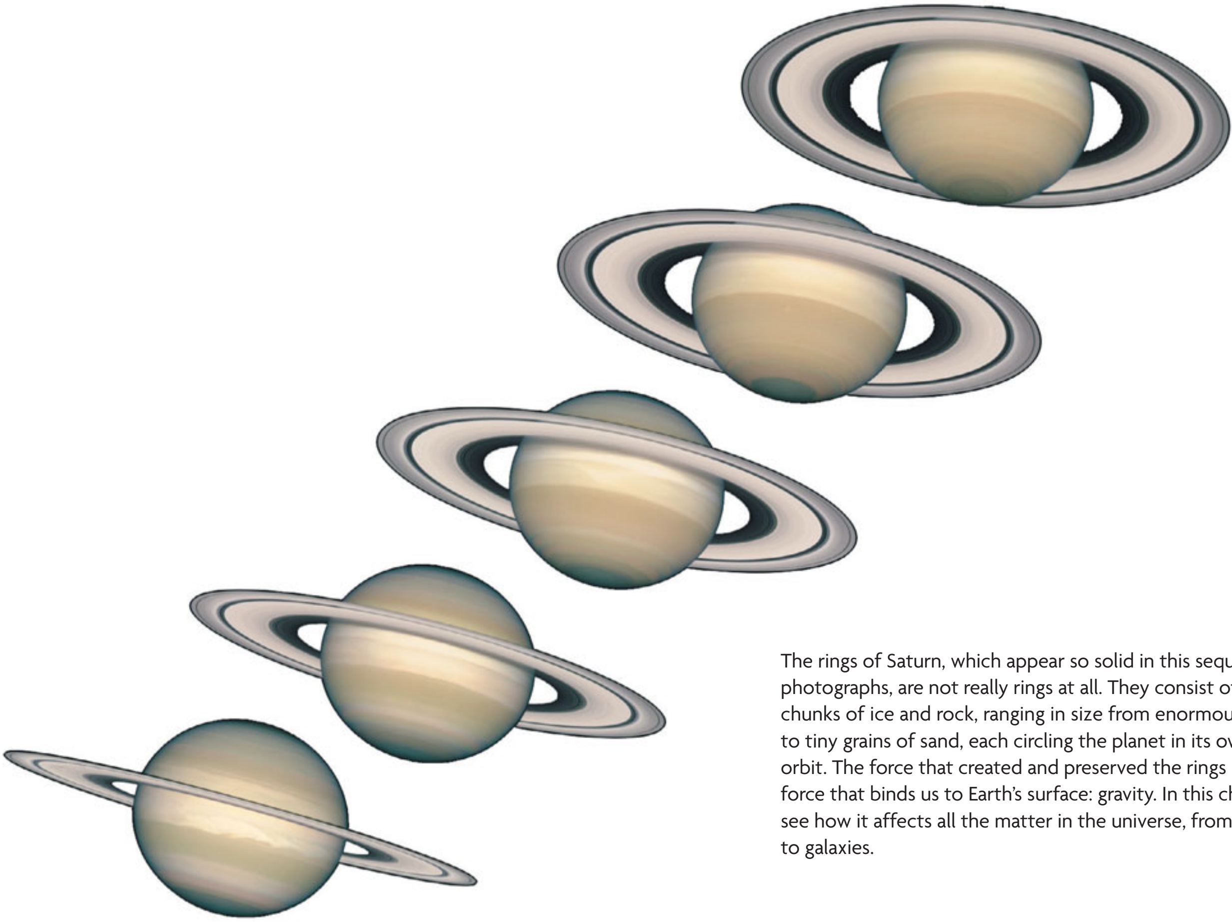
# 12 Gravity



The rings of Saturn, which appear so solid in this sequence of photographs, are not really rings at all. They consist of countless chunks of ice and rock, ranging in size from enormous boulders to tiny grains of sand, each circling the planet in its own individual orbit. The force that created and preserved the rings is the same force that binds us to Earth's surface: gravity. In this chapter we'll see how it affects all the matter in the universe, from dust motes to galaxies.

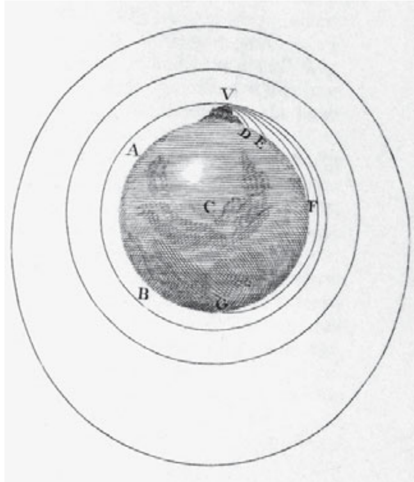


The study of gravity has always been a central theme in physics, from Galileo's early experiments on free fall in the seventeenth century, to Einstein's general theory of relativity in the early years of the twentieth century, and Stephen Hawking's work on black holes in recent years. Perhaps the grandest milestone in this endeavor, however, was the discovery by Newton of the **universal law of gravitation**. With just one simple equation to describe the force of gravity, Newton was able to determine the orbits of planets, moons, and comets, and to explain such earthly phenomena as the tides and the fall of an apple.

Before Newton's work, it was generally thought that the heavens were quite separate from the Earth, and that they obeyed their own "heavenly" laws. Newton showed, on the contrary, that the same law of gravity that operates on the surface of the Earth applies to the Moon and to other astronomical objects. As a result of Newton's efforts, physics expanded its realm of applicability to natural phenomena throughout the universe.

So successful was Newton's law of gravitation that Edmond Halley (1656–1742) was able to use it to predict the return of the comet that today bears his name. Though he did not live to see its return in 1758, the fact that the comet did reappear when predicted was an event unprecedented in human history. Roughly a hundred years later, Newton's theory of gravity scored an even more impressive success. Astronomers observing the planet Uranus noticed small deviations in its orbit, which they thought might be due to the gravitational tug of a previously unknown planet. Using Newton's law to calculate the predicted position of the new planet—now called Neptune—it was found on the very first night of observations, September 23, 1846. The fact that Neptune was precisely where the law of gravitation said it should be still stands as one of the most astounding triumphs in the history of science.

Today, Newton's law of gravitation is used to determine the orbits that take spacecraft from the Earth to various destinations within our solar system and beyond. Appropriately enough, spacecraft were even sent to view Halley's comet at close range in 1986. In addition, the law allows us to calculate with pinpoint accuracy the time of solar eclipses and other astronomical events in the distant past and remote future. This incredibly powerful and precise law of nature is the subject of this chapter.




▲ In this illustration from his great work, the *Principia*, published in 1687, Newton presents a "thought experiment" to show the connection between free fall and orbital motion. Imagine throwing a projectile horizontally from the top of a mountain. The greater the initial speed of the projectile, the farther it travels in free fall before striking the ground. In the absence of air resistance, a great enough initial speed could result in the projectile circling the Earth and returning to its starting point. Thus, an object orbiting the Earth is actually in free fall—it simply has a large horizontal speed.

