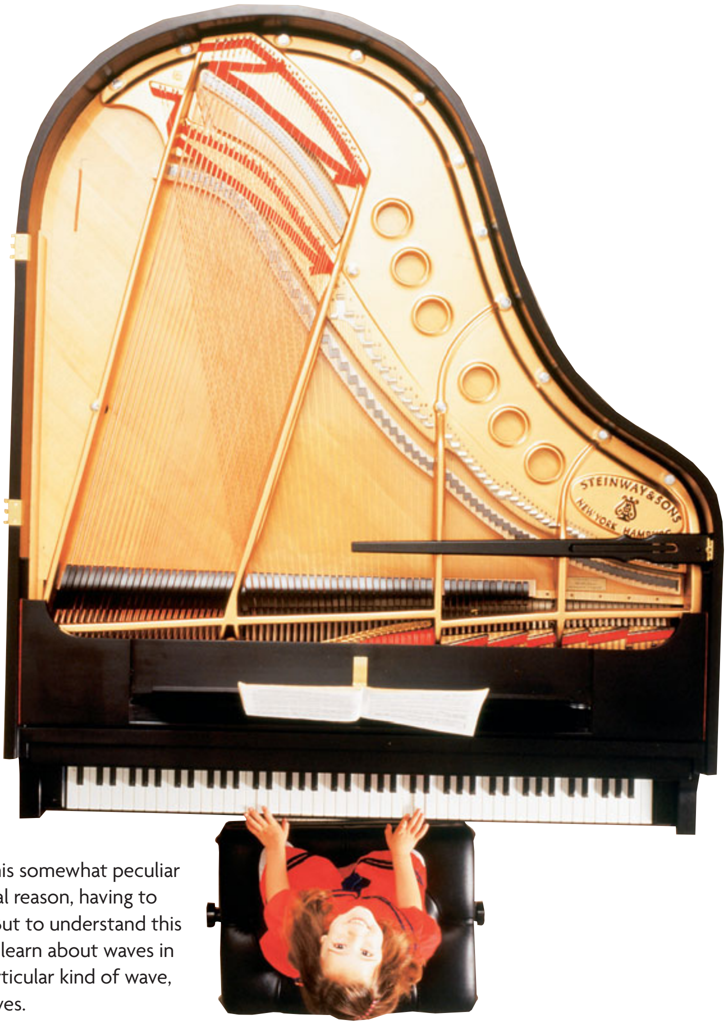
14 Waves and Sound



Have you ever wondered why a grand piano has this somewhat peculiar shape? It's not just tradition—there's also a physical reason, having to do with the way vibrating strings produce sound. But to understand this and other aspects of sound, it is first necessary to learn about waves in general—for sound, as we shall see, is merely a particular kind of wave, though one that has a special importance in our lives.

In the last chapter, we studied the behavior of an oscillator. Here, we consider the behavior of a series of oscillators that are connected to one another. Connecting oscillators leads to an assortment of new phenomena, including waves on a string, water waves, and sound. In this chapter, we focus our attention on the behavior of such waves, and in particular on the way they propagate, their speed of propagation, and their interactions with one another. Later, in Chapter 25, we shall see that light is also a type of wave, and that it displays many of the same phenomena exhibited by the waves considered in this chapter.

14–1 Types of Waves

Consider a group of swings in a playground swing set. We know that each swing by itself behaves like a simple pendulum; that is, like an oscillator. Now, let's connect the swings to one another. To be specific, suppose we tie a rope from the seat of the first swing to its neighbor, and then another rope from the second swing to the third swing, and so on. When the swings are at rest—in equilibrium— the connecting ropes have no effect. If you now sit in the first swing and begin oscillating—thus "disturbing" the equilibrium—the connecting ropes cause the other swings along the line to start oscillating as well. You have created a traveling disturbance.

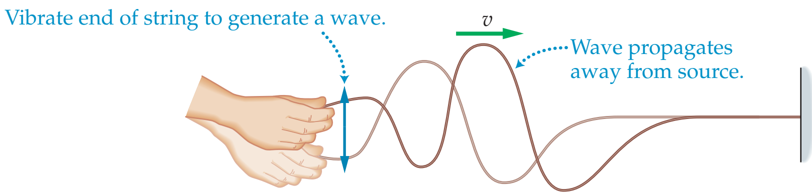
In general, a disturbance that propagates from one place to another is referred to as a **wave.** Waves propagate with well-defined speeds determined by the properties of the material through which they travel. In addition, waves carry energy. For example, part of the energy you put into sound waves when you speak is carried to the ears of others, where some of the sound energy is converted into electrical energy carried by nerve impulses to the brain which, in turn, creates the sensation of hearing.

It is important to distinguish between the motion of the wave itself and the motion of the individual particles that make up the wave. Common examples include the waves that propagate through a field of wheat. The individual wheat stalks sway back and forth as a wave passes, but they do not change their location. Similarly, a "wave" at a ball game may propagate around the stadium more quickly than a person can run, but the individual people making up the wave simply stand and sit in one place. From these simple examples it is clear that waves can come in a variety of types. We discuss some of the more common types in this section. In addition, we show how the speed of a wave is related to some of its basic properties.

▲ A wave can be viewed as a disturbance that propagates through space. Although the wave itself moves steadily in one direction, the particles that create the wave do not share in this motion. Instead, they oscillate back and forth about their equilibrium positions. The water in an ocean wave, for example, moves mainly up and down—as it passes, you bob up and down with it rather than being carried onto the shore. Similarly, the people in a human "wave" at a ballpark simply stand or raise their arms in place—they do not travel around the stadium.

Transverse Waves

Perhaps the easiest type of wave to visualize is a wave on a string, as illustrated in **Figure 14–1**. To generate such a wave, start by tying one end of a long string or rope to a wall. Pull on the free end with your hand, producing a tension in the string, and then move your hand up and down. As you do so, a wave will travel along the string toward the wall. In fact, if your hand moves up and down with simple harmonic motion, the wave on the string will have the shape of a sine or a cosine; we refer to such a wave as a **harmonic wave.**

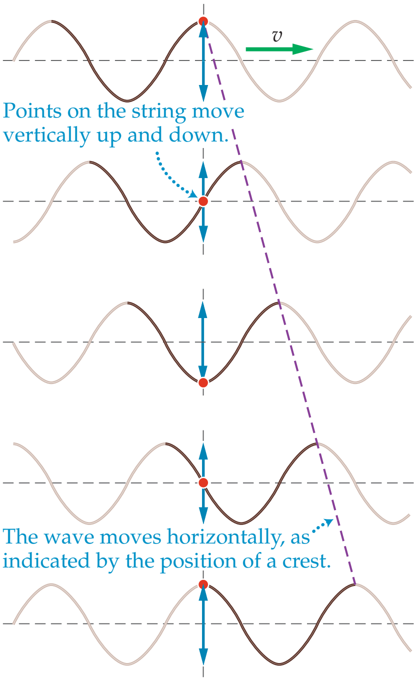


◀ **FIGURE 14–1 A wave on a string**
Vibrating one end of a string with an up-and-down motion generates a wave that travels away from its point of origin.

Note that the wave travels in the horizontal direction, even though your hand oscillates vertically about one spot. In fact, if you look at any point on the string, it too moves vertically up and down, with no horizontal motion at all. This is shown in **Figure 14–2**, where we see the location of an individual point on a string as a wave travels past. Notice, in particular, that the displacement of particles in a string is at right angles to the direction of propagation of the wave. A wave with this property is called a **transverse wave:**

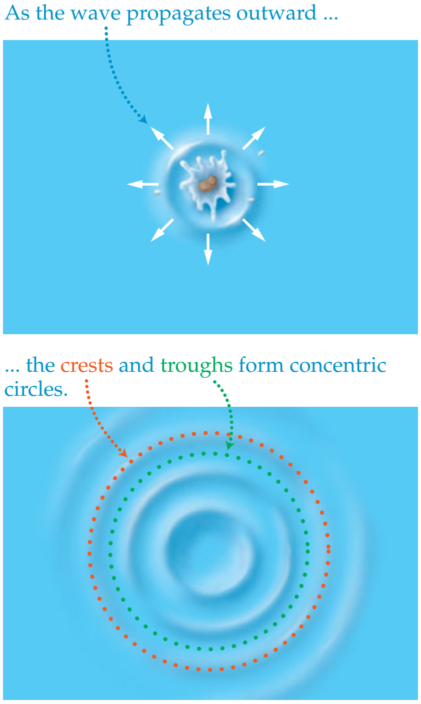
> In a transverse wave, the displacement of individual particles is at *right angles* to the direction of propagation of the wave.

Other examples of transverse waves include light and radio waves. These will be discussed in detail in Chapter 25.



▲ **FIGURE 14–2 The motion of a wave on a string**

As a wave on a string moves horizontally, all points on the string vibrate in the vertical direction, as indicated by the blue arrow.



▲ **FIGURE 14–4 Water waves from a disturbance**

An isolated disturbance in a pool of water, caused by a pebble dropped into the water, creates waves that propagate symmetrically away from the disturbance. The crests and troughs form concentric circles on the surface of the water as they move outward.

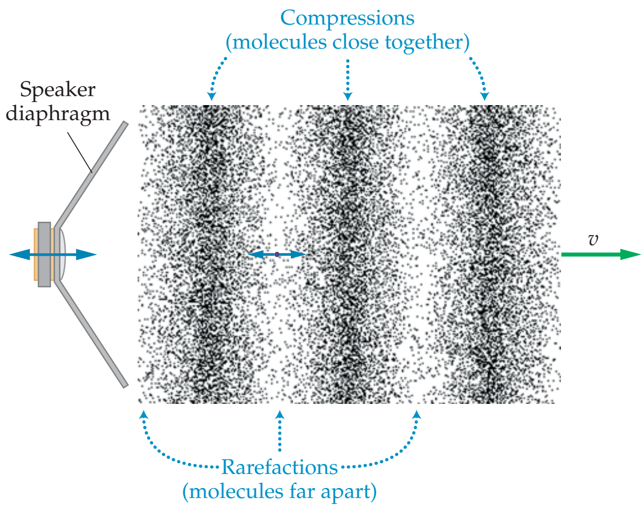
Longitudinal Waves

Longitudinal waves differ from transverse waves in the way that particles in the wave move. In particular, a longitudinal wave is defined as follows:

> In a longitudinal wave, the displacement of individual particles is parallel to the direction of propagation of the wave.

The classic example of a longitudinal wave is sound. When you speak, for example, the vibrations in your vocal cords create a series of compressions and expansions (rarefactions) in the air. The same kind of situation occurs with a loudspeaker, as illustrated in **Figure 14–3**. Here we see a speaker diaphragm vibrating horizontally with simple harmonic motion. As it moves to the right it compresses the air momentarily; as it moves to the left it rarefies the air. A series of compressions and rarefactions then travel horizontally away from the loudspeaker with the speed of sound.

Figure 14–3 also indicates the motion of an individual particle in the air as a sound wave passes. Note that the particle moves back and forth horizontally; that is, in the same direction as the propagation of the wave. The particle does not travel with the wave—each individual particle simply oscillates about a given position in space.



▲ **FIGURE 14–3 Sound produced by a speaker**

As the diaphragm of a speaker vibrates back and forth, it alternately compresses and rarefies the surrounding air. These regions of high and low density propagate away from the speaker with the speed of sound. Individual particles in the air oscillate back and forth about a given position, as indicated by the blue arrow.

Water Waves

If a pebble is dropped into a pool of water, a series of concentric waves move away from the drop point. This is illustrated in **Figure 14–4**. To visualize the movement of the water as a wave travels by, place a small piece of cork into the water. As a wave passes, the motion of the cork will trace out the motion of the water itself, as indicated in **Figure 14–5**.

Notice that the cork moves in a roughly circular path, returning to approximately its starting point. Thus, each element of water moves both vertically and horizontally as the wave propagates by in the horizontal direction. In this sense, a water wave is a combination of both transverse and longitudinal waves. This makes the water wave more difficult to analyze. Hence, most of our results will refer to the simpler cases of purely transverse and purely longitudinal waves.

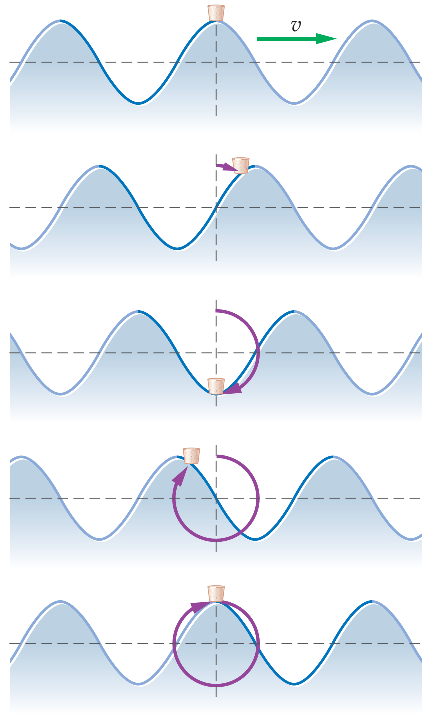


FIGURE 14–5 The motion of a water wave

As a water wave passes a given point, a molecule (or a small piece of cork) moves in a roughly circular path. This means that the water molecules move both vertically and horizontally. In this sense, the water wave has characteristics of both transverse and longitudinal waves.

Wavelength, Frequency, and Speed

A simple wave can be thought of as a regular, rhythmic disturbance that propagates from one point to another, repeating itself both in *space* and in *time*. We now show that the repeat length and the repeat time of a wave are directly related to its speed of propagation.

We begin by considering the snapshots of a wave shown in **Figure 14–6**. Points on the wave corresponding to maximum upward displacement are referred to as **crests;** points corresponding to maximum downward displacement are called **troughs.** The distance from one crest to the next, or from one trough to the next, is the repeat length—or **wavelength,** λ—of the wave.

Definition of Wavelength, λ

λ = distance over which a wave repeats

SI unit: m

Similarly, the repeat time—or **period,** T—of a wave is the time required for one wavelength to pass a given point, as illustrated in Figure 14–6. Closely related to the period of a wave is its **frequency,** f, which, as with oscillations, is defined by the relation $f = 1/T$.

Combining these observations, we see that a wave travels a distance λ in the time T. Applying the definition of speed—distance divided by time—it follows that the speed of a wave is

Speed of a Wave

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = \lambda f \quad \text{14–1}$$

SI unit: m/s

This result applies to all waves.

EXERCISE 14–1

Sound waves travel in air with a speed of 343 m/s. The lowest frequency sound we can hear is 20.0 Hz; the highest frequency is 20.0 kHz. Find the wavelength of sound for frequencies of 20.0 Hz and 20.0 kHz.

SOLUTION

Solve Equation 14–1 for λ:

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{20.0\ \text{s}^{-1}} = 17.2\ \text{m}$$

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{20{,}000\ \text{s}^{-1}} = 1.72\ \text{cm}$$

14–2 Waves on a String

In this section we consider some of the basic properties of waves traveling on a string, a rope, a wire, or any similar linear medium.

The Speed of a Wave on a String

The speed of a wave is determined by the properties of the medium through which it propagates. In the case of a string of length L, there are two basic

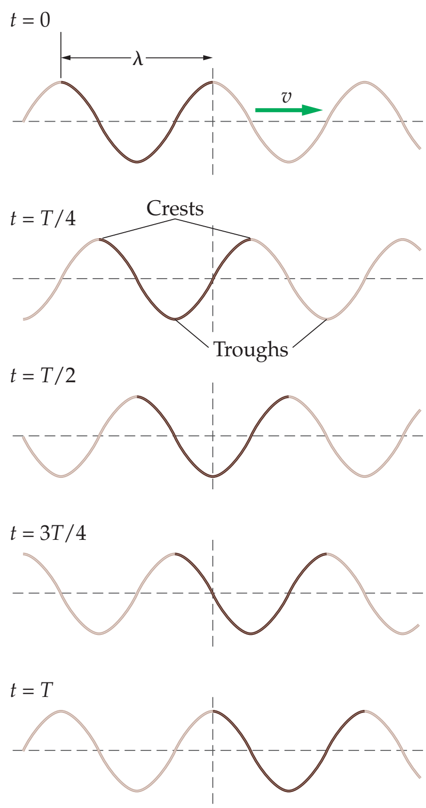


FIGURE 14–6 The speed of a wave

A wave repeats over a distance equal to the wavelength, λ. The time necessary for a wave to move one wavelength is the period, T. Thus, the speed of a wave is $v = \lambda/T = \lambda f$.

characteristics that determine the speed of a wave: (i) the tension in the string, and (ii) the mass of the string.

Let's begin with the tension, which is the force F transmitted through the string (we will use F for the tension rather than T, to avoid confusion between the tension and the period). Clearly, there must be a tension in a string in order for it to propagate a wave. Imagine, for example, that a string lies on a smooth floor with both ends free. If you take one end into your hand and shake it, the portions of the string near your hand will oscillate slightly, but no wave will travel to the other end of the string. If someone else takes hold of the other end of the string and pulls enough to set up a tension, then any movement you make on your end will propagate to the other end. In fact, if the tension is increased—so that the string becomes less slack—waves will travel through the string more rapidly.

Next, we consider the mass m of the string. A heavy string responds slowly to a given disturbance because of its inertia. Thus, if you try sending a wave through a kite string or a large rope, both under the same tension, you will find that the wave in the rope travels more slowly. In general, the heavier a rope or string the slower the speed of waves in it. Of course, the total mass of a string doesn't really matter; a longer string has more mass, but its other properties are basically the same. What is important is the mass of the string per length. We give this quantity the label μ:

Definition of Mass per Length, μ

μ = mass per length = m/L

SI unit: kg/m

To summarize, we expect the speed v to increase with the tension F and decrease with the mass per length, μ. Assuming these are the only factors determining the speed of a wave on a string, we can obtain the dependence of v on F and μ using dimensional analysis (see Chapter 1, Section 3). First, we identify the dimensions of v, F, and μ:

$$[v] = \text{m/s}$$
$$[F] = \text{N} = \text{kg}\cdot\text{m/s}^2$$
$$[\mu] = \text{kg/m}$$

Next, we seek a combination of F and μ that has the dimensions of v; namely, m/s. Suppose, for example, that v depends on F to the power a and μ to the power b. Then, we have

$$v = F^a \mu^b$$

In terms of dimensions, this equation is

$$\text{m/s} = (\text{kg}\cdot\text{m/s}^2)^a(\text{kg/m})^b = \text{kg}^{a+b}\text{m}^{a-b}\text{s}^{-2a}$$

Comparing dimensions, we see that kg does not appear on the left side of the equation; therefore, we conclude that $a + b = 0$ so that kg does not appear on the right side of the equation. Hence, $a = -b$. Looking at the time dimension, s, we see that on the left we have s^{-1}; thus on the right side we must have $-2a = -1$, or $a = \frac{1}{2}$. It follows that $b = -a = -\frac{1}{2}$. This gives the following result:

Speed of a Wave on a String, v

$$v = \sqrt{\frac{F}{\mu}} \qquad 14\text{–}2$$

SI unit: m/s

As expected, the speed increases with F and decreases with μ.

Dimensional analysis does not guarantee that this is the complete, final result; there could be a dimensionless factor like $\frac{1}{2}$ or 2π left unaccounted for. It turns out, however, that a complete analysis based on Newton's laws gives precisely the same result.

EXERCISE 14–2

A 5.0-m length of rope, with a mass of 0.52 kg, is pulled taut with a tension of 46 N. Find the speed of waves on the rope.

SOLUTION

First, calculate the mass per length, μ:

$$\mu = m/L = (0.52\text{ kg})/(5.0\text{ m}) = 0.10\text{ kg/m}$$

Now, substitute μ and F into Equation 14–2:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{46\text{ N}}{0.10\text{ kg/m}}} = 21\text{ m/s}$$

PROBLEM-SOLVING NOTE

Mass Versus Mass-per-Length

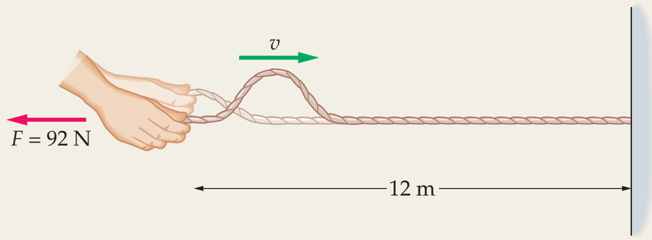
To find the mass of a string, multiply its mass per length, μ, by its length L. That is, $m = \mu L$.

EXAMPLE 14–1 A WAVE ON A ROPE

A 12-m rope is pulled tight with a tension of 92 N. When one end of the rope is given a "thunk" it takes 0.45 s for the disturbance to propagate to the other end. What is the mass of the rope?

PICTURE THE PROBLEM

Our sketch shows a wave pulse traveling with a speed v from one end of the rope to the other, a distance of 12 m. The tension in the rope is 92 N, and the travel time of the pulse is 0.45 s.



STRATEGY

We know that the speed of waves (disturbances) on a rope is determined by the tension and the mass per length. Thus, we first calculate the speed of the wave with the information given in the problem statement. Next, we solve for the mass per length, then multiply by the length to get the mass.

SOLUTION

1. Calculate the speed of the wave: $v = \frac{d}{t} = \frac{12\text{ m}}{0.45\text{ s}} = 27\text{ m/s}$
2. Use $v = \sqrt{F/\mu}$ to solve for the mass per length: $\mu = F/v^2$
3. Substitute numerical values for F and v: $\mu = \frac{F}{v^2} = \frac{92\text{ N}}{(27\text{ m/s})^2} = 0.13\text{ kg/m}$
4. Multiply μ by $L = 12$ m to find the mass: $m = \mu L = (0.13\text{ kg/m})(12\text{ m}) = 1.6\text{ kg}$

INSIGHT

Note that the speed of a wave on this rope (about 60 mi/h) is comparable to the speed of a car on a highway. This speed could be increased even further by pulling harder on the rope, thus increasing its tension.

PRACTICE PROBLEM

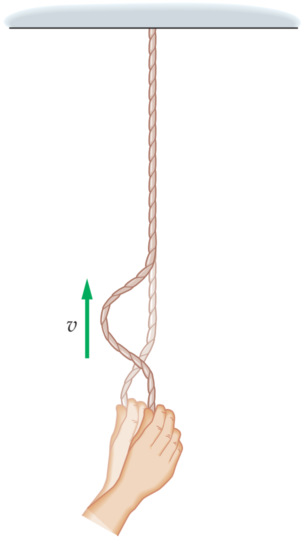
If the tension in this rope is doubled, how long will it take for the thunk to travel from one end to the other? [**Answer:** In this case the wave speed is $v = 38$ m/s; hence the time is $t = 0.32$ s.]

Some related homework problems: Problem 14, Problem 15, Problem 16

In the following Conceptual Checkpoint, we consider the speed of a wave on a vertical rope of finite mass.

CONCEPTUAL CHECKPOINT 14–1 SPEED OF A WAVE

A rope of length L and mass M hangs from a ceiling. If the bottom of the rope is given a gentle wiggle, a wave will travel to the top of the rope. As the wave travels upward does its speed **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

The speed of the wave is determined by the tension in the rope and its mass per length. The mass per length is the same from bottom to top, but not the tension. In particular, the tension at any point in the rope is equal to the weight of rope below that point. Thus, the tension increases from almost zero near the bottom to essentially Mg near the top. Since the tension increases with height, so too does the speed, according to Equation 14–2.

ANSWER

(a) The speed increases.

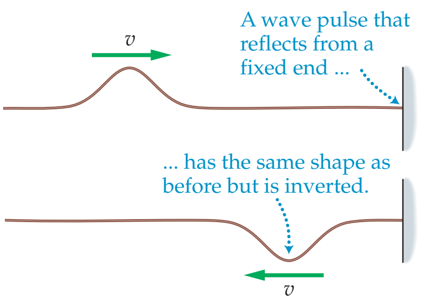


FIGURE 14–7 A reflected wave pulse: fixed end

A wave pulse on a string is inverted when it reflects from an end that is tied down.

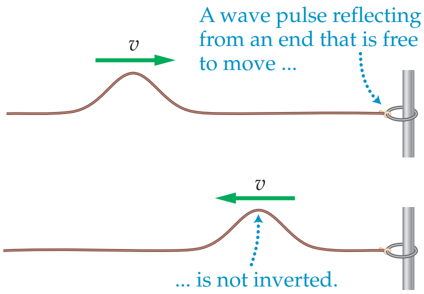


FIGURE 14–8 A reflected wave pulse: free end

A wave pulse on a string whose end is free to move is reflected without inversion.

Reflections

Thus far we have discussed only the situation in which a wave travels along a string; but at some point the wave must reach the end of the string. What happens then? Clearly, we expect the wave to be reflected, but the precise way in which the reflection occurs needs to be considered.

