel-plate capacitors are discussed further in the next chapter and will be of particular interest in Chapters 21 and 24, when we consider electric circuits.

EXAMPLE 19-6 DANGLING BY A THREAD

The electric field between the plates of a parallel-plate capacitor is horizontal, uniform, and has a magnitude E . A small object of mass 0.0250 kg and charge $-3.10\text{ }\mu\text{C}$ is suspended by a thread between the plates, as shown in the sketch. The thread makes an angle of 10.5° with the vertical. Find (a) the tension in the thread and (b) the magnitude of the electric field.

PICTURE THE PROBLEM

Our sketch shows the thread making an angle $\theta = 10.5^\circ$ with the vertical. The inset to the right shows the free-body diagram for the suspended object, as well as our choice of positive x and y directions. Note that we label the charge of the object $-q$, where $q = 3.10\text{ }\mu\text{C}$, in order to clearly indicate its sign.

STRATEGY

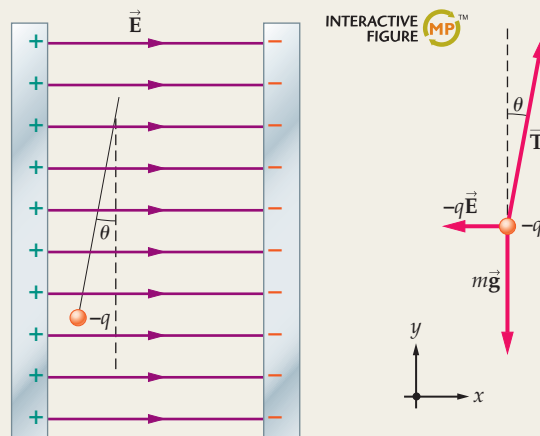
The relevant physical principle in this problem is that because the object is at rest, the net force acting on it must vanish. Thus, setting the x and y components of the net force equal to zero yields two conditions, which can be used to solve for the two unknowns, T and E .

SOLUTION

1. Set the net force in the x direction equal to zero:
2. Set the net force in the y direction equal to zero:

Part (a)

3. Because we know all the quantities in the y force equation except for the tension, we use it to solve for T :



$$-qE + T \sin \theta = 0$$

$$T \cos \theta - mg = 0$$

$$T = \frac{mg}{\cos \theta} = \frac{(0.0250\text{ kg})(9.81\text{ m/s}^2)}{\cos(10.5^\circ)} = 0.249\text{ N}$$

Part (b)

4. Now use the x force equation to find the magnitude of the electric field, E :

$$E = \frac{T \sin \theta}{q} = \frac{(0.249 \text{ N}) \sin(10.5^\circ)}{3.10 \times 10^{-6} \text{ C}} = 1.46 \times 10^4 \text{ N/C}$$

INSIGHT

As expected, the negatively charged object is attracted to the positively charged plate. This means that the electric force exerted on it is opposite in direction to the electric field.

PRACTICE PROBLEM

Suppose the electric field between the plates is $2.50 \times 10^4 \text{ N/C}$, but the charge of the object and the angle of the thread with the vertical are the same as before. Find the tension in the thread and the mass of the object. [Answer: $T = 0.425 \text{ N}$, $m = 0.0426 \text{ kg}$]

Some related homework problems: Problem 60, Problem 94

The fact that a charge experiences a force when it passes between two charged plates finds application in a wide variety of devices. For example, the image you see on many television screens is produced when a beam of electrons strikes the screen from behind and illuminates individual red, blue, or green *pixels*. Which pixels are illuminated and which remain dark is controlled by parallel charged plates that deflect the electron beam up or down and left or right. Thus, sending the appropriate electrical signals to the deflection plates makes the beam of electrons “paint” any desired picture on the screen.

Similar deflection plates are used in an ink-jet printer. In this case, the beam in question is not a beam of electrons but rather a beam of electrically charged ink droplets. The beam of droplets can be deflected as desired, so that individual letters can be constructed from a series of closely spaced dots on the page. A typical printer might produce as many as 600 dots per inch—that is, 600 dpi.

19-6 Shielding and Charging by Induction

In a perfect conductor there are enormous numbers of electrons completely free to move about within the conductor. This simple fact has some rather interesting consequences. Consider, for example, a solid metal sphere attached to an insulating base as in **Figure 19-18**. Suppose a positive charge Q is placed on the sphere. The question is: How does this charge distribute itself on the sphere when it is in equilibrium—that is, when all the charges are at rest? In particular, does the charge spread itself uniformly throughout the volume of the sphere, or does it concentrate on the surface?

The answer is that the charge concentrates on the surface, as shown in **Figure 19-18 (a)**, but let’s investigate why this should be the case. First, assume the opposite—that the charge is spread uniformly throughout the sphere’s volume, as indicated in **Figure 19-18 (b)**. If this were the case, a charge at location A would experience an outward force due to the spherical distribution of charge between it and the center of the sphere. Since charges are free to move, the charge at A would respond to this force by moving toward the surface. Clearly, then, a uniform distribution of charge within the sphere’s volume is not in equilibrium. In fact, the argument that a charge at point A will move toward the surface can be applied to any charge within the sphere. Thus, the net result is that *all* the excess charge Q moves onto the surface of the sphere which, in turn, allows the individual charges to be spread as far from one another as possible.

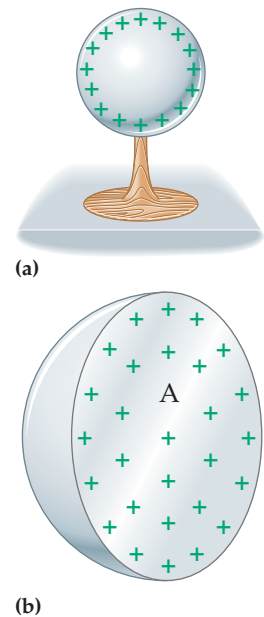
The preceding result holds no matter what the shape of the conductor. In general,

Excess Charge on a Conductor

Excess charge placed on a conductor, whether positive or negative, moves to the exterior surface of the conductor.

REAL-WORLD PHYSICS

Television screens and ink-jet printers



▲ FIGURE 19-18 Charge distribution on a conducting sphere

(a) A charge placed on a conducting sphere distributes itself uniformly on the surface of the sphere; none of the charge is within the volume of the sphere. (b) If the charge were distributed uniformly throughout the volume of a sphere, individual charges, like that at point A , would experience a force due to other charges in the volume. Since charges are free to move in a conductor, they will respond to these forces by moving as far from one another as possible—that is, to the surface of the conductor.

We specify the exterior surface in this statement because a conductor may contain one or more cavities. When an excess charge is applied to such a conductor, all the charge ends up on the exterior surface, and none on the interior surfaces.

Electrostatic Shielding

The ability of electrons to move freely within a conductor has another important consequence; namely, the electric field within a conductor vanishes.

Zero Field within a Conductor

When electric charges are at rest, the electric field within a conductor is zero; $E = 0$.

By *within* a conductor, we mean a location in the actual material of the conductor, as opposed to a location in a cavity within the material.

The best way to see the validity of this statement is to again consider the opposite. If there were a nonzero field within a conductor, electrons would move in response to the field. They would continue to move until finally the field was reduced to zero, at which point the system would be in equilibrium and no more charges would move. Thus, equilibrium and $E = 0$ within a conductor go hand in hand.

A straightforward extension of this idea explains the phenomenon of **shielding**, in which a conductor “shields” its interior from external electric fields. For example, in **Figure 19–19 (a)** we show an uncharged, conducting metal sphere placed in an electric field. Because the positive ions in the metal do not move, the field tends to move negative charges to the left and leave excess positive charges on the right; hence, it causes the sphere to have an **induced** negative charge on its left half and an induced positive charge on its right half. The total charge on the sphere, of course, is still zero. Since field lines end on (–) charges and begin on (+) charges, the external electric field ends on the left half of the sphere and starts up again on the right half. In between, within the conductor, the field is zero, as expected. Thus, the conductor has shielded its interior from the applied electric field.

Shielding occurs whether the conductor is solid, as in **Figure 19–19**, or hollow. In fact, even a thin sheet of metal foil formed into the shape of a box will shield its interior from external electric fields. This effect is put to use in numerous electrical devices, which often have a metal foil or wire mesh enclosure surrounding the sensitive electrical circuits. In this way, a given device can be isolated from the effects of other nearby devices that might otherwise interfere with its operation.

Notice also in **Figure 19–19** that the field lines bend slightly near the surface of the sphere. In fact, on closer examination, as in **Figure 19–19 (b)**, we see that the field lines always contact the surface at right angles. This is true for any conductor:

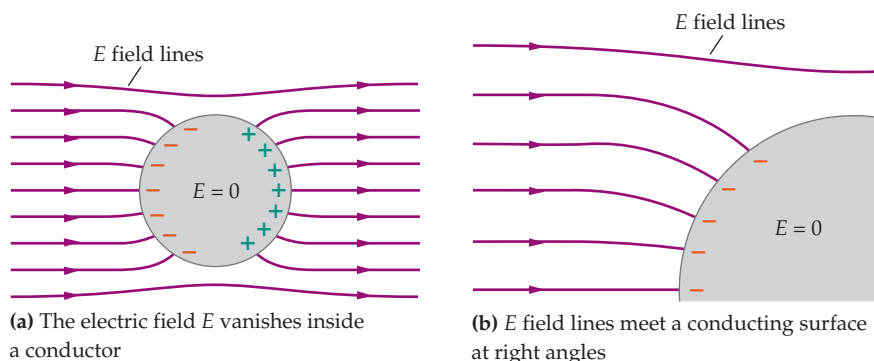
Electric Fields at Conductor Surfaces

Electric field lines contact conductor surfaces at right angles.

If an electric field contacted a conducting surface at an angle other than 90° , the result would be a component of force parallel to the surface. This would result in a movement of electrons and, hence, would not correspond to equilibrium. Instead, electrons would move until the parallel component of the electric field was canceled.

► **FIGURE 19–19** Electric field near a conducting surface

(a) When an uncharged conductor is placed in an electric field, the field induces opposite charges on opposite sides of the conductor. The net charge on the conductor is still zero, however. The induced charges produce a field within the conductor that *exactly* cancels the external field, leading to $E = 0$ inside the conductor. This is an example of electrical shielding. (b) Electric field lines meet the surface of a conductor at right angles.