## 12–1 Newton's Law of Universal Gravitation

It's ironic, but the first fundamental force of nature to be recognized as such, **gravity**, is also the weakest of the fundamental forces. Still, it is the force most apparent to us in our everyday lives, and is the force responsible for the motion of the Moon, the Earth, and the planets. Yet the connection between falling objects on Earth and planets moving in their orbits was not known before Newton.

The flash of insight that came to Newton—whether it was due to seeing an apple fall to the ground or not—is simply this: The force causing an apple to accelerate downward is the same force causing the Moon to move in a circular path around the Earth. To put it another way, Newton was the first to realize that the Moon is *constantly falling* toward the Earth, though without ever getting closer to it, and that it falls for the same reason that an apple falls. This is illustrated in a classic drawing due to Newton, shown to the right.

To be specific, in the case of the apple the motion is linear as it accelerates downward toward the center of the Earth. In the case of the Moon the motion is circular with constant speed. As discussed in Section 6–5, an object in uniform circular motion accelerates toward the center of the circle. It follows, therefore, that the Moon *also* accelerates toward the center of the Earth. In fact, the force responsible for the Moon's centripetal acceleration is the Earth's gravitational attraction, the same force responsible for the fall of the apple.

To describe the force of gravity, Newton proposed the following simple law:

**Newton's Law of Universal Gravitation**

The force of gravity between any two point objects of mass $m_1$ and $m_2$ is attractive and of magnitude

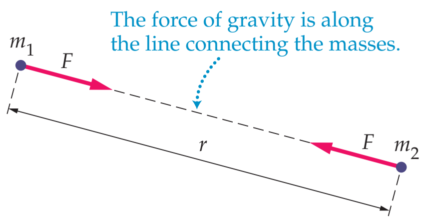
$$F = G\frac{m_1 m_2}{r^2} \qquad 12\text{–}1$$

In this expression, $r$ is the distance between the masses, and $G$ is a constant referred to as the **universal gravitation constant**. Its value is

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2} \qquad 12\text{–}2$$

The force is directed along the line connecting the masses, as indicated in **Figure 12–1**.

Note that each mass experiences a force of the same magnitude, $F = Gm_1m_2/r^2$, but acting in opposite directions. That is, the force of gravity between two objects forms an action-reaction pair.




▲ **FIGURE 12–1 Gravitational force between point masses**

Two point masses, $m_1$ and $m_2$, separated by a distance $r$ exert equal and opposite attractive forces on one another. The magnitude of the forces, $F$, is given by Equation 12–1.

According to Newton's law, all objects in the universe attract all other objects in the universe by way of the gravitational interaction. It is in this sense that the force law is termed "universal." Thus, the net gravitational force acting on you is due not only to the planet on which you stand, which is certainly responsible for the majority of the net force, but also to people nearby, planets, and even stars in far-off galaxies. In short, everything in the universe "feels" everything else, thanks to gravity.

The fact that $G$ is such a small number means that the force of gravity between objects of human proportions is imperceptibly small. This is shown in the following Exercise.

## EXERCISE 12–1

A man takes his dog for a walk on a deserted beach. Treating people and dogs as point objects for the moment, find the force of gravity between the 105-kg man and his 11.2-kg dog when they are separated by a distance of **(a)** 1.00 m and **(b)** 10.0 m.

**SOLUTION**

**a.** Substituting numerical values into Equation 12–1 yields

$$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(105\ \mathrm{kg})(11.2\ \mathrm{kg})}{(1.00\ \mathrm{m})^2} = 7.84 \times 10^{-8}\ \mathrm{N}$$

**b.** Repeating the calculation for $r = 10.0$ m gives

$$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(105\ \mathrm{kg})(11.2\ \mathrm{kg})}{(1.00\ \mathrm{m})^2} = 7.84 \times 10^{-10}\ \mathrm{N}$$

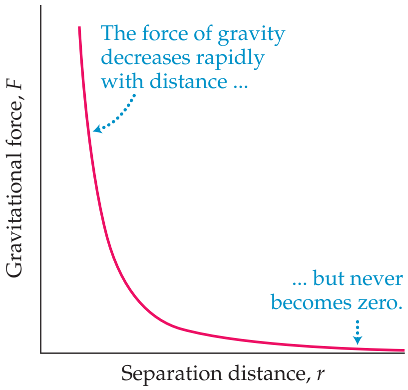



**FIGURE 12–2 Dependence of the gravitational force on separation distance, *r***

The $1/r^2$ dependence of the gravitational force means that it decreases rapidly with distance. Still, it never completely vanishes. For this reason, we say that gravity is a force of infinite range; that is, every mass in the universe experiences a nonzero force from every other mass in the universe, no matter how far away.

The forces found in Exercise 12–1 are imperceptibly small. In comparison, the force exerted by the Earth on the man is 1030 N and the force exerted on the dog is 110 N—these forces are several orders of magnitude greater than the force between the man and the dog. In general, gravitational forces are significant only when large masses, such as the Earth or the Moon, are involved.

Exercise 12–1 also illustrates how rapidly the force of gravity decreases with distance. In particular, since $F$ varies as $1/r^2$, it is said to have an **inverse square dependence** on distance. Thus, for example, an increase in distance by a factor of 10 results in a decrease in the force by a factor of $10^2 = 100$. A plot of the force of gravity versus distance is given in **Figure 12–2**. Note that even though the force diminishes rapidly with distance, it never completely vanishes; thus, we say that gravity is a force of infinite range.

Note also that the force of gravity between two masses depends on the product of the masses, $m_1$ times $m_2$. With this type of dependence, it follows that if either mass is doubled, the force of gravity is doubled as well. This would not be the case, for example, if the force of gravity depended on the *sum* of the masses, $m_1 + m_2$.

Finally, if a given mass is acted on by gravitational interactions with a number of other masses, the net force acting on it is simply the vector sum of each of the forces individually. This property of gravity is referred to as **superposition**. As an example, superposition implies that the net gravitational force exerted on you at this moment is the vector sum of the force exerted by the Earth, plus the force exerted by the Moon, plus the force exerted by the Sun, and so on. The following Example illustrates superposition.

**PROBLEM-SOLVING NOTE**

**Net Gravitational Force**

To find the net gravitational force acting on an object, you should (i) resolve each of the forces acting on the object into components and (ii) add the forces component by component.