Suppose, for example, that the far end of a string is anchored firmly into a wall, as shown in **Figure 14–7**. If you give a flick to your end of the string, you set up a wave "pulse" that travels toward the far end. When it reaches the end, it exerts an upward force on the wall, trying to pull it up into the pulse. Since the end is tied down, however, the wall exerts an equal and opposite downward force to keep the end at rest. Thus, the wall exerts a downward force on the string that is just the *opposite* of the upward force you exerted when you created the pulse. As a result, the reflection is an inverted, or upside-down, pulse, as indicated in Figure 14–7. We shall encounter this same type of inversion under reflection when we consider the reflection of light in Chapter 28.

Another way to tie off the end of the string is shown in **Figure 14–8**. In this case, the string is tied to a small ring that slides vertically with little friction on a vertical pole. In this way, the string still has a tension in it, since it pulls on the ring, but it is also free to move up and down.

Consider a pulse moving along such a string, as in Figure 14–8. When the pulse reaches the end, it lifts the ring upward and then lowers it back down. In fact, the pulse flicks the far end of the string in the *same* way that you flicked it when you created the pulse. Therefore, the far end of the string simply creates a new pulse, identical to the first, only traveling in the opposite direction. This is illustrated in the figure.

Thus, when waves reflect, they may or may not be inverted, depending on how the reflection occurs.

*14–3 Harmonic Wave Functions

If a wave is generated by oscillating one end of a string with simple harmonic motion, the waves will have the shape of a sine or a cosine. This is shown in **Figure 14–9**, where the y direction denotes the vertical displacement of the string, and $y = 0$ corresponds to the flat string with no wave present. In what follows, we consider the mathematical formula that describes y as a function of time, t, and position, x, for such a harmonic wave.

First, note that the harmonic wave in Figure 14–9 repeats when x increases by an amount equal to the wavelength, λ. Thus, the dependence of the wave on x must be of the form

$$y(x) = A\cos\left(\frac{2\pi}{\lambda}x\right) \qquad \text{14–3}$$

To see that this is the correct dependence, note that replacing x with $x + \lambda$ gives the same value for y:

$$y(x+\lambda) = A\cos\left[\frac{2\pi}{\lambda}(x+\lambda)\right] = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\right) = A\cos\left(\frac{2\pi}{\lambda}x\right) = y(x)$$

It follows that Equation 14–3 describes a vertical displacement that repeats with a wavelength λ, as desired for a wave.

This is only part of the "wave function," however, since we have not yet described the way the wave changes with time. This is illustrated in Figure 14–9, where we see a harmonic wave at time $t = 0$, $t = T/4$, $t = T/2$, $t = 3T/4$, and $t = T$. Note that the peak in the wave that was originally at $x = 0$ at $t = 0$ moves to $x = \lambda/4$, $x = \lambda/2$, $x = 3\lambda/4$, and $x = \lambda$ for the times just given. Thus, the position x of this peak can be written as follows:

$$x = \lambda\frac{t}{T}$$

Equivalently, we can say that the peak that was at $x = 0$ is now at the location given by

$$x - \lambda\frac{t}{T} = 0$$

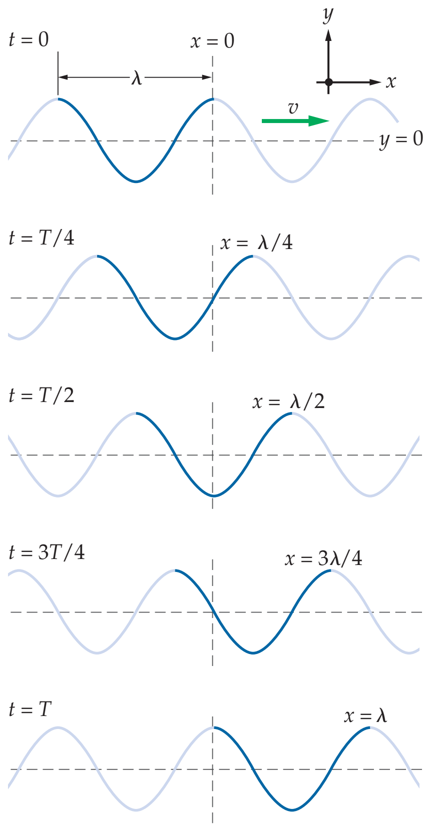
Similarly, the peak that was originally at $x = \lambda$ at $t = 0$ is at the following position at the general time t:

$$x - \lambda\frac{t}{T} = \lambda$$

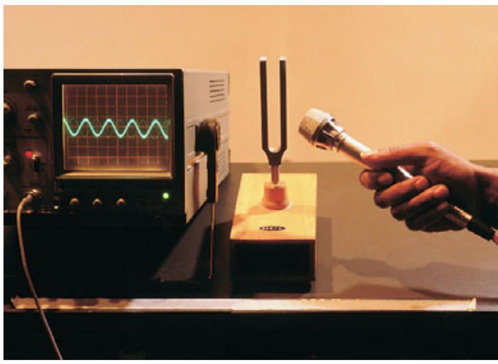
In general, if the position of a given point on a wave at $t = 0$ is $x(0)$, and its position at the time t is $x(t)$, the relation between these positions is $x(t) - \lambda t/T = x(0)$. Therefore, to take into account the time dependence of a wave, we replace $x = x(0)$ in Equation 14–3 with $x(0) = x - \lambda t/T$. This yields the harmonic wave function:

$$y(x,t) = A\cos\left[\frac{2\pi}{\lambda}\left(x - \lambda\frac{t}{T}\right)\right] = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \qquad \text{14–4}$$

Note that the wave function, $y(x, t)$, depends on both time and position, and that the wave repeats whenever position increases by the wavelength, λ, or time increases by the period, T.



▲ FIGURE 14–9 A harmonic wave moving to the right
As a wave moves, the peak that was at $x = 0$ at time $t = 0$ moves to the position $x = \lambda t/T$ at the time t.

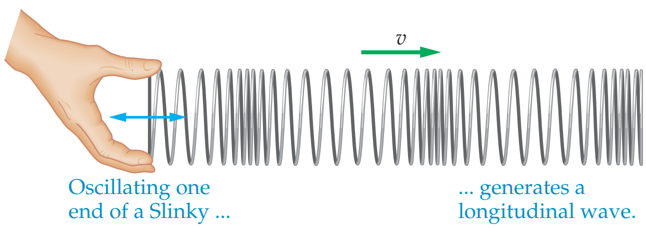


▲ An oscilloscope connected to a microphone can be used to display the wave form of a pure tone, created here by a tuning fork. The trace on the screen shows that the wave form is sinusoidal.

14–4 Sound Waves

The first thing we do when we come into this world is make a sound. It is many years later before we realize that sound is a wave propagating through the air at a speed of about 770 mi/h. More years are required to gain an understanding of the physics of a sound wave.

A useful mechanical model of a sound wave is provided by a Slinky. If we oscillate one end of a Slinky back and forth horizontally, as in **Figure 14–10**, we send out a longitudinal wave that also travels in the horizontal direction. The wave



◀ FIGURE 14–10 A wave on a Slinky
If one end of a Slinky is oscillated back and forth, a series of longitudinal waves are produced. These Slinky waves are analogous to sound waves.

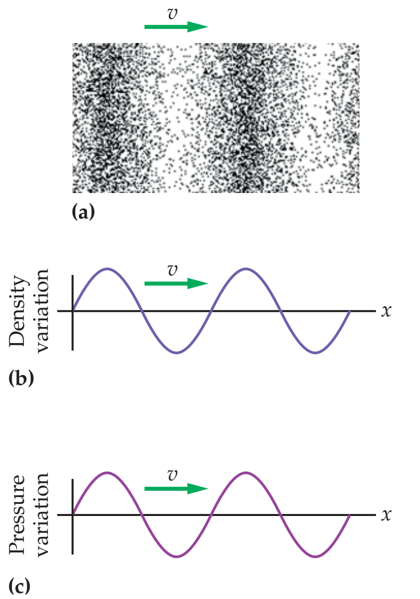


FIGURE 14–11 Wave properties of sound
A sound wave moving through the air **(a)** produces a wavelike disturbance in the **(b)** density and **(c)** pressure of the air.

consists of regions where the coils of the Slinky are compressed alternating with regions where the coils are more widely spaced.

In close analogy with the Slinky model, a speaker produces sound waves by oscillating a diaphragm back and forth horizontally, as we saw in Figure 14–3. Just as with the Slinky, a wave travels away from the source horizontally. The wave consists of compressed regions alternating with rarefied regions.

At first glance, the sound wave seems very different from the wave on a string. In particular, the sound wave doesn't seem to have the nice, sinusoidal shape of a wave. Certainly, Figure 14–3 gives no hint of such a wavelike shape.

If we plot the appropriate quantities, however, the classic wave shape emerges. For example, in **Figure 14–11 (a)** we plot the rarefactions and compressions of a typical sound wave, while in **Figure 14–11 (b)** we plot the fluctuations in the density of the air versus x. Clearly, the density oscillates in a wavelike fashion. Similarly, **Figure 14–11 (c)** shows a plot of the fluctuations in the pressure of the air as a function of x. In regions where the density is high, the pressure is also high; and where the density is low, the pressure is low. Thus, pressure versus position again shows that a sound wave has the usual wavelike properties.

Just like the speed of a wave on a string, the speed of sound is determined by the properties of the medium through which it propagates. In air, under normal atmospheric pressure and temperature, the speed of sound is approximately the following:

Speed of Sound in Air (at room temperature, 20 °C)

$v = 343\ \text{m/s} \approx 770\ \text{mi/h}$

SI unit: m/s

When we refer to the speed of sound in this text we will always assume the value is 343 m/s, unless stated specifically otherwise.

As we shall see in Chapter 17, where we study the kinetic theory of gases, the speed of sound in air is directly related to the speed of the molecules themselves. Did you know, for example, that the air molecules colliding with your body at this moment have speeds that are essentially the speed of sound? As the air is heated the molecules will move faster, and hence the speed of sound also increases with temperature.

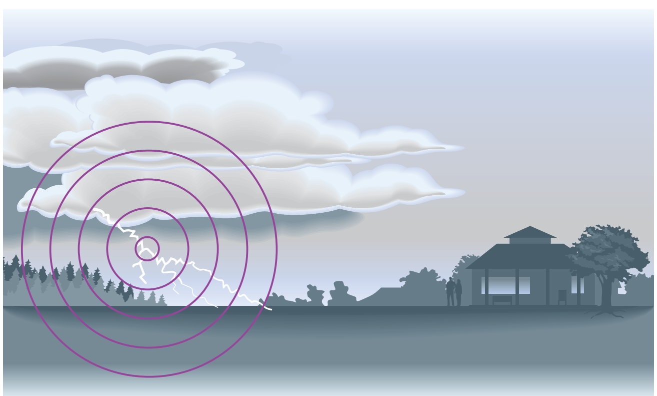
In a solid, the speed of sound is determined in part by how stiff the material is. The stiffer the material, the faster the sound wave, just as having more tension in a string causes a faster wave. Thus the speed of sound in plastic is rather high (2680 m/s), and in steel it is greater still (5960 m/s). Both speeds are much higher than the speed in air, which is certainly a "squishy" material in comparison. Table 14–1 gives a sampling of sound speed in a range of different materials.

TABLE 14–1 Speed of Sound in Various Materials

Material	Speed (m/s)
Aluminum	6420
Granite	6000
Steel	5960
Pyrex glass	5640
Copper	5010
Plastic	2680
Fresh water (20 °C)	1482
Fresh water (0 °C)	1402
Hydrogen (0 °C)	1284
Helium (0 °C)	965
Air (20 °C)	343
Air (0 °C)	331

CONCEPTUAL CHECKPOINT 14–2 HOW FAR TO THE LIGHTNING?

Five seconds after a brilliant flash of lightning, thunder shakes the house. Was the lightning **(a)** about a mile away, **(b)** much closer than a mile, or **(c)** much farther away than a mile?



REASONING AND DISCUSSION

As mentioned, the speed of sound is 343 m/s, which is just over 1000 ft/s. Thus, in five seconds sound travels slightly more than one mile. This gives rise to the following popular rule of thumb: The distance to a lightning strike (in miles) is the time for the thunder to arrive (in seconds) divided by 5.

Notice that we have neglected the travel time of light in our discussion. This is because light propagates with such a high speed (approximately 186,000 mi/s) that its travel time is about a million times less than that of sound.

ANSWER

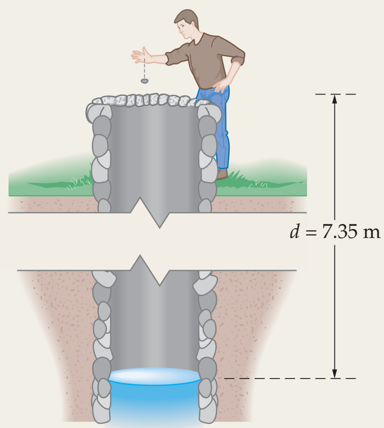
(a) The lightning was about a mile away.

EXAMPLE 14–2 WISHING WELL

You drop a stone from rest into a well that is 7.35 m deep. How long does it take before you hear the splash?

PICTURE THE PROBLEM

Our sketch shows the well into which the stone is dropped. Notice that the depth of the well is $d = 7.35$ m. After the stone falls a distance d, the sound from the splash rises the same distance d before it is heard.



STRATEGY

The time until the splash is heard is the sum of (i) the time, t_1, for the stone to drop a distance d, and (ii) the time, t_2, for sound to travel a distance d.

For the time of drop, we use one-dimensional kinematics with an initial velocity $v = 0$, since the stone is dropped from rest, and an acceleration g. Therefore, the relationship between distance and time for the stone is $d = \frac{1}{2}gt_1^2$, with $g = 9.81\ \text{m/s}^2$.

For the sound wave, we use $d = vt_2$, with $v = 343$ m/s.

SOLUTION

1. Calculate the time for the stone to drop:

$$d = \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(7.35\ \text{m})}{9.81\ \text{m/s}^2}} = 1.22\ \text{s}$$

2. Calculate the time for sound to travel a distance d:

$$d = vt_2$$

$$t_2 = \frac{d}{v} = \frac{7.35\ \text{m}}{343\ \text{m/s}} = 0.0214\ \text{s}$$

3. Sum the times found above:

$$t = t_1 + t_2 = 1.22\ \text{s} + 0.0214\ \text{s} = 1.24\ \text{s}$$

INSIGHT

Note that the time of travel for the sound is quite small, only a couple hundredths of a second. It is still nonzero, however, and must be taken into account to obtain the correct total time.

In addition, notice that we use the same speed for a sound wave whether it is traveling horizontally, vertically upward, or vertically downward—its speed is independent of its direction of motion. As a result, the waves emanating from a source of sound propagate outward in a spherical pattern, with the wave crests forming concentric spheres around the source.

PRACTICE PROBLEM

You drop a stone into a well and hear the splash 1.47 s later. How deep is the well? [**Answer:** 10.2 m]

Some related homework problems: Problem 30, Problem 31

The Frequency of a Sound Wave

When we hear a sound, its frequency makes a great impression on us; in fact, the frequency determines the **pitch** of a sound. For example, the keys on a piano produce sound with frequencies ranging from 55 Hz for the key farthest to the left to 4187 Hz for the rightmost key. Similarly, as you hum a song you change the shape and size of your vocal chords slightly to change the frequency of the sound you produce.


▶ Many animal species use sound waves with frequencies that are too high (ultrasonic) or too low (infrasonic) for human ears to detect. Bats, for example, navigate in the dark and locate their prey by means of a system of biological sonar. They emit a continuous stream of ultrasonic sounds and detect the echoes from objects around them. Blue whales, by contrast, communicate over long distances by means of infrasonic sounds.

The frequency range of human hearing extends well beyond the range of a piano, however. As a rule of thumb, humans can hear sounds between 20 Hz on the low-frequency end and 20,000 Hz on the high-frequency end. Sounds with frequencies above this range are referred to as **ultrasonic,** while those with frequencies lower than 20 Hz are classified as **infrasonic.** Though we are unable to hear ultrasound and infrasound, these frequencies occur commonly in nature, and are used in many technological applications as well.

REAL-WORLD PHYSICS
Ultrasonic sounds in nature

For example, bats and dolphins produce ultrasound almost continuously as they go about their daily lives. By listening to the echoes of their calls—that is, by using *echolocation*—they are able to navigate about their environment and detect their prey. As a defense mechanism, some of the insects that are preyed upon by bats have the ability to hear the ultrasonic frequency of a hunting bat and take evasive action. For instance, the praying mantis has a specialized ultrasound receptor on its abdomen that allows it to take cover in response to an approaching bat. More dramatically, certain moths fold their wings in flight and drop into a precipitous dive toward the ground when they hear a bat on the prowl.

REAL-WORLD PHYSICS: BIO
Medical applications of ultrasound: ultrasonic scans

Medical applications of ultrasound are also common. Perhaps the most familiar is the ultrasound scan that is used to image a fetus in the womb. By sending bursts of ultrasound into the body and measuring the time delay of the resulting echoes—the technological equivalent of echolocation—it is possible to map out the location of structures that lie hidden beneath the skin. In addition to imaging the interior of a body, ultrasound can also produce changes within the body that would otherwise require surgery. For example, in a technique called *shock wave lithotripsy* (SWL), an intense beam of ultrasound is concentrated onto a kidney stone that must be removed. After being hit with as many as 1000 to 3000 pulses of sound (at 23 joules per pulse), the stone is fractured into small pieces that the body can then eliminate on its own.

REAL-WORLD PHYSICS: BIO
Medical applications of ultrasound: shock wave lithotripsy

As for infrasound, it has been discovered in recent years that elephants can communicate with one another using sounds with frequencies as low as 15 Hz. In fact, it may be that *most* elephant communication is infrasonic. These sounds, which humans feel as vibration rather than hear as sound, can carry over an area of about thirty square kilometers on the dry African savanna. And elephants are not alone in this ability. Whales, such as the blue and the finback, produce powerful infrasonic calls as well. Since sound generally travels farther in water than in air, the whale calls can be heard by others of their species over distances of thousands of kilometers.

REAL-WORLD PHYSICS
Infrasonic communication among animals

One final example of infrasound is related to a dramatic event that occurred in southern New Mexico about a decade ago. At 12:47 in the afternoon of October 10, 1997, a meteor shining as bright as the full Moon streaked across the sky for a few brief moments. The event was observed not just visually, however, but with

REAL-WORLD PHYSICS
Infrasound produced by meteors

▲ Ultrasound is used in medicine both as an imaging medium and for therapeutic purposes. Ultrasound scans, or sonograms, are created by beaming ultrasonic pulses into the body and measuring the time required for the echoes to return. This technique is commonly used to evaluate heart function (echocardiograms) and to visualize the fetus in the uterus, as shown above (left). In shock wave lithotripsy (right), pulses of high-frequency sound waves are used to shatter kidney stones into fragments that can be passed in the urine.

infrasound as well. An array of special microphones at the Los Alamos National Laboratory—originally designed to listen for clandestine nuclear weapons tests—heard the infrasonic boom created by the meteor. By tracking the sonic signals of such meteors it may be possible to recover fragments that manage to reach the ground. The Los Alamos detector is in constant operation, and it detects about ten rather large objects (2 m or more in diameter) entering the Earth's atmosphere each year.

It should be noted, in light of the wide range of frequencies observed in sound, that the speed of sound is the same for all frequencies. Thus, in the relation

$$v = \lambda f$$

the speed v remains fixed. For example, if the frequency of a wave is doubled, its wavelength is halved, so that the speed v stays the same. The fact that different frequencies travel with the same speed is evident when we listen to an orchestra in a large room. Different instruments are producing sounds of different frequencies, but we hear the sounds at the same time. Otherwise, listening to music from a distance would be quite a different and inharmonious experience.

14–5 Sound Intensity

The noise made by a jackhammer is much louder than the song of a sparrow. On this we can all agree. But how do we express such an observation physically? What physical quantity determines the loudness of a sound? We address these questions in this section, and we also present a quantitative scale by which loudness may be measured.

Intensity

The loudness of a sound is determined by its **intensity;** that is, by the amount of energy that passes through a given area in a given time. This is illustrated in Figure 14–12. If the energy E passes through the area A in the time t, the intensity, I, of the wave carrying the energy is

$$I = \frac{E}{At}$$

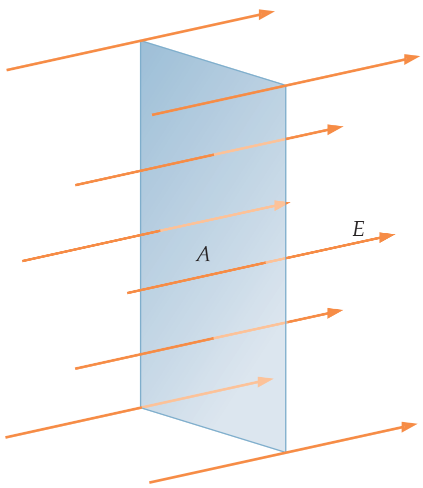
Recalling that power is energy per time, $P = E/t$, we can express the intensity as follows:

Definition of Intensity, *I*

$$I = \frac{P}{A} \qquad 14\text{–}5$$

SI unit: W/m^2

The units are those of power (watts, W) divided by area (meters squared, m^2).



▲ **FIGURE 14–12 Intensity of a wave**
If a wave carries an energy E through an area A in the time t, the corresponding intensity is $I = E/At = P/A$, where $P = E/t$ is the power.

TABLE 14–2 Sound Intensities (W/m^2)

Loudest sound produced in laboratory	10^9
Saturn V rocket at 50 m	10^8
Rupture of the eardrum	10^4
Jet engine at 50 m	10
Threshold of pain	1
Rock concert	10^{-1}
Jackhammer at 1 m	10^{-3}
Heavy street traffic	10^{-5}
Conversation at 1 m	10^{-6}
Classroom	10^{-7}
Whisper at 1 m	10^{-10}
Normal breathing	10^{-11}
Threshold of hearing	10^{-12}

Though we have introduced the concept of intensity in terms of sound, it applies to all types of waves. For example, the intensity of light from the Sun as it reaches the Earth's upper atmosphere is about 1380 W/m^2. If this intensity could be heard as sound, it would be painfully loud—roughly the equivalent of four jet airplanes taking off simultaneously. By comparison, the intensity of microwaves in a microwave oven is even greater, about 6000 W/m^2, whereas the intensity of a whisper is an incredibly tiny 10^{-10} W/m^2. A selection of representative intensities is given in Table 14–2.

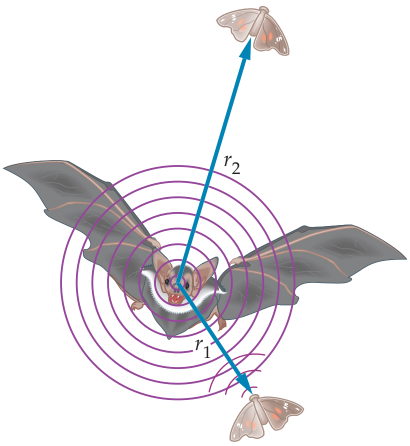
EXERCISE 14–3

A loudspeaker puts out 0.15 W of sound through a square area 2.0 m on each side. What is the intensity of this sound?

SOLUTION

Applying Equation 14–5, with $A = (2.0\ \text{m})^2$, we find

$$I = \frac{P}{A} = \frac{0.15\ \text{W}}{(2.0\ \text{m})^2} = 0.038\ \text{W/m}^2$$



▲ **FIGURE 14–13 Echolocation**

Two moths, at distances r_1 and r_2, hear the sonar signals sent out by a bat. The intensity of the signal decreases with the square of the distance from the bat. The bat, in turn, hears the echoes sent back by the moths. It can then use the direction and intensity of the returning echoes to locate its prey.

When we listen to a source of sound, such as a person speaking or a radio playing a song, we notice that the loudness of the sound decreases as we move away from the source. This means that the intensity of the sound is also decreasing. The reason for this reduction in intensity is simply that the energy emitted per time by the source spreads out over a larger area—just as spreading a certain amount of jam over a larger piece of bread reduces the intensity of the taste.

In **Figure 14–13** we show a source of sound (a bat) and two observers (moths) listening at the distances r_1 and r_2. Notice that the waves emanating from the bat propagate outward spherically, with the wave crests forming a series of concentric spheres. Assuming no reflections of sound, and a power output by the bat equal to P, the intensity detected by the first moth is

$$I_1 = \frac{P}{4\pi r_1^2}$$

In writing this expression, we have used the fact that the area of a sphere of radius r is $A = 4\pi r^2$. Similarly, the second moth hears the same sound with an intensity of

$$I_2 = \frac{P}{4\pi r_2^2}$$

The power P is the same in each case—it simply represents the amount of sound emitted by the bat. Solving for the intensity at moth 2 in terms of the intensity at moth 1 we find

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 \qquad 14\text{–}6$$

In words, the intensity falls off with the square of the distance; doubling the distance reduces the intensity by a factor of 4.