Further examples of field lines near conducting surfaces are shown in **Figure 19-20**. Notice that the field lines are more densely packed near a sharp point, indicating that the field is more intense in such regions. This effect illustrates the basic principle behind the operation of lightning rods. If you look closely, you will notice that all lightning rods have a sharply pointed tip. During an electrical storm, the electric field at the tip becomes so intense that electric charge is given off into the atmosphere. In this way, a lightning rod acts to discharge the area near a house—by giving off a steady stream of charge—thus preventing a strike by a bolt of lightning, which transfers charge in one sudden blast. Sharp points on the rigging of a ship at sea can also give off streams of charge during a storm, often producing glowing lights referred to as Saint Elmo's fire.

The same principle is used to clean the air we breathe, in devices known as *electrostatic precipitators*. In an electrostatic precipitator, smoke and other airborne particles in a smokestack are given a charge as they pass by sharply pointed electrodes—like lightning rods—within the stack. Once the particles are charged, they are removed from the air by charged plates that exert electrostatic forces on them. The resultant emission from the smokestack contains drastically reduced amounts of potentially harmful particulates.



▲ In a dramatic science-museum demonstration of electrical shielding (left), the metal bars of a cage provide excellent protection from an artificially generated lightning bolt. A more practical safeguard is the lightning rod (right). Lightning rods always have sharp points, because that is where the electric field of a conductor is most intense. At the tip, the field can become so strong that charge leaks away into the atmosphere rather than building up to levels that will attract a lightning strike. If a strike does occur, it is conducted to the ground through the lightning rod, rather than through some part of the building itself.

One final note regarding shielding is that it works in one direction only: A conductor shields its interior from external fields, but it does not shield the external world from fields within it. This phenomenon is illustrated in **Figure 19-21** for the case of an uncharged conductor. First, the charge $+Q$ in the cavity induces a charge $-Q$ on the interior surface, in order for the field in the conductor to be zero. Since the conductor is uncharged, a charge $+Q$ will be induced on its exterior surface. As a result, the external world will experience a field due, ultimately, to the charge $+Q$ within the cavity of the conductor.

Charging by Induction

One way to charge an object is to touch it with a charged rod; but since electric forces can act at a distance, it is also possible to charge an object without making direct physical contact. This type of charging is referred to as **charging by induction**.

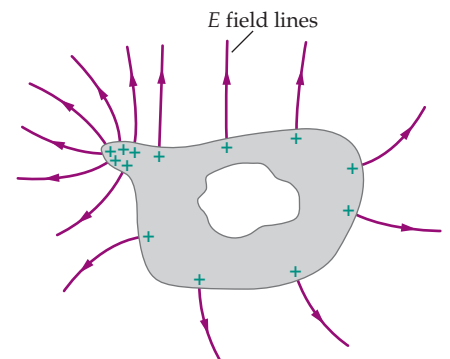
REAL-WORLD PHYSICS

Lightning rods and Saint Elmo's fire



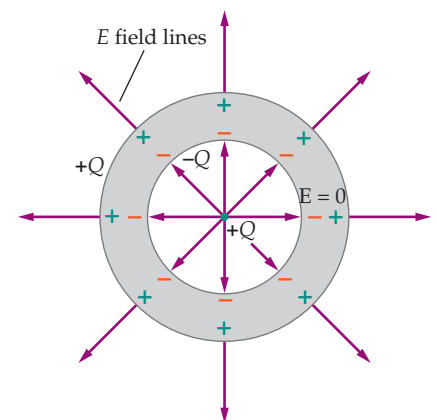
REAL-WORLD PHYSICS

Electrostatic precipitation



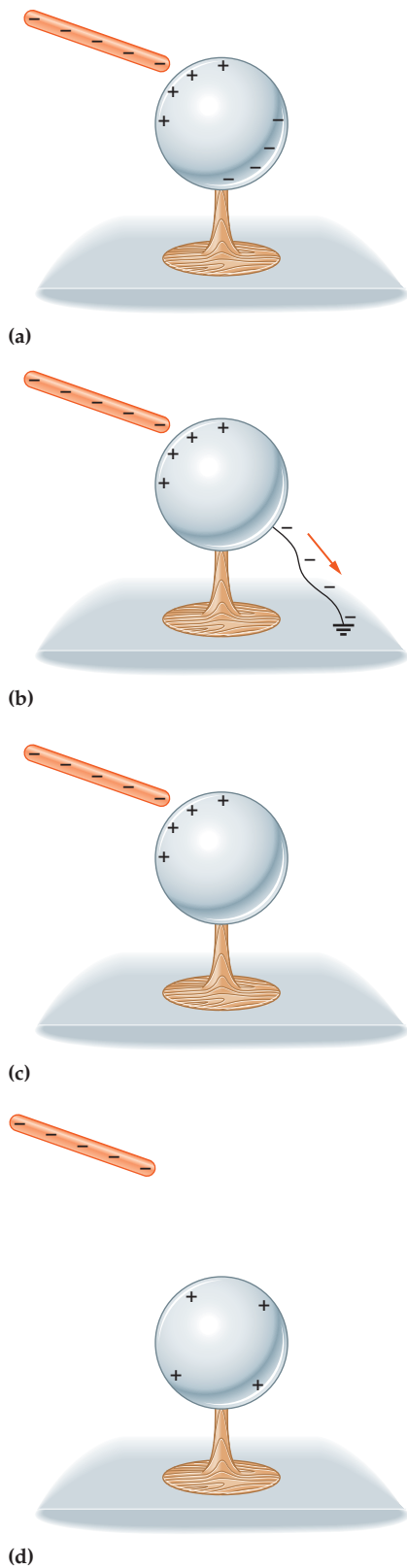
▲ **FIGURE 19-20** Intense electric field near a sharp point

Electric charges and field lines are more densely packed near a sharp point. This means that the electric field is more intense in such regions as well. (Note that there are no electric charges on the interior surface surrounding the cavity.)



▲ **FIGURE 19-21** Shielding works in only one direction

A conductor does not shield the external world from charges it encloses. Still, the electric field is zero within the conductor itself.



◀ **FIGURE 19–22** Charging by induction

(a) A charged rod induces + and – charges on opposite sides of the conductor. (b) When the conductor is grounded, charges that are repelled by the rod enter the ground. There is now a net charge on the conductor. (c) Removing the grounding wire, with the charged rod still in place, traps the net charge on the conductor. (d) The charged rod can now be removed, and the conductor retains a charge that is opposite in sign to that on the charged rod.

To see how this type of charging works, consider an uncharged metal sphere on an insulating base. If a negatively charged rod is brought close to the sphere without touching it, as in **Figure 19–22 (a)**, electrons in the sphere are repelled. An induced positive charge is produced on the near side of the sphere, and an induced negative charge on the far side. At this point the sphere is still electrically neutral, however.

The key step in this charging process, shown in **Figure 19–22 (b)**, is to connect the sphere to the ground using a conducting wire. As one might expect, this is referred to as **grounding** the sphere, and is indicated by the symbol \equiv . (A table of electrical symbols can be found in Appendix D.) Since the ground is a fairly good conductor of electricity, and since the Earth can receive or give up practically unlimited numbers of electrons, the effect of grounding the sphere is that the electrons repelled by the charged rod enter the ground. Now the sphere has a net positive charge. With the rod kept in place, the grounding wire is removed, as in **Figure 19–22 (c)**, trapping the net positive charge on the sphere. The rod can now be pulled away, as in **Figure 19–22 (d)**.

Notice that the *induced* charge on the sphere is opposite in sign to the charge on the rod. In contrast, when the sphere is charged by *touch* as in **Figure 19–6**, it acquires a charge with the same sign as the charge on the rod.

19–7 Electric Flux and Gauss’s Law

In this section, we introduce the idea of an electric flux and show that it can be used to calculate the electric field. The precise connection between electric flux and the charges that produce the electric field is provided by Gauss’s law.

Electric Flux

Consider a uniform electric field \vec{E} , as in **Figure 19–23 (a)**, passing through an area A that is perpendicular to the field. Looking at the electric field lines with their arrows, we can easily imagine a “flow” of electric field through the area. Though there is no actual flow, of course, the analogy is a useful one. It is with this in mind that we define an **electric flux**, Φ , for this case as follows:

$$\Phi = EA$$

On the other hand, if the area A is parallel to the field lines, as in **Figure 19–23 (b)**, none of the \vec{E} lines pierce the area, and hence there is no flux of electric field:

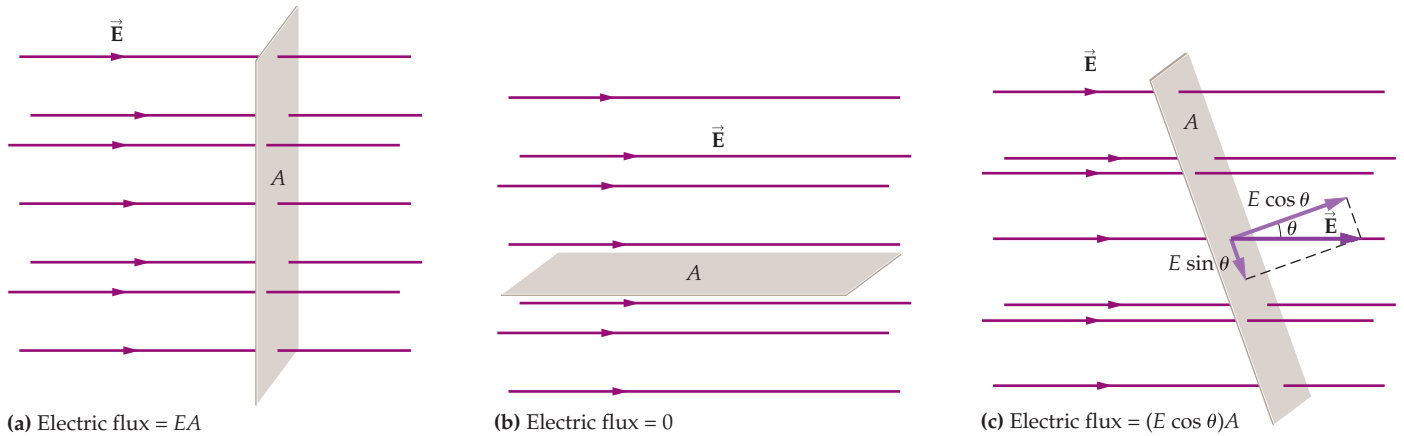
$$\Phi = 0$$

In an intermediate case, as shown in **Figure 19–23 (c)**, the \vec{E} lines pierce the area A at an angle θ away from the perpendicular. As a result, the component of \vec{E} perpendicular to the surface is $E \cos \theta$, and the component parallel to the surface is $E \sin \theta$. Since only the perpendicular component of \vec{E} causes a flux (the parallel component does not pierce the area), the flux in the general case is the following:

Definition of Electric Flux, Φ

$$\Phi = EA \cos \theta$$

SI unit: $\text{N} \cdot \text{m}^2/\text{C}$



▲ **FIGURE 19-23** Electric flux

(a) When an electric field \vec{E} passes perpendicularly through the plane of an area A , the electric flux is $\Phi = EA$. (b) When the plane of an area is parallel to \vec{E} , so that no field lines “pierce” the area, the electric flux is zero, $\Phi = 0$. (c) When the normal to the plane of an area is tilted at an angle θ away from the electric field \vec{E} , $E \cos \theta$, only the perpendicular component of \vec{E} , $E \cos \theta$, contributes to the electric flux. Thus, the flux is $\Phi = (E \cos \theta)A$.