## EXAMPLE 12–1 HOW MUCH FORCE IS WITH YOU?

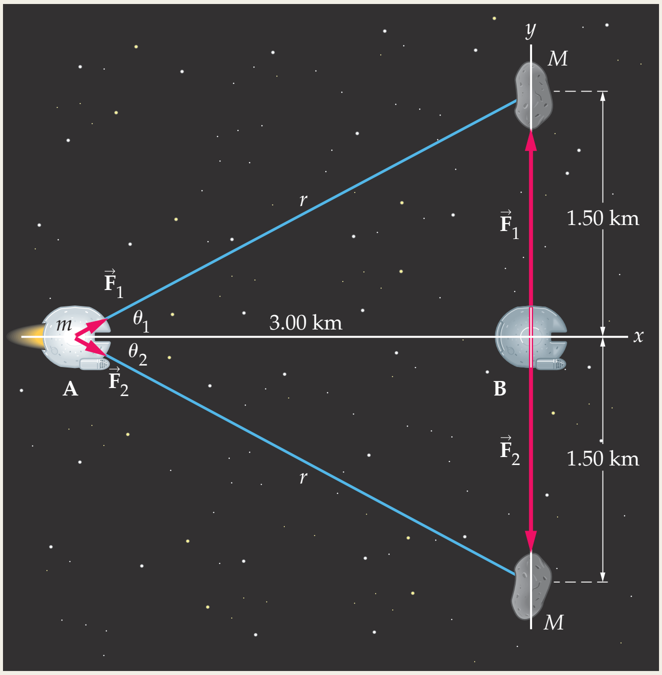
As part of a daring rescue attempt, the *Millennium Eagle* passes between a pair of twin asteroids, as shown. If the mass of the spaceship is $2.50 \times 10^7$ kg and the mass of each asteroid is $3.50 \times 10^{11}$ kg, find the net gravitational force exerted on the *Millennium Eagle* **(a)** when it is at location A and **(b)** when it is at location B. Treat the spaceship and the asteroids as if they were point objects.

**PICTURE THE PROBLEM**

Our sketch shows the spaceship as it follows a path between the twin asteroids. The relevant distances and masses are indicated, as are the two points of interest, A and B. Note that at location A the force $\vec{\mathbf{F}}_1$ points above the $x$ axis at the angle $\theta_1$ (to be determined); the force $\vec{\mathbf{F}}_2$ points below the $x$ axis at the angle $\theta_2 = -\theta_1$, as can be seen by symmetry. At location B, the two forces act in opposite directions.

**STRATEGY**

To find the net gravitational force exerted on the spaceship, we first determine the magnitude of the force exerted on it by each asteroid. This is done by using Equation 12–1 and the distances given in our sketch. Next, we resolve these forces into $x$ and $y$ components. Finally, we sum the force components to find the net force.




**SOLUTION**

**Part (a)**

1. Use the Pythagorean theorem to find the distance $r$ from point A to each asteroid. Also, refer to the sketch to find the angle between $\vec{\mathbf{F}}_1$ and the $x$ axis. The angle between $\vec{\mathbf{F}}_2$ and the $x$ axis has the same magnitude but the opposite sign:

$$r = \sqrt{(3.00 \times 10^3\ \text{m})^2 + (1.50 \times 10^3\ \text{m})^2} = 3350\ \text{m}$$

$$\theta_1 = \tan^{-1}\left(\frac{1.50 \times 10^3\ \text{m}}{3.00 \times 10^3\ \text{m}}\right) = \tan^{-1}(0.500) = 26.6°$$

$$\theta_2 = -\theta_1 = -26.6°$$

2. Use $r$ and Equation 12–1 to calculate the magnitude of the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ at point A:

$$F_1 = F_2 = G\frac{mM}{r^2}$$

$$= (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.50 \times 10^7\ \text{kg})(3.50 \times 10^{11}\ \text{kg})}{(3350\ \text{m})^2}$$

$$= 52.0\ \text{N}$$

3. Use the values of $\theta_1$ and $\theta_2$ found in Step 1 to calculate the $x$ and $y$ components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$:

$$F_{1,x} = F_1 \cos\theta_1 = (52.0\ \text{N})\cos 26.6° = 46.5\ \text{N}$$

$$F_{1,y} = F_1 \sin\theta_1 = (52.0\ \text{N})\sin 26.6° = 23.3\ \text{N}$$

$$F_{2,x} = F_2 \cos\theta_2 = (52.0\ \text{N})\cos(-26.6°) = 46.5\ \text{N}$$

$$F_{2,y} = F_2 \sin\theta_2 = (52.0\ \text{N})\sin(-26.6°) = -23.3\ \text{N}$$

4. Add the components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ to find the components of the net force, $\vec{\mathbf{F}}$:

$$F_x = F_{1,x} + F_{2,x} = 93.0\ \text{N}$$

$$F_y = F_{1,y} + F_{2,y} = 0$$

**Part (b)**

5. Use Equation 12–1 to find the magnitude of the forces exerted on the spaceship by the asteroids at location B:

$$F_1 = F_2 = G\frac{mM}{r^2}$$

$$= (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.50 \times 10^7\ \text{kg})(3.50 \times 10^{11}\ \text{kg})}{(1.50 \times 10^3\ \text{m})^2}$$

$$= 259\ \text{N}$$

6. Use the fact that $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ have equal magnitudes and point in opposite directions to determine the net force, $\vec{\mathbf{F}}$, acting on the spaceship:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

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**INSIGHT**

We find that the net force at location A is in the positive $x$ direction, as one would expect by symmetry. At location B, where the force exerted by each asteroid is about 5 times greater than it is at location A, the *net* force is zero since the attractive forces exerted by the two asteroids are equal and opposite, and thus cancel. Note that the forces in our sketch have been drawn in correct proportion.

Rocket scientists often use the gravitational force between astronomical objects and spacecraft to accelerate the spacecraft and send them off to distant parts of the solar system. In fact, this gravitational attraction makes possible the "slingshot" effect illustrated in Figure 9–31.

**PRACTICE PROBLEM**

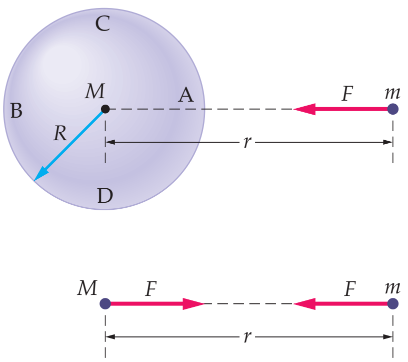
Find the net gravitational force acting on the spaceship when it is at the location $x = 5.00 \times 10^3$ m, $y = 0$. **[Answer:** 41.0 N in the negative $x$ direction]

*Some related homework problems: Problem 9, Problem 11, Problem 12*

## 12–2 Gravitational Attraction of Spherical Bodies

Newton's law of gravity applies to point objects. How, then, do we calculate the force of gravity for an object of finite size? In general, the approach is to divide the finite object into a collection of small mass elements, then use superposition and the methods of calculus to determine the net gravitational force. For an arbitrary shape, this calculation can be quite difficult. For objects with a uniform spherical shape, however, the final result is remarkably simple, as was shown by Newton.

### Uniform Sphere




▲ **FIGURE 12–3 Gravitational force between a point mass and a sphere**

The force is the same as if all the mass of the sphere were concentrated at its center.