To summarize, the intensity a distance r from a point source of power P is

Intensity with Distance from a Point Source

$$I = \frac{P}{4\pi r^2} \qquad 14\text{–}7$$

SI unit: W/m^2

This result assumes that no sound is reflected—which could increase the amount of energy passing through a given area—that no sound is absorbed, and that the sound propagates outward spherically. These assumptions are applied in the next Example.

PROBLEM-SOLVING NOTE

Intensity Variation with Distance

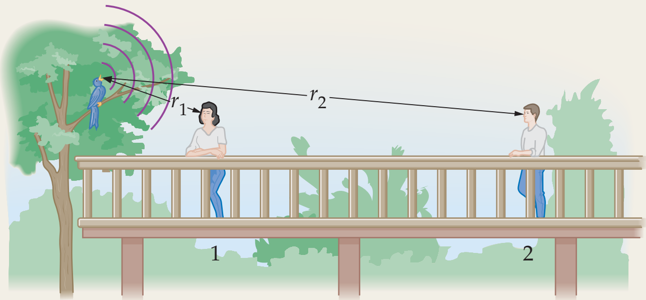
Suppose the intensity of a point source is I_1 at a distance r_1. This is enough information to find its intensity at any other distance. For example, to find the intensity I_2 at a distance r_2 we use the relation $I_2 = (r_1/r_2)^2 I_1$.

EXAMPLE 14–3 THE POWER OF SONG

Two people relaxing on a deck listen to a songbird sing. One person, only 1.00 m from the bird, hears the sound with an intensity of $2.80 \times 10^{-6}\ \mathrm{W/m^2}$. **(a)** What intensity is heard by the second person, who is 4.25 m from the bird? Assume that no reflected sound is heard by either person. **(b)** What is the power output of the bird's song?

PICTURE THE PROBLEM

Our sketch shows the two observers, one at a distance of $r_1 = 1.00$ m from the bird, the other at a distance of $r_2 = 4.25$ m. The sound emitted by the bird is assumed to spread out spherically, with no reflections.



STRATEGY

a. The two intensities are related by Equation 14–6, with $r_1 = 1.00$ m and $r_2 = 4.25$ m.

b. The power output can be obtained from the definition of intensity, $I = P/A$. We can calculate P for either observer, noting that $A = 4\pi r^2$.

SOLUTION

Part (a)

1. Substitute numerical values into Equation 14–6:

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{1.00\ \mathrm{m}}{4.25\ \mathrm{m}}\right)^2 (2.80 \times 10^{-6}\ \mathrm{W/m^2}) = 1.55 \times 10^{-7}\ \mathrm{W/m^2}$$

Part (b)

2. Solve $I = P/A$ for the power, P, using data for observer 1:

$$I_1 = P/A_1$$
$$P = I_1 A_1 = (2.80 \times 10^{-6}\ \mathrm{W/m^2})[4\pi(1.00\ \mathrm{m})^2] = 3.52 \times 10^{-5}\ \mathrm{W}$$

3. As a check, repeat the calculation for observer 2:

$$I_2 = P/A_2$$
$$P = I_2 A_2 = (1.55 \times 10^{-7}\ \mathrm{W/m^2})[4\pi(4.25\ \mathrm{m})^2] = 3.52 \times 10^{-5}\ \mathrm{W}$$

INSIGHT

The intensity at observer 1 is $4.25^2 = 18.1$ times the intensity at observer 2. Even so, the bird only *seems* to be about 2.5 times louder to observer 1. The connection between intensity and perceived (subjective) loudness is discussed in detail later in this section.

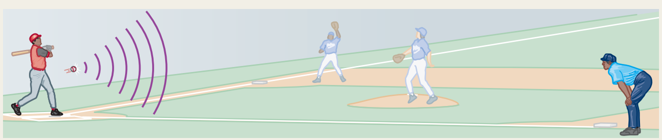
PRACTICE PROBLEM

If the intensity at observer 2 were $7.40 \times 10^{-7}\ \mathrm{W/m^2}$, how far would he be from the bird? [**Answer:** $r_2 = 1.95$ m]

Some related homework problems: Problem 36, Problem 41

ACTIVE EXAMPLE 14–1 THE BIG HIT: FIND THE INTENSITY

Ken Griffey, Jr., connects with a fast ball and sends it out of the park. A fan in the outfield bleachers, 140 m away, hears the hit with an intensity of $3.80 \times 10^{-7}\ \mathrm{W/m^2}$. Assuming no reflected sounds, what is the intensity heard by the first-base umpire, 90 ft (27.4 m) away from home plate?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Label the data given in the problem. Let the umpire be observer 1 and the fan be observer 2:

$r_1 = 27.4$ m
$r_2 = 140$ m
$I_2 = 3.80 \times 10^{-7}\ \mathrm{W/m^2}$

2. Solve Equation 14–6 for I_1: $I_1 = (r_2/r_1)^2 I_2$

3. Substitute numerical values: $I_1 = 9.9 \times 10^{-6}\ \mathrm{W/m^2}$

CONTINUED ON NEXT PAGE

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INSIGHT

For the fan, the sound from the hit is somewhat less intense than normal conversation. For the umpire it is comparable to the sound of a busy street.

YOUR TURN

Find the distance at which the sound of the hit has the intensity of a whisper. Refer to Table 14–2 for the necessary information.

(Answers to **Your Turn** *problems are given in the back of the book.)*

REAL-WORLD PHYSICS: BIO

Human perception of sound intensity

Human Perception of Sound

Hearing, like most of our senses, is incredibly versatile and sensitive. We can detect sounds that are about a million times fainter than a typical conversation, and listen to sounds that are a million times louder before experiencing pain. In addition, we are able to hear sounds over a wide range of frequencies, from 20 Hz to 20,000 Hz.

When detecting the faintest of sounds, our hearing is more sensitive than one would ever guess. For example, a faint sound, with an intensity of about 10^{-11} W/m^2, causes a displacement of molecules in the air of about 10^{-10} m. This displacement is roughly the diameter of an atom!

Equally interesting is the way we perceive the loudness of a sound. As an example, suppose you hear a sound of intensity I_1. Next, you listen to a second sound of intensity I_2, and this sound seems to be "twice as loud" as the first. If the two intensities are measured, it turns out that I_2 is about 10 times I_1. Similarly, a third sound, twice as loud as I_2, has an intensity I_3 that is 10 times greater than I_2. Thus, $I_2 = 10\,I_1$ and $I_3 = 10\,I_2 = 100\,I_1$.

Our perception of sound, then, is such that uniform increases in loudness correspond to intensities that increase by multiplicative factors. For this reason, a convenient scale to measure loudness depends on the logarithm of intensity, as we discuss next.

PROBLEM-SOLVING NOTE

Intensity Versus Intensity Level

When reading a problem statement, be sure to note carefully whether it refers to the intensity, I, or to the intensity level, β. These two quantities have similar names but completely different meanings and units, as indicated in the following table:

Physical quantity	Physical meaning	Units
Intensity, I	Energy per time per area	W/m^2
Intensity level, β	A measure of relative loudness	dB

Intensity Level and Decibels

In the study of sound, loudness is measured by the **intensity level** of a wave. Designated by the symbol β, the intensity level is defined as follows:

Definition of Intensity Level, β

$$\beta = (10\text{ dB})\log(I/I_0) \qquad 14\text{–}8$$

SI unit: decibel (dB), which is dimensionless

In this expression, log indicates the logarithm to the base 10, and I_0 is the intensity of the faintest sounds that can be heard. Experiments show this lowest detectable intensity to be

$$I_0 = 10^{-12}\text{ W/m}^2$$

Note that β is dimensionless; the only dimensions that enter into the definition are those of intensity, and they cancel in the logarithm. Still, just as with radians, it is convenient to label the values of intensity level with a name. The name we use—the bel—honors the work of Alexander Graham Bell (1847–1922), the inventor of the telephone. Since the bel is a fairly large unit, it is more common to measure β in units that are one-tenth of a bel. This unit is referred to as the **decibel,** and its abbreviation is dB.

To get a feeling for the decibel scale, let's start with the faintest sounds. If a sound has an intensity $I = I_0$, the corresponding intensity level is

$$\beta = (10\text{ dB})\log(I_0/I_0) = 10\log(1) = 0$$

Increasing the intensity by a factor of 10 makes the sound seem twice as loud. In terms of decibels, we have

$$\beta = (10\text{ dB})\log(10I_0/I_0) = (10\text{ dB})\log(10) = 10\text{ dB}$$

Going up in intensity by another factor of 10 doubles the loudness of the sound again, and yields

$$\beta = (10\text{ dB})\log(100I_0/I_0) = (10\text{ dB})\log(100) = 20\text{ dB}$$

Thus, *the loudness of a sound doubles with each increase in intensity level of 10 dB*. The *smallest* increase in intensity level that can be detected by the human ear is about 1 dB.

The intensity of a number of independent sound sources is simply the sum of the individual intensities. We use this fact in the following Example.

PROBLEM-SOLVING NOTE

Calculating the Intensity Level

When determining the intensity level β, be sure to use the base 10 logarithm (log), as opposed to the "natural," or base e, logarithm (ln).

EXAMPLE 14–4 PASS THE PACIFIER

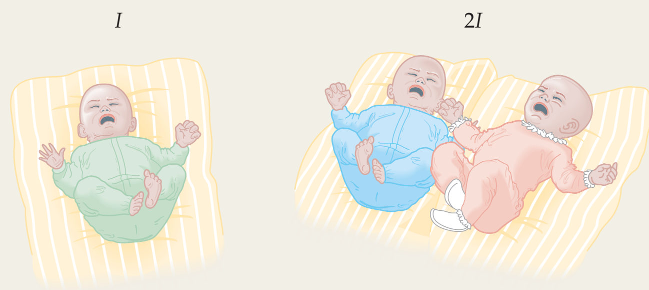
A crying child emits sound with an intensity of $8.0 \times 10^{-6}\text{ W/m}^2$. Find **(a)** the intensity level in decibels for the child's sounds, and **(b)** the intensity level for this child and its twin, both crying with identical intensities.

PICTURE THE PROBLEM

We consider the crying sounds of either one or two children. Each child emits sound with an intensity $I = 8.0 \times 10^{-6}\text{ W/m}^2$. If two children are crying together, the total intensity of their sound is $2I$.

STRATEGY

The intensity level, β, is obtained by applying Equation 14–8.



(a) (b)

SOLUTION

Part (a)

1. Calculate β for $I = 8.0 \times 10^{-6}\text{ W/m}^2$:

$$\beta = (10\text{ dB})\log(I/I_0)$$
$$= (10\text{ dB})\log\left(\frac{8.0 \times 10^{-6}\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right) = (10\text{ dB})\log(8.0 \times 10^6)$$
$$= (10\text{ dB})\log(8.0) + (10\text{ dB})\log(10^6) = 69\text{ dB}$$

Part (b)

2. Repeat the calculation with I replaced by $2I$:

$$\beta = (10\text{ dB})\log(2I/I_0)$$
$$= (10\text{ dB})\log(2) + (10\text{ dB})\log(I/I_0)$$
$$= 3.0\text{ dB} + 69\text{ dB} = 72\text{ dB}$$

INSIGHT

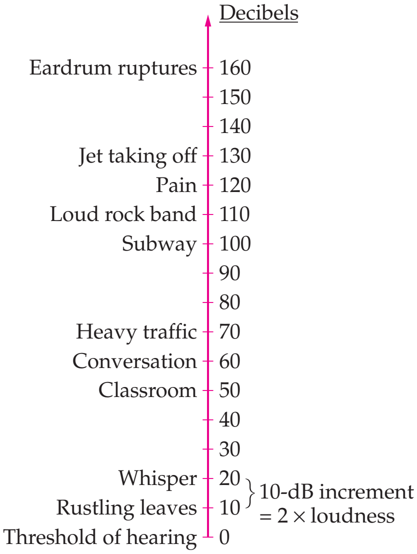
Note that the intensity level is increased by $(10\text{ dB})\log(2) = 3$ dB. This is a general rule: When the intensity is doubled, the intensity level, β, increases by 3 dB. Similarly, when the intensity is halved, β decreases by 3 dB.

PRACTICE PROBLEM

What is the intensity level of four identically crying quadruplets? [**Answer:** $\beta = 75$ dB]

Some related homework problems: Problem 38, Problem 39

Even though a change of 3 dB is relatively small—after all, a change of 10 dB is required to make a sound seem twice as loud—it still requires changing the intensity by a factor of two. For example, suppose a large nursery in a hospital has so many crying babies that the intensity level is 6 dB above the safe value, as determined by OSHA (Occupational Safety and Health Administration). To reduce



▲ **FIGURE 14–14 Representative intensity levels for common sounds**

the level by 6 dB it would be necessary to remove three-quarters of the children, leaving only one-quarter the original number. To our ears, however, the nursery will *sound* only 40 percent quieter!

Figure 14–14 shows the decibel scale with representative values indicated for a variety of common sounds.

14–6 The Doppler Effect

One of the most common physical phenomena involving sound is the change in pitch of a train whistle or a car horn as the vehicle moves past us. This change in pitch, due to the relative motion between a source of sound and the receiver, is called the **Doppler effect,** after the Austrian physicist Christian Doppler (1803–1853). If you listen carefully to the Doppler effect, you will notice that the pitch increases when the observer and the source are moving closer together, and decreases when the observer and source are separating.

One of the more fascinating aspects of the Doppler effect is the fact that it applies to all wave phenomena, not just to sound. In particular, the frequency of light is also Doppler-shifted when there is relative motion between the source and receiver. For light, this change in frequency means a change in color. In fact, most distant galaxies are observed to be red-shifted in the color of their light, which means they are moving away from the Earth. Some galaxies, however, are moving toward us, and their light shows a blue shift.

In the remainder of this section, we focus on the Doppler effect in sound waves. We show that the effect is different depending on whether the observer or the source is moving. Finally, both observer and source may be in motion, and we present results for such cases as well.

Moving Observer

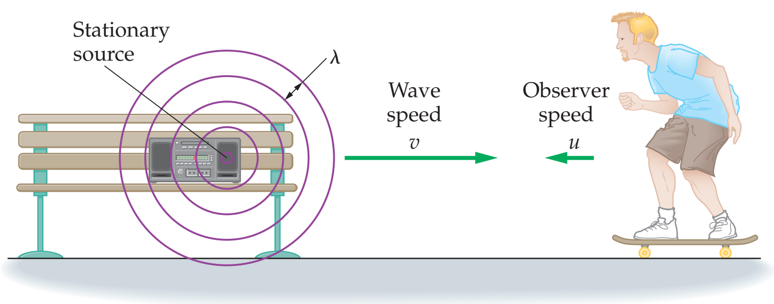
In **Figure 14–15** we see a stationary source of sound in still air. The radiated sound is represented by the circular patterns of compressions moving away from the source with a speed v. The distance between the compressions is the wavelength, λ, and the frequency of the sound is f. As for any wave, these quantities are related by

$$v = \lambda f$$

For an observer moving toward the source with a speed u, as in Figure 14–15, the sound *appears* to have a higher speed, $v + u$ (though, of course, the speed of sound relative to the air is always the same). As a result, more compressions move past the observer in a given time than if the observer had been at rest. To the observer, then, the sound has a frequency, f', that is higher than the frequency of the source, f.

We can find the frequency f' by first noting that the wavelength of the sound does not change—it is still λ. The speed, however, has increased to $v' = v + u$. Thus, we can solve $v' = \lambda f'$ for the frequency, which yields

$$f' = \frac{v'}{\lambda} = \frac{v + u}{\lambda}$$



▶ **FIGURE 14–15 The Doppler effect: a moving observer**

Sound waves from a stationary source form concentric circles moving outward with a speed v. To the observer, who moves toward the source with a speed u, the waves are moving with a speed $v + u$.

Finally, we recall from Equation 14–1 that $\lambda = v/f$, and hence

$$f' = \frac{v + u}{(v/f)} = \left(\frac{v + u}{v}\right)f = (1 + u/v)f$$

Note that f' is greater than f. This is the Doppler effect.

If the observer had been moving away from the source with a speed u, the sound would *appear* to the observer to have the reduced speed $v' = v - u$. Repeating the calculation just given, we find that

$$f' = \frac{v'}{\lambda} = \frac{v - u}{\lambda} = (1 - u/v)f$$

In this case the Doppler effect results in f' being less than f.

Combining these results, we have

Doppler Effect for Moving Observer

$$f' = (1 \pm u/v)f \qquad 14\text{–}9$$

SI unit: $1/\text{s} = \text{s}^{-1}$

In this expression u and v are speeds, and hence are always positive. The appropriate signs are obtained by using the *plus* sign when the observer moves *toward* the source, and the *minus* sign when the observer moves *away from* the source.

PROBLEM-SOLVING NOTE

Using the Correct Sign for the Doppler Effect

When an observer approaches a source, the frequency heard by the observer is greater than the frequency of the source; that is, $f' > f$. This means that we must use the plus sign in $f' = (1 \pm u/v)f$. Similarly, we must use the minus sign when the observer moves away from the source.

EXAMPLE 14–5 A MOVING OBSERVER

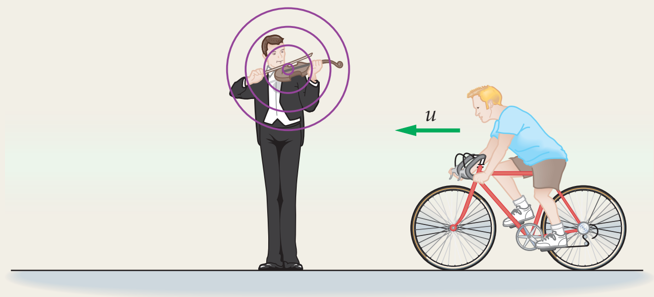
A street musician sounds the A string of his violin, producing a tone of 440 Hz. What frequency does a bicyclist hear as he **(a)** approaches and **(b)** recedes from the musician with a speed of 11.0 m/s?

PICTURE THE PROBLEM

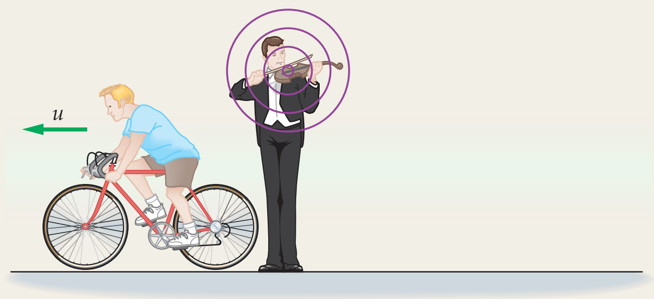
The sketch shows a stationary source of sound and a moving observer. In part (a) the observer approaches the source with a speed $u = 11.0$ m/s; in part (b) the observer has passed the source and is moving away with the same speed.

STRATEGY

The frequency heard by the observer is given by Equation 14–9, with the plus sign for part (a) and the minus sign for part (b).



(a)



(b)

SOLUTION

Part (a)

1. Apply Equation 14–9 with the plus sign and $u = 11.0$ m/s: $f' = (1 + u/v)f = \left(1 + \frac{11.0\text{ m/s}}{343\text{ m/s}}\right)(440\text{ Hz}) = 454\text{ Hz}$

Part (b)

2. Now use the minus sign in Equation 14–9: $f' = (1 - u/v)f = \left(1 - \frac{11.0\text{ m/s}}{343\text{ m/s}}\right)(440\text{ Hz}) = 426\text{ Hz}$

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INSIGHT

As the bicyclist passes the musician, the observed frequency of sound decreases, giving a "wow" effect. The difference in frequency is about 1 semitone, the frequency difference between adjacent notes on the piano. See Table 14–3 on p. 481 for semitones in the vicinity of middle C.

PRACTICE PROBLEM

If the bicyclist hears a frequency of 451 Hz when approaching the musician, what is his speed? [**Answer:** $u = 8.58$ m/s]

Some related homework problems: Problem 47, Problem 50

Moving Source

With a stationary observer and a moving source, the Doppler effect is not due to the sound wave appearing to have a higher or lower speed, as when the observer moves. On the contrary, the speed of a wave is determined by the properties of the medium through which it propagates. Thus, once the source emits a sound wave, it travels through the medium with its characteristic speed v regardless of what the source is doing.

By way of analogy, consider a water wave. The speed of such waves is the same whether they are created by a rock dropped into the water or by a stick moved rapidly through the water. To take an extreme case, the waves coming to the beach from a slow-moving tugboat move with the same speed as the waves produced by a 100-mph speed boat. The same is true of sound waves.

Consider, then, a source moving toward an observer with a speed u, as shown in **Figure 14–16**. If the frequency of the source is f, it emits one compression every T seconds, where $T = 1/f$. Therefore, during one cycle of the wave a compression travels a distance vT while the source moves a distance uT. As a result, the next compression is emitted a distance $vT - uT$ behind the previous compression, as illustrated in **Figure 14–17**. This means that the wavelength in the forward direction is

$$\lambda' = vT - uT = (v - u)T$$

Now, the speed of the wave is still v, as mentioned, hence

$$v = \lambda' f'$$

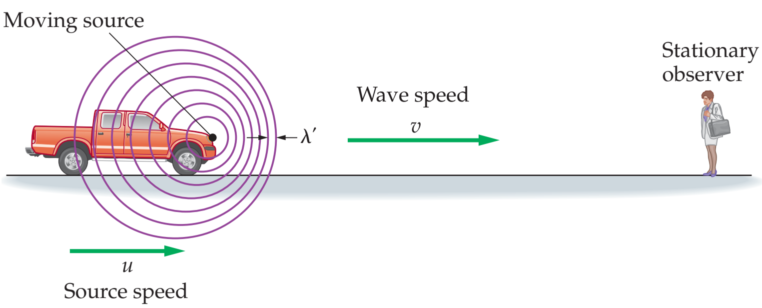


FIGURE 14–16 The Doppler effect: a moving source
Sound waves from a moving source are bunched up in the forward direction, causing a shorter wavelength and a higher frequency.

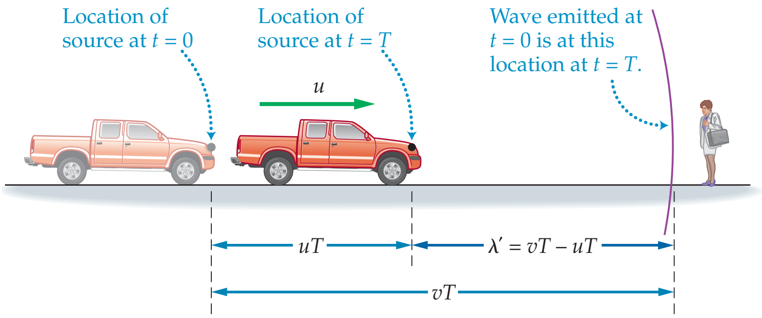


FIGURE 14–17 The Doppler-shifted wavelength
During one period, T, the wave emitted at $t = 0$ moves through a distance vT. In the same time, the source moves toward the observer through the distance uT. At the time $t = T$ the next wave is emitted from the source; hence, the distance between the waves (the wavelength) is $\lambda' = vT - uT$.

Solving for the new frequency, f', we find

$$f' = \frac{v}{\lambda'} = \frac{v}{(v - u)T}$$

Finally, recalling that $T = 1/f$, we have

$$f' = \frac{v}{(v - u)(1/f)} = \frac{v}{v - u}f = \left(\frac{1}{1 - u/v}\right)f$$

Note that f' is greater than f, as expected.

In the reverse direction, the wavelength is increased by the amount uT. Thus,

$$\lambda' = vT + uT = (v + u)T$$

It follows that the Doppler-shifted frequency is

$$f' = \frac{v}{(v + u)T} = \left(\frac{1}{1 + u/v}\right)f$$

This is less than the source frequency, f.

Finally, we can combine these results to yield

Doppler Effect for Moving Source

$$f' = \left(\frac{1}{1 \mp u/v}\right)f \qquad \text{14–10}$$

SI unit: $1/s = s^{-1}$

As before, u and v are positive quantities. The *minus* sign is used when the source moves *toward* the observer, and the *plus* sign when the source moves *away from* the observer.