Finally, if the surface through which the flux is calculated is *closed*, the sign of the flux is as follows:

- The flux is *positive* for field lines that *leave* the enclosed volume of the surface.
- The flux is *negative* for field lines that *enter* the enclosed volume of the surface.

Gauss's Law

As a simple example of electric flux, consider a positive point charge q and a spherical surface of radius r centered on the charge, as in **Figure 19-24**. The electric field on the surface of the sphere has the constant magnitude

$$E = k \frac{q}{r^2}$$

Since the electric field is everywhere perpendicular to the spherical surface, it follows that the electric flux is simply E times the area $A = 4\pi r^2$ of the sphere:

$$\Phi = EA = \left(k \frac{q}{r^2}\right)(4\pi r^2) = 4\pi kq$$

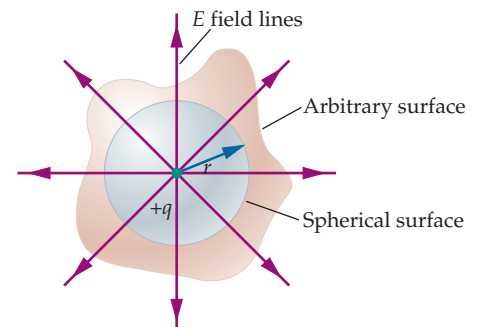
We will often find it convenient to express k in terms of another constant, ϵ_0 , as follows: $k = 1/(4\pi\epsilon_0)$. This new constant, which we call the **permittivity of free space**, is

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \quad 19-12$$

In terms of ϵ_0 , the flux through the spherical surface reduces to

$$\Phi = 4\pi kq = \frac{q}{\epsilon_0}$$

Thus, we find the very simple result that the electric flux through a sphere that encloses a charge q is the charge divided by the permittivity of free space, ϵ_0 . This is a nice result, but what makes it truly remarkable is that it is equally true for *any* surface that encloses the charge q . For example, if one were to calculate the electric flux through the closed irregular surface also shown in **Figure 19-24**—which would be a difficult task—the result, nonetheless, would still be simply q/ϵ_0 . This, in fact, is a special case of Gauss's law:



▲ **FIGURE 19-24** Electric flux for a point charge

The electric flux through the spherical surface surrounding a positive point charge q is $\Phi = EA = (kq/r^2)(4\pi r^2) = q/\epsilon_0$. The electric flux through an arbitrary surface is the same as for the sphere. The calculation of the flux, however, would be much more difficult for this surface.

Gauss's Law

If a charge q is enclosed by an arbitrary surface, the total electric flux through the surface, Φ , is

$$\Phi = \frac{q}{\epsilon_0}$$

19-13

SI unit: $\text{N} \cdot \text{m}^2/\text{C}$

Note that we use q rather than $|q|$ in Equation 19–13. This is because the electric flux can be positive or negative, depending on the sign of the charge. In particular, if the charge q is positive, the field lines leave the enclosed volume and the flux is positive; if the charge is negative, the field lines enter the enclosed volume and the flux is negative.

CONCEPTUAL CHECKPOINT 19–6 SIGN OF THE ELECTRIC FLUX

Consider the surface S shown in the sketch. Is the electric flux through this surface (a) negative, (b) positive, or (c) zero?

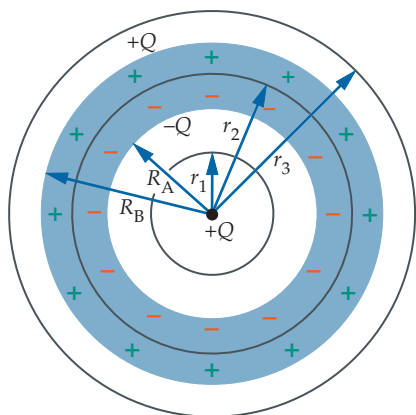
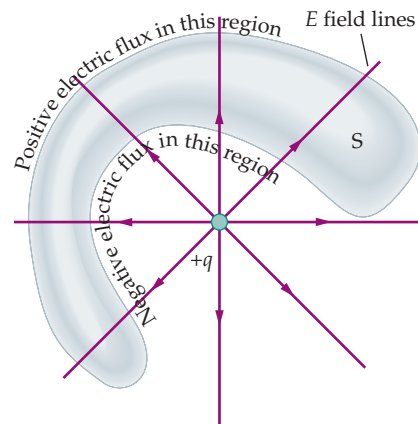
REASONING AND DISCUSSION

Because the surface S encloses no charge, the net electric flux through it must be zero, by Gauss's law. The fact that a charge $+q$ is nearby is irrelevant, because it is outside the volume enclosed by the surface.

We can explain why the flux vanishes in another way. Notice that the flux on portions of S near the charge is negative, since field lines enter the enclosed volume there. On the other hand, the flux is positive on the outer portions of S where field lines exit the volume. The combination of these positive and negative contributions is a net flux of zero.

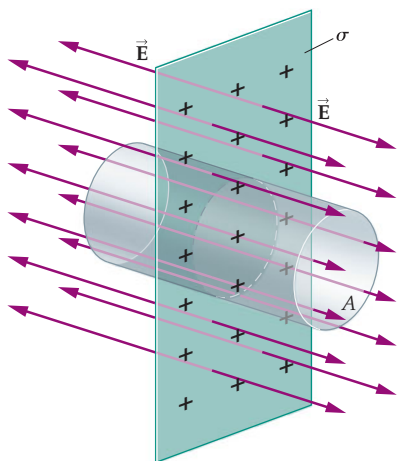
ANSWER

(c) The electric flux through the surface S is zero.



▲ **FIGURE 19–25** Gauss's law applied to a spherical shell

A simple system with three different Gaussian surfaces.



▲ **FIGURE 19–26** Gauss's law applied to a sheet of charge

A charged sheet of infinite extent and the Gaussian surface used to calculate the electric field.

Although Gauss's law holds for an arbitrarily complex surface, its real utility is seen when the system in question has a simple symmetry. For example, consider a point charge $+Q$ in the center of a hollow, conducting spherical shell, as illustrated in **Figure 19–25**. The shell has an inside radius R_A and an outside radius R_B , and is uncharged. To calculate the field inside the cavity, where $r < R_A$, we consider an imaginary spherical surface—a so-called **Gaussian surface**—with radius r_1 , and centered on the charge $+Q$, as indicated in Figure 19–25. The electric flux through this Gaussian surface is $\Phi = E(4\pi r_1^2) = Q/\epsilon_0$. Therefore, the magnitude of the electric field, as expected, is

$$E = \frac{Q}{4\pi\epsilon_0 r_1^2} = k \frac{Q}{r_1^2}$$

Note, in particular, that the charges on the spherical shell do not affect the electric flux through this Gaussian surface, since they are not contained within the surface.

Next, consider a Gaussian surface within the shell, with $R_A < r_2 < R_B$, as in Figure 19–25. Since the field within a conductor is zero, $E = 0$, it follows that the electric flux for this surface is zero: $\Phi = EA = 0$. This means that the net charge within the Gaussian surface is also zero; that is, the induced charge on the inner surface of the shell is $-Q$.

Finally, consider a spherical Gaussian surface that encloses the entire spherical shell, with a radius $r_3 > R_B$. In this case, also shown in Figure 19–25, the flux is $\Phi = E(4\pi r_3^2) = (\text{enclosed charge})/\epsilon_0$. What is the enclosed charge? Well, we know that the spherical shell is uncharged—the induced charges of $+Q$ and $-Q$ on its outer and inner surfaces sum to zero—hence the total enclosed charge is simply $+Q$ from the charge at the center of the shell. Therefore, $\Phi = E(4\pi r_3^2) = Q/\epsilon_0$, and

$$E = \frac{Q}{4\pi\epsilon_0 r_3^2} = k \frac{Q}{r_3^2}$$

Note that the field outside the shell is the same as if the shell were not present, showing that the conducting shell does not shield the external world from charges within it, in agreement with our conclusions in the previous section.

Gaussian surfaces do not need to be spherical, however. Consider, for example, a thin sheet of charge that extends to infinity, as in **Figure 19–26**. We expect the

field to be at right angles to the sheet because, by symmetry, there is no reason for it to tilt in one direction more than in any other direction. Hence, we choose our Gaussian surface to be a cylinder, as in Figure 19-26. With this choice, no field lines pierce the curved surface of the cylinder. The electric flux through this Gaussian surface, then, is due solely to the contributions of the two end caps, each of area A . Hence, the flux is $\Phi = E(2A)$. If the charge per area on the sheet is σ , the enclosed charge is σA , and hence Gauss's law states that

$$\Phi = E(2A) = \frac{(\sigma A)}{\epsilon_0}$$

Canceling the area, we find

$$E = \frac{\sigma}{2\epsilon_0}$$

Note that E does not depend in any way on the distance from the sheet, as was mentioned in Section 19-5.

We conclude this chapter with an additional example of Gauss's law in action.

PROBLEM-SOLVING NOTE

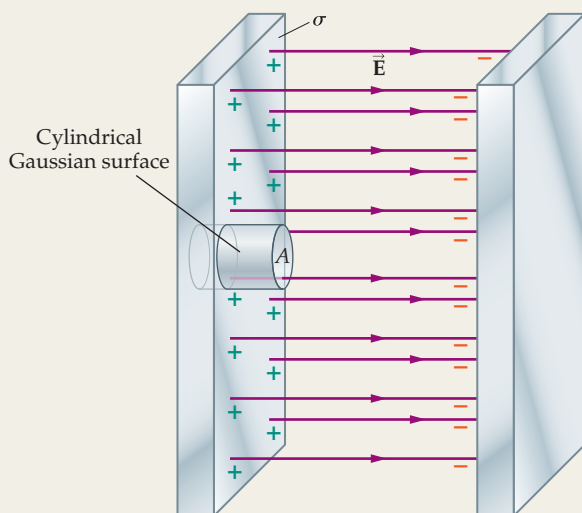
Applying Gauss's Law



Gauss's law is useful only when the electric field is constant on a given surface. It is only in such cases that the electric flux can be calculated with ease.