Consider a uniform sphere of radius $R$ and mass $M$, as in **Figure 12–3**. A point object of mass $m$ is brought near the sphere, though still outside it at a distance $r$ from its center. The object experiences a relatively strong attraction from mass near the point A, and a weaker attraction from mass near point B. In both cases the force is along the line connecting the mass $m$ and the center of the sphere; that is, along the $x$ axis. In addition, mass at the points C and D exert a net force that is also along the $x$ axis—just as in the case of the twin asteroids in Example 12–1. Thus, the symmetry of the sphere guarantees that the net force it exerts on $m$ is directed toward the sphere's center. The magnitude of the force exerted by the sphere must be calculated with the methods of calculus—which Newton invented and then applied to this problem. As a result of his calculations, Newton was able to show that **the net force exerted by the sphere on the mass $m$ is the same as if all the mass of the sphere were concentrated at its center.** That is, the force between the mass $m$ and the sphere of mass $M$ has a magnitude that is simply

$$F = G\frac{mM}{r^2} \qquad 12\text{–}3$$

Let's apply this result to the case of a mass $m$ on the surface of the Earth. If the mass of the Earth is $M_E$, and its radius is $R_E$, it follows that the force exerted on $m$ by the Earth is

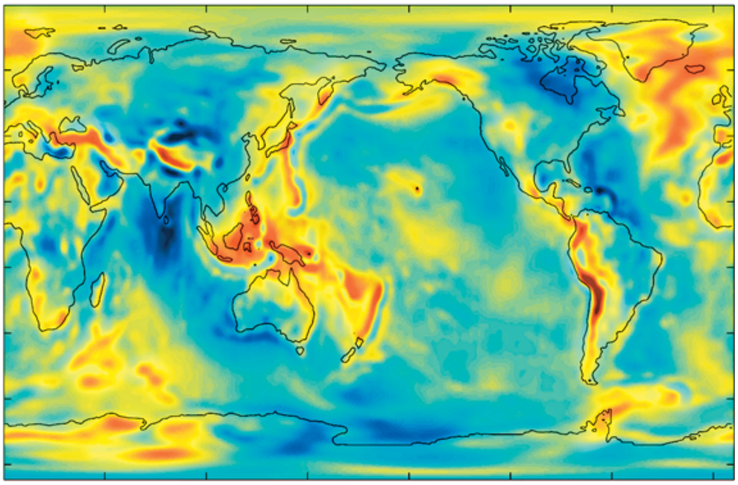
$$F = G\frac{mM_E}{{R_E}^2} = m\left(\frac{GM_E}{{R_E}^2}\right)$$

We also know, however, that the gravitational force experienced by a mass $m$ on the Earth's surface is simply $F = mg$, where $g$ is the acceleration due to gravity. Therefore, we see that

$$m\left(\frac{GM_E}{{R_E}^2}\right) = mg$$

or

$$g = \frac{GM_E}{{R_E}^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\ \mathrm{kg})}{(6.37 \times 10^6\ \mathrm{m})^2} = 9.81\ \mathrm{m/s^2} \qquad 12\text{–}4$$



This global model of the Earth's gravitational strength was constructed from a combination of surface gravity measurements and satellite tracking data. It shows how the acceleration of gravity varies from the value at an idealized "sea level" that takes into account the Earth's nonspherical shape. (The Earth is somewhat flattened at the poles—its radius is greatest at the equator.) Gravity is strongest in the red areas and weakest in the dark blue areas.

This result can be extended to objects above the Earth's surface, and hence farther from the center of the Earth, as we show in the next Example.

## EXAMPLE 12–2 THE DEPENDENCE OF GRAVITY ON ALTITUDE

**REAL-WORLD PHYSICS**

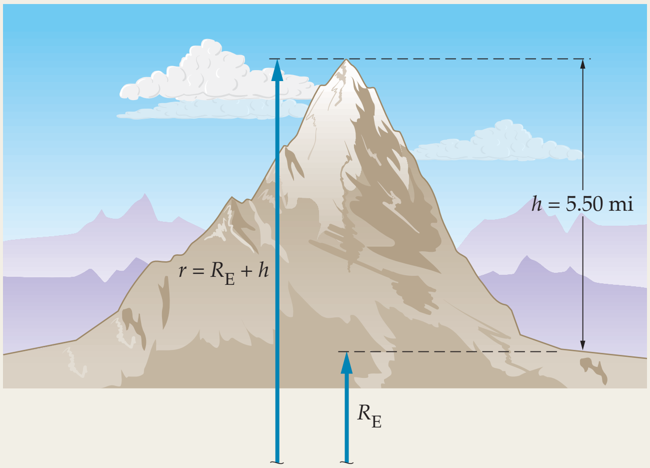
If you climb to the top of Mt. Everest, you will be about 5.50 mi above sea level. What is the acceleration due to gravity at this altitude?

**PICTURE THE PROBLEM**

At the top of the mountain, your distance from the center of the Earth is $r = R_E + h$, where $h = 5.50$ mi is the altitude.

**STRATEGY**

First, use $F = GmM_E/r^2$ to find the force due to gravity on the mountaintop. Then, set $F = mg_h$ to find the acceleration $g_h$ at the height $h$.




**SOLUTION**

1. Calculate the force $F$ due to gravity at a height $h$ above the Earth's surface:

$$F = G\frac{mM_E}{(R_E + h)^2}$$

2. Set $F$ equal to $mg_h$ and solve for $g_h$:

$$F = G\frac{mM_E}{(R_E + h)^2} = mg_h$$

$$g_h = G\frac{M_E}{(R_E + h)^2}$$

3. Factor out $R_E{}^2$ from the denominator, and use the fact that $GM_E/R_E{}^2 = g$:

$$g_h = \left(\frac{GM_E}{R_E{}^2}\right)\frac{1}{\left(1 + \dfrac{h}{R_E}\right)^2} = \frac{g}{\left(1 + \dfrac{h}{R_E}\right)^2}$$

4. Substitute numerical values, with $h = 5.50 \text{ mi} = (5.50 \text{ mi})(1609 \text{ m/mi}) = 8850$ m, and $R_E = 6.37 \times 10^6$ m:

$$g_h = \frac{g}{\left(1 + \dfrac{h}{R_E}\right)^2} = \frac{9.81 \text{ m/s}^2}{\left(1 + \dfrac{8850 \text{ m}}{6.37 \times 10^6 \text{ m}}\right)^2} = 9.78 \text{ m/s}^2$$

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**INSIGHT**

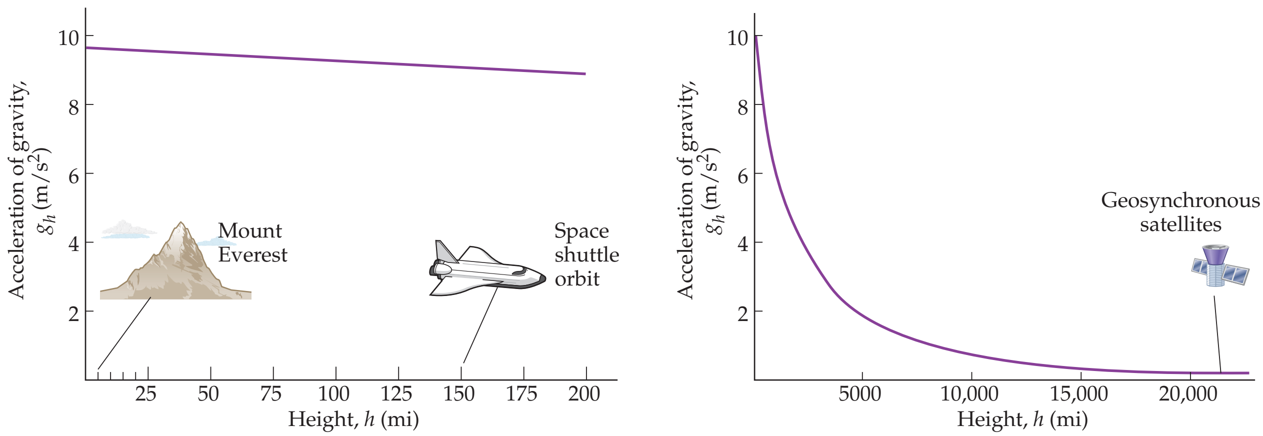
As expected, the acceleration due to gravity is less as one moves farther from the center of the Earth. Thus, if you were to climb to the top of Mt. Everest, you would lose weight—not only because of the physical exertion required for the climb, but also because of the reduced gravity. In particular, a person with a mass of 60 kg (about 130 lb) would lose about half a pound of weight just by standing on the summit of the mountain.