PROBLEM-SOLVING NOTE

Using Correct Signs

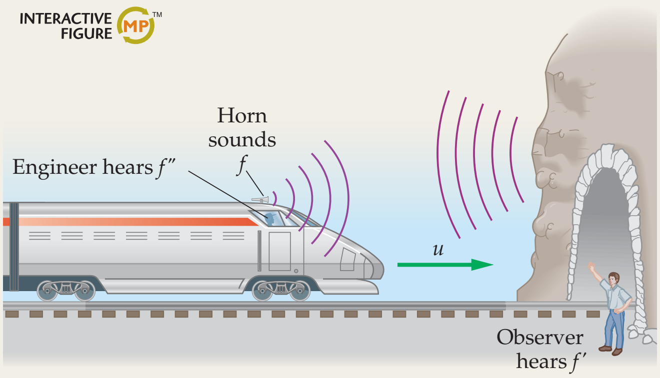
When a source approaches an observer, the frequency heard by the observer is greater than the frequency of the source; that is, $f' > f$. This means that we must use the minus sign in Equation 14–10, $f' = f/(1 - u/v)$, since this makes the denominator less than one. Similarly, use the plus sign when the source moves away from the observer.

EXAMPLE 14–6 WHISTLE STOP

A train sounds its whistle as it approaches a tunnel in a cliff. The whistle produces a tone of 650.0 Hz, and the train travels with a speed of 21.2 m/s. **(a)** Find the frequency heard by an observer standing near the tunnel entrance. **(b)** The sound from the whistle reflects from the cliff back to the engineer in the train. What frequency does the engineer hear?

PICTURE THE PROBLEM

The train moves with a speed $u = 21.2$ m/s and emits sound with a frequency $f = 650.0$ Hz. The observer near the tunnel hears the Doppler-shifted frequency f', and the engineer on the train hears the reflected sound at an even higher frequency f''.



STRATEGY

Two Doppler shifts are involved in this problem. The first is due to the motion of the train toward the cliff. This shift causes the observer at the cliff to hear sound with a higher frequency f', given by Equation 14–10 with the minus sign. The reflected sound has the same frequency, f'.

The second shift is due to the engineer moving toward the reflected sound. Thus, the engineer hears a frequency f'' that is greater than f'. We find f'' using Equation 14–9 with the plus sign.

SOLUTION

Part (a)

1. Use Equation 14–10, with the minus sign, to Doppler shift from f to f'.

$$f' = \left(\frac{1}{1 - u/v}\right)f = \left(\frac{1}{1 - \frac{21.2\text{ m/s}}{343\text{ m/s}}}\right)(650.0\text{ Hz})$$

$$= \left(\frac{1}{1 - 0.0618}\right)(650.0\text{ Hz}) = 693\text{ Hz}$$

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Part (b)

2. Now use Equation 14–9, with the plus sign, to Doppler shift from f' to f''.

$$f'' = (1 + u/v)f' = \left(1 + \frac{21.2\ \text{m/s}}{343\ \text{m/s}}\right)(693\ \text{Hz})$$
$$= (1 + 0.0618)(693\ \text{Hz}) = 736\ \text{Hz}$$

INSIGHT

Note that the reflected sound has the same frequency f' heard by the stationary observer, since all the cliff does is reverse the direction of motion of the sound heard at the cliff. Therefore, the cliff acts as a stationary source of sound at the frequency f'. The engineer's motion toward this stationary source results in the Doppler shift from f' to f''.

PRACTICE PROBLEM

If the stationary observer hears a frequency of 700.0 Hz, what are **(a)** the speed of the train and **(b)** the frequency heard by the engineer? [**Answer:** (a) $u = 24.5$ m/s, (b) $f'' = 750$ Hz]

Some related homework problems: Problem 45, Problem 103

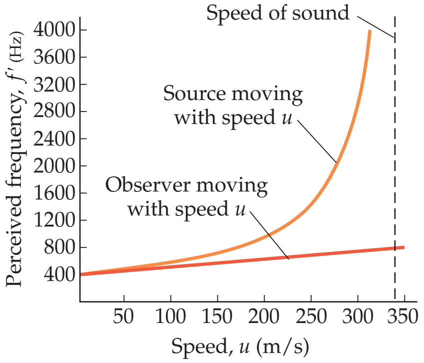


FIGURE 14–18 Doppler-shifted frequency versus speed for a 400-Hz sound source

The upper curve corresponds to a moving source, the lower curve to a moving observer. Notice that while the two cases give similar results for low speed, the high-speed behavior is quite different. In fact, the Doppler frequency for the moving source grows without limit for speeds near the speed of sound, while the Doppler frequency for the moving observer is relatively small. If a source moves faster than the speed of sound, the sound it produces is perceived not as a pure tone, but as a shock wave, commonly referred to as a sonic boom.

Now that we have obtained the Doppler effect for both moving observers and moving sources, it is interesting to compare the results. **Figure 14–18** shows the Doppler-shifted frequency versus speed for a 400-Hz source of sound. The upper curve corresponds to a moving source, the lower curve to a moving observer. Notice that while the two cases give similar results for low speed, the high-speed behavior is quite different. In fact, the Doppler frequency for the moving source grows without limit for speeds near the speed of sound, while the Doppler frequency for the moving observer is relatively small.

These results can be understood both in terms of mathematics—by simply comparing Equations 14–9 and 14–10—and physically. In physical terms, recall that a moving observer encounters wave crests separated by the wavelength, as indicated in Figure 14–15. Doubling the speed of the observer simply reduces the time required to move from one crest to the next by a factor of 2, which doubles the frequency. Thus, in general, the frequency is proportional to the speed, as we see in the lower curve in Figure 14–18. In contrast, when the source moves, as in Figure 14–16, the wave crests become "bunched up" in the forward direction, since the source is almost keeping up with the propagating waves. As the speed of the source approaches the speed of sound, the separation between crests approaches zero. Consequently, the frequency with which the crests pass a stationary observer approaches infinity, as indicated by the upper curve in Figure 14–18.

General Case

The results derived earlier in this section can be combined to give the Doppler effect for situations in which both observer and source move. Letting u_s be the speed of the source, and u_o be the speed of the observer, we have

Doppler Effect for Moving Source and Observer

$$f' = \left(\frac{1 \pm u_o/v}{1 \mp u_s/v}\right)f \qquad \text{14–11}$$

SI unit: $1/\text{s} = \text{s}^{-1}$

As in the previous cases, u_s, u_o, and v are positive quantities. In the numerator, the plus sign corresponds to the case in which the observer moves in the direction of the source, whereas the minus sign indicates motion in the opposite direction. In the denominator, the minus sign corresponds to the case in which the source moves in the direction of the observer, whereas the plus sign indicates motion in the opposite direction.

EXERCISE 14–4

A car moving at 18 m/s sounds its 550-Hz horn. A bicyclist, traveling with a speed of 7.2 m/s, moves toward the approaching car. What frequency is heard by the bicyclist?

SOLUTION

We use Equation 14–11 with $u_s = 18$ m/s and $u_o = 7.2$ m/s. Since the source and observer are approaching, we use the plus sign in the numerator and the minus sign in the denominator:

$$f' = \left(\frac{1 + u_o/v}{1 - u_s/v}\right)f = \left(\frac{1 + 7.2/343}{1 - 18/343}\right)(550\text{ Hz}) = 590\text{ Hz}$$

REAL-WORLD PHYSICS

Radar guns

The Doppler effect is used in an amazing variety of technological applications. Perhaps one of the most familiar of these is the "radar gun" which is used to measure the speed of a pitched baseball or a car breaking the speed limit. Though the radar gun uses radio waves rather than sound waves, the basic physical principle is the same—by measuring the Doppler-shifted frequency of waves reflected from an object, it is possible to determine its speed. Doppler radar, used in weather forecasting, applies this same technology to tracking the motion of precipitation caused by storm clouds.

REAL-WORLD PHYSICS: BIO

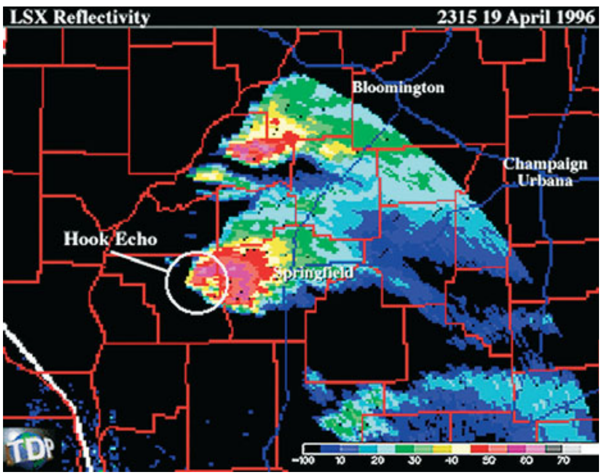
Measuring the speed of blood flow

In medicine, the Doppler shift is used to measure the speed of blood flow in an artery or in the heart itself. In this application, a beam of ultrasound is directed toward an artery in a patient. Some of the sound is reflected back by red blood cells moving through the artery. The reflected sound is detected, and its frequency is used to determine the speed of blood flow. If this information is color coded, with different colors indicating different speeds and directions of flow, an impressive image of blood flow can be constructed.

REAL-WORLD PHYSICS

Red shift of distant galaxies

Finally, the Doppler effect applies to the light of distant galaxies as well. For example, if a galaxy moves away from us—as most do—the light we observe from that galaxy has a lower frequency than if the galaxy were at rest relative to our galaxy, the Milky Way. Since red light has the lowest frequency of visible light, we refer to this reduction in frequency as a "red shift." Thus, by measuring the red shift of a galaxy, we can determine its speed relative to us. Such measurements show that the more distant a galaxy, the greater its speed relative to us—a result known as Hubble's law. This is just what one would expect from an explosion, or



▲ Many familiar and not-so-familiar devices utilize the Doppler effect. Doppler radar, now widely used at airports and for weather forecasting, makes it possible to determine the speed and direction of winds in a distant storm by measuring the Doppler shift they produce—winds shift the radar frequency upward if they blow toward the source and downward if they blow away from the source. The image at left is the Doppler radar scan of a severe thunderstorm that struck the town of Ogden, Illinois, on April 19, 1996. Reddish colors indicate winds blowing toward the radar station, bluish colors indicate winds blowing away. The hook-shaped echo marked on the image is characteristic of tornadoes in the making. In the photo at right, a medical technician uses a Doppler blood flow meter instead of a stethoscope while measuring the blood pressure of a patient.

"Big Bang," in which rapidly moving pieces travel farther in a given amount of time. Thus the Doppler effect, and red-shift measurements, provide strong evidence for the Big Bang and an expanding universe.

14–7 Superposition and Interference

So far we have considered only a single wave at a time. In this section we turn our attention to the way waves combine when more than one is present. As we shall see, the behavior of waves is quite simple in this respect.

Superposition

The combination of two or more waves to form a resultant wave is referred to as **superposition.** When waves are of small amplitude, they superpose in the simplest of ways—they just add. For example, consider two waves on a string, as in Figure 14–19, described by the wave functions y_1 and y_2. If these two waves are present on the same string at the same time, the result is a wave given by

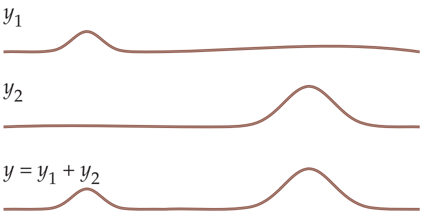
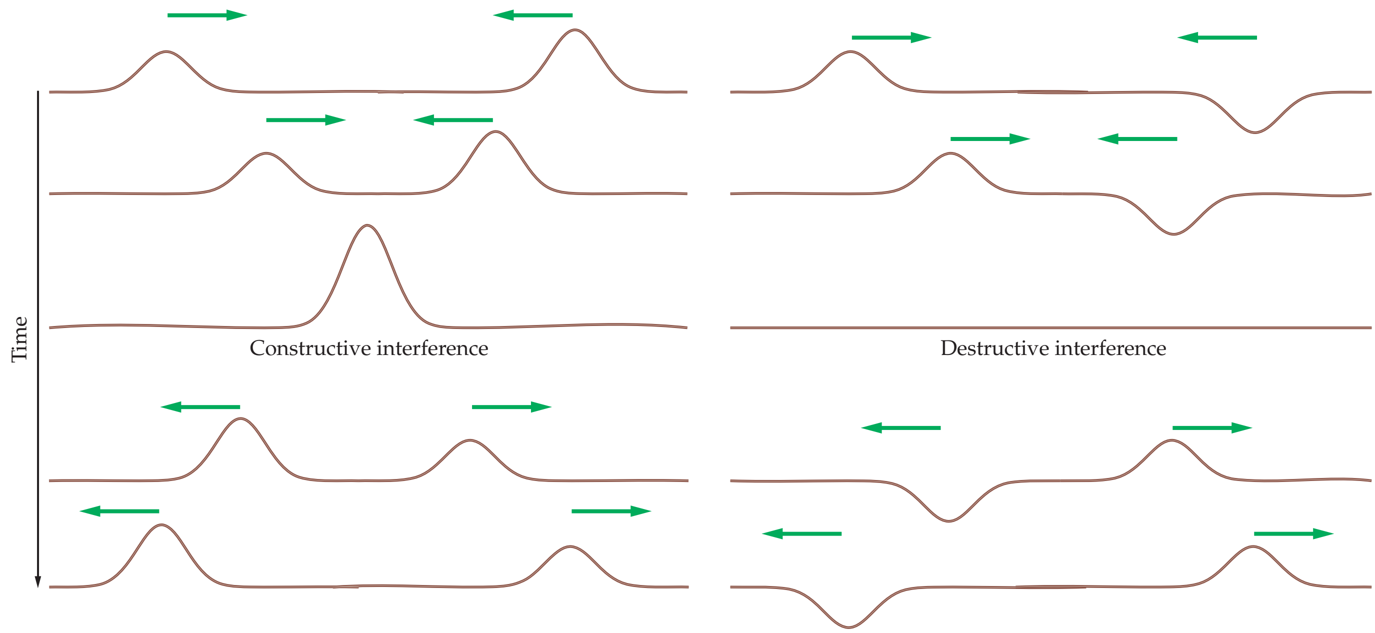


FIGURE 14–19 Wave superposition
Waves of small amplitude superpose (that is, combine) by simple addition.

$$y = y_1 + y_2$$

To see how superposition works as a function of time, consider a string with two wave pulses on it, one traveling in each direction as shown in Figure 14–20 (a). When the pulses arrive in the same region, they add, as stated. This is illustrated in Figure 14–20 (a). The question is, "What do the pulses look like after they have passed through one another? Does their interaction change them in any way?"

The answer is that the waves are completely unaffected by their interaction. This is also shown in Figure 14–20 (a). After the wave pulses pass through one another they continue on as if nothing had happened. It is like listening to an orchestra, where many different instruments are playing simultaneously, and their sounds are combining throughout the concert hall. Even so, you can still hear individual instruments, each making its own sound as if the others were not present.



(a) Two waves combine constructively

(b) Two waves combine destructively

FIGURE 14–20 Interference
Wave pulses superpose as they pass through one another. Afterward, the pulses continue on unchanged. In **(a),** the pulses combine to give a larger amplitude. This is an example of *constructive interference*. When a positive pulse superposes with a negative pulse **(b),** the result is *destructive interference*. In this case, with symmetrical pulses, there is one moment of complete cancellation.

CONCEPTUAL CHECKPOINT 14–3 AMPLITUDE OF A RESULTANT WAVE

Since waves add, does the resultant wave y always have a greater amplitude than the individual waves y_1 and y_2?

REASONING AND DISCUSSION

The wave y is the sum of y_1 and y_2, but remember that y_1 and y_2 are sometimes positive and sometimes negative. Thus, if y_1 is positive at a given time, for example, and y_2 is negative, the sum $y_1 + y_2$ can be zero or even negative. For example, if y_1 and y_2 both have the amplitude A, the amplitude of y can take any value from 0 to $2A$.

ANSWER

No. The amplitude of y can be greater than, less than, or equal to the amplitudes of y_1 and y_2.

Interference

As simple as the principle of superposition is, it still leads to interesting consequences. For example, consider the wave pulses on a string shown in Figure 14–20 (a). When they combine, the resulting pulse has an amplitude equal to the sum of the amplitudes of the individual pulses. This is referred to as **constructive interference.**

On the other hand, two pulses like those in **Figure 14–20 (b)** may combine. When this happens, the positive displacement of one wave adds to the negative displacement of the other to create a net displacement of zero. That is, the pulses momentarily cancel one another. This is **destructive interference.**

It is important to note that the waves don't simply disappear when they experience destructive interference. For example, in Figure 14–20 (b) the wave pulses continue on unchanged after they interact. This makes sense from an energy point of view—after all, each wave carries energy, hence the waves, along with their energy, cannot simply vanish. In fact, when the string is flat in Figure 14–20 (b) it has its greatest transverse speed—just like a swing has its highest speed when it is in its equilibrium position. Therefore, the energy of the wave is still present at this instant of time—it is just in the form of the kinetic energy of the string.

It should also be noted that interference is not limited to waves on a string; all waves exhibit interference effects. In fact, you could say that interference is one of the key characteristics that define waves. In general, when waves combine, they form **interference patterns** that include regions of both constructive and destructive interference. An example is shown in **Figure 14–21**, where two circular waves are interfering. Note the regions of constructive interference separated by regions of destructive interference.

To understand the formation of such patterns, consider a system of two identical sources, as in **Figure 14–22**. Each source sends out waves consisting of alternating crests and troughs. We set up the system so that when one source emits a crest, the other emits a crest as well. Sources that are synchronized like this are said to be **in phase.**

Now, at a point like A, the distance to each source is the same. Thus, if the wave from one source produces a crest at point A, so too does the wave from the other source. As a result, with crest combining with crest, the interference at A is constructive.

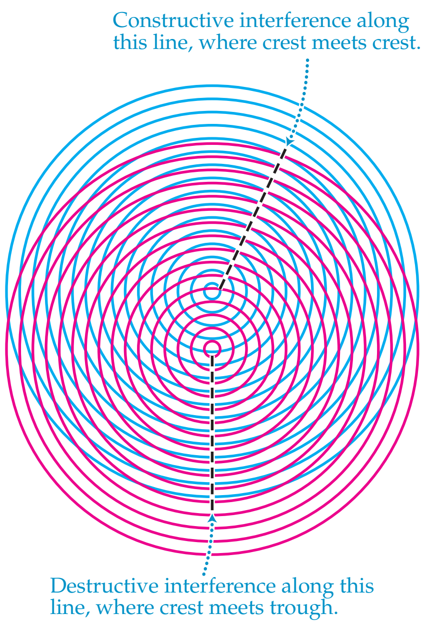


FIGURE 14–21 Interference of circular waves

Interference pattern formed by the superposition of two sets of circular waves. The light radial "rays" are regions where crest meets crest and trough meets trough (constructive interference). The dark areas in between the light rays are regions where the crest of one wave overlaps the trough of another wave (destructive interference).

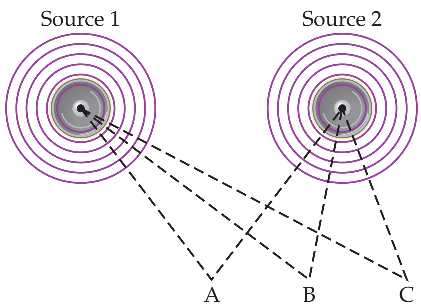


FIGURE 14–22 Interference with two sources

Suppose the two sources emit waves in phase. At point A the distance to each source is the same, and, hence, crest meets crest. The result is constructive interference. At B the distance from source 1 is greater than that from source 2 by half a wavelength. The result is crest meeting trough and destructive interference. Finally, at C the distance from source 1 is one wavelength greater than the distance from source 2. Hence, we find constructive interference at C. If the sources had been opposite in phase, then A and C would be points of destructive interference, and B would be a point of constructive interference.

Next consider point B. At this location the wave from source 1 must travel a greater distance than the wave from source 2. If the extra distance is half a wavelength, it follows that when the wave from source 2 produces a crest at B the wave from source 1 produces a trough. As a result, the waves combine to give destructive interference at B. At point C, on the other hand, the distance from source 1 is one wavelength greater than the distance from source 2. Hence the waves are in phase again at C, with crest meeting crest for constructive interference.

In general, then, we can say that constructive and destructive interference occur under the following conditions for two sources that are in phase:

> Constructive interference occurs when the path length from the two sources differs by $0, \lambda, 2\lambda, 3\lambda, \ldots$.
>
> Destructive interference occurs when the path length from the two sources differs by $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$.

Other path length differences result in intermediate degrees of interference, between the extremes of destructive and constructive interference.

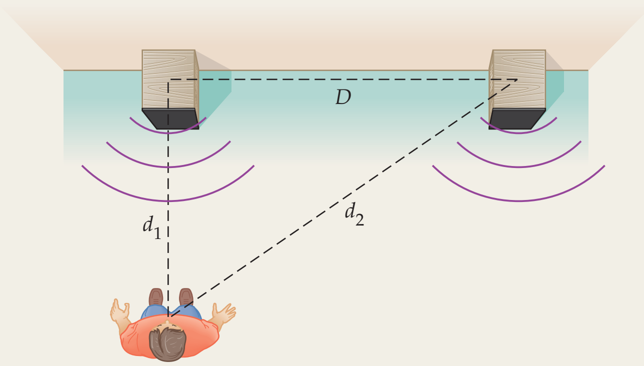
A specific example of interference patterns is provided by sound, using speakers that emit sound in phase with the same frequency. This situation is analogous to the two sources in Figure 14–22. As a result, constructive and destructive interference is to be expected, depending on the path length from each speaker. This is illustrated in the next Example.

EXAMPLE 14–7 SOUND OFF

Two speakers separated by a distance of 4.30 m emit sound of frequency 221 Hz. The speakers are in phase with one another. A person listens from a location 2.80 m directly in front of one of the speakers. Does the person hear constructive or destructive interference?

PICTURE THE PROBLEM

Our sketch shows the two speakers emitting sound in phase at the frequency $f = 221$ Hz. The speakers are separated by the distance $D = 4.30$ m, and the observer is a distance $d_1 = 2.80$ m directly in front of one of the speakers.



STRATEGY

The type of interference depends on whether the difference in path length, $d_2 - d_1$, is one or more wavelengths or an odd multiple of half a wavelength. Thus, we begin by calculating the wavelength, λ. Next, we find d_2, and compare the difference in path length to λ.

SOLUTION

1. Calculate the wavelength of this sound, using $v = \lambda f$. As usual, let $v = 343$ m/s be the speed of sound:
$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{221 \text{ Hz}} = 1.55 \text{ m}$$

2. Find the path length d_2:
$$d_2 = \sqrt{D^2 + d_1{}^2} = \sqrt{(4.30 \text{ m})^2 + (2.80 \text{ m})^2} = 5.13 \text{ m}$$

3. Determine the difference in path length, $d_2 - d_1$:
$$d_2 - d_1 = 5.13 \text{ m} - 2.80 \text{ m} = 2.33 \text{ m}$$

4. Divide λ into $d_2 - d_1$ to find the number of wavelengths that fit into the path difference:
$$\frac{d_2 - d_1}{\lambda} = \frac{2.33 \text{ m}}{1.55 \text{ m}} = 1.50$$

INSIGHT

Since the path difference is $3\lambda/2$, we expect destructive interference. In the ideal case, the person would hear no sound. As a practical matter, some sound will be reflected from objects in the vicinity, resulting in a finite sound intensity.

PRACTICE PROBLEM

We know that 221 Hz gives destructive interference. What is the lowest frequency that gives constructive interference for the case described in this Example? [**Answer:** Set $\lambda = d_2 - d_1 = 2.33$ m. This gives $f = 147$ Hz.]

Some related homework problems: Problem 61, Problem 63

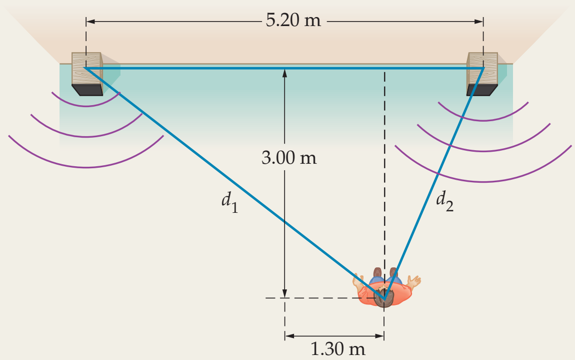
REAL-WORLD PHYSICS
Connecting speakers in phase

It is possible to connect a speaker with its wires reversed, which can result in a set of speakers that have **opposite phase.** In this case, as one speaker emits a compression the other sends out a rarefaction. When you set up a stereo system, it is important to be sure the wires are connected in a consistent fashion so that your speakers will be in phase.

If the two speakers in Figure 14–22 have opposite phase, for example, the conditions for constructive and destructive interference are changed, as are the interference patterns. For example, at point A, where the distances from the two speakers are the same, the wave from one speaker is a compression when the wave from the other speaker is a rarefaction. Thus, point A is now a point of destructive interference rather than constructive interference. In general, then, the conditions for constructive and destructive interference are simply reversed—a path difference of $0, \lambda, 2\lambda, \ldots$ results in destructive interference, a path difference of $\lambda/2, 3\lambda/2, \ldots$ results in constructive interference.