ACTIVE EXAMPLE 19-3 FIND THE ELECTRIC FIELD

Use the cylindrical Gaussian surface shown in the diagram to calculate the electric field between the metal plates of a parallel-plate capacitor. Each plate has a charge per area of magnitude σ .



SOLUTION (Test your understanding by performing the calculations indicated in each step.)

- | | |
|--|-------------------------|
| 1. Calculate the electric flux through the curved surface of the cylinder: | 0 |
| 2. Calculate the electric flux through the end caps of the cylinder: | $0 + EA$ |
| 3. Determine the charge enclosed by the cylinder: | σA |
| 4. Apply Gauss's law to find the field, E : | $E = \sigma/\epsilon_0$ |

INSIGHT

Note that the electric field is zero within the metal of the plates (because they are conductors). It is for this reason that the electric flux through the left end cap of the Gaussian surface is zero.

YOUR TURN

Suppose we extend the Gaussian surface so that its right end cap is within the metal of the right plate. The left end of the Gaussian surface remains in its original location. What is the electric flux through this new Gaussian surface? Explain.

(Answers to Your Turn problems are given in the back of the book.)

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of a conserved quantity, like energy (Chapter 8) or momentum (Chapter 9), appears again in Section 19–1, where we show that electric charge is also a conserved quantity.

Coulomb's law for the electrostatic force between two charges, Equation 19–5, is virtually identical to Newton's law of universal gravitation between two masses (Chapter 12), but with electric charge replacing mass.

The electric force, like all forces, is a vector quantity. Therefore, the material on vector addition (Chapter 3) again finds use in Sections 19–3, 19–4, and 19–5.

LOOKING AHEAD

The electric force is conservative, and hence it has an associated electric potential energy. We will determine this potential energy in Chapter 20. We will also point out the close analogy between the electric potential energy and the gravitational potential energy of Chapter 12.

When electric charge flows from one location to another, it produces an electric current. We consider electric circuits with direct current (dc) in Chapter 21, and circuits with alternating current (ac) in Chapter 24.

Coulomb's law comes up again in atomic physics, where it plays a key role in the Bohr model of the hydrogen atom in Section 31–3.

CHAPTER SUMMARY

19–1 ELECTRIC CHARGE

Electric charge is one of the fundamental properties of matter. Electrons have a negative charge, $-e$, and protons have a positive charge, $+e$. An object with zero net charge, like a neutron, is said to be electrically neutral.

Magnitude of an Electron's Charge

The charge on an electron has the following magnitude:

$$e = 1.60 \times 10^{-19} \text{ C} \quad 19-1$$

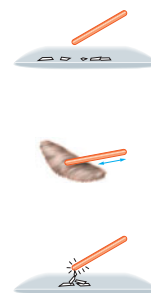
The SI unit of charge is the coulomb, C.

Charge Conservation

The total charge in the universe is constant.

Charge Quantization

Charge comes in quantized amounts that are always integer multiples of e .



19–2 INSULATORS AND CONDUCTORS

An insulator does not allow electrons within it to move from atom to atom. In conductors, each atom gives up one or more electrons that are then free to move throughout the material. Semiconductors have properties that are intermediate between those of insulators and conductors.

19–3 COULOMB'S LAW

Electric charges exert forces on one another along the line connecting them: Like charges repel, opposite charges attract.

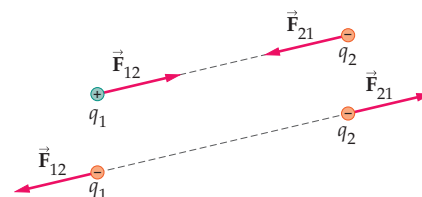
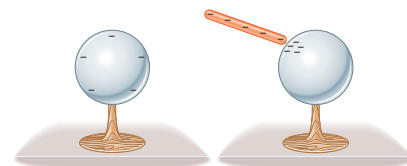
Coulomb's Law

The magnitude of the force between two point charges, q_1 and q_2 , separated by a distance r is

$$F = k \frac{|q_1| |q_2|}{r^2} \quad 19-5$$

The constant k in this expression is

$$k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \quad 19-6$$



Superposition

The electric force on one charge due to two or more other charges is the vector sum of each individual force.

Spherical Charge Distributions

A spherical distribution of charge, when viewed from outside, behaves the same as an equivalent point charge at the center of the sphere.

19–4 THE ELECTRIC FIELD

The electric field is the force per charge at a given location in space.

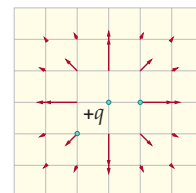
Direction of \vec{E}

\vec{E} points in the direction of the force experienced by a *positive* test charge.

Point Charge

The electric field a distance r from a point charge q has a magnitude given by

$$E = k \frac{|q|}{r^2} \quad 19-10$$

**Superposition**

The total electric field due to two or more charges is given by the vector sum of the fields due to each charge individually.

19–5 ELECTRIC FIELD LINES

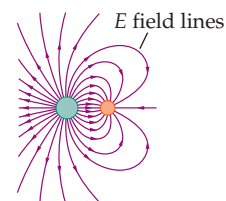
The electric field can be visualized by drawing lines according to a given set of rules.

Rules for Drawing Electric Field Lines

Electric field lines (1) point in the direction of the electric field vector \vec{E} at all points; (2) start at $+$ charges or infinity; (3) end at $-$ charges or infinity; and (4) are more dense the greater the magnitude of \vec{E} .

Parallel-Plate Capacitor

A parallel-plate capacitor consists of two oppositely charged, conducting parallel plates separated by a finite distance. The field between the plates is uniform in direction (perpendicular to the plates) and magnitude.

**19–6 SHIELDING AND CHARGING BY INDUCTION**

Ideal conductors have a range of interesting behaviors that arise because they have an enormous number of electrons that are free to move.

Excess Charge

Any excess charge placed on a conductor moves to its exterior surface.

Zero Field in a Conductor

The electric field within a conductor in equilibrium is zero.

Shielding

A conductor shields a cavity within it from external electric fields.

Electric Fields at Conductor Surfaces

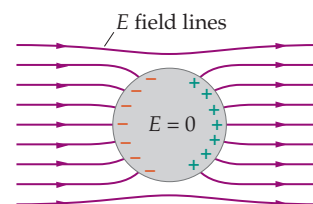
Electric field lines contact conductor surfaces at right angles.

Charging by Induction

A conductor can be charged without direct physical contact with another charged object. This is charging by induction.

Grounding

Connecting a conductor to the ground is referred to as grounding. The ground itself is a good conductor, and it can give up or receive an unlimited number of electrons.

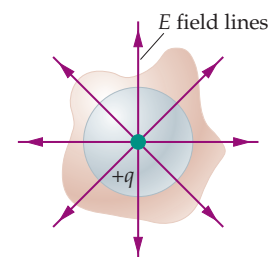
**19–7 ELECTRIC FLUX AND GAUSS'S LAW**

Gauss's law relates the charge enclosed by a given surface to the electric flux through the surface.

Electric Flux

If an area A is tilted at an angle θ to an electric field \vec{E} , the electric flux through the area is

$$\Phi = EA \cos \theta \quad 19-11$$



Gauss's Law

Gauss's law states that if a charge q is enclosed by a surface, the electric flux through the surface is

$$\Phi = \frac{q}{\epsilon_0} \quad 19-13$$

The constant appearing in this equation is the permittivity of free space, ϵ_0 :

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \quad 19-12$$

Gauss's law is used to calculate the electric field in highly symmetric systems.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the electric force exerted by one or more point charges.	The magnitude of the electric force between point charges is $F = k q_1 q_2 /r^2$; the direction of the force is given by the expression "opposites attract, likes repel." When more than one charge exerts a force on a given charge, the net force is the vector sum of the individual forces.	Examples 19-1, 19-2, 19-3 Active Examples 19-1, 19-2
Find the electric force due to a spherical distribution of charge.	For points outside a spherical distribution of charge, the spherical distribution behaves the same as a point charge of the same amount at the center of the sphere.	Active Example 19-2
Calculate the force exerted by an electric field.	An electric field, \vec{E} , produces a force, $\vec{F} = q\vec{E}$, on a point charge q . The direction of the force is in the direction of the field if the charge is positive and opposite to the field if the charge is negative.	Example 19-4
Find the electric field due to one or more point charges.	The electric field due to a point charge q has a magnitude given by $E = k q /r^2$. The direction of the field is radially outward if q is positive, and radially inward if q is negative. When a group of point charges is being considered, the total electric field is the vector sum of the individual fields.	Example 19-5
Calculate the electric field using Gauss's law.	The electric field can be found by setting the electric flux through a given surface equal to the charge enclosed by the surface divided by ϵ_0 .	Active Example 19-3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com



(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

- When an object that was neutral becomes charged, does the total charge of the universe change? Explain.
- The fact that the electron has a negative charge and the proton has a positive charge is due to a convention established by Benjamin Franklin. Would there have been any significant consequences if Franklin had chosen the opposite convention? Is there any advantage to naming charges plus and minus as opposed to, say, A and B?
- Explain why a comb that has been rubbed through your hair attracts small bits of paper, even though the paper is uncharged.
- Small bits of paper are attracted to an electrically charged comb, but as soon as they touch the comb they are strongly repelled. Explain this behavior.
- A charged rod is brought near a suspended object, which is repelled by the rod. Can we conclude that the suspended object is charged? Explain.
- A charged rod is brought near a suspended object, which is attracted to the rod. Can we conclude that the suspended object is charged? Explain.
- Describe some of the similarities and differences between Coulomb's law and Newton's law of gravity.
- A point charge $+Q$ is fixed at a height H above the ground. Directly below this charge is a small ball with a charge $-q$ and a mass m . When the ball is at a height h above the ground, the net force (gravitational plus electrical) acting on it is zero. Is this a stable equilibrium for the object? Explain.
- Four identical point charges are placed at the corners of a square. A fifth point charge placed at the center of the square experiences zero net force. Is this a stable equilibrium for the fifth charge? Explain.
- A proton moves in a region of constant electric field. Does it follow that the proton's velocity is parallel to the electric field? Does it follow that the proton's acceleration is parallel to the electric field? Explain.
- Describe some of the differences between charging by induction and charging by contact.
- A system consists of two charges of equal magnitude and opposite sign separated by a distance d . Since the total electric

charge of this system is zero, can we conclude that the electric field produced by the system is also zero? Does your answer depend on the separation d ? Explain.

13. The force experienced by charge 1 at point A is different in direction and magnitude from the force experienced by charge 2 at point B. Can we conclude that the electric fields at points A and B are different? Explain.
14. Can an electric field exist in a vacuum? Explain.
15. Explain why electric field lines never cross.