A plot of $g_h$ as a function of $h$ is shown in **Figure 12–4 (a)**. The plot indicates the altitude of Mt. Everest and the orbit of the space shuttle. **Figure 12–4 (b)** shows $g_h$ out to the orbit of communications and weather satellites, which orbit at an altitude of roughly 22,300 mi.

**PRACTICE PROBLEM**

Find the acceleration due to gravity at the altitude of the space shuttle's orbit, 250 km above the Earth's surface. [**Answer:** $g_h = 9.08\ \mathrm{m/s^2}$, a reduction of only 7.44% compared to the acceleration of gravity on the surface of the Earth.]

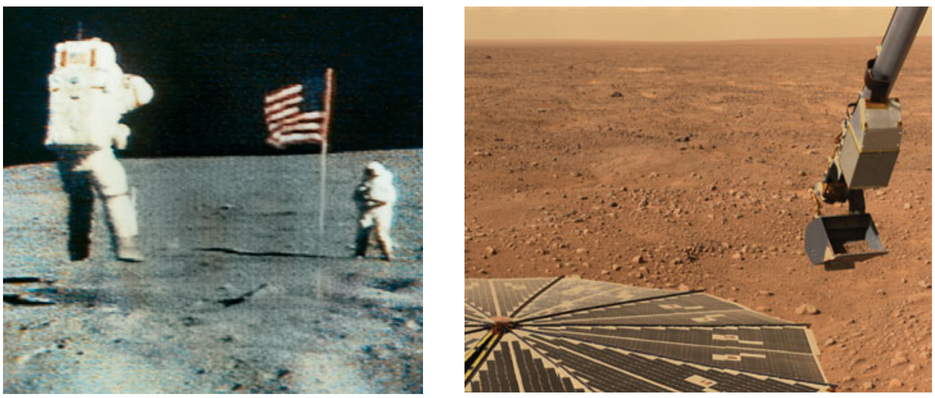
*Some related homework problems: Problem 15, Problem 17*




**(a)** Acceleration of gravity near the Earth's surface **(b)** Acceleration of gravity far from the Earth

**FIGURE 12–4** ▲ **The acceleration due to gravity at a height $h$ above the Earth's surface**
**(a)** In this plot, the peak of Mt. Everest is at about $h = 5.50$ mi, and the space shuttle orbit is at roughly $h = 150$ mi. **(b)** This shows the decrease in the acceleration of gravity from the surface of the Earth to an altitude of about 25,000 mi. The orbit of geosynchronous satellites—ones that orbit above a fixed point on the Earth—is at roughly $h = 22{,}300$ mi.

Equation 12–4 can be used to calculate the acceleration due to gravity on other objects in the solar system besides the Earth. For example, to calculate the acceleration due to gravity on the Moon, $g_m$, we simply use the mass and radius of the Moon in Equation 12–4. Once $g_m$ is known, the weight of an object of mass $m$ on the Moon is found by using $W_m = mg_m$.



▶ (Left) The weak lunar gravity permits astronauts, even encumbered by their massive space suits, to bound over the Moon's surface. The low gravitational pull, only about one-sixth that of Earth, is a consequence not only of the Moon's smaller size, but also of its lower average density. (Right) The force of gravity on the surface of Mars is only about 38% of its strength on Earth. This was an important factor in designing NASA's Phoenix Mars Lander, shown here lifting a scoop of dirt on its 16th Martian day after landing in May 2008.

## EXERCISE 12–2

**a.** Find the acceleration due to gravity on the surface of the Moon.

**b.** The lunar rover had a mass of 225 kg. What was its weight on the Earth and on the Moon? (*Note:* The mass of the Moon is $M_m = 7.35 \times 10^{22}$ kg and its radius is $R_m = 1.74 \times 10^6$ m.)

**SOLUTION**

**a.** For the Moon, the acceleration due to gravity is

$$g_m = \frac{GM_m}{R_m^{\,2}} = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(7.35 \times 10^{22}\ \mathrm{kg})}{(1.74 \times 10^6\ \mathrm{m})^2} = 1.62\ \mathrm{m/s^2}$$

This is about one-sixth the acceleration due to gravity on the Earth.

**b.** On the Earth, the rover's weight was

$$W = mg = (225\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) = 2210\ \mathrm{N}$$

On the Moon, its weight was

$$W_m = mg_m = (225\ \mathrm{kg})(1.62\ \mathrm{m/s^2}) = 365\ \mathrm{N}$$

As expected, this is roughly one-sixth its Earth weight.

The replacement of a sphere with a point mass at its center can be applied to many physical systems. For example, the force of gravity between two spheres of finite size is the same as if *both* were replaced by point masses. Thus, the gravitational force between the Earth, with mass $M_E$, and the Moon, with mass $M_m$, is

$$F = G\frac{M_E M_m}{r^2}$$

The distance $r$ in this expression is the center-to-center distance between the Earth and the Moon, as shown in **Figure 12–5**. It follows, then, that in many calculations involving the solar system, moons and planets can be treated as point objects.

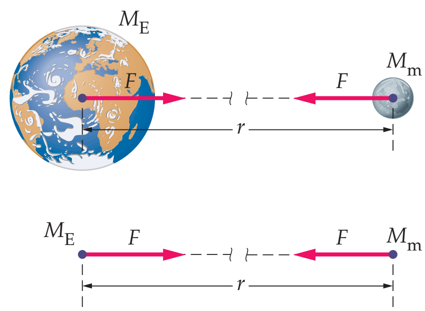



**FIGURE 12–5 Gravitational force between the Earth and the Moon**

The force is the same as if both the Earth and the Moon were point masses. (The sizes of the Earth and Moon are in correct proportion in this figure, but the separation between the two should be much greater than that shown here. In reality, it is about 30 times the diameter of the Earth, and so would be about 2 ft on this scale.)

## Weighing the Earth

The British physicist Henry Cavendish performed an experiment in 1798 that is often referred to as "weighing the Earth." What he did, in fact, was measure the value of the universal gravitation constant, $G$, that appears in Newton's law of gravity. As we have pointed out before, $G$ is a very small number; hence a sensitive experiment is needed for its measurement. It is because of this experimental difficulty that $G$ was not measured until more than 100 years after Newton published the law of gravitation.