ACTIVE EXAMPLE 14–2 OPPOSITE PHASE INTERFERENCE

The speakers shown to the right have opposite phase. They are separated by a distance of 5.20 m and emit sound with a frequency of 104 Hz. A person stands 3.00 m in front of the speakers and 1.30 m to one side of the center line between them. What type of interference occurs at the person's location?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the wavelength:	$\lambda = 3.30$ m
2. Find the path length d_1:	$d_1 = 4.92$ m
3. Find the path length d_2:	$d_2 = 3.27$ m
4. Calculate the path length difference, $d_1 - d_2$:	$d_1 - d_2 = 1.65$ m
5. Divide the path length difference by the wavelength:	$(d_1 - d_2)/\lambda = 0.500$

INSIGHT

The path difference is half a wavelength, and the speakers have opposite phase. As a result, the person experiences constructive interference.

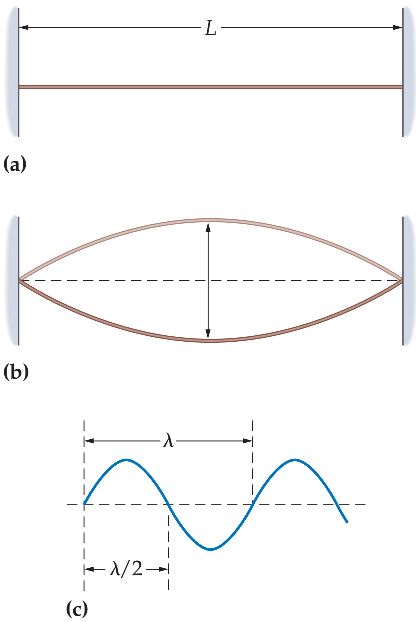
YOUR TURN

Find the next higher frequency for which constructive interference occurs at the person's location.

(Answers to **Your Turn** *problems are given in the back of the book.)*

REAL-WORLD PHYSICS
Active noise reduction

Destructive interference can be used to reduce the intensity of noise in a variety of situations, such as a factory, a busy office, or even the cabin of an airplane. The process, referred to as Active Noise Reduction (ANR), begins with a microphone that picks up the noise to be reduced. The signal from the microphone is then reversed in phase and sent to a speaker. As a result, the speaker emits sound that is opposite in phase to the incoming noise—in effect, the speaker produces "anti-noise." In this way, the noise is *actively* canceled by destructive interference, rather than simply reduced by absorption. The effect when wearing a pair of ANR headphones can be as much as a 30-dB reduction in the intensity level of noise.



▲ **FIGURE 14–23 A standing wave**
(a) A string is tied down at both ends. **(b)** If the string is plucked in the middle, a standing wave results. This is the fundamental mode of oscillation of the string. **(c)** The fundamental consists of one-half (1/2) a wavelength between the two ends of the string. Hence, its wavelength is $2L$.

14–8 Standing Waves

If you have ever plucked a guitar string, or blown across the mouth of a pop bottle to create a tone, you have generated **standing waves.** In general, a standing wave is one that oscillates with time, but remains fixed in its location. It is in this sense that the wave is said to be "standing."

In some respects, a standing wave can be considered as resulting from constructive interference of a wave with itself. As one might expect, then, standing waves occur only if specific conditions are satisfied. We explore these conditions in this section for two cases: (i) waves on a string and (ii) sound waves in a hollow, cylindrical structure.

Waves on a String

We begin by considering a string of length L that is tied down at both ends, as in **Figure 14–23 (a)**. If you pluck this string in the middle it vibrates as shown in **Figure 14–23 (b)**. This is referred to as the **fundamental mode** of vibration for this string or, also, as the **first harmonic.** Clearly, the string assumes a wavelike shape, but because of the boundary conditions—the ends tied down—the wave stays in place.

As is clear from **Figure 14–23 (c)**, the fundamental mode corresponds to half a wavelength of a usual wave on a string. One can think of the fundamental as being formed by this wave reflecting back and forth between the walls holding the string. If the frequency is just right, the reflections combine to give constructive interference and the fundamental is formed; if the frequency differs from the fundamental frequency, the reflections result in destructive interference and a standing wave does not result.

We can find the frequency of the fundamental, or first harmonic, as follows: First use the fact that the wavelength of the first harmonic is twice the distance between the walls. Thus,

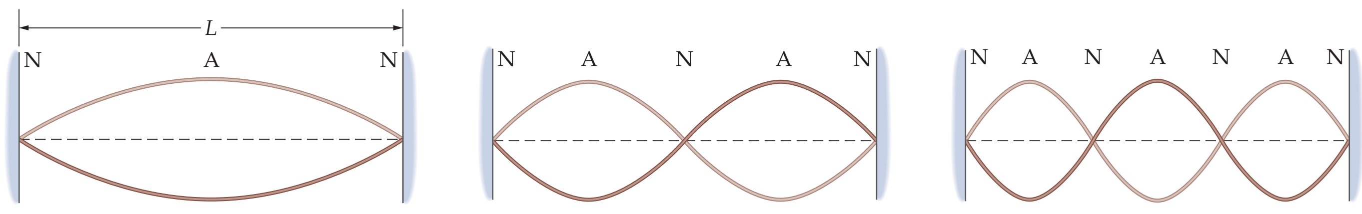
$$\lambda_1 = 2L$$

If the speed of waves on the string is v, it follows that the frequency of the first harmonic, f_1, is determined by $v = \lambda_1 f_1 = (2L)f_1$. Therefore,

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

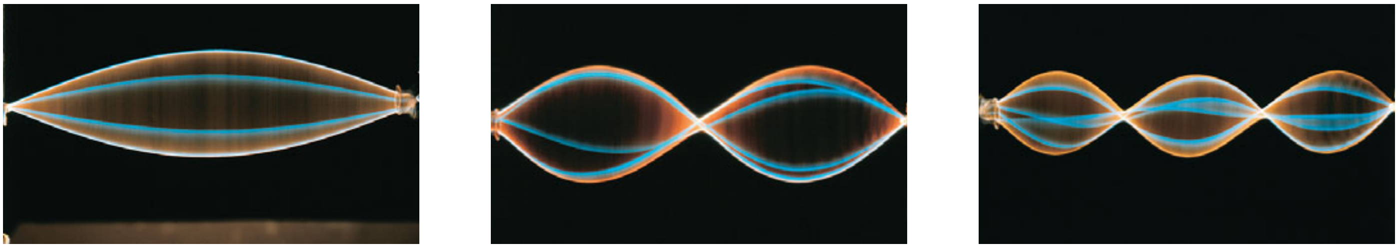
Note that the frequency of the first harmonic increases with the speed of the waves, and decreases as the string is lengthened.

The first harmonic is not the only standing wave that can exist on a string, however. In fact, there are an infinite number of standing wave modes—or **harmonics**—for any given string, with frequencies that are integer multiples of the first harmonic. To find the higher harmonics, note that the two ends of the string must remain fixed. Points on a standing wave that stay fixed are referred to as **nodes.** Halfway between any two nodes is a point on the wave that has a maximum displacement, as indicated in **Figure 14–24**. Such a point is called an **antinode.** Referring to Figure 14–24 (a), then, we see that the first harmonic consists of two nodes (N) and one antinode (A); the sequence is N-A-N.



(a) First harmonic (fundamental) **(b) Second harmonic** **(c) Third harmonic**

▲ **FIGURE 14–24 Harmonics**
The first three harmonics for a string tied down at both ends. Note that an extra half wavelength is added to go from one harmonic to the next. **(a)** $\lambda/2 = L, \lambda = 2L$; **(b)** $\lambda = L$; **(c)** $3\lambda/2 = L, \lambda = 2L/3$.



▲ The string in these multiflash photographs vibrates in one of three different standing wave patterns, each with its own characteristic frequency. The lowest frequency standing wave—the fundamental, or first harmonic—is shown in the photograph at left. In this case, there are only two nodes, one at each end of the string where it is tied down. If the length of the string is L, we see that the wavelength of the fundamental is twice this length, or $\lambda_1 = 2L$. Thus, if waves have a speed v on this string, their frequency is $f_1 = v/2L$. Higher harmonics are produced by adding one node at a time to the standing wave pattern. The second harmonic, shown in the middle photograph, has a node at either end and one in the middle. In this case the wavelength is $\lambda_2 = L$ and the frequency is $f_2 = v/L = 2f_1$. The photograph at right shows the third harmonic, where $\lambda_3 = 2L/3$ and $f_3 = v/(2L/3) = 3v/2L = 3f_1$. In general, the nth harmonic on a string tied down at both ends is $f_n = nf_1$.

The *second* harmonic can be constructed by including one more half wavelength in the standing wave, as in Figure 14–24 (b). This mode has the sequence N-A-N-A-N, and has one complete wavelength between the walls; that is, $\lambda_2 = L$. Therefore, the frequency of the second harmonic, f_2, is

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$$

Similarly, the *third* harmonic again includes one more half wavelength, as in Figure 14–24 (c). Now there are one-and-a-half wavelengths in the length L, and hence $(3/2)\lambda_3 = L$, or $\lambda_3 = 2L/3$. The corresponding third-harmonic frequency, f_3, is

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2}{3}L} = 3\frac{v}{2L} = 3f_1$$

Note that the frequencies of the harmonics are increasing in integer steps; that is, each harmonic has a frequency that is an integer multiple of the first-harmonic frequency. Clearly, then, the sequence of standing waves is characterized by the following:

Standing Waves on a String

First harmonic (fundamental) frequency and wavelength:

$$f_1 = \frac{v}{2L}$$
$$\lambda_1 = 2L \qquad 14\text{–}12$$

Frequency and wavelength of the nth harmonic, with $n = 1, 2, 3, \ldots$:

$$f_n = nf_1 = n\frac{v}{2L}$$
$$\lambda_n = \lambda_1/n = 2L/n \qquad 14\text{–}13$$

▲ The photos above show a time sequence (from left to right) as a square metal plate that is initially at rest is vibrated vertically about its center. Initially the plate is covered with a uniform coating of salt crystals. As the plate is vibrated, however, a standing wave develops. The salt makes the wave pattern visible by collecting at the nodes, where the plate is at rest. Clearly, standing waves on a two-dimensional plate can be much more complex than the standing waves on a string.

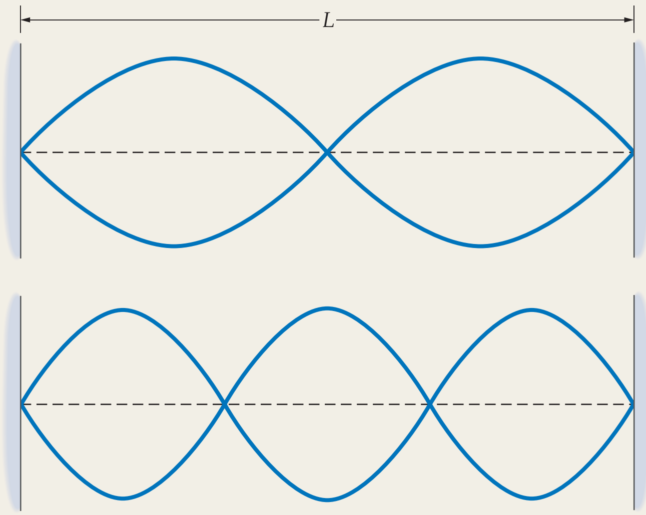
Note that the difference in frequency between any two successive harmonics is equal to the first-harmonic frequency, f_1, and that n represents the number of half wavelengths in the standing wave.

EXAMPLE 14–8 IT'S FUNDAMENTAL

One of the harmonics on a string 1.30 m long has a frequency of 15.60 Hz. The next higher harmonic has a frequency of 23.40 Hz. Find **(a)** the fundamental frequency, and **(b)** the speed of waves on this string.

PICTURE THE PROBLEM

The problem statement does not tell us directly which two harmonics have the given frequencies. We do know, however, that they are *successive* harmonics of the string. For example, if one harmonic has one node between the two ends of the string, the next harmonic has two nodes. Our sketch illustrates this case, which turns out to be appropriate for this problem.



STRATEGY

a. We know from Equation 14–13 that the frequencies of successive harmonics increase by f_1. That is, $f_2 = f_1 + f_1 = 2f_1$, $f_3 = f_2 + f_1 = 3f_1$, $f_4 = f_3 + f_1 = 4f_1$, ... Therefore, we can find the fundamental frequency, f_1, by taking the difference between the given frequencies.

b. Once the fundamental frequency is determined, we can find the speed of waves in the string from the relation $f_1 = v/2L$.

SOLUTION

Part (a)

1. The fundamental frequency is the difference between the two given frequencies: $f_1 = 23.40\ \text{Hz} - 15.60\ \text{Hz} = 7.80\ \text{Hz}$

Part (b)

2. Solve $f_1 = v/2L$ for the speed, v: $f_1 = v/2L$; $v = 2Lf_1$

3. Substitute numerical values: $v = 2Lf_1 = 2(1.30\ \text{m})(7.80\ \text{Hz}) = 20.3\ \text{m/s}$

INSIGHT

Now that we know the fundamental frequency, we can identify the harmonics given in the problem statement. First, $15.6\ \text{Hz} = 2(7.80\ \text{Hz})$, so this is the second harmonic. The next mode, $23.4\ \text{Hz} = 3(7.80\ \text{Hz})$, is the third harmonic, as expected.

PRACTICE PROBLEM

Suppose the tension in this string is increased until the speed of the waves is 22.0 m/s. What are the frequencies of the first three harmonics in this case? [**Answer:** $f_1 = 8.46$ Hz, $f_2 = 16.9$ Hz, $f_3 = 25.4$ Hz]

Some related homework problems: Problem 72, Problem 73

When a guitar string is plucked or a piano string is struck, it vibrates primarily in its fundamental mode, with smaller contributions coming from the higher harmonics. It follows that notes of different pitch can be produced by using strings of different length. Recalling that the fundamental frequency for a string of length L is $f_1 = v/2L$, we see that long strings produce low frequencies and short strings produce high frequencies—all other variables remaining the same.

REAL-WORLD PHYSICS
The shape of a piano

This fact accounts for the general shape of a piano. Note that the strings shorten toward the right side of the piano, where the notes are of higher frequency. Similarly, a double bass is a larger instrument with longer strings than a violin, as one would expect by the different frequencies the instruments produce. To tune a stringed instrument, the tension in the strings is adjusted—since changing the length of the instrument is impractical. This in turn varies the speed v of waves on the string, and hence the fundamental frequency $f_1 = v/2L$ can be adjusted as desired.

The human ear responds to frequency in a rather interesting and unexpected way. In particular, frequencies that seem to increase by the same amount are in fact increasing by the same multiplicative factor. For example, if three frequencies, f_1, f_2, and f_3, sound equally spaced to our ears, you might think that f_2 is greater than f_1 by a certain amount, x, and that f_3 is greater than f_2 by the same amount. Mathematically, we would write this as $f_2 = f_1 + x$ and $f_3 = f_2 + x = f_1 + 2x$. In fact, when we measure the frequencies and compare, we find that f_2 is greater than f_1 by a multiplicative factor x, and that f_3 is greater than f_2 by the same factor; that is $f_2 = xf_1$ and $f_3 = xf_2 = x^2 f_1$.

REAL-WORLD PHYSICS: BIO
Human perception of pitch

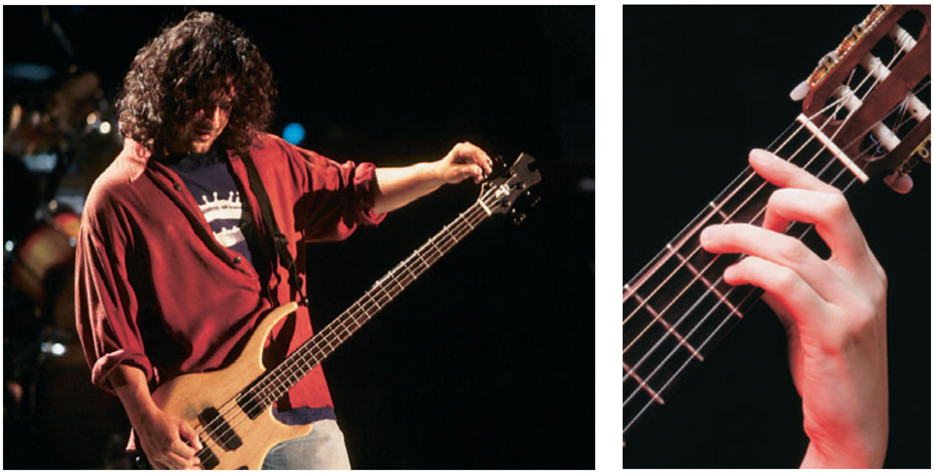
For instance, middle C on the piano has a frequency of 261.7 Hz. If we move up one octave to the next C, the frequency is 523.3 Hz; going up one more octave, the next C is 1047 Hz. Note that with each octave the frequency doubles; that is, it goes up by a multiplicative factor of 2. Since there are 12 semitones in one octave of the chromatic scale, the frequency increase from one semitone to the next is $(2)^{1/12}$. The frequencies for a full chromatic octave are given in Table 14–3.

TABLE 14–3 Chromatic Musical Scale

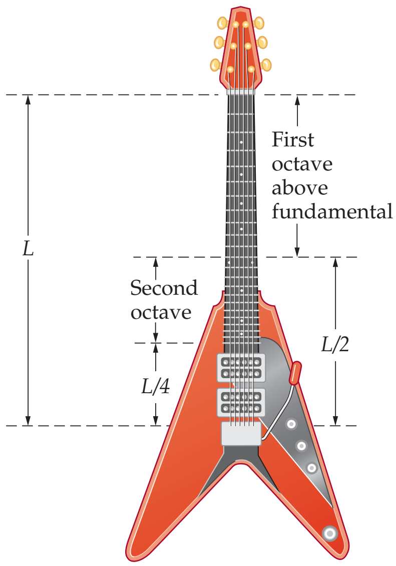
Note	Frequency (Hz)
Middle C	261.7
C# (C-sharp) D♭ (D-flat)	277.2
D	293.7
D#, E♭	311.2
E	329.7
F	349.2
F#, G♭	370.0
G	392.0
G#, A♭	415.3
A	440.0
A#, B♭	466.2
B	493.9
C	523.3

On a guitar two full octaves and more can be produced on a single string by pressing the string down against frets to effectively change its length. Notice that the separation between frets is not uniform. In particular, suppose the unfretted string has a fundamental frequency of 250 Hz. Since one octave up on the scale would be twice the frequency, 500 Hz, the length of the string must be halved to produce that note. To go to the next octave, and double the frequency again to 1000 Hz, the string must be shortened by a factor of two again, to one quarter its original length. This is illustrated in **Figure 14–25**. Since the distance between successive octaves is decreasing—in this case from $L/2$ to $L/4$—it follows that the spacing between frets must decrease as one goes to higher notes. As a result, the frets on a guitar are always more closely spaced as one moves toward the base of the neck.

REAL-WORLD PHYSICS
Frets on a guitar



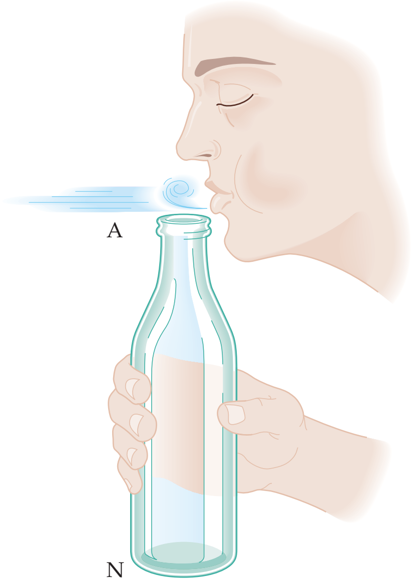
▲ Three factors determine the pitch of a vibrating string: mass per unit length, μ; tension, F; and length, L. In an instrument such as a guitar, the first of these factors is fixed once the strings are put on. (Note in the photos that the strings vary in thickness; other things being equal, the heavier the string, the lower the pitch.) The second factor, the tension, can be varied by means of pegs that the player uses to tune the instrument (left), adjusting the pitch of each "open" string to its correct value. The third factor, the length of the string, is the only one that the performer controls while playing. Pressing a string against one of the frets (right), changes its effective length—the length of string that is free to vibrate—and thus the note that is produced.



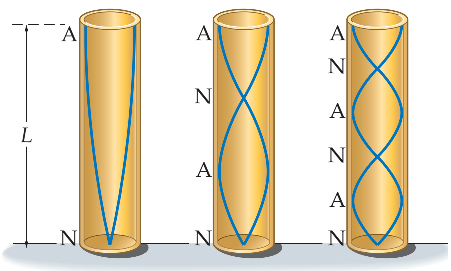
▲ **FIGURE 14–25 Frets on a guitar**
To go up one octave from the fundamental, the effective length of a guitar string must be halved. To increase one more octave, it is necessary to halve the length of the string again. Thus, the distance between frets is not uniform; they are more closely spaced near the base of the neck.

Vibrating Columns of Air

If you blow across the open end of a pop bottle, as in **Figure 14–26**, you hear a tone of a certain frequency. If you pour some water into the bottle and repeat the experiment, the sound you hear has a higher frequency. In both cases you have excited the fundamental mode of the column of air within the bottle. When water was added to the bottle, however, the column of air was shortened, leading to a higher frequency—in the same way that a shortened string has a higher frequency.



▲ **FIGURE 14–26 Exciting a standing wave**
When air is blown across the open top of a soda pop bottle, the turbulent air flow can cause an audible standing wave. The standing wave will have an antinode, A, at the top (where the air is moving) and a node, N, at the bottom (where the air cannot move).



▲ **FIGURE 14–27 Standing waves in a pipe that is open at one end**
The first three harmonics for waves in a column of air of length L that is open at one end: **(a)** $\lambda/4 = L$, $\lambda = 4L$; **(b)** $3\lambda/4 = L$, $\lambda = 4L/3$; **(c)** $5\lambda/4 = L$, $\lambda = 4L/5$.

Let's examine the situation more carefully. When you blow across the opening in the bottle, the result is a swirling movement of air that excites rarefactions and compressions, as illustrated in the figure. For this reason, the opening is an antinode (A) for sound waves. On the other hand, the bottom of the bottle is closed, preventing movement of the air; hence, it must be a node (N). Any standing wave in the bottle must have a node at the bottom and an antinode at the top.

The lowest frequency standing wave that is consistent with these conditions is shown in **Figure 14–27 (a)**. If we plot the density variation of the air for this wave, we see that one-quarter of a wavelength fits into the column of air in the bottle. Thus, if the length of the bottle is L, the first harmonic (fundamental) has a wavelength satisfying the following:

$$\tfrac{1}{4}\lambda = L$$
$$\lambda = 4L$$

The first-harmonic frequency, f_1, is given by

$$v = \lambda f_1$$

Solving for f_1 we find

$$f_1 = \frac{v}{\lambda} = \frac{v}{4L}$$

This is half the corresponding fundamental frequency for a wave on a string.

The next harmonic is produced by adding half a wavelength, just as in the case of the string. Thus, if the fundamental is represented by N-A, the second harmonic can be written as N-A-N-A. Since the distance from a node to an antinode is a quarter of a wavelength, we see that three-quarters (3/4) of a wavelength fits into the bottle for this mode. This is shown in **Figure 14–27 (b)**. Therefore, $3\lambda/4 = L$, and, hence,

$$\lambda = \tfrac{4}{3}L$$

As a result, the frequency is

$$\frac{v}{\lambda} = \frac{v}{\tfrac{4}{3}L} = 3\frac{v}{4L} = 3f_1$$

Notice that this is the *third* harmonic of the pipe, since its frequency is three times f_1.

Similarly, the next-higher harmonic is represented by N-A-N-A-N-A, as indicated in **Figure 14–27 (c)**. In this case, $5\lambda/4 = L$, and the frequency is

$$\frac{v}{\lambda} = \frac{v}{\tfrac{4}{5}L} = 5\frac{v}{4L} = 5f_1$$

This is the *fifth* harmonic of the pipe.