16. Charge q_1 is inside a closed Gaussian surface; charge q_2 is just outside the surface. Does the electric flux through the surface depend on q_1 ? Does it depend on q_2 ? Explain.
17. In the previous question, does the electric field at a point on the Gaussian surface depend on q_1 ? Does it depend on q_2 ? Explain.
18. Gauss's law can tell us how much charge is contained within a Gaussian surface. Can it tell us where inside the surface it is located? Explain.
19. Explain why Gauss's law is not very useful in calculating the electric field of a charged disk.

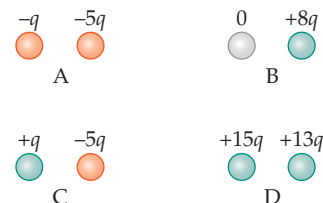
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** denotes an integrated problem, with both conceptual and numerical parts; **BIO** identifies problems of biological or medical interest; **CE** indicates a conceptual exercise. **Predict/Explain** problems ask for two responses: (a) your prediction of a physical outcome, and (b) the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.

SECTION 19-1 ELECTRIC CHARGE

1. • **CE Predict/Explain** An electrically neutral object is given a positive charge. (a) In principle, does the object's mass increase, decrease, or stay the same as a result of being charged? (b) Choose the *best explanation* from among the following:
 - I. To give the object a positive charge we must remove some of its electrons; this will reduce its mass.
 - II. Since electric charges have mass, giving the object a positive charge will increase its mass.
 - III. Charge is conserved, and therefore the mass of the object will remain the same.
2. • **CE Predict/Explain** An electrically neutral object is given a negative charge. (a) In principle, does the object's mass increase, decrease, or stay the same as a result of being charged? (b) Choose the *best explanation* from among the following:
 - I. To give the object a negative charge we must give it more electrons, and this will increase its mass.
 - II. A positive charge increases an object's mass; a negative charge decreases its mass.
 - III. Charge is conserved, and therefore the mass of the object will remain the same.
3. • **CE** (a) Based on the materials listed in Table 19-1, is the charge of the rubber balloon shown on page 655 more likely to be positive or negative? Explain. (b) If the charge on the balloon is reversed, will the stream of water deflect toward or away from the balloon? Explain.
4. • **CE** This problem refers to the information given in Table 19-1. (a) If rabbit fur is rubbed against glass, what is the sign of the charge each acquires? Explain. (b) Repeat part (a) for the case of glass and rubber. (c) Comparing the situations described in parts (a) and (b), in which case is the magnitude of the triboelectric charge greater? Explain.
5. • Find the net charge of a system consisting of 4.9×10^7 electrons.
6. • Find the net charge of a system consisting of (a) 6.15×10^6 electrons and 7.44×10^6 protons or (b) 212 electrons and 165 protons.
7. • How much negative electric charge is contained in 2 moles of carbon?
8. • Find the total electric charge of 1.5 kg of (a) electrons and (b) protons.

9. • A container holds a gas consisting of 1.85 moles of oxygen molecules. One in a million of these molecules has lost a single electron. What is the net charge of the gas?
10. • **The Charge on Adhesive Tape** When adhesive tape is pulled from a dispenser, the detached tape acquires a positive charge and the remaining tape in the dispenser acquires a negative charge. If the tape pulled from the dispenser has $0.14 \mu\text{C}$ of charge per centimeter, what length of tape must be pulled to transfer 1.8×10^{13} electrons to the remaining tape?
11. •• **CE** Four pairs of conducting spheres, all with the same radius, are shown in Figure 19-27, along with the net charge placed on them initially. The spheres in each pair are now brought into contact, allowing charge to transfer between them. Rank the pairs of spheres in order of increasing magnitude of the charge transferred. Indicate ties where appropriate.

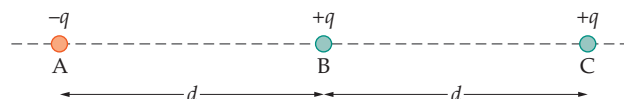


▲ FIGURE 19-27 Problem 11

12. •• A system of 1525 particles, each of which is either an electron or a proton, has a net charge of $-5.456 \times 10^{-17} \text{ C}$. (a) How many electrons are in this system? (b) What is the mass of this system?

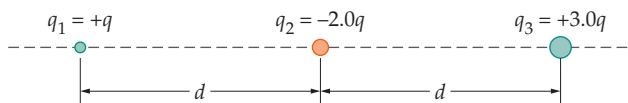
SECTION 19-3 COULOMB'S LAW

13. • **CE** A charge $+q$ and a charge $-q$ are placed at opposite corners of a square. Will a third point charge experience a greater force if it is placed at one of the empty corners of the square, or at the center of the square? Explain.
14. • **CE** Repeat the previous question, this time with charges $+q$ and $+q$ at opposite corners of a square.
15. • **CE** Consider the three electric charges, A, B, and C, shown in Figure 19-28. Rank the charges in order of increasing magnitude of the net force they experience. Indicate ties where appropriate.



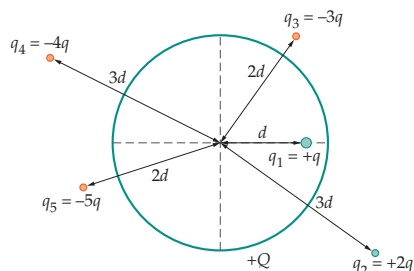
▲ FIGURE 19-28 Problems 15 and 46

16. • **CE Predict/Explain** Suppose the charged sphere in Active Example 19–2 is made from a conductor, rather than an insulator. (a) Do you expect the magnitude of the force between the point charge and the conducting sphere to be greater than, less than, or equal to the force between the point charge and an insulating sphere? (b) Choose the *best explanation* from among the following:
- The conducting sphere will allow the charges to move, resulting in a greater force.
 - The charge of the sphere is the same whether it is conducting or insulating, and therefore the force is the same.
 - The charge on a conducting sphere will move as far away as possible from the point charge. This results in a reduced force.
17. • At what separation is the electrostatic force between a $+11.2\text{-}\mu\text{C}$ point charge and a $+29.1\text{-}\mu\text{C}$ point charge equal in magnitude to 1.57 N ?
18. • The attractive electrostatic force between the point charges $+8.44 \times 10^{-6}\text{ C}$ and Q has a magnitude of 0.975 N when the separation between the charges is 1.31 m . Find the sign and magnitude of the charge Q .
19. • If the speed of the electron in Example 19–1 were $7.3 \times 10^5\text{ m/s}$, what would be the corresponding orbital radius?
20. • **IP** Two point charges, the first with a charge of $+3.13 \times 10^{-6}\text{ C}$ and the second with a charge of $-4.47 \times 10^{-6}\text{ C}$, are separated by 25.5 cm . (a) Find the magnitude of the electrostatic force experienced by the positive charge. (b) Is the magnitude of the force experienced by the negative charge greater than, less than, or the same as that experienced by the positive charge? Explain.
21. • When two identical ions are separated by a distance of $6.2 \times 10^{-10}\text{ m}$, the electrostatic force each exerts on the other is $5.4 \times 10^{-9}\text{ N}$. How many electrons are missing from each ion?
22. • A sphere of radius 4.22 cm and uniform surface charge density $+12.1\text{ }\mu\text{C/m}^2$ exerts an electrostatic force of magnitude $46.9 \times 10^{-3}\text{ N}$ on a point charge of $+1.95\text{ }\mu\text{C}$. Find the separation between the point charge and the center of the sphere.
23. • Given that $q = +12\text{ }\mu\text{C}$ and $d = 16\text{ cm}$, find the direction and magnitude of the net electrostatic force exerted on the point charge q_1 in **Figure 19–29**.



▲ **FIGURE 19–29** Problems 23, 26, 27, and 58

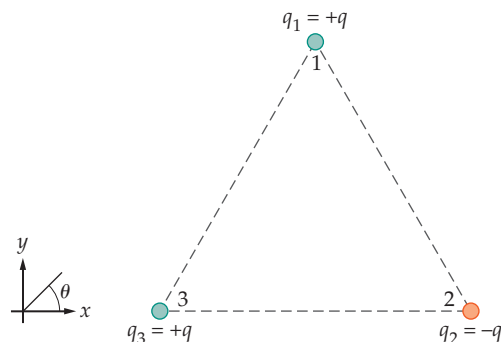
24. •• **CE** Five point charges, $q_1 = +q$, $q_2 = +2q$, $q_3 = -3q$, $q_4 = -4q$, and $q_5 = -5q$, are placed in the vicinity of an insulating spherical shell with a charge $+Q$, distributed uniformly over its surface, as indicated in **Figure 19–30**. Rank the point



▲ **FIGURE 19–30** Problem 24

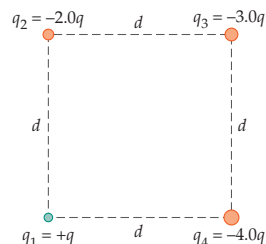
charges in order of increasing magnitude of the force exerted on them by the sphere. Indicate ties where appropriate.

25. •• **CE** Three charges, $q_1 = +q$, $q_2 = -q$, and $q_3 = +q$, are at the vertices of an equilateral triangle, as shown in **Figure 19–31**. (a) Rank the three charges in order of increasing magnitude of the electric force they experience. Indicate ties where appropriate. (b) Give the direction angle, θ , of the net electric force experienced by charge 1. Note that θ is measured counterclockwise from the positive x axis. (c) Repeat part (b) for charge 2. (d) Repeat part (b) for charge 3.



▲ **FIGURE 19–31** Problems 25 and 82

26. •• **IP** Given that $q = +12\text{ }\mu\text{C}$ and $d = 19\text{ cm}$, (a) find the direction and magnitude of the net electrostatic force exerted on the point charge q_2 in **Figure 19–29**. (b) How would your answers to part (a) change if the distance d were tripled?
27. •• Suppose the charge q_2 in **Figure 19–29** can be moved left or right along the line connecting the charges q_1 and q_3 . Given that $q = +12\text{ }\mu\text{C}$, find the distance from q_1 where q_2 experiences a net electrostatic force of zero. (The charges q_1 and q_3 are separated by a fixed distance of 32 cm .)
28. •• Find the orbital radius for which the kinetic energy of the electron in Example 19–1 is 1.51 eV . (Note: $1\text{ eV} = 1\text{ electron volt} = 1.6 \times 10^{-19}\text{ J}$.)
29. •• A point charge $q = -0.35\text{ nC}$ is fixed at the origin. Where must a proton be placed in order for the electric force acting on it to be exactly opposite to its weight? (Let the y axis be vertical and the x axis be horizontal.)
30. •• A point charge $q = -0.35\text{ nC}$ is fixed at the origin. Where must an electron be placed in order for the electric force acting on it to be exactly opposite to its weight? (Let the y axis be vertical and the x axis be horizontal.)
31. •• Find the direction and magnitude of the net electrostatic force exerted on the point charge q_2 in **Figure 19–32**. Let $q = +2.4\text{ }\mu\text{C}$ and $d = 33\text{ cm}$.