In the Cavendish experiment, illustrated in **Figure 12–6**, two masses $m$ are suspended from a thin thread. Near each suspended mass is a large stationary mass $M$, as shown. Each suspended mass is attracted by the force of gravity toward the large mass near it; hence the rod holding the suspended masses tends to rotate and twist the thread. The angle through which the thread twists can be measured by bouncing a beam of light from a mirror attached to the thread. If the force required to twist the thread through a given angle is known (from previous experiments), a measurement of the twist angle gives the magnitude of the force of gravity. Finally, knowing the masses $m$ and $M$, and the distance between their centers, $r$, we can use Equation 12–1 to solve for $G$. Cavendish found $6.754 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$, in good agreement with the currently accepted value given in Equation 12–2.

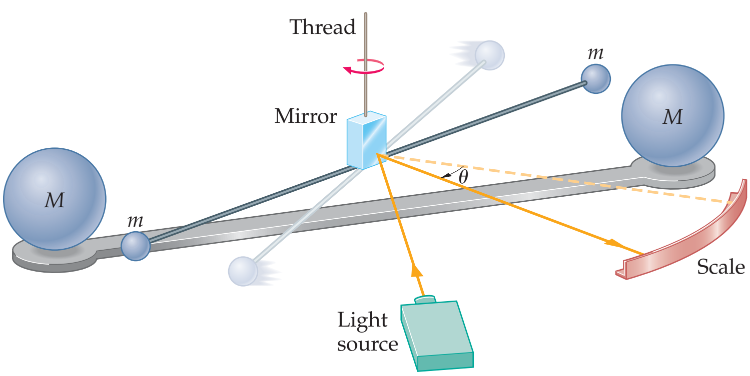



**FIGURE 12–6 The Cavendish experiment**

The gravitational attraction between the masses $m$ and $M$ causes the rod and the suspending thread to twist. Measurement of the twist angle allows for a direct measurement of the gravitational force.

To see why Cavendish is said to have weighed the Earth, recall that the force of gravity on the surface of the Earth, $mg$, can be written as follows:

$$mg = G\frac{mM_E}{R_E^2}$$

Canceling $m$ and solving for $M_E$ yields

$$M_E = \frac{gR_E^2}{G} \qquad \text{12–5}$$

Before the Cavendish experiment, the quantities $g$ and $R_E$ were known from direct measurement, but $G$ had yet to be determined. When Cavendish measured $G$, he didn't actually "weigh" the Earth, of course. Instead, he calculated its mass, $M_E$.

### EXERCISE 12–3

Use $M_E = gR_E^2/G$ to calculate the mass of the Earth.

**SOLUTION**

Substituting numerical values, we find

$$M_E = \frac{gR_E^2}{G} = \frac{(9.81\ \mathrm{m/s^2})(6.37 \times 10^6\ \mathrm{m})^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 5.97 \times 10^{24}\ \mathrm{kg}$$

As soon as Cavendish determined the mass of the Earth, geologists were able to use the result to calculate its average density; that is, its average mass per volume. Assuming a spherical Earth of radius $R_E$, its total volume is

$$V_E = \tfrac{4}{3}\pi R_E^3 = \tfrac{4}{3}\pi(6.37 \times 10^6\ \mathrm{m})^3 = 1.08 \times 10^{21}\ \mathrm{m^3}$$

Dividing this into the total mass yields the average density, $\rho$:

$$\rho = \frac{M_E}{V_E} = \frac{5.97 \times 10^{24}\ \mathrm{kg}}{1.08 \times 10^{21}\ \mathrm{m^3}} = 5530\ \mathrm{kg/m^3} = 5.53\ \mathrm{g/cm^3}$$

This is an interesting result because typical rocks found near the surface of the Earth, such as granite, have a density of only about $3.00\ \mathrm{g/cm^3}$. We conclude, then, that the interior of the Earth must have a greater density than its surface. In fact, by analyzing the propagation of seismic waves around the world, we now know that the Earth has a rather complex interior structure, including a solid inner core with a density of about $15.0\ \mathrm{g/cm^3}$ (see Section 10–5).

**REAL-WORLD PHYSICS**

**The internal structure of the Earth and the Moon**

A similar calculation for the Moon yields an average density of about $3.33\ \mathrm{g/cm^3}$, essentially the same as the density of the lunar rocks brought back during the Apollo program. Hence, it is likely that the Moon does not have an internal structure similar to that of the Earth.

Since $G$ is a universal constant—with the same value everywhere in the universe—it can be used to calculate the mass of other bodies in the solar system as well. This is illustrated in the following Example.

### EXAMPLE 12–3 MARS ATTRACTS!

After landing on Mars, an astronaut performs a simple experiment by dropping a rock. A quick calculation using the drop height and the time of fall yields a value of $3.73\ \mathrm{m/s^2}$ for the rock's acceleration. **(a)** Find the mass of Mars, given that its radius is $R_M = 3.39 \times 10^6\ \mathrm{m}$. **(b)** What is the acceleration of gravity due to Mars at a distance $2R_M$ from the center of the planet?

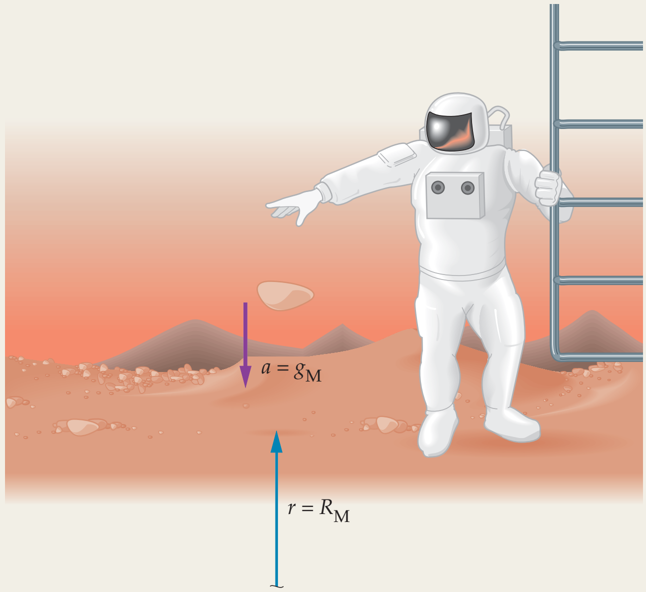
**PICTURE THE PROBLEM**

Our sketch shows an astronaut dropping a rock to the ground on the surface of Mars. If the acceleration of the rock is measured, we find $g_M = 3.73\ \mathrm{m/s^2}$, where the subscript M refers to Mars. In addition, we indicate the radius of Mars in our sketch, where $R_M = 3.39 \times 10^6\ \mathrm{m}$.

**STRATEGY**

**a.** Since the acceleration of gravity is $g_M$ on the surface of Mars, it follows that the force of gravity on an object of mass $m$ is $F = mg_M$. This force is also given by Newton's law of gravity—that is, $F = GmM_M/R_M{}^2$. Setting these expressions for the force equal to one another yields the mass of Mars, $M_M$.