Clearly, the progression of harmonics for a column of air that is closed at one end and open at the other end is described by the following frequencies and wavelengths:

Standing Waves in a Column of Air Closed at One End

$$f_1 = \frac{v}{4L}$$

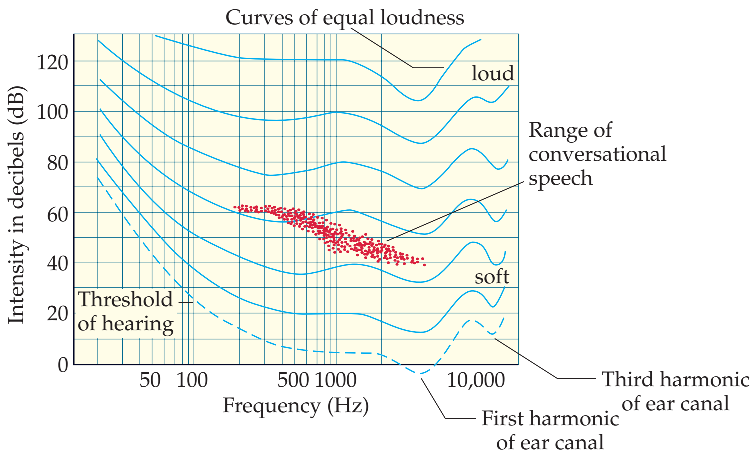
$$f_n = nf_1 = n\frac{v}{4L} \qquad n = 1, 3, 5, \ldots \qquad \text{14–14}$$

$$\lambda_n = \lambda_1/n = 4L/n$$

Note that only the odd harmonics are present in this case, as opposed to waves on a string, in which all integer harmonics occur.

REAL-WORLD PHYSICS: BIO
Human hearing and the ear canal

The human ear canal is an example of a column of air that is closed at one end (the eardrum) and open at the other end. Standing waves in the ear canal can lead to an increased sensitivity of hearing. This is illustrated in **Figure 14–28**, which shows "curves of equal loudness" as a function of frequency. Where these curves dip downward, sounds of lower intensity seem just as loud as sounds of higher intensity at other frequencies. The two prominent dips near 3500 Hz and 11,000 Hz are due to standing waves in the ear canal corresponding to Figures 14–27 (a) and (b), respectively.



▲ **FIGURE 14–28 Human response to sound**

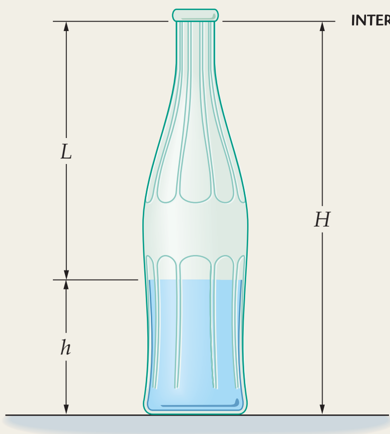
The human ear is more sensitive to some frequencies of sound than to others. For example, every point on a "curve of equal loudness" seems just as loud to us as any other point, even though the corresponding physical intensities may be quite different. To illustrate, note that the threshold of hearing is not equal to 0 dB for all frequencies. In fact, it is approximately 25 dB at 100 Hz, about 5 dB at 1000 Hz, and is even slightly negative near 3500 Hz. Thus, regions where the curves dip downward correspond to increased sensitivity of the ear—in fact, near 3500 Hz we can hear sounds that are a thousand times less intense than sounds at 100 Hz. The two most prominent dips occur near 3500 Hz and 11,000 Hz, corresponding to standing waves in the ear canal analogous to those shown in Figure 14–27 (a) and (b), respectively. (See Problem 71.)

EXAMPLE 14–9 POP MUSIC

An empty soda pop bottle is to be used as a musical instrument in a band. In order to be tuned properly, the fundamental frequency of the bottle must be 440.0 Hz. **(a)** If the bottle is 26.0 cm tall, how high should it be filled with water to produce the desired frequency? Treat the bottle as a pipe that is closed at one end (the surface of the water) and open at the other end. **(b)** What is the frequency of the next higher harmonic for this bottle?

PICTURE THE PROBLEM

In our sketch, we label the height of the bottle with $H = 26.0$ cm, and the unknown height of water with h. Clearly, then, the length of the vibrating column of air is $L = H - h$.



STRATEGY

a. Given the frequency of the fundamental ($f_1 = 440.0$ Hz) and the speed of sound in air ($v = 343$ m/s), we can use $f_1 = v/4L$ to solve for the length L of the air column. The height of water is then $h = H - L$.

b. The next higher harmonic for a pipe open at one end is the third harmonic ($n = 3$ in Equation 14–14). Thus, the next higher harmonic frequency for this bottle is $f_3 = 3f_1$.

SOLUTION

Part (a)

1. Solve $f_1 = v/4L$ for the length L: $\quad f_1 = v/4L \quad \text{or} \quad L = v/4f_1$

2. Substitute numerical values: $$L = \frac{v}{4f_1} = \frac{343 \text{ m/s}}{4(440.0 \text{ Hz})} = 0.195 \text{ m}$$

3. Use $h = H - L$ to find the height of the water: $\quad h = H - L = 0.260 \text{ m} - 0.195 \text{ m} = 0.065 \text{ m} = 6.5 \text{ cm}$

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Part (b)

4. Calculate the frequency of the third harmonic (the next highest) with $f_3 = 3f_1$: $f_3 = 3f_1 = 3(440.0\text{ Hz}) = 1320\text{ Hz}$

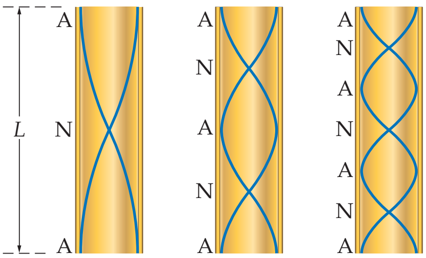
INSIGHT

If more water is added to the bottle, the air column will shorten and the fundamental frequency will become higher than 440.0 Hz. All higher harmonics would be increased in frequency as well.

PRACTICE PROBLEM

Calculate the fundamental frequency if the water level is increased to 7.00 cm. [**Answer:** $f_1 = 451\text{ Hz}$]

Some related homework problems: Problem 68, Problem 75



(a) First harmonic (fundamental) **(b)** Second harmonic **(c)** Third harmonic

FIGURE 14–29 Standing waves in a pipe that is open at both ends

The first three harmonics for waves in a column of air of length L that is open at both ends: **(a)** $\lambda/2 = L$, $\lambda = 2L$; **(b)** $\lambda = L$; **(c)** $3\lambda/2 = L$, $\lambda = 2L/3$.

It is also possible to excite standing waves in columns of air that are open at both ends, as illustrated in **Figure 14–29**. In this case there is an antinode at each end of the column. Hence, the first harmonic, or fundamental, is A-N-A, as shown in Figure 14–29 (a). Note that half a wavelength fits into the pipe, thus

$$f_1 = \frac{v}{2L}$$

This is the same as the corresponding result for a wave on a string.

The next harmonic is A-N-A-N-A, which fits one complete wavelength in the pipe. This harmonic is shown in Figure 14–29 (b), and has the frequency

$$f_2 = \frac{v}{L} = 2f_1$$

This is the *second* harmonic of the pipe. The rest of the harmonics continue in exactly the same manner as for waves on a string, with all integer harmonics present. Thus, the frequencies and wavelengths in a column of air open at both ends are as follows:

Standing Waves in a Column of Air Open at Both Ends

$$f_1 = \frac{v}{2L}$$

$$f_n = nf_1 = n\frac{v}{2L} \quad n = 1, 2, 3, \ldots \qquad \text{14–15}$$

$$\lambda_n = \lambda_1/n = 2L/n$$

CONCEPTUAL CHECKPOINT 14–4 TALKING WITH HELIUM

If you fill your lungs with helium and speak, you sound something like Donald Duck. From this observation, we can conclude that the speed of sound in helium must be **(a)** less than, **(b)** the same as, or **(c)** greater than the speed of sound in air.

REASONING AND DISCUSSION

When we speak with helium, our words are higher pitched. Looking at Equation 14–15, we see that for the frequency to increase, while the length of the vocal chords remains the same, the speed of sound must be higher.

ANSWER

(c) The speed of sound is greater in helium than in air.

REAL-WORLD PHYSICS

Organ pipes

A pipe organ uses a variety of pipes of different length, with some being open at both ends, others open at one end only. When a key is pressed on the console of the organ, air is forced through a given pipe. By accurately adjusting the length of the pipe it can be given the desired tone. In addition, since open and closed pipes have different harmonic frequencies, they sound distinctly different to the ear, even if they have the same fundamental frequency. Thus, by judiciously choosing both the length and the type of a pipe, an organ can be given a range of different sounds, allowing it to mimic a trumpet, a trombone, a clarinet, and so on.

Standing waves have also been observed in the Sun. Like an enormous, low-frequency musical instrument, the Sun vibrates once roughly every five minutes, a result of the roiling nuclear reactions that take place within its core. One of the goals of SOHO, the Solar and Heliospheric Observatory, is to study these solar vibrations in detail. By observing the variety of standing waves produced in the Sun, we can learn more about its internal structure and dynamics.


▲ Blowing across the mouth of a bottle (Figure 14–26) sets the air column within the bottle vibrating, producing a tone. This principle is put to use in the pipe organ. A large organ may have hundreds of pipes of different lengths, some open at both ends and some at only one, affording the performer great control over the tonal quality of the sound produced, as well as its pitch.

14–9 Beats

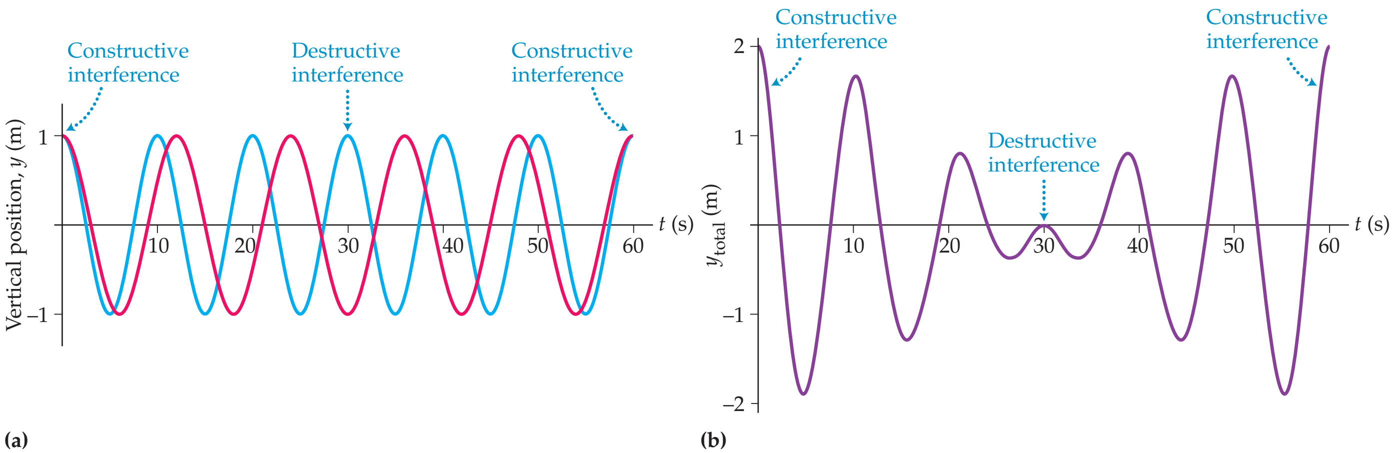
An interference pattern, such as that shown in Figure 14–21, is a snapshot at a given time, showing locations where constructive and destructive interference occur. It is an interference pattern in space. **Beats,** on the other hand, can be thought of as an interference pattern in time.

To be specific, imagine plucking two guitar strings that have slightly different frequencies. If you listen carefully, you notice that the sound produced by the strings is not constant in time. In fact, the intensity increases and decreases with a definite period. These fluctuations in intensity are the beats, and the frequency of successive maximum intensities is the **beat frequency.**

As an example, suppose two waves, with frequencies $f_1 = 1/T_1$ and $f_2 = 1/T_2$, interfere at a given, fixed location. At this location, each wave moves up and down with simple harmonic motion, as described by Equation 13–2. Applying this result to the vertical position, y, of each wave yields the following:

$$
\begin{aligned}
y_1 &= A\cos\left(\frac{2\pi}{T_1}t\right) = A\cos(2\pi f_1 t) \\
y_2 &= A\cos\left(\frac{2\pi}{T_2}t\right) = A\cos(2\pi f_2 t)
\end{aligned}
\qquad \text{14–16}
$$

These equations are plotted in **Figure 14–30 (a)**, with $A = 1$ m, and their superposition, $y_{total} = y_1 + y_2$, is shown in **Figure 14–30 (b)**.



▲ **FIGURE 14–30 Interference of two waves with slightly different frequencies**

(a) A plot of the two waves, y_1 (blue) and y_2 (red), given in Equations 14–16. **(b)** The resultant wave y_{total} for the two waves shown in part (a). Note the alternately constructive and destructive interference leading to beats.

Note that at the time $t = 0$, both y_1 and y_2 are equal to A; thus their superposition gives $2A$. Since the waves have different frequencies, however, they do not stay in phase. At a later time, t_1, we find that $y_1 = A$ and $y_2 = -A$; their superposition gives zero at this time. At a still later time, $t_2 = 2t_1$, the waves are again in phase and add to give $2A$. Thus, a person listening to these two waves hears a sound whose amplitude and loudness vary with time; that is, the person hears beats.

Superposing these waves mathematically, we find

$$\begin{aligned} y_{total} &= y_1 + y_2 \\ &= A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t) \\ &= 2A\cos\left(2\pi\frac{f_1 - f_2}{2}t\right)\cos\left(2\pi\frac{f_1 + f_2}{2}t\right) \end{aligned} \quad \text{14–17}$$

The final step in the expression follows from the trigonometric identities given in Appendix A. The first part of y_{total} is

$$2A\cos\left(2\pi\frac{f_1 - f_2}{2}t\right)$$

This gives the slowly varying amplitude of the beats, as indicated in **Figure 14–31**. Since a loud sound is heard whenever this term is equal to $-2A$ or $2A$—which happens twice during any given oscillation—the beat frequency is

Definition of Beat Frequency

$$f_{beat} = |f_1 - f_2| \quad \text{14–18}$$

SI unit: $1/s = s^{-1}$

Finally, the rapid oscillations within each beat are due to the second part of y_{total}:

$$\cos\left(2\pi\frac{f_1 + f_2}{2}t\right)$$

Note that these oscillations have a frequency that is the average of the two input frequencies.

These results apply to any type of wave. In particular, if two sound waves produce beats, your ear will hear the average frequency with a loudness that varies with the beat frequency. For example, suppose the two guitar strings mentioned at the beginning of this section have the frequencies 438 Hz and 442 Hz. If you sound them simultaneously, you will hear the average frequency, 440 Hz, increasing

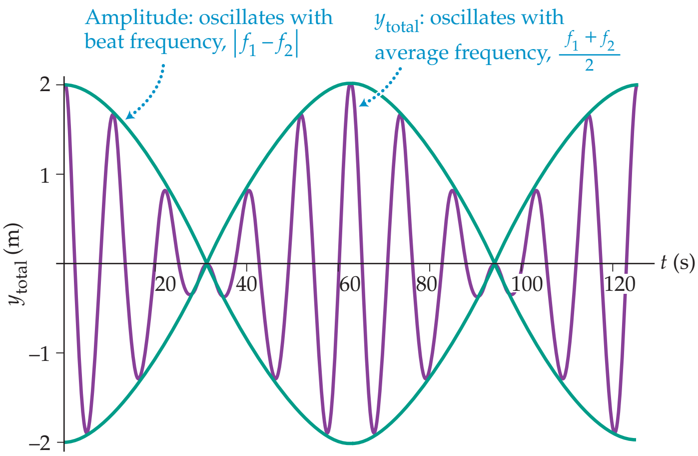


FIGURE 14–31 Beats
Beats can be understood as oscillations at the average frequency, modulated by a slowly varying amplitude.

and decreasing in loudness with a beat frequency of 4 Hz. This means that you will hear maximum loudness 4 times a second. If the frequencies are brought closer together, the beat frequency will be less and fewer maxima will be heard each second.

Clearly, beats can be used to tune a musical instrument to a desired frequency. To tune a guitar string to 440 Hz, for example, the string can be played simultaneously with a 440-Hz tuning fork. Listening to the beats, the tension in the string can be increased or decreased until the beat frequency becomes vanishingly small. This technique applies only to frequencies that are reasonably close to begin with, since the maximum beat frequencies the ear can detect are about 15 to 20 Hz.

PROBLEM-SOLVING NOTE

Calculating the Beat Frequency

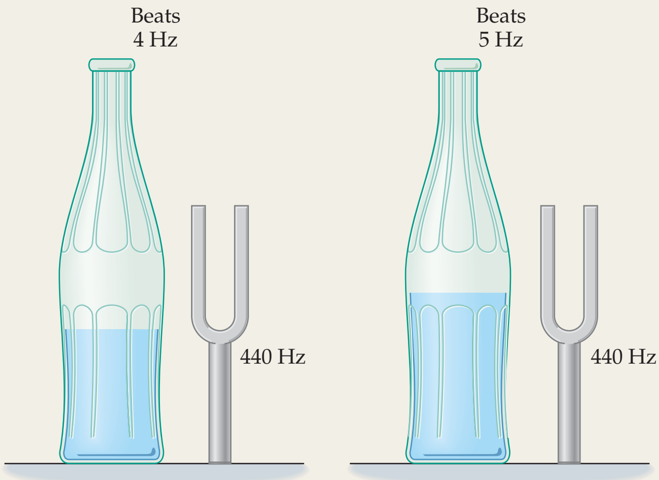
The beat frequency of two waves is the *magnitude* of the difference in their frequencies. Thus, the beat frequency is always positive.

EXAMPLE 14–10 GETTING A TUNE-UP

An experimental way to tune the soda pop bottle in Example 14–9 is to compare its frequency with that of a 440-Hz tuning fork. Initially, a beat frequency of 4 Hz is heard. As a small amount of water is added to that already present, the beat frequency increases steadily to 5 Hz. What were the initial and final frequencies of the bottle?

PICTURE THE PROBLEM

Our sketch shows the before and after situations for this problem. With the low water level the beat frequency is 4 Hz, with the higher level it is 5 Hz.



STRATEGY/SOLUTION

The fact that the initial beat frequency is 4 Hz means the initial frequency of the bottle is either 436 Hz or 444 Hz.

As water is added, we know from Example 14–9 that the bottle's frequency will increase. We also know that the new beat frequency is 5 Hz, and hence the final frequency is either 435 Hz or 445 Hz. Only 445 Hz satisfies the condition that the frequency must have increased.

Therefore, the initial frequency is 444 Hz, and the final frequency is 445 Hz.

INSIGHT

In this case, the initial frequency was too high. To tune the bottle properly, it is necessary to lower the water level.

PRACTICE PROBLEM

Suppose the initial beat frequency was 4 Hz and that adding a small amount of water caused the beat frequency to decrease steadily to 2 Hz. What were the initial and final frequencies in this case? [**Answer:** Initial frequency, 436 Hz; final frequency, 438 Hz]

Some related homework problems: Problem 82, Problem 84

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of frequency, first introduced in terms of oscillations (Chapter 13), is applied here to waves and sound in Sections 14–1 and 14–4, to the Doppler effect in 14–6, to standing waves in 14–8, and to beats in 14–9.

We use power (Chapter 7) in our definition of the intensity of a wave in Section 14–5.

Basic concepts from kinematics (Chapter 2) are used to derive the Doppler effect in Section 14–6.

LOOKING AHEAD

We next encounter waves when we study electricity and magnetism. In fact, we shall see in Chapter 25 that light is an electromagnetic wave, with both the electric and magnetic fields propagating much like a wave on a string.

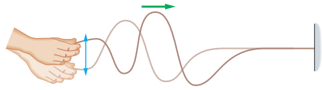
We return to the wave properties of light in Chapter 28, where we see that superposition and interference play a similar role for light as they do for sound.

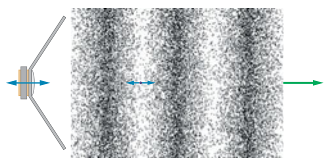
Another type of wave behavior is known as diffraction. This concept is also developed in Chapter 28.

CHAPTER SUMMARY

14–1 TYPES OF WAVES

A wave is a propagating disturbance.





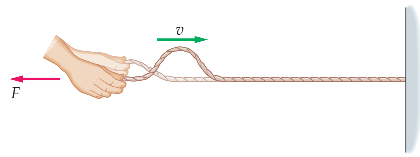
Transverse Waves and Longitudinal Waves
In a transverse wave individual particles move at right angles to the direction of wave propagation. In a longitudinal wave individual particles move in the same direction as the wave propagation.

Wavelength, Frequency, and Speed
The wavelength, λ, frequency, f, and speed, v, of a wave are related by

$$v = \lambda f \qquad 14\text{–}1$$

14–2 WAVES ON A STRING

Transverse waves can propagate on a string held taut with a tension force, F.



Mass per Length
The mass per length of a string is $\mu = m/L$.

Speed of a Wave on a String
The speed of a wave on a string with a tension force F and a mass per length μ is

$$v = \sqrt{\frac{F}{\mu}} \qquad 14\text{–}2$$

Reflections
If the end of a string is fixed, the reflection of a wave is inverted. If the end of a string is free to move transversely, waves are reflected with no inversion.

*14–3 HARMONIC WAVE FUNCTIONS

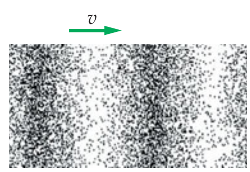
A harmonic wave has the shape of a sine or a cosine.

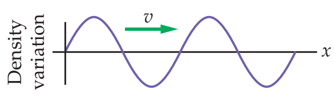
Wave Function
A harmonic wave of wavelength λ and period T is described by the following expression:

$$y(x, t) = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \qquad 14\text{–}4$$

14–4 SOUND WAVES

A sound wave is a longitudinal wave of compressions and rarefactions that can travel through the air, as well as through other gases, liquids, and solids.



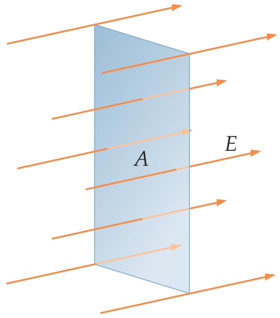


Speed of Sound
The speed of sound in air, under typical conditions, is $v = 343$ m/s.

Frequency of Sound
The frequency of sound determines its pitch. High-pitched sounds have high frequencies; low-pitched sounds have low frequencies.

Human Hearing Range
Human hearing extends from 20 Hz to 20,000 Hz.

14–5 SOUND INTENSITY



The loudness of a sound is determined by its intensity.

Intensity
Intensity, I, is a measure of the amount of energy per time that passes through a given area. Since energy per time is power, P, the intensity of a wave is

$$I = \frac{P}{A} \qquad 14\text{–}5$$

Point Source
If a point source emits sound with a power P, and there are no reflections, the intensity a distance r from the source is

$$I = \frac{P}{4\pi r^2} \qquad 14\text{–}7$$

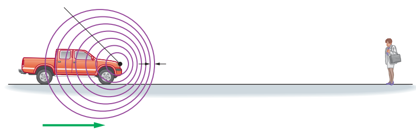
Human Perception of Loudness
The intensity of a sound must be increased by a factor of 10 in order for it to seem twice as loud to our ears.

Intensity Level and Decibels
The intensity level, β, of a sound gives an indication of how loud it sounds to our ears. The intensity level is defined as follows:

$$\beta = 10 \log(I/I_0) \qquad 14\text{–}8$$

The value of β is given in decibels.

14–6 THE DOPPLER EFFECT



The change in frequency due to relative motion between a source and a receiver is called the Doppler effect.

Moving Observer
Suppose an observer is moving with a speed u relative to a stationary source. If the frequency of the source is f, and the speed of the waves is v, the frequency f' detected by the observer is

$$f' = (1 \pm u/v)f \qquad 14\text{–}9$$

The plus sign applies to the observer approaching the source, and the minus sign to the observer receding from the source.

Moving Source
If the source is moving with a speed u and the observer is at rest, the observed frequency is

$$f' = \left(\frac{1}{1 \mp u/v}\right)f \qquad 14\text{–}10$$

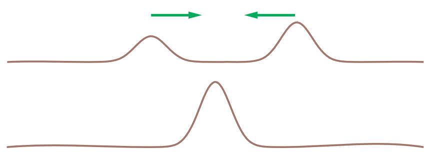
The minus sign applies to the source approaching the observer, and the plus sign to the source receding from the observer.

General Case
If the observer moves with a speed u_o and the source moves with a speed u_s, the Doppler effect gives

$$f' = \left(\frac{1 \pm u_o/v}{1 \mp u_s/v}\right)f \qquad 14\text{–}11$$

The meaning of the plus and minus signs is the same as for the moving-observer and moving-source cases given above.

14–7 SUPERPOSITION AND INTERFERENCE



Waves can combine to give a variety of effects.