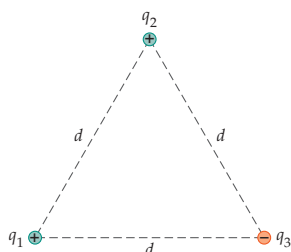


▲ **FIGURE 19–32** Problems 31 and 32

32. •• **IP** (a) Find the direction and magnitude of the net electrostatic force exerted on the point charge q_3 in **Figure 19–32**. Let

$q = +2.4 \mu\text{C}$ and $d = 27 \text{ cm}$. (b) How would your answers to part (a) change if the distance d were doubled?

33. ••IP Two point charges lie on the x axis. A charge of $+9.9 \mu\text{C}$ is at the origin, and a charge of $-5.1 \mu\text{C}$ is at $x = 10.0 \text{ cm}$. (a) At what position x would a third charge q_3 be in equilibrium? (b) Does your answer to part (a) depend on whether q_3 is positive or negative? Explain.
34. •• A system consists of two positive point charges, q_1 and $q_2 > q_1$. The total charge of the system is $+62.0 \mu\text{C}$, and each charge experiences an electrostatic force of magnitude 85.0 N when the separation between them is 0.270 m . Find q_1 and q_2 .
35. ••IP The point charges in Figure 19–33 have the following values: $q_1 = +2.1 \mu\text{C}$, $q_2 = +6.3 \mu\text{C}$, $q_3 = -0.89 \mu\text{C}$. (a) Given that the distance d in Figure 19–33 is 4.35 cm , find the direction and magnitude of the net electrostatic force exerted on the point charge q_1 . (b) How would your answers to part (a) change if the distance d were doubled? Explain.



▲ FIGURE 19–33 Problems 35, 36, 48, and 59

36. •• Referring to Problem 35, suppose that the magnitude of the net electrostatic force exerted on the point charge q_2 in Figure 19–33 is 0.65 N . (a) Find the distance d . (b) What is the direction of the net force exerted on q_2 ?
37. ••IP (a) If the nucleus in Example 19–1 had a charge of $+2e$ (as would be the case for a nucleus of helium), would the speed of the electron be greater than, less than, or the same as that found in the Example? Explain. (Assume the radius of the electron's orbit is the same.) (b) Find the speed of the electron for a nucleus of charge $+2e$.
38. •• Four point charges are located at the corners of a square with sides of length a . Two of the charges are $+q$, and two are $-q$. Find the magnitude and direction of the net electric force exerted on a charge $+Q$, located at the center of the square, for each of the following two arrangements of charge: (a) The charges alternate in sign ($+q, -q, +q, -q$) as you go around the square; (b) the two positive charges are on the top corners, and the two negative charges are on the bottom corners.
39. ••IP Two identical point charges in free space are connected by a string 7.6 cm long. The tension in the string is 0.21 N . (a) Find the magnitude of the charge on each of the point charges. (b) Using the information given in the problem statement, is it possible to determine the sign of the charges? Explain. (c) Find the tension in the string if $+1.0 \mu\text{C}$ of charge is transferred from one point charge to the other. Compare with your result from part (a).
40. ••• Two spheres with uniform surface charge density, one with a radius of 7.2 cm and the other with a radius of 4.7 cm , are separated by a center-to-center distance of 33 cm . The spheres have a combined charge of $+55 \mu\text{C}$ and repel one another with a force of 0.75 N . What is the surface charge density on each sphere?
41. ••• Point charges, q_1 and q_2 , are placed on the x axis, with q_1 at $x = 0$ and q_2 at $x = d$. A third point charge, $+Q$, is placed at

$x = 3d/4$. If the net electrostatic force experienced by the charge $+Q$ is zero, how are q_1 and q_2 related?

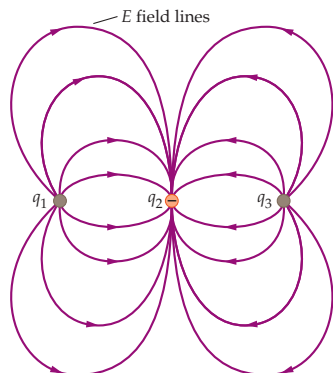
SECTION 19–4 THE ELECTRIC FIELD

42. •CE Two electric charges are separated by a finite distance. Somewhere between the charges, on the line connecting them, the net electric field they produce is zero. (a) Do the charges have the same or opposite signs? Explain. (b) If the point of zero field is closer to charge 1, is the magnitude of charge 1 greater than or less than the magnitude of charge 2? Explain.
43. • What is the magnitude of the electric field produced by a charge of magnitude $7.50 \mu\text{C}$ at a distance of (a) 1.00 m and (b) 2.00 m ?
44. • A $+5.0\text{-}\mu\text{C}$ charge experiences a 0.44-N force in the positive y direction. If this charge is replaced with a $-2.7\text{-}\mu\text{C}$ charge, what force will it experience?
45. • Two point charges lie on the x axis. A charge of $+6.2 \mu\text{C}$ is at the origin, and a charge of $-9.5 \mu\text{C}$ is at $x = 10.0 \text{ cm}$. What is the net electric field at (a) $x = -4.0 \text{ cm}$ and at (b) $x = +4.0 \text{ cm}$?
46. ••CE The electric field on the dashed line in Figure 19–28 vanishes at infinity, but also at two different points a finite distance from the charges. Identify the regions in which you can find $E = 0$ at a finite distance from the charges: region 1, to the left of point A; region 2, between points A and B; region 3, between points B and C; region 4, to the right of point C.
47. •• An object with a charge of $-3.6 \mu\text{C}$ and a mass of 0.012 kg experiences an upward electric force, due to a uniform electric field, equal in magnitude to its weight. (a) Find the direction and magnitude of the electric field. (b) If the electric charge on the object is doubled while its mass remains the same, find the direction and magnitude of its acceleration.
48. ••IP Figure 19–33 shows a system consisting of three charges, $q_1 = +5.00 \mu\text{C}$, $q_2 = +5.00 \mu\text{C}$, and $q_3 = -5.00 \mu\text{C}$, at the vertices of an equilateral triangle of side $d = 2.95 \text{ cm}$. (a) Find the magnitude of the electric field at a point halfway between the charges q_1 and q_2 . (b) Is the magnitude of the electric field halfway between the charges q_2 and q_3 greater than, less than, or the same as the electric field found in part (a)? Explain. (c) Find the magnitude of the electric field at the point specified in part (b).
49. •• Two point charges of equal magnitude are 7.5 cm apart. At the midpoint of the line connecting them, their combined electric field has a magnitude of 45 N/C . Find the magnitude of the charges.
50. ••IP A point charge $q = +4.7 \mu\text{C}$ is placed at each corner of an equilateral triangle with sides 0.21 m in length. (a) What is the magnitude of the electric field at the midpoint of any of the three sides of the triangle? (b) Is the magnitude of the electric field at the center of the triangle greater than, less than, or the same as the magnitude at the midpoint of a side? Explain.
51. •••IP Four point charges, each of magnitude q , are located at the corners of a square with sides of length a . Two of the charges are $+q$, and two are $-q$. The charges are arranged in one of the following two ways: (1) The charges alternate in sign ($+q, -q, +q, -q$) as you go around the square; (2) the top two corners of the square have positive charges ($+q, +q$), and the bottom two corners have negative charges ($-q, -q$). (a) In which case will the electric field at the center of the square have the greatest magnitude? Explain. (b) Calculate the electric field at the center of the square for each of these two cases.
52. ••• The electric field at the point $x = 5.00 \text{ cm}$ and $y = 0$ points in the positive x direction with a magnitude of 10.0 N/C . At the point $x = 10.0 \text{ cm}$ and $y = 0$ the electric field points in the positive x direction with a magnitude of 15.0 N/C . Assuming this

electric field is produced by a single point charge, find (a) its location and (b) the sign and magnitude of its charge.

SECTION 19-5 ELECTRIC FIELD LINES

53. • **IP** The electric field lines surrounding three charges are shown in **Figure 19-34**. The center charge is $q_2 = -10.0 \mu\text{C}$. (a) What are the signs of q_1 and q_3 ? (b) Find q_1 . (c) Find q_3 .



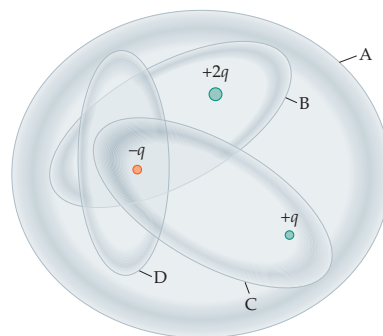
▲ **FIGURE 19-34** Problems 53 and 56

54. • Make a qualitative sketch of the electric field lines produced by two equal positive charges, $+q$, separated by a distance d .
55. • Make a qualitative sketch of the electric field lines produced by two charges, $+q$ and $-q$, separated by a distance d .
56. •• Referring to **Figure 19-34**, suppose q_2 is not known. Instead, it is given that $q_1 + q_2 = -2.5 \mu\text{C}$. Find q_1 , q_2 , and q_3 .
57. •• Make a qualitative sketch of the electric field lines produced by the four charges, $+q$, $-q$, $+q$, and $-q$, arranged clockwise on the four corners of a square with sides of length d .
58. •• Sketch the electric field lines for the system of charges shown in **Figure 19-29**.
59. •• Sketch the electric field lines for the system of charges described in **Problem 35**.
60. •• Suppose the magnitude of the electric field between the plates in **Example 19-6** is changed, and a new object with a charge of $-2.05 \mu\text{C}$ is attached to the string. If the tension in the string is 0.450 N , and the angle it makes with the vertical is 16° , what are (a) the mass of the object and (b) the magnitude of the electric field?

SECTION 19-7 ELECTRIC FLUX AND GAUSS'S LAW

61. • **CE Predict/Explain** Gaussian surface 1 has twice the area of Gaussian surface 2. Both surfaces enclose the same charge Q . (a) Is the electric flux through surface 1 greater than, less than, or the same as the electric flux through surface 2? (b) Choose the *best explanation* from among the following:
- Gaussian surface 2 is closer to the charge, since it has the smaller area. It follows that it has the greater electric flux.
 - The two surfaces enclose the same charge, and hence they have the same electric flux.
 - Electric flux is proportional to area. As a result, Gaussian surface 1 has the greater electric flux.
62. • **CE** Suppose the conducting shell in **Figure 19-25**—which has a point charge $+Q$ at its center—has a nonzero net charge. How much charge is on the inner and outer surface of the shell when the net charge of the shell is (a) $-2Q$, (b) $-Q$, and (c) $+Q$?