**b.** Set $F = ma$ equal to $F = GmM_M/(2R_M)^2$ and solve for the acceleration, $a$.




**SOLUTION**

**Part (a)**

1. Set $mg_M$ equal to $GmM_M/R_M{}^2$:

$$mg_M = G\frac{mM_M}{R_M{}^2}$$

2. Cancel $m$ and solve for the mass of Mars:

$$M_M = \frac{g_M R_M{}^2}{G}$$

3. Substitute numerical values:

$$M_M = \frac{(3.73\ \mathrm{m/s^2})(3.39\times10^6\ \mathrm{m})^2}{6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}} = 6.43\times10^{23}\ \mathrm{kg}$$

**Part (b)**

4. Apply Newton's law of gravity with $r = 2R_M$. Use the fact that $g_M = GM_M/R_M{}^2$ from Step 1 to simplify the calculation:

$$ma = G\frac{mM_M}{(2R_M)^2} \quad \text{or}$$

$$a = G\frac{M_M}{(2R_M)^2} = \tfrac{1}{4}\left(G\frac{M_M}{R_M{}^2}\right) = \tfrac{1}{4}(g_M) = \tfrac{1}{4}(3.73\ \mathrm{m/s^2}) = 0.933\ \mathrm{m/s^2}$$

**INSIGHT**

The important point here is that the universal gravitation constant, $G$, applies as well on Mars as on Earth, or any other object. Therefore, knowledge of the size and acceleration of gravity of an astronomical body is sufficient to determine its mass.

**PRACTICE PROBLEM**

If the radius of Mars were reduced to $3.00 \times 10^6$ m, with its mass remaining the same, would the acceleration of gravity on Mars increase, decrease, or stay the same? Check your answer by calculating the acceleration of gravity for this case. [**Answer:** The acceleration of gravity increases to $4.77\ \mathrm{m/s^2}$.]

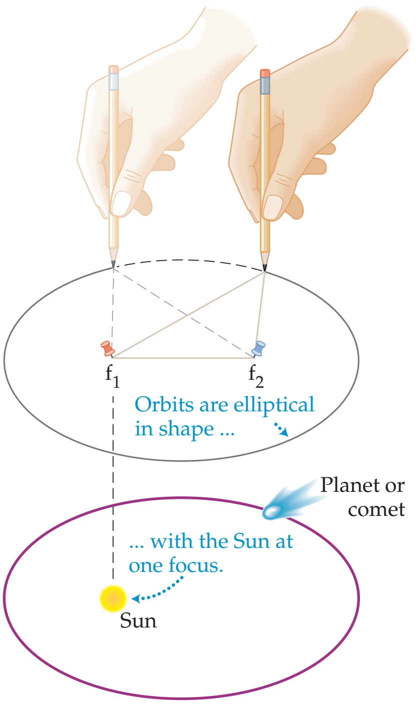
*Some related homework problems: Problem 20, Problem 21*

## 12–3 Kepler's Laws of Orbital Motion

If you go outside each clear night and observe the position of Mars with respect to the stars, you will find that its apparent motion across the sky is rather complex. Instead of moving on a simple curved path, it occasionally reverses direction (this is known as *retrograde motion*). A few months later it reverses direction yet again and resumes its original direction of motion. Other planets exhibit similar odd behavior.

The Danish astronomer Tycho Brahe (1546–1601) followed the paths of the planets, and Mars in particular, for many years, even though the telescope had not yet been invented. He used, instead, an elaborate sighting device to plot the precise position of the planets. Brahe was joined in his work by Johannes Kepler (1571–1630) in 1600, and after Brahe's death, Kepler inherited his astronomical observations.

Kepler made good use of Brahe's life work, extracting from his carefully collected data the three laws of orbital motion we know today as Kepler's laws. These laws make it clear that the Sun and the planets do not orbit the Earth, as Ptolemy—the ancient Greek astronomer—claimed, but rather that the Earth, along with the other planets, orbit the Sun, as proposed by Copernicus (1473–1543).




▲ **FIGURE 12–7 Drawing an ellipse**
To draw an ellipse, put two tacks in a piece of cardboard. The tacks define the "foci" of the ellipse. Now connect a length of string to the two tacks, and use a pencil and the string to sketch out a smooth closed curve, as shown. This closed curve is an ellipse. In a planetary orbit a planet follows an elliptical path, with the Sun at one focus. Nothing is at the other focus.

Why the planets obey Kepler's laws no one knew—not even Kepler—until Newton considered the problem decades after Kepler's death. Newton was able to show that each of Kepler's laws follows as a direct consequence of the universal law of gravitation. In the remainder of this section we consider Kepler's three laws one at a time, and point out the connection between them and the law of gravitation.

## Kepler's First Law

Kepler tried long and hard to find a circular orbit around the Sun that would match Brahe's observations of Mars. After all, up to that time everyone from Ptolemy to Copernicus believed that celestial objects moved in circular paths of one sort or another. Though the orbit of Mars was exasperatingly close to being circular, the small differences between a circular path and the experimental observations just could not be ignored. Eventually, after a great deal of hard work and disappointment over the loss of circular orbits, Kepler discovered that Mars followed an orbit that was elliptical rather than circular. The same applied to the other planets. This observation became Kepler's first law:

> Planets follow elliptical orbits, with the Sun at one focus of the ellipse.

This is a fine example of the scientific method in action. Though Kepler expected and wanted to find circular orbits, he would not allow himself to ignore the data. If Brahe's observations had not been so accurate, Kepler probably would have chalked up the small differences between the data and a circular orbit to error. As it was, he had to discard a treasured—but incorrect—theory, and move on to an unexpected, but ultimately correct, view of nature.

Kepler's first law is illustrated in **Figure 12–7**, along with a definition of an ellipse in terms of its two foci. In the case where the two foci merge, as in **Figure 12–8**, the ellipse reduces to a circle. Thus, a circular orbit *is* allowed by Kepler's first law, but only as a special case.

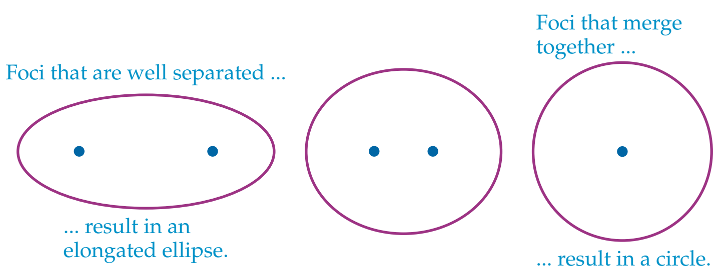
Newton was able to show that, because the force of gravity decreases with distance as $1/r^2$, closed orbits must have the form of ellipses or circles, as stated in Kepler's first law. He also showed that orbits that are not closed—say the orbit of a comet that passes by the Sun once and then leaves the solar system—are either parabolic or hyperbolic.