Superposition
When two or more waves occupy the same location at the same time they simply add, $y_{total} = y_1 + y_2$.

Constructive Interference
Waves that add to give a larger amplitude exhibit constructive interference.

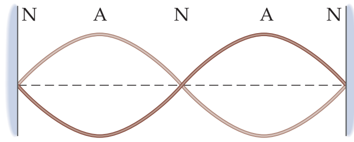
Destructive Interference
Waves that add to give a smaller amplitude exhibit destructive interference.

Interference Patterns
Waves that overlap can create patterns of constructive and destructive interference. These are referred to as interference patterns.

In Phase/Opposite Phase
Two sources are in phase if they both emit crests at the same time.

Sources have opposite phase if one emits a crest at the same time the other emits a trough.

14–8 STANDING WAVES



Standing waves oscillate in a fixed location.

Waves on a String
The fundamental, or first harmonic, corresponds to half a wavelength fitting into the length of the string. The fundamental for waves of speed v on a string of length L is

$$f_1 = \frac{v}{2L} \qquad \lambda_1 = 2L \qquad \text{14–12}$$

The higher harmonics, with $n = 1, 2, 3, \ldots$, are described by

$$f_n = nf_1 = n(v/2L) \qquad \lambda_n = \lambda_1/n = 2L/n \qquad \text{14–13}$$

Vibrating Columns of Air
The harmonics for a column of air closed at one end are

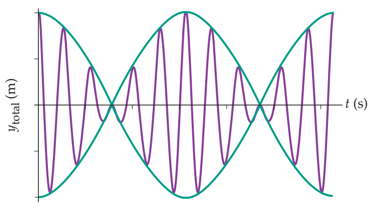
$$f_n = nf_1 = n(v/4L) \quad n = 1, 3, 5, \ldots \qquad \lambda_n = \lambda_1/n = 4L/n \qquad \text{14–14}$$

The harmonics for a column of air open at both ends are

$$f_n = nf_1 = n(v/2L) \quad n = 1, 2, 3, \ldots \qquad \lambda_n = \lambda_1/n = 2L/n \qquad \text{14–15}$$

In both of these expressions the speed of sound is v and the length of the column is L.

14–9 BEATS



Beats occur when waves of slightly different frequencies interfere.

They can be thought of as interference patterns in time. To the ear, beats are perceived as an alternating loudness and softness to the sound.

Beat Frequency
If waves of frequencies f_1 and f_2 interfere, the beat frequency is

$$f_{beat} = |f_1 - f_2| \qquad \text{14–18}$$

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the speed of a wave on a string, or relate the speed of a wave to the mass of a string.	The speed of a wave on a string is related to the tension in the string, F, and the string's mass per length, $\mu = m/L$, by the expression $v = \sqrt{F/\mu}$.	Example 14–1
Relate the intensity of a sound wave to its intensity level.	The intensity level of a sound wave, β, depends on the logarithm of the wave's intensity, I. The relation between β and I is $\beta = 10 \log(I/I_0)$, where $I_0 = 10^{-12}$ W/m^2.	Example 14–4
Calculate the Doppler shift for a moving source or observer.	If an observer and a source of sound with frequency f approach one another, the frequency heard by the observer is greater than f. If the source and observer recede from one another, the frequency heard by the observer is less than f.	Examples 14–5, 14–6
Calculate the beat frequency.	The beat frequency produced when sounds of frequency f_1 and f_2 are heard simultaneously is the magnitude of the difference in frequencies: $f_{beat} = |f_1 - f_2|$.	Example 14–10

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A long nail has been driven halfway into the side of a barn. How should you hit the nail with a hammer to generate a longitudinal wave? How should you hit it to generate a transverse wave?
2. What type of wave is exhibited by "amber waves of grain"?
3. At a ball game, a "wave" circulating through the stands can be an exciting event. What type of wave (longitudinal or transverse) are we talking about? Is it possible to change the type of wave? Explain how people might move their bodies to accomplish this.
4. In a classic TV commercial, a group of cats feed from bowls of cat food that are lined up side by side. Initially there is one cat for each bowl. When an additional cat is added to the scene, it runs to a bowl at the end of the line and begins to eat. The cat that was there originally moves to the next bowl, displacing that cat, which moves to the next bowl, and so on down the line. What type of wave have the cats created? Explain.
5. Describe how the sound of a symphony played by an orchestra would be altered if the speed of sound depended on the frequency of sound.
6. A "radar gun" is often used to measure the speed of a major league pitch by reflecting a beam of radio waves off a moving ball. Describe how the Doppler effect can give the speed of the ball from a measurement of the frequency of the reflected beam.
7. When you drive a nail into a piece of wood, you hear a tone with each blow of the hammer. In fact, the tone increases in pitch as the nail is driven farther into the wood. Explain.
8. Explain the function of the sliding part of a trombone.
9. When you tune a violin string, what causes its frequency to change?
10. On a guitar, some strings are single wires, others are wrapped with another wire to increase the mass per length. Which type of string would you expect to be used for a low-frequency note? Explain.
11. As a string oscillates in its fundamental mode, there are times when it is completely flat. Is the energy of oscillation zero at these times? Explain.
12. On a rainy day, while driving your car, you notice that your windshield wipers are moving in synchrony with the wiper blades of the car in front of you. After several cycles, however your wipers and the wipers of the other car are moving opposite to one another. A short time later the wipers are synchronous again. What wave phenomena do the wipers illustrate? Explain.
13. To play a C major chord on the piano, you hit the C, E, and G keys simultaneously. When you do so, you hear no beats. Why? (Refer to Table 14–3.)

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•,••,•••) are used to indicate the level of difficulty.*

SECTION 14–1 TYPES OF WAVES

1. • A wave travels along a stretched horizontal rope. The vertical distance from crest to trough for this wave is 13 cm and the horizontal distance from crest to trough is 28 cm. What are **(a)** the wavelength and **(b)** the amplitude of this wave?

2. • A surfer floating beyond the breakers notes 14 waves per minute passing her position. If the wavelength of these waves is 34 m, what is their speed?

3. • The speed of surface waves in water decreases as the water becomes shallower. Suppose waves travel across the surface of a lake with a speed of 2.0 m/s and a wavelength of 1.5 m. When these waves move into a shallower part of the lake, their speed decreases to 1.6 m/s, though their frequency remains the same. Find the wavelength of the waves in the shallower water.

4. • **Tsunami** A tsunami traveling across deep water can have a speed of 750 km/h and a wavelength of 310 km. What is the frequency of such a wave?

5. •• **IP** A 4.5-Hz wave with an amplitude of 12 cm and a wavelength of 27 cm travels along a stretched horizontal string. **(a)** How far does a given peak on the wave travel in a time interval of 0.50 s? **(b)** How far does a knot on the string travel in the same time interval? **(c)** How would your answers to parts (a) and (b) change if the amplitude of the wave were halved? Explain.

6. •• **Deepwater Waves** The speed of a deepwater wave with a wavelength λ is given approximately by $v = \sqrt{g\lambda/2\pi}$. Find the speed and frequency of a deepwater wave with a wavelength of 4.5 m.

7. •• **Shallow-Water Waves** In shallow water of depth d the speed of waves is approximately $v = \sqrt{gd}$. Find the speed and frequency of a wave with wavelength 0.75 cm in water that is 2.6 cm deep.

SECTION 14–2 WAVES ON A STRING

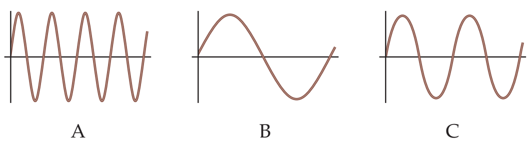
8. • **CE** Consider a wave on a string with constant tension. If the frequency of the wave is doubled, by what multiplicative factor does **(a)** the speed and **(b)** the wavelength change?

9. • **CE** Suppose you would like to double the speed of a wave on a string. By what multiplicative factor must you increase the tension?

10. • **CE Predict/Explain** Two strings are made of the same material and have equal tensions. String 1 is thick; string 2 is thin. **(a)** Is the speed of waves on string 1 greater than, less than, or equal to the speed of waves on string 2? **(b)** Choose the *best explanation* from among the following:
    - **I.** Since the strings are made of the same material, the wave speeds will also be the same.
    - **II.** A thick string implies a large mass per length and a slow wave speed.
    - **III.** A thick string has a greater force constant, and therefore a greater wave speed.

11. • **CE Predict/Explain** Two strings are made of the same material and have waves of equal speed. String 1 is thick; string 2 is thin. **(a)** Is the tension in string 1 greater than, less than, or equal to the tension in string 2? **(b)** Choose the *best explanation* from among the following:
  - **I.** String 1 must have a greater tension to compensate for its greater mass per length.
  - **II.** String 2 will have a greater tension because it is thinner than string 1.
  - **III.** Equal wave speeds implies equal tensions.

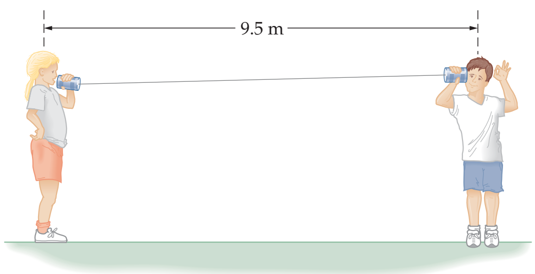
12. • **CE** The three waves, A, B and C, shown in **Figure 14–32** propagate on strings with equal tensions and equal mass per length. Rank the waves in order of increasing **(a)** frequency, **(b)** wavelength, and **(c)** speed. Indicate ties where appropriate.



▲ **FIGURE 14–32** Problem 12

13. • Waves on a particular string travel with a speed of 16 m/s. By what factor should the tension in this string be changed to produce waves with a speed of 32 m/s?

14. •• A brother and sister try to communicate with a string tied between two tin cans (**Figure 14–33**). If the string is 9.5 m long, has a mass of 32 g, and is pulled taut with a tension of 8.6 N, how much time does it take for a wave to travel from one end of the string to the other?



▲ **FIGURE 14–33** Problems 14 and 15

15. •• **IP** **(a)** Suppose the tension is increased in the previous problem. Does a wave take more, less, or the same time to travel from one end to the other? Calculate the time of travel for tensions of **(b)** 9.0 N and **(c)** 10.0 N.

16. •• **IP** A 5.2-m wire with a mass of 87 g is attached to the mast of a sailboat. If the wire is given a "thunk" at one end, it takes 0.094 s for the resulting wave to reach the other end. **(a)** What is the tension in the wire? **(b)** Would the tension found in part (a) be larger or smaller if the mass of the wire is greater than 87 g? **(c)** Calculate the tension for a 97-g wire.

17. •• Two steel guitar strings have the same length. String A has a diameter of 0.50 mm and is under 410.0 N of tension. String B has a diameter of 1.0 mm and is under a tension of 820.0 N. Find the ratio of the wave speeds, v_A/v_B, in these two strings.

18. ••• Use dimensional analysis to show how the speed v of a wave on a string of circular cross section depends on the tension in the string, T, the radius of the string, R, and its mass per volume, ρ.

*SECTION 14–3 HARMONIC WAVE FUNCTIONS

19. • Write an expression for a harmonic wave with an amplitude of 0.16 m, a wavelength of 2.1 m, and a period of 1.8 s. The wave is transverse, travels to the right, and has a displacement of 0.16 m at $t = 0$ and $x = 0$.

20. • Write an expression for a transverse harmonic wave that has a wavelength of 2.6 m and propagates to the right with a speed of 14.3 m/s. The amplitude of the wave is 0.11 m, and its displacement at $t = 0$ and $x = 0$ is 0.11 m.

21. •• **CE** The vertical displacement of a wave on a string is described by the equation $y(x, t) = A\sin(Bx - Ct)$, in which A, B, and C are positive constants. **(a)** Does this wave propagate in the positive or negative x direction? **(b)** What is the wavelength of this wave? **(c)** What is the frequency of this wave? **(d)** What is the smallest positive value of x where the displacement of this wave is zero at $t = 0$?

22. •• **CE** The vertical displacement of a wave on a string is described by the equation $y(x, t) = A\sin(Bx + Ct)$, in which A, B, and C are positive constants. **(a)** Does this wave propagate in the positive or negative x direction? **(b)** What is the physical meaning of the constant A? **(c)** What is the speed of this wave? **(d)** What is the smallest positive time t for which the wave has zero displacement at the point $x = 0$?

23. •• **IP** A wave on a string is described by the following equation:

$$y = (15\text{ cm})\cos\left(\frac{\pi}{5.0\text{ cm}}x - \frac{\pi}{12\text{ s}}t\right)$$

(a) What is the amplitude of this wave? **(b)** What is its wavelength? **(c)** What is its period? **(d)** What is its speed? **(e)** In which direction does the wave travel?

24. •• Consider the wave function given in the previous problem. Sketch this wave from $x = 0$ to $x = 10$ cm for the following times: **(a)** $t = 0$; **(b)** $t = 3.0$ s; **(c)** $t = 6.0$ s. **(d)** What is the least amount of time required for a given point on this wave to move from $y = 0$ to $y = 15$ cm? Verify your answer by referring to the sketches for parts (a), (b), and (c).

25. •• **IP** Four waves are described by the following equations, in which all distances are measured in centimeters and all times are measured in seconds:

$$y_A = 10\cos(3x - 4t)$$
$$y_B = 10\cos(5x + 4t)$$
$$y_C = 20\cos(-10x + 60t)$$
$$y_D = 20\cos(-4x - 20t)$$

(a) Which of these waves travel in the $+x$ direction? **(b)** Which of these waves travel in the $-x$ direction? **(c)** Which wave has the highest frequency? **(d)** Which wave has the greatest wavelength? **(e)** Which wave has the greatest speed?

SECTION 14–4 SOUND WAVES

26. • At Zion National Park a loud shout produces an echo 1.80 s later from a colorful sandstone cliff. How far away is the cliff?

27. • **BIO Dolphin Ultrasound** Dolphins of the open ocean are classified as Type II Odontocetes (toothed whales). These animals use ultrasonic "clicks" with a frequency of about 55 kHz to navigate and find prey. **(a)** Suppose a dolphin sends out a series of clicks that are reflected back from the bottom of the ocean 75 m below. How much time elapses before the dolphin hears the echoes of the clicks? (The speed of sound in seawater is approximately 1530 m/s.) **(b)** What is the wavelength of 55-kHz sound in the ocean?

28. • The lowest note on a piano is A, four octaves below the A given in Table 14–3. The highest note on a piano is C, four octaves above middle C. Find the frequencies and wavelengths (in air) of these notes.

29. •• **IP** A sound wave in air has a frequency of 425 Hz. **(a)** What is its wavelength? **(b)** If the frequency of the sound is increased, does its wavelength increase, decrease, or stay the same? Explain. **(c)** Calculate the wavelength for a sound wave with a frequency of 475 Hz.

30. •• **IP** When you drop a rock into a well, you hear the splash 1.5 seconds later. **(a)** How deep is the well? **(b)** If the depth of the well were doubled, would the time required to hear the splash be greater than, less than, or equal to 3.0 seconds? Explain.

31. •• A rock is thrown downward into a well that is 8.85 m deep. If the splash is heard 1.20 seconds later, what was the initial speed of the rock?

SECTION 14–5 SOUND INTENSITY

32. • **CE** If the distance to a point source of sound is doubled, by what multiplicative factor does the intensity change?

33. • The intensity level of sound in a truck is 92 dB. What is the intensity of this sound?

34. • The distance to a point source is decreased by a factor of three. **(a)** By what multiplicative factor does the intensity increase? **(b)** By what additive amount does the intensity level increase?

35. • Sound 1 has an intensity of 38.0 W/m^2. Sound 2 has an intensity level that is 2.5 dB greater than the intensity level of sound 1. What is the intensity of sound 2?

36. •• A bird-watcher is hoping to add the white-throated sparrow to her "life list" of species. How far could she be from the bird described in Example 14–3 and still be able to hear it? Assume no reflections or absorption of the sparrow's sound.

37. •• Residents of Hawaii are warned of the approach of a tsunami by sirens mounted on the tops of towers. Suppose a siren produces a sound that has an intensity level of 120 dB at a distance of 2.0 m. Treating the siren as a point source of sound, and ignoring reflections and absorption, find the intensity level heard by an observer at a distance of **(a)** 12 m and **(b)** 21 m from the siren. **(c)** How far away can the siren be heard?

38. •• In a pig-calling contest, a caller produces a sound with an intensity level of 110 dB. How many such callers would be required to reach the pain level of 120 dB?

39. •• **IP** Twenty violins playing simultaneously with the same intensity combine to give an intensity level of 82.5 dB. **(a)** What is the intensity level of each violin? **(b)** If the number of violins is increased to 40, will the combined intensity level be more than, less than, or equal to 165 dB? Explain.

40. •• **BIO The Human Eardrum** The radius of a typical human eardrum is about 4.0 mm. Find the energy per second received by an eardrum when it listens to sound that is **(a)** at the threshold of hearing and **(b)** at the threshold of pain.

41. ••• A point source of sound that emits uniformly in all directions is located in the middle of a large, open field. The intensity at Brittany's location directly north of the source is twice that at Phillip's position due east of the source. What is the distance between Brittany and Phillip if Brittany is 12.5 m from the source?

SECTION 14–6 THE DOPPLER EFFECT

42. • **CE Predict/Explain** A horn produces sound with frequency f_0. Let the frequency you hear when you are at rest and the horn moves toward you with a speed u be f_1; let the frequency you hear when the horn is at rest and you move toward it with a speed u be f_2. **(a)** Is f_1 greater than, less than, or equal to f_2? **(b)** Choose the *best explanation* from among the following:
   I. A moving observer encounters wave crests more often than a stationary observer, leading to a higher frequency.
   II. The relative speeds are the same in either case. Therefore, the frequencies will be the same as well.
   III. A moving source causes the wave crests to "bunch up," leading to a higher frequency than for a moving observer.

43. • **CE** You are heading toward an island in your speedboat when you see a friend standing on shore at the base of a cliff. You sound the boat's horn to get your friend's attention. Let the wavelength of the sound produced by the horn be λ_1, the wavelength as heard by your friend be λ_2, and the wavelength of the echo as heard on the boat be λ_3. Rank these wavelengths in order of increasing length. Indicate ties where appropriate.

44. • A person with perfect pitch sits on a bus bench listening to the 450-Hz horn of an approaching car. If the person detects a frequency of 470 Hz, how fast is the car moving?

45. • A train moving with a speed of 31.8 m/s sounds a 136-Hz horn. What frequency is heard by an observer standing near the tracks as the train approaches?

46. • In the previous problem, suppose the stationary observer sounds a horn that is identical to the one on the train. What frequency is heard from this horn by a passenger in the train?

47. • **BIO** A bat moving with a speed of 3.25 m/s and emitting sound of 35.0 kHz approaches a moth at rest on a tree trunk. **(a)** What frequency is heard by the moth? **(b)** If the speed of the bat is increased, is the frequency heard by the moth higher or lower? **(c)** Calculate the frequency heard by the moth when the speed of the bat is 4.25 m/s.

48. • A motorcycle and a police car are moving toward one another. The police car emits sound with a frequency of 502 Hz and has a speed of 27.0 m/s. The motorcycle has a speed of 13.0 m/s. What frequency does the motorcyclist hear?

49. • In the previous problem, suppose that the motorcycle and the police car are moving in the same direction, with the motorcycle in the lead. What frequency does the motorcyclist hear in this case?

50. •• Hearing the siren of an approaching fire truck, you pull over to the side of the road and stop. As the truck approaches, you hear a tone of 460 Hz; as the truck recedes, you hear a tone of 410 Hz. How much time will it take for the truck to get from your position to the fire 5.0 km away, assuming it maintains a constant speed?

51. •• With what speed must you approach a source of sound to observe a 15% change in frequency?

52. •• **IP** A particular jet engine produces a tone of 495 Hz. Suppose that one jet is at rest on the tarmac while a second identical jet flies overhead at 82.5% of the speed of sound. The pilot of each jet listens to the sound produced by the engine of the other jet. **(a)** Which pilot hears a greater Doppler shift? Explain. **(b)** Calculate the frequency heard by the pilot in the moving jet. **(c)** Calculate the frequency heard by the pilot in the stationary jet.

53\. •• **IP** Two bicycles approach one another, each traveling with a speed of 8.50 m/s. **(a)** If bicyclist A beeps a 315-Hz horn, what frequency is heard by bicyclist B? **(b)** Which of the following would cause the greater increase in the frequency heard by bicyclist B: (i) bicyclist A speeds up by 1.50 m/s, or (ii) bicyclist B speeds up by 1.50 m/s? Explain.

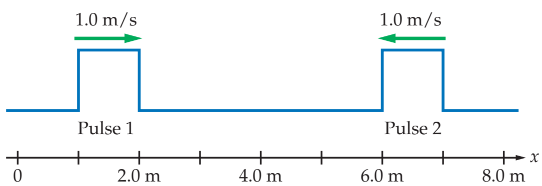
54\. •• A train on one track moves in the same direction as a second train on the adjacent track. The first train, which is ahead of the second train and moves with a speed of 36.8 m/s, blows a horn whose frequency is 124 Hz. If the frequency heard on the second train is 135 Hz, what is its speed?

55\. •• Two cars traveling with the same speed move directly away from one another. One car sounds a horn whose frequency is 205 Hz and a person in the other car hears a frequency of 192 Hz. What is the speed of the cars?

56\. ••• **The Bullet Train** The Shinkansen, the Japanese "bullet" train, runs at high speed from Tokyo to Nagoya. Riding on the Shinkansen, you notice that the frequency of a crossing signal changes markedly as you pass the crossing. As you approach the crossing, the frequency you hear is f; as you recede from the crossing the frequency you hear is $2f/3$. What is the speed of the train?

SECTION 14–7 SUPERPOSITION AND INTERFERENCE

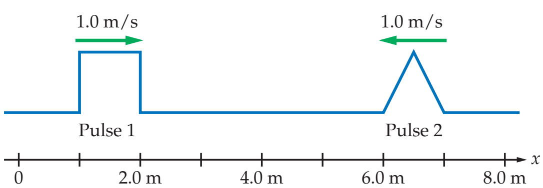
57\. • Two wave pulses on a string approach one another at the time $t = 0$, as shown in **Figure 14–34**. Each pulse moves with a speed of 1.0 m/s. Make a careful sketch of the resultant wave at the times $t = 1.0$ s, 2.0 s, 2.5 s, 3.0 s, and 4.0 s, assuming that the superposition principle holds for these waves.



▲ **FIGURE 14–34** Problems 57 and 58

58\. • Suppose pulse 2 in Problem 57 is inverted, so that it is a downward deflection of the string rather than an upward deflection. Repeat Problem 57 in this case.

59\. • Two wave pulses on a string approach one another at the time $t = 0$, as shown in **Figure 14–35**. Each pulse moves with a speed of 1.0 m/s. Make a careful sketch of the resultant wave at the times $t = 1.0$ s, 2.0 s, 2.5 s, 3.0 s, and 4.0 s, assuming that the superposition principle holds for these waves.



▲ **FIGURE 14–35** Problems 59 and 60

60\. • Suppose pulse 2 in Problem 59 is inverted, so that it is a downward deflection of the string rather than an upward deflection. Repeat Problem 59 in this case.

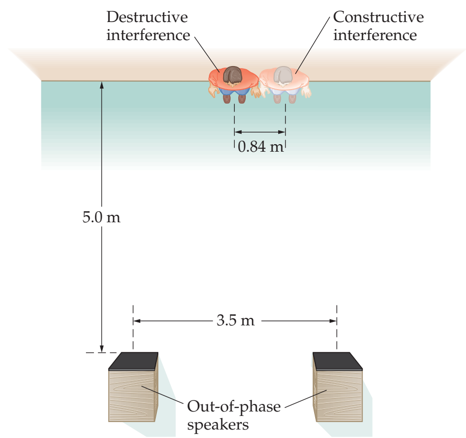
61\. •• A pair of in-phase stereo speakers is placed side by side, 0.85 m apart. You stand directly in front of one of the speakers, 1.1 m from the speaker. What is the lowest frequency that will produce constructive interference at your location?

62\. •• **IP** Two violinists, one directly behind the other, play for a listener directly in front of them. Both violinists sound concert A (440 Hz). **(a)** What is the smallest separation between the violinists that will produce destructive interference for the listener? **(b)** Does this smallest separation increase or decrease if the violinists produce a note with a higher frequency? **(c)** Repeat part (a) for violinists who produce sounds of 540 Hz.

63\. •• Two loudspeakers are placed at either end of a gymnasium, both pointing toward the center of the gym and equidistant from it. The speakers emit 266-Hz sound that is in phase. An observer at the center of the gym experiences constructive interference. How far toward either speaker must the observer walk to first experience destructive interference?