63. • **CE** Rank the Gaussian surfaces shown in **Figure 19-35** in order of increasing electric flux, starting with the most negative. Indicate ties where appropriate.

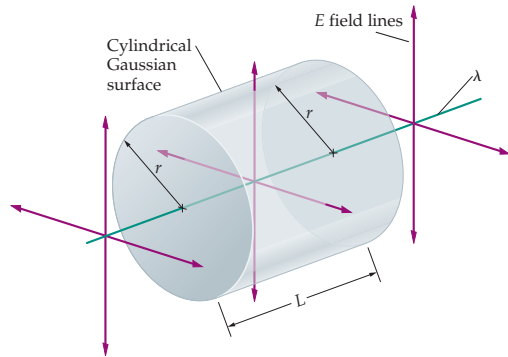


▲ **FIGURE 19-35** Problems 63 and 77

64. • A uniform electric field of magnitude $25,000 \text{ N/C}$ makes an angle of 37° with a plane surface of area 0.0153 m^2 . What is the electric flux through this surface?
65. • A surface encloses the charges $q_1 = 3.2 \mu\text{C}$, $q_2 = 6.9 \mu\text{C}$, and $q_3 = -4.1 \mu\text{C}$. Find the electric flux through this surface.
66. • **IP** A uniform electric field of magnitude $6.00 \times 10^3 \text{ N/C}$ points upward. An empty, closed shoe box has a top and bottom that are 35.0 cm by 25.0 cm , vertical ends that are 25.0 cm by 20.0 cm , and vertical sides that are 20.0 cm by 35.0 cm . (a) Which side of the box has the greatest positive electric flux? Which side has the greatest negative electric flux? Which sides have zero electric flux? (b) Calculate the electric flux through each of the six sides of the box.
67. • **BIO Nerve Cells** Nerve cells are long, thin cylinders along which electrical disturbances (nerve impulses) travel. The cell membrane of a typical nerve cell consists of an inner and an outer wall separated by a distance of $0.10 \mu\text{m}$. The electric field within the cell membrane is $7.0 \times 10^5 \text{ N/C}$. Approximating the cell membrane as a parallel-plate capacitor, determine the magnitude of the charge density on the inner and outer cell walls.
68. •• The electric flux through each of the six sides of a rectangular box are as follows:
- $$\begin{aligned} \Phi_1 &= +150.0 \text{ N} \cdot \text{m}^2/\text{C}; & \Phi_2 &= +250.0 \text{ N} \cdot \text{m}^2/\text{C}; \\ \Phi_3 &= -350.0 \text{ N} \cdot \text{m}^2/\text{C}; & \Phi_4 &= +175.0 \text{ N} \cdot \text{m}^2/\text{C}; \\ \Phi_5 &= -100.0 \text{ N} \cdot \text{m}^2/\text{C}; & \Phi_6 &= +450.0 \text{ N} \cdot \text{m}^2/\text{C}. \end{aligned}$$
- How much charge is in this box?
69. •• Consider a spherical Gaussian surface and three charges: $q_1 = 1.61 \mu\text{C}$, $q_2 = -2.62 \mu\text{C}$, and $q_3 = 3.91 \mu\text{C}$. Find the electric flux through the Gaussian surface if it completely encloses (a) only charges q_1 and q_2 , (b) only charges q_2 and q_3 , and (c) all three charges. (d) Suppose a fourth charge, Q , is added to the situation described in part (c). Find the sign and magnitude of Q required to give zero electric flux through the surface.
70. ••• A thin wire of infinite extent has a charge per unit length of λ . Using the cylindrical Gaussian surface shown in **Figure 19-36**, show that the electric field produced by this wire at a radial distance r has a magnitude given by

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

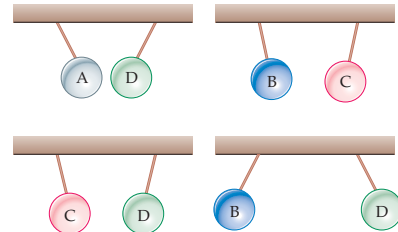
Note that the direction of the electric field is always radially away from the wire.



▲ FIGURE 19-36 Problems 70 and 87

GENERAL PROBLEMS

71. • **CE Predict/Explain** An electron and a proton are released from rest in space, far from any other objects. The particles move toward each other, due to their mutual electrical attraction. (a) When they meet, is the kinetic energy of the electron greater than, less than, or equal to the kinetic energy of the proton? (b) Choose the *best explanation* from among the following:
- The proton has the greater mass. Since kinetic energy is proportional to mass, it follows that the proton will have the greater kinetic energy.
 - The two particles experience the same force, but the light electron moves farther than the massive proton. Therefore, the work done on the electron, and hence its kinetic energy, is greater.
 - The same force acts on the two particles. Therefore, they will have the same kinetic energy and energy will be conserved.
72. • **CE Predict/Explain** In Conceptual Checkpoint 19-3, suppose the charge to be placed at either point A or point B is $+q$ rather than $-q$. (a) Is the magnitude of the net force experienced by the movable charge at point A greater than, less than, or equal to the magnitude of the net force at point B? (b) Choose the *best explanation* from among the following:
- Point B is farther from the two fixed charges. As a result, the net force at point B is less than at point A.
 - The net force at point A cancels, just as it does in Conceptual Checkpoint 19-3. Therefore, the nonzero net force at point B is greater in magnitude than the zero net force at point A.
 - The net force is greater in magnitude at point A because at that location the movable charge experiences a net repulsion from each of the fixed charges.
73. • **CE** An electron (charge $= -e$) orbits a helium nucleus (charge $= +2e$). Is the magnitude of the force exerted on the helium nucleus by the electron greater than, less than, or the same as the magnitude of the force exerted on the electron by the helium nucleus? Explain.
74. • **CE** In the operating room, technicians and doctors must take care not to create an electric spark, since the presence of the oxygen gas used during an operation increases the risk of a deadly fire. Should the operating-room personnel wear shoes that are conducting or nonconducting? Explain.
75. • **CE** Under normal conditions, the electric field at the surface of the Earth points downward, into the ground. What is the sign of the electric charge on the ground?
76. • **CE** Two identical spheres are made of conducting material. Initially, sphere 1 has a net charge of $+35Q$ and sphere 2 has a net charge of $-26Q$. If the spheres are now brought into contact, what is the final charge on sphere 1? Explain.
77. • **CE** A Gaussian surface for the charges shown in Figure 19-35 has an electric flux equal to $+3q/\epsilon_0$. Which charges are contained within this Gaussian surface?
78. • A proton is released from rest in a uniform electric field of magnitude 1.08×10^5 N/C. Find the speed of the proton after it has traveled (a) 1.00 cm and (b) 10.0 cm.
79. • **BIO Ventricular Fibrillation** If a charge of 0.30 C passes through a person's chest in 1.0 s, the heart can go into ventricular fibrillation—a nonrhythmic “fluttering” of the ventricles that results in little or no blood being pumped to the body. If this rate of charge transfer persists for 4.5 s, how many electrons pass through the chest?
80. • A point charge at the origin of a coordinate system produces the electric field $\mathbf{E} = (36,000 \text{ N/C})\hat{x}$ on the x axis at the location $x = -0.75$ m. Determine the sign and magnitude of the charge.
81. •• **CE** Four lightweight, plastic spheres, labeled A, B, C, and D, are suspended from threads in various combinations, as illustrated in Figure 19-37. It is given that the net charge on sphere D is $+Q$, and that the other spheres have net charges of $+Q$, $-Q$, or 0. From the results of the four experiments shown in Figure 19-37, and the fact that the spheres have equal masses, determine the net charge of (a) sphere A, (b) sphere B, and (c) sphere C.



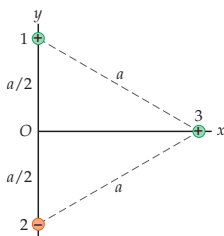
▲ FIGURE 19-37 Problem 81

82. •• Find (a) the direction and (b) the magnitude of the net electric field at the center of the equilateral triangle in Figure 19-31. Give your answers in terms of the angle θ , as defined in Figure 19-31, and E , the magnitude of the electric field produced by any one of the charges at the center of the triangle.
83. •• At the moment, the number of electrons in your body is essentially the same as the number of protons, giving you a net charge of zero. Suppose, however, that this balance of charges is off by 1% in both you and your friend, who is 1 meter away. Estimate the magnitude of the electrostatic force each of you experiences, and compare it with your weight.
84. •• A small object of mass 0.0150 kg and charge $3.1 \mu\text{C}$ hangs from the ceiling by a thread. A second small object, with a charge of $4.2 \mu\text{C}$, is placed 1.2 m vertically below the first charge. Find (a) the electric field at the position of the upper charge due to the lower charge and (b) the tension in the thread.
85. •• **IP** Consider a system of three point charges on the x axis. Charge 1 is at $x = 0$, charge 2 is at $x = 0.20$ m, and charge 3 is at $x = 0.40$ m. In addition, the charges have the following values: $q_1 = -19 \mu\text{C}$, $q_2 = q_3 = +19 \mu\text{C}$. (a) The electric field vanishes at some point on the x axis between $x = 0.20$ m and $x = 0.40$ m. Is the point of zero field (i) at $x = 0.30$ m, (ii) to the left of $x = 0.30$ m, or (iii) to the right of $x = 0.30$ m? Explain. (b) Find the point where $E = 0$ between $x = 0.20$ m and $x = 0.40$ m.
86. •• **IP** Consider the system of three point charges described in the previous problem. (a) The electric field vanishes at two different points on the x axis. One point is between $x = 0.20$ m and $x = 0.40$ m. Is the second point located to the left of charge 1 or to the right of charge 3? Explain. (b) Find the value of x at the second point where $E = 0$.

87. •• The electric field at a radial distance of 47.7 cm from the thin charged wire shown in Figure 19–36 has a magnitude of 35,400 N/C. (a) Using the result given in Problem 70, what is the magnitude of the charge per length on this wire? (b) At what distance from the wire is the magnitude of the electric field equal to $\frac{1}{2}(35,400 \text{ N/C})$?

88. •• A system consisting entirely of electrons and protons has a net charge of $1.84 \times 10^{-15} \text{ C}$ and a net mass of $4.56 \times 10^{-23} \text{ kg}$. How many (a) electrons and (b) protons are in this system?

89. •• **IP** Three charges are placed at the vertices of an equilateral triangle of side $a = 0.93 \text{ m}$, as shown in Figure 19–38. Charges 1 and 3 are $+7.3 \mu\text{C}$; charge 2 is $-7.3 \mu\text{C}$. (a) Find the magnitude and direction of the net force acting on charge 3. (b) If charge 3 is moved to the origin, will the net force acting on it there be greater than, less than, or equal to the net force found in part (a)? Explain. (c) Find the net force on charge 3 when it is at the origin.



▲ FIGURE 19–38 Problems 89 and 90

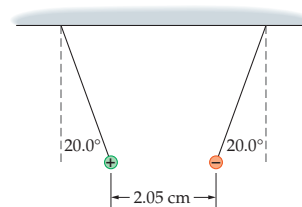
90. •• **IP** Consider the system of three charges described in the previous problem and shown in Figure 19–38. (a) Do you expect the net force acting on charge 1 to have a magnitude greater than, less than, or the same as the magnitude of the net force acting on charge 2? Explain. (b) Find the magnitude of the net force acting on charge 1. (c) Find the magnitude of the net force acting on charge 2.

91. •• **IP BIO Cell Membranes** The cell membrane in a nerve cell has a thickness of $0.12 \mu\text{m}$. (a) Approximating the cell membrane as a parallel-plate capacitor with a surface charge density of $5.9 \times 10^{-6} \text{ C/m}^2$, find the electric field within the membrane. (b) If the thickness of the membrane were doubled, would your answer to part (a) increase, decrease, or stay the same? Explain.

92. •• A square with sides of length L has a point charge at each of its four corners. Two corners that are diagonally opposite have charges equal to $+2.25 \mu\text{C}$; the other two diagonal corners have charges Q . Find the magnitude and sign of the charges Q such that each of the $+2.25\text{-}\mu\text{C}$ charges experiences zero net force.

93. •• **IP** Suppose a charge $+Q$ is placed on the Earth, and another charge $+Q$ is placed on the Moon. (a) Find the value of Q needed to “balance” the gravitational attraction between the Earth and the Moon. (b) How would your answer to part (a) change if the distance between the Earth and the Moon were doubled? Explain.

94. •• Two small plastic balls hang from threads of negligible mass. Each ball has a mass of 0.14 g and a charge of magnitude q . The balls are attracted to each other, and the threads attached to the balls make an angle of 20.0° with the vertical, as shown in Figure 19–39. Find (a) the magnitude of the electric force acting on each ball, (b) the tension in each of the threads, and (c) the magnitude of the charge on the balls.



▲ FIGURE 19–39 Problem 94

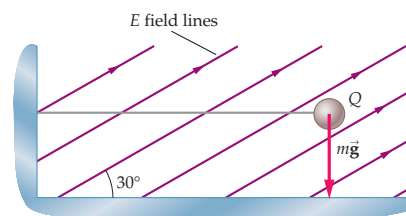
95. •• A small sphere with a charge of $+2.44 \mu\text{C}$ is attached to a relaxed horizontal spring whose force constant is 89.2 N/m . The spring extends along the x axis, and the sphere rests on a frictionless surface with its center at the origin. A point charge $Q = -8.55 \mu\text{C}$ is now moved slowly from infinity to a point $x = d > 0$ on the x axis. This causes the small sphere to move to the position $x = 0.124 \text{ m}$. Find d .

96. •• Twelve identical point charges q are equally spaced around the circumference of a circle of radius R . The circle is centered at the origin. One of the twelve charges, which happens to be on the positive x axis, is now moved to the center of the circle. Find (a) the direction and (b) the magnitude of the net electric force exerted on this charge.

97. •• **BIO Nerve Impulses** When a nerve impulse propagates along a nerve cell, the electric field within the cell membrane changes from $7.0 \times 10^5 \text{ N/C}$ in one direction to $3.0 \times 10^5 \text{ N/C}$ in the other direction. Approximating the cell membrane as a parallel-plate capacitor, find the magnitude of the change in charge density on the walls of the cell membrane.

98. •• **IP The Electric Field of the Earth** The Earth produces an approximately uniform electric field at ground level. This electric field has a magnitude of 110 N/C and points radially inward, toward the center of the Earth. (a) Find the surface charge density (sign and magnitude) on the surface of the Earth. (b) Given that the radius of the Earth is $6.38 \times 10^6 \text{ m}$, find the total electric charge on the Earth. (c) If the Moon had the same amount of electric charge distributed uniformly over its surface, would its electric field at the surface be greater than, less than, or equal to 110 N/C ? Explain.

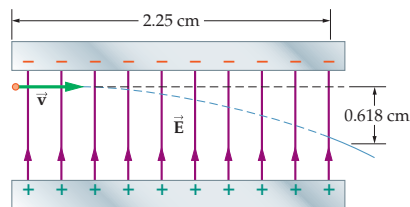
99. •• An object of mass $m = 3.1 \text{ g}$ and charge $Q = +48 \mu\text{C}$ is attached to a string and placed in a uniform electric field that is inclined at an angle of 30.0° with the horizontal (Figure 19–40). The object is in static equilibrium when the string is horizontal. Find (a) the magnitude of the electric field and (b) the tension in the string.



▲ FIGURE 19–40 Problem 99

100. •• Four identical charges, $+Q$, occupy the corners of a square with sides of length a . A fifth charge, q , can be placed at any desired location. Find the location of the fifth charge, and the value of q , such that the net electric force acting on each of the original four charges, $+Q$, is zero.

101. •• Figure 19–41 shows an electron entering a parallel-plate capacitor with a speed of $5.45 \times 10^6 \text{ m/s}$. The electric field of the



▲ FIGURE 19-41 Problem 101

capacitor has deflected the electron downward by a distance of 0.618 cm at the point where the electron exits the capacitor. Find (a) the magnitude of the electric field in the capacitor and (b) the speed of the electron when it exits the capacitor.

102. ••• Two identical conducting spheres are separated by a fixed center-to-center distance of 45 cm and have different charges. Initially, the spheres attract each other with a force of 0.095 N. The spheres are now connected by a thin conducting wire. After the wire is removed, the spheres are positively charged and repel one another with a force of 0.032 N. Find (a) the final and (b) the initial charges on the spheres.

PASSAGE PROBLEMS

Bumblebees and Static Cling

Have you ever pulled clothes from a dryer only to have them “cling” together? Have you ever walked across a carpet and had a “shocking” experience when you touched a doorknob? If so, you already know a lot about static electricity.

Ben Franklin showed that the same kind of spark we experience on a carpet, when scaled up in size, is responsible for bolts of lightning. His insight led to the invention of lightning rods to conduct electricity safely away from a building into the ground. Today, we employ static electricity in many technological applications, ranging from photocopiers to electrostatic precipitators that clean emissions from smokestacks. We even use electrostatic sating machines to give potato chips the salty taste we enjoy!

Living organisms also use static electricity—in fact, static electricity plays an important role in the pollination process. Imagine a bee busily flitting from flower to flower. As air rushes over its body and wings it acquires an electric charge—just as you do when your feet rub against a carpet. A bee might have only 93.0 pC of charge, but that’s more than enough to attract grains of pollen from a distance, like a charged comb attracting bits of paper. The result is a bee covered with grains of pollen, as illustrated in the accompanying photo, unwittingly transporting pollen from one flower to another. So, the next time you experience annoying static cling in your clothes, just remember that the same force helps pollinate the plants that we all need for life on Earth.



▲ A white-tailed bumblebee with static cling. (Problems 103, 104, 105, and 106)

103. • How many electrons must be transferred away from a bee to produce a charge of +93.0 pC?
 A. 1.72×10^{-9} B. 5.81×10^8
 C. 1.02×10^{20} D. 1.49×10^{29}
104. • Suppose two bees, each with a charge of 93.0 pC, are separated by a distance of 1.20 cm. Treating the bees as point charges, what is the magnitude of the electrostatic force experienced by the bees? (In comparison, the weight of a 0.140-g bee is 1.37×10^{-3} N.)
 A. 6.01×10^{-17} N B. 6.48×10^{-9} N
 C. 5.40×10^{-7} N D. 5.81×10^{-3} N
105. • The force required to detach a grain of pollen from an avocado stigma is approximately 4.0×10^{-8} N. What is the maximum distance at which the electrostatic force between a bee and a grain of pollen is sufficient to detach the pollen? Treat the bee and pollen as point charges, and assume the pollen has a charge opposite in sign and equal in magnitude to the bee.
 A. 4.7×10^{-7} m B. 1.9 mm
 C. 4.4 cm D. 220 m
106. • The Earth produces an electric field of magnitude 110 N/C. What force does this electric field exert on a bee carrying a charge of 93.0 pC? (Again, for comparison, the weight of a bee is approximately 1.37×10^{-3} N.)
 A. 1.76×10^{-17} N B. 8.45×10^{-13} N
 C. 1.02×10^{-8} N D. 1.13×10^{-6} N

INTERACTIVE PROBLEMS

107. •• IP Referring to Example 19-5 Suppose $q_1 = +2.90 \mu\text{C}$ is no longer at the origin, but is now on the y axis between $y = 0$ and $y = 0.500$ m. The charge $q_2 = +2.90 \mu\text{C}$ is at $x = 0$ and $y = 0.500$ m, and point 3 is at $x = y = 0.500$ m. (a) Is the magnitude of the net electric field at point 3, which we call E_{net} , greater than, less than, or equal to its previous value? Explain. (b) Is the angle θ that E_{net} makes with the x axis greater than, less than, or equal to its previous value? Explain. Find the new values of (c) E_{net} and (d) θ if q_1 is at $y = 0.250$ m.
108. •• IP Referring to Example 19-5 In this system, the charge q_1 is at the origin, the charge q_2 is at $x = 0$ and $y = 0.500$ m, and point 3 is at $x = y = 0.500$ m. Suppose that $q_1 = +2.90 \mu\text{C}$, but that q_2 is increased to a value greater than $+2.90 \mu\text{C}$. As a result, do (a) E_{net} and (b) θ increase, decrease, or stay the same? Explain. If $E_{\text{net}} = 1.66 \times 10^5$ N/C, find (c) q_2 and (d) θ .
109. •• IP Referring to Example 19-6 The magnitude of the charge is changed until the angle the thread makes with the vertical is $\theta = 15.0^\circ$. The electric field is 1.46×10^4 N/C and the mass of the object is 0.0250 kg. (a) Is the new magnitude of q greater than or less than its previous value? Explain. (b) Find the new value of q .
110. •• Referring to Example 19-6 Suppose the magnitude of the electric field is adjusted to give a tension of 0.253 N in the thread. This will also change the angle the thread makes with the vertical. (a) Find the new value of E . (b) Find the new angle between the thread and the vertical.