64\. •• **IP** **(a)** In the previous problem, does the required distance increase, decrease, or stay the same if the frequency of the speakers is lowered? **(b)** Calculate the distance to the first position of destructive interference if the frequency emitted by the speakers is lowered to 238 Hz.

65\. •• Two speakers with opposite phase are positioned 3.5 m apart, both pointing toward a wall 5.0 m in front of them (**Figure 14–36**). An observer standing against the wall midway between the speakers hears destructive interference. If the observer hears constructive interference after moving 0.84 m to one side along the wall, what is the frequency of the sound emitted by the speakers?



▲ **FIGURE 14–36** Problem 65

66\. •• Suppose, in Example 14–7, that the speakers have opposite phase. What is the lowest frequency that gives destructive interference in this case?

SECTION 14–8 STANDING WAVES

67\. • **CE Predict/Explain** When you blow across the opening of a partially filled 2-L soda pop bottle you hear a tone. **(a)** If you take a sip of the pop and blow across the opening again, does the tone you hear have a higher frequency, a lower frequency, or the same frequency as before? **(b)** Choose the *best explanation* from among the following:

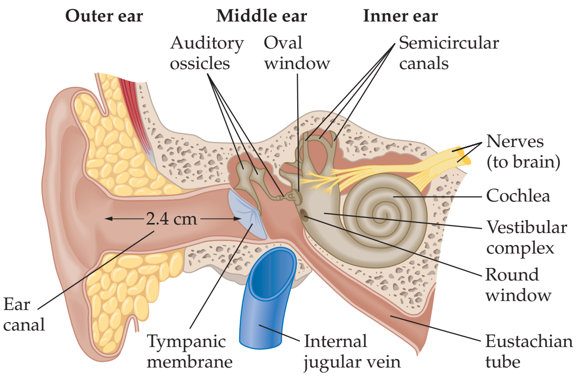
- **I.** The same pop bottle will give the same frequency regardless of the amount of pop it contains.
- **II.** The greater distance from the top of the bottle to the level of the pop results in a higher frequency.
- **III.** A lower level of pop results in a longer column of air, and hence a lower frequency.

68. • An organ pipe that is open at both ends is 3.5 m long. What is its fundamental frequency?

69. • A string 1.5 m long with a mass of 2.6 g is stretched between two fixed points with a tension of 93 N. Find the frequency of the fundamental on this string.

70. •• **CE** A string is tied down at both ends. Some of the standing waves on this string have the following frequencies: 100 Hz, 200 Hz, 250 Hz, and 300 Hz. It is also known that there are no standing waves with frequencies between 250 Hz and 300 Hz. **(a)** What is the fundamental frequency of this string? **(b)** What is the frequency of the third harmonic?

71. •• **IP BIO Standing Waves in the Human Ear** The human ear canal is much like an organ pipe that is closed at one end (at the tympanic membrane or eardrum) and open at the other (**Figure 14–37**). A typical ear canal has a length of about 2.4 cm. **(a)** What are the fundamental frequency and wavelength of the ear canal? **(b)** Find the frequency and wavelength of the ear canal's third harmonic. (Recall that the third harmonic in this case is the standing wave with the second-lowest frequency.) **(c)** Suppose a person has an ear canal that is shorter than 2.4 cm. Is the fundamental frequency of that person's ear canal greater than, less than, or the same as the value found in part (a)? Explain. [Note that the frequencies found in parts (a) and (b) correspond closely to the frequencies of enhanced sensitivity in Figure 14–28.]



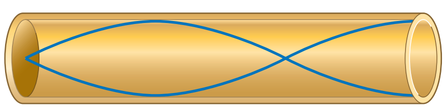
▲ **FIGURE 14–37** Problem 71

72. •• A guitar string 66 cm long vibrates with a standing wave that has three antinodes. **(a)** Which harmonic is this? **(b)** What is the wavelength of this wave?

73. •• **IP** A 12.5-g clothesline is stretched with a tension of 22.1 N between two poles 7.66 m apart. What is the frequency of **(a)** the fundamental and **(b)** the second harmonic? **(c)** If the tension in the clothesline is increased, do the frequencies in parts (a) and (b) increase, decrease, or stay the same? Explain.

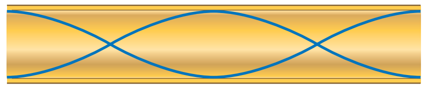
74. •• **IP (a)** In the previous problem, will the frequencies increase, decrease, or stay the same if a more massive rope is used? **(b)** Repeat Problem 73 for a clothesline with a mass of 15.0 g.

75. •• The organ pipe in **Figure 14–38** is 2.75 m long. **(a)** What is the frequency of the standing wave shown in the pipe? **(b)** What is the fundamental frequency of this pipe?



▲ **FIGURE 14–38** Problem 75

76. •• The frequency of the standing wave shown in **Figure 14–39** is 202 Hz. **(a)** What is the fundamental frequency of this pipe? **(b)** What is the length of the pipe?



▲ **FIGURE 14–39** Problem 76

77. ••• An organ pipe open at both ends has a harmonic with a frequency of 440 Hz. The next higher harmonic in the pipe has a frequency of 495 Hz. Find **(a)** the frequency of the fundamental and **(b)** the length of the pipe.

SECTION 14–9 BEATS

78. • **CE** When guitar strings A and B are plucked at the same time, a beat frequency of 2 Hz is heard. If string A is tightened, the beat frequency increases to 3 Hz. Which of the two strings had the lower frequency initially?

79. • **CE Predict/Explain (a)** Is the beat frequency produced when a 245-Hz tone and a 240-Hz tone are played together greater than, less than, or equal to the beat frequency produced when a 140-Hz tone and a 145-Hz tone are played together? **(b)** Choose the *best explanation* from among the following:

I. The beat frequency is determined by the difference in frequencies and is independent of their actual values.
II. The higher frequencies will produce a higher beat frequency.
III. The percentage change in frequency for 240 and 245 Hz is less than for 140 and 145 Hz, resulting in a lower beat frequency.

80. • Two tuning forks have frequencies of 278 Hz and 292 Hz. What is the beat frequency if both tuning forks are sounded simultaneously?

81. • **Tuning a Piano** To tune middle C on a piano, a tuner hits the key and at the same time sounds a 261-Hz tuning fork. If the tuner hears 3 beats per second, what are the possible frequencies of the piano key?

82. • Two musicians are comparing their clarinets. The first clarinet produces a tone that is known to be 441 Hz. When the two clarinets play together they produce eight beats every 2.00 seconds. If the second clarinet produces a higher pitched tone than the first clarinet, what is the second clarinet's frequency?

83. •• **IP** Two strings that are fixed at each end are identical, except that one is 0.560 cm longer than the other. Waves on these strings propagate with a speed of 34.2 m/s, and the fundamental frequency of the shorter string is 212 Hz. **(a)** What beat frequency is produced if each string is vibrating with its fundamental frequency? **(b)** Does the beat frequency in part (a) increase or decrease if the longer string is increased in length? **(c)** Repeat part (a), assuming that the longer string is 0.761 cm longer than the shorter string.

84. •• **IP** A tuning fork with a frequency of 320.0 Hz and a tuning fork of unknown frequency produce beats with a frequency of 4.5 Hz. If the frequency of the 320.0-Hz fork is lowered slightly by placing a bit of putty on one of its tines, the new beat frequency is 7.5 Hz. **(a)** Which tuning fork has the lower frequency? Explain. **(b)** What is the final frequency of the 320.0-Hz tuning fork? **(c)** What is the frequency of the other tuning fork?

85. •• Identical cellos are being tested. One is producing a fundamental frequency of 130.9 Hz on a string that is 1.25 m long and has a mass of 109 g. On the second cello the same string is fingered to reduce the length that can vibrate. If the beat frequency produced by these two strings is 4.33 Hz, what is the vibrating length of the second string?

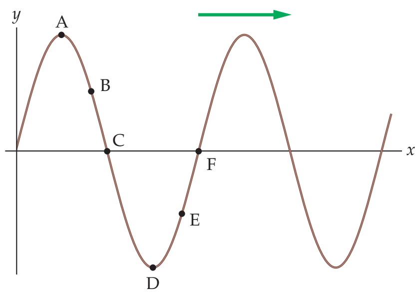
86. ••• A friend in another city tells you that she has two organ pipes of different lengths, one open at both ends, the other open at one end only. In addition, she has determined that the beat frequency caused by the second-lowest frequency of each pipe is equal to the beat frequency caused by the third-lowest frequency of each pipe. Her challenge to you is to calculate the length of the organ pipe that is open at both ends, given that the length of the other pipe is 1.00 m.

GENERAL PROBLEMS

87. • **CE** A harmonic wave travels along a string. **(a)** At a point where the displacement of the string is greatest, is the kinetic energy of the string a maximum or a minimum? Explain. **(b)** At a point where the displacement of the string is zero, is the kinetic energy of the string a maximum or a minimum? Explain.

88. • **CE** A harmonic wave travels along a string. **(a)** At a point where the displacement of the string is greatest, is the potential energy of the string a maximum or a minimum? Explain. **(b)** At a point where the displacement of the string is zero, is the potential energy of the string a maximum or a minimum? Explain.

89. • **CE** **Figure 14–40** shows a wave on a string moving to the right. For each of the points indicated on the string, A–F, state whether it is (**I**, moving upward; **II**, moving downward; or **III**, instantaneously at rest) at the instant pictured.



▲ **FIGURE 14–40** Problem 89

90. • **CE** You stand near the tracks as a train approaches with constant speed. The train is operating its horn continuously, and you listen carefully to the sound it makes. For each of the following properties of the sound, state whether it increases, decreases, or stays the same as the train gets closer: **(a)** the intensity; **(b)** the frequency; **(c)** the wavelength; **(d)** the speed.

91. • Sitting peacefully in your living room one stormy day, you see a flash of lightning through the windows. Eight and a half seconds later thunder shakes the house. Estimate the distance from your house to the bolt of lightning.

92. • The fundamental of an organ pipe that is closed at one end and open at the other end is 261.6 Hz (middle C). The second harmonic of an organ pipe that is open at both ends has the same frequency. What are the lengths of these two pipes?

93. • **The Loudest Animal** The loudest sound produced by a living organism on Earth is made by the bowhead whale (*Balaena mysticetus*). These whales can produce a sound that, if heard in air at a distance of 3.00 m, would have an intensity level of 127 dB. This is roughly the equivalent of 5000 trumpeting elephants. How far away can you be from a 127-dB sound and still just barely hear it?(Assume a point source, and ignore reflections and absorption.)

94. • **Hearing a Good Hit** Physicist Robert Adair, once appointed the "official physicist to the National League" by the commissioner of baseball, believes that the "crack of the bat" can tell an outfielder how well the ball has been hit. According to Adair, a good hit makes a sound of 510 Hz, while a poor hit produces a sound of 170 Hz. What is the difference in wavelength of these sounds?

95. • A standing wave of 603 Hz is produced on a string that is 1.33 m long and fixed on both ends. If the speed of waves on this string is 402 m/s, how many antinodes are there in the standing wave?

96. • **BIO** **Measuring Hearing Loss** To determine the amount of temporary hearing loss loud music can cause in humans, researchers studied a group of 20 adult females who were exposed to 110-dB music for 60 minutes. Eleven of the 20 subjects showed a 20.0-dB reduction in hearing sensitivity at 4000 Hz. What is the intensity corresponding to the threshold of hearing for these subjects?

97. •• **BIO** **Hearing a Pin Drop** The ability to hear a "pin drop" is the sign of sensitive hearing. Suppose a 0.55-g pin is dropped from a height of 28 cm, and that the pin emits sound for 1.5 s when it lands. Assuming all of the mechanical energy of the pin is converted to sound energy, and that the sound radiates uniformly in all directions, find the maximum distance from which a person can hear the pin drop. (This is the ideal maximum distance, but atmospheric absorption and other factors will make the actual maximum distance considerably smaller.)

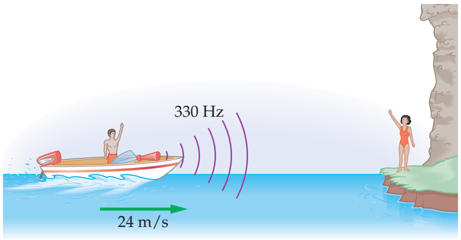
98. •• A machine shop has 120 equally noisy machines that together produce an intensity level of 92 dB. If the intensity level must be reduced to 82 dB, how many machines must be turned off?

99. •• **IP** When you blow across the top of a soda pop bottle you hear a fundamental frequency of 206 Hz. Suppose the bottle is now filled with helium. **(a)** Does the fundamental frequency increase, decrease, or stay the same? Explain. **(b)** Find the new fundamental frequency. (Assume that the speed of sound in helium is three times that in air.)

100. •• **Speed of a Tsunami** Tsunamis can have wavelengths between 100 and 400 km. Since this is much greater than the average depth of the oceans (about 4.3 km), the ocean can be considered as shallow water for these waves. Using the speed of waves in shallow water of depth d given in Problem 7, find the typical speed for a tsunami. (*Note*: In the open ocean, tsunamis generally have an amplitude of less than a meter, allowing them to pass ships unnoticed. As they approach shore, however, the water depth decreases and the waves slow down. This can result in an increase of amplitude to as much as 37 m or more.)

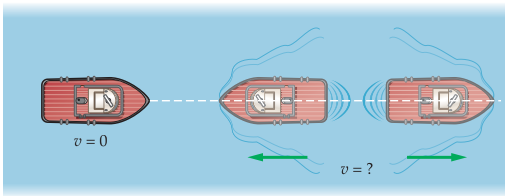
101. •• Two trains with 124-Hz horns approach one another. The slower of the two trains has a speed of 26 m/s. What is the speed of the fast train if an observer standing near the tracks between the trains hears a beat frequency of 4.4 Hz?

102. •• **IP** Jim is speeding toward James Island with a speed of 24 m/s when he sees Betsy standing on shore at the base of a cliff (**Figure 14–41**). Jim sounds his 330-Hz horn. **(a)** What frequency does Betsy hear? **(b)** Jim can hear the echo of his horn reflected back to him by the cliff. Is the frequency of this echo greater than or equal to the frequency heard by Betsy? Explain. **(c)** Calculate the frequency Jim hears in the echo from the cliff.



▲ **FIGURE 14–41** Problem 102

103. •• Two ships in a heavy fog are blowing their horns, both of which produce sound with a frequency of 175.0 Hz (**Figure 14–42**). One ship is at rest; the other moves on a straight line that passes through the one at rest. If people on the stationary ship hear a beat frequency of 3.5 Hz, what are the two possible speeds and directions of motion of the moving ship?



▲ **FIGURE 14–42** Problem 103

104. •• **BIO Cracking Your Knuckles** When you "crack" a knuckle, you cause the knuckle cavity to widen rapidly. This, in turn, allows the synovial fluid to expand into a larger volume. If this expansion is sufficiently rapid, it causes a gas bubble to form in the fluid in a process known as *cavitation*. This is the mechanism responsible for the cracking sound. (Cavitation can also cause pits in rapidly rotating ship's propellers.) If a "crack" produces a sound with an intensity level of 57 dB at your ear, which is 18 cm from the knuckle, how far from your knuckle can the "crack" be heard? Assume the sound propagates uniformly in all directions, with no reflections or absorption.

105. •• A steel guitar string has a tension T, length L, and diameter D. Give the multiplicative factor by which the fundamental frequency of the string changes under the following conditions: **(a)** The tension in the string is increased by a factor of 4. The diameter is D and the length is L. **(b)** The diameter of the string is increased by a factor of 3. The tension is T and the length is L. **(c)** The length of the string is halved. The tension is T and the diameter is D.

106. •• A Slinky has a mass of 0.28 kg and negligible length. When it is stretched 1.5 m, it is found that transverse waves travel the length of the Slinky in 0.75 s. **(a)** What is the force constant, k, of the Slinky? **(b)** If the Slinky is stretched farther, will the time required for a wave to travel the length of the Slinky increase, decrease, or stay the same? Explain. **(c)** If the Slinky is stretched 3.0 m, how much time does it take a wave to travel the length of the Slinky? (The Slinky stretches like an ideal spring, and propagates transverse waves like a rope with variable tension.)

107. •• **IP BIO OSHA Noise Standards** OSHA, the Occupational Safety and Health Administration, has established standards for workplace exposure to noise. According to OSHA's Hearing Conservation Standard, the permissible noise exposure per day is 95.0 dB for 4 hours or 105 dB for 1 hour. Assuming the eardrum is 9.5 mm in diameter, find the energy absorbed by the eardrum **(a)** with 95.0 dB for 4 hours and **(b)** with 105 dB for 1 hour. **(c)** Is OSHA's safety standard simply a measure of the amount of energy absorbed by the ear? Explain.

108. •• **IP Thundersticks at Ball Games** "Thundersticks" are a popular noisemaking device at many sporting events. A typical thunderstick is a hollow plastic tube about 82 cm long and 8.5 cm in diameter. When two thundersticks are hit sharply together, they produce a copious amount of noise. **(a)** Which dimension, the length or diameter, is more important in determining the frequency of the sound emitted by the thundersticks? Explain. **(b)** Estimate the characteristic frequency of the thunderstick's sound. **(c)** Suppose a single pair of thundersticks produces sound with an intensity level of 95 dB. What is the intensity level of 1200 pairs of thundersticks clapping simultaneously?

109. •• An organ pipe 2.5 m long is open at one end and closed at the other end. What is the linear distance between a node and the adjacent antinode for the third harmonic in this pipe?

110. •• Two identical strings with the same tension vibrate at 631 Hz. If the tension in one of the strings is increased by 2.25%, what is the resulting beat frequency?

111. •• **The Sound of a Black Hole** Astronomers using the Chandra X-ray Observatory have discovered that the Perseus Black Hole, some 250 million light years away, produces sound waves in the gaseous halo that surrounds it. The frequency of this sound is the same as the frequency of the 59th B-flat below the B-flat given in Table 14–3. How long does it take for this sound wave to complete one cycle? Give your answer in years.

112. •• **BIO The Love Song of the Midshipman Fish** When the plainfin midshipman fish (*Porichthys notatus*) migrates from deep Pacific waters to the west coast of North America each summer, the males begin to sing their "love song," which some describe as sounding like a low-pitched motorboat. Houseboat residents and shore dwellers are kept awake for nights on end by the amorous fish. The love song consists of a single note, the second A flat below middle C. **(a)** If the speed of sound in seawater is 1531 m/s, what is the wavelength of the midshipman's song? **(b)** What is the wavelength of the sound after it emerges into the air? (Information on the musical scale is given in Table 14–3.)

113. ••• **IP** A rope of length L and mass M hangs vertically from a ceiling. The tension in the rope is only that due to its own weight. **(a)** Suppose a wave starts near the bottom of the rope and propagates upward. Does the speed of the wave increase, decrease, or stay the same as it moves up the rope? Explain. **(b)** Show that the speed of waves a height y above the bottom of the rope is $v = \sqrt{gy}$.

114. ••• Experiments on water waves show that the speed of waves in shallow water is independent of their wavelength (see Problem 7). Using this observation and dimensional analysis, determine how the speed v of shallow-water waves depends on the depth of the water, d, the mass per volume of water, ρ, and the acceleration of gravity, g.

115. ••• A deepwater wave of wavelength λ has a speed given approximately by $v = \sqrt{g\lambda/2\pi}$. Find an expression for the period of a deepwater wave in terms of its wavelength. (Note the similarity of your result to the period of a pendulum.)

116. ••• **Beats and Standing Waves** In Problem 63, suppose the observer walks toward one speaker with a speed of 1.35 m/s. **(a)** What frequency does the observer hear from each speaker? **(b)** What beat frequency does the observer hear? **(c)** How far must the observer walk to go from one point of constructive interference to the next? **(d)** How many times per second does the observer hear maximum loudness from the speakers? Compare your result with the beat frequency from part (b).

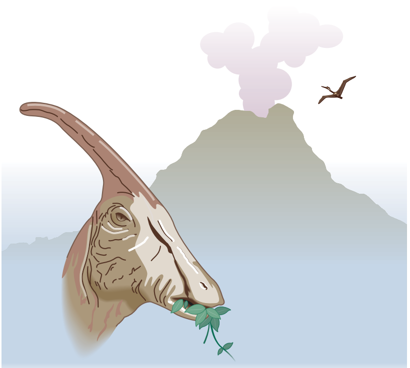
PASSAGE PROBLEMS

BIO The Sound of a Dinosaur

Modern-day animals make extensive use of sounds in their interactions with others. Some sounds are meant primarily for members of the same species, like the cooing calls of a pair of doves, the long-range infrasound communication between elephants, or the songs of the hump-backed whale. Other sounds may be used as a threat to other species, such as the rattle of a rattlesnake or the roar of a lion.

There is little doubt that extinct animals used sounds in much the same ways. But how can we ever hear the call of a long-vanished animal like a dinosaur when sounds don't fossilize? In some cases, basic physics may have the answer.

Consider, for example, the long-crested, duck-billed dinosaur *Parasaurolophus walkeri*, which roamed the Earth 75 million years ago. This dinosaur possessed the largest crest of any duck bill—so long, in fact, that there was a notch in *P. walkeri*'s spine to make room for the crest when its head was tilted backward. Many paleontologists believe the air passages in the dinosaur's crest acted like bent organ pipes open at both ends, and that they produced sounds *P. walkeri* used to communicate with others of its kind. As air was forced through the passages, the predominant sound they produced would be the fundamental standing wave, with a small admixture of higher harmonics as well. The frequencies of these standing waves can be determined with basic physical principles. **Figure 14–43** presents a plot of the lowest ten harmonics of a pipe that is open at both ends as a function of the length of the pipe.



The long crest of *Parasaurolophus walkeri* played a key role in its communications with others.

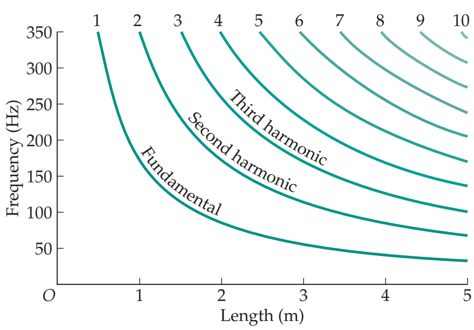


FIGURE 14–43 Standing wave frequencies as a function of length for a pipe open at both ends. The first ten harmonics ($n = 1$–10) are shown. (Problems 117, 118, 119, and 120)

117. • Suppose the air passages in a certain *P. walkeri* crest produce a bent tube 2.7 m long. What is the fundamental frequency of this tube, assuming the bend has no effect on the frequency? (For comparison, a typical human hearing range is 20 Hz to 20 kHz.)

A. 0.0039 Hz **B.** 32 Hz

C. 64 Hz **D.** 130 Hz

118. • Paleontologists believe the crest of a female *P. walkeri* was probably shorter than the crest of a male. If this was the case, would the fundamental frequency of a female be greater than, less than, or equal to the fundamental frequency of a male?

119. • Suppose the fundamental frequency of a particular female was 74 Hz. What was the length of the air passages in this female's crest?

A. 1.2 m **B.** 2.3 m

C. 2.7 m **D.** 4.6 m

120. • As a young *P. walkeri* matured, the air passages in its crest might increase in length from 1.5 m to 2.7 m, causing a decrease in the standing wave frequencies. Referring to Figure 14–43, do you expect the change in the fundamental frequency to be greater than, less than, or equal to the change in the second harmonic frequency?

INTERACTIVE PROBLEMS

121. •• **IP Referring to Example 14–6** Suppose the engineer adjusts the speed of the train until the sound he hears reflected from the cliff is 775 Hz. The train's whistle still produces a tone of 650.0 Hz. **(a)** Is the new speed of the train greater than, less than, or equal to 21.2 m/s? Explain. **(b)** Find the new speed of the train.

122. •• **Referring to Example 14–6** Suppose the train is backing away from the cliff with a speed of 18.5 m/s and is sounding its 650.0-Hz whistle. **(a)** What is the frequency heard by the observer standing near the tunnel entrance? **(b)** What is the frequency heard by the engineer?

123. •• **IP Referring to Example 14–9** Suppose we add more water to the soda pop bottle. **(a)** Does the fundamental frequency increase, decrease, or stay the same? Explain. **(b)** Find the fundamental frequency if the height of water in the bottle is increased to 7.5 cm. The height of the bottle is still 26.0 cm.

124. •• **IP Referring to Example 14–9** The speed of sound increases slightly with temperature. **(a)** Does the fundamental frequency of the bottle increase, decrease, or stay the same as the air heats up on a warm day? Explain. **(b)** Find the fundamental frequency if the speed of sound in air increases to 348 m/s. Assume the bottle is 26.0 cm tall, and that it contains water to a depth of 6.5 cm.