## Kepler's Second Law

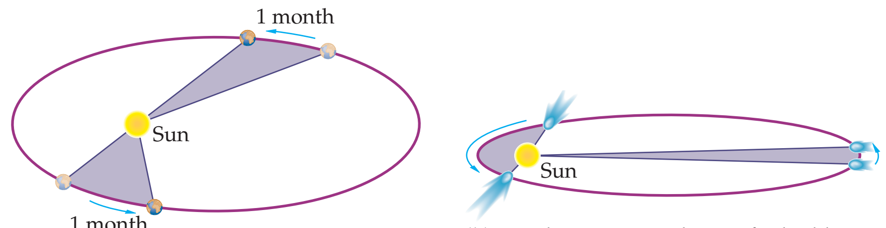
When Kepler plotted the position of a planet on its elliptical orbit, indicating at each position the time the planet was there, he made an interesting observation. First, draw a line from the Sun to a planet at a given time. Then a certain time later—perhaps a month—draw a line again from the Sun to the new position of the planet. The result is that the planet has "swept out" a wedge-shaped area, as indicated in **Figure 12–9 (a)**. If this procedure is repeated when the planet is on a different part of its orbit, another wedge-shaped area is generated. Kepler's observation was that the areas of these two wedges are equal:

> As a planet moves in its orbit, it sweeps out an equal amount of area in an equal amount of time.

Kepler's second law follows from the fact that the force of gravity on a planet is directly toward the Sun. As a result, gravity exerts zero torque about the Sun,




▶ **FIGURE 12–8 The circle as a special case of the ellipse**
As the two foci of an ellipse approach one another, the ellipse becomes more circular. In the limit that the foci merge, the ellipse becomes a circle.




**(a)** Equal areas in equal times

**(b)** Equal areas in equal times for highly elliptical orbit

**FIGURE 12–9 Kepler's second law**

**(a)** The second law states that a planet sweeps out equal areas in equal times. **(b)** In a highly elliptical orbit, the long, thin area is equal to the broad, fan-shaped area.

which means that the angular momentum of a planet in its orbit must be conserved. As Newton showed, conservation of angular momentum is equivalent to the equal-area law stated by Kepler.

## CONCEPTUAL CHECKPOINT 12–1 COMPARE SPEEDS

The Earth's orbit is slightly elliptical. In fact, the Earth is closer to the Sun during the northern hemisphere winter than it is during the summer. Is the speed of the Earth during winter **(a)** greater than, **(b)** less than, or **(c)** the same as its speed during summer?

**REASONING AND DISCUSSION**

According to Kepler's second law, the area swept out by the Earth per month is the same in winter as it is in summer. In winter, however, the radius from the Sun to the Earth is less than it is in summer. Therefore, if this smaller radius is to sweep out the same area, the Earth must move more rapidly.

**ANSWER**

**(a)** The speed of the Earth is greater during the winter.

Though we have stated the first two laws in terms of planets, they apply equally well to any object orbiting the Sun. For example, a comet might follow a highly elliptical orbit, as in **Figure 12–9 (b)**. When it is near the Sun, it moves very quickly, for the reason discussed in Conceptual Checkpoint 12–1, sweeping out a broad wedge-shaped area in a month's time. Later in its orbit, the comet is far from the Sun and moving slowly. In this case, the area it sweeps out in a month is a long, thin wedge. Still, the two wedges have equal areas.

## Kepler's Third Law

Finally, Kepler studied the relation between the mean distance of a planet from the Sun, $r$, and its period—that is, the time, $T$, it takes for the planet to complete one orbit. **Figure 12–10** shows a plot of period versus distance for the planets of the solar system. Kepler tried to "fit" these results to a simple dependence between $T$ and $r$. If he tried a linear fit—that is, $T$ proportional to $r$ (the bottom curve in Figure 12–10)—he found that the period did not increase rapidly enough with distance. On the other hand, if he tried $T$ proportional to $r^2$ (the top curve in Figure 12–10), the period increased too rapidly. Splitting the difference, and trying $T$ proportional to $r^{3/2}$, yields a good fit (the middle curve in Figure 12–10). This is Kepler's third law:

> The period, $T$, of a planet increases as its mean distance from the Sun, $r$, raised to the 3/2 power. That is,
>
> $$T = (\text{constant})r^{3/2} \qquad 12\text{–}6$$

It is straightforward to derive this result for the special case of a circular orbit. Consider, then, a planet orbiting the Sun at a distance $r$, as in **Figure 12–11**. Since the planet moves in a circular path, a centripetal force must act on it, as we saw in Section 6–5. In addition, this force must be directed toward the center of the circle; that is, toward the Sun. It is as if you were to swing a ball on the end of a string in a circle above your head, as in Figure 6–12 (p. 169). In order for the ball to move in a circular path, you have to exert a force on the ball toward the center of the circular path. This force is exerted through the string. In the case of a planet orbiting the Sun, the centripetal force is provided by the force of gravity between the Sun and the planet.

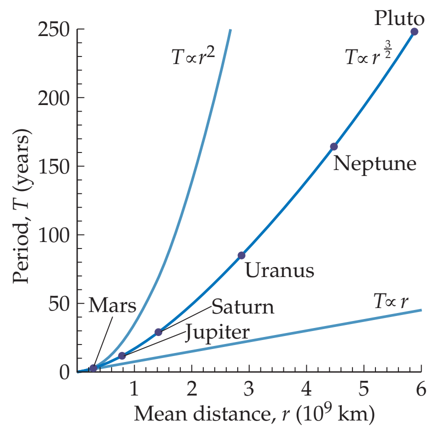



**FIGURE 12–10 Kepler's third law and some near misses**

These plots represent three possible mathematical relationships between period of revolution, $T$ (in years), and mean distance from the Sun, $r$ (in kilometers). The lower curve shows $T = (\text{constant})r$; the upper curve is $T = (\text{constant})r^2$. The middle curve, which fits the data, is $T = (\text{constant})r^{3/2}$. This is Kepler's third law.

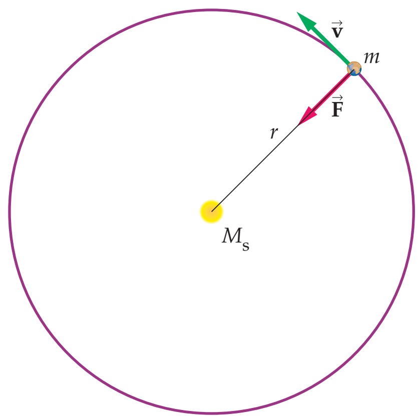



**FIGURE 12–11 Centripetal force on a planet in orbit**

As a planet revolves about the Sun in a circular orbit of radius $r$, the force of gravity between it and the Sun, $F = GmM_s/r^2$, provides the required centripetal force.

If the planet has a mass $m$, and the Sun has a mass $M_s$, the force of gravity between them is

$$F = G\frac{mM_s}{r^2}$$

Now, this force creates the centripetal acceleration of the planet, $a_{cp}$, which, according to Equation 6–15, is

$$a_{cp} = \frac{v^2}{r}$$

Thus, the centripetal force necessary for the planet to orbit is $ma_{cp}$:

$$F = ma_{cp} = m\frac{v^2}{r}$$

Since the speed of the planet, $v$, is the circumference of the orbit, $2\pi r$, divided by the time to complete an orbit, $T$, we have

$$F = m\frac{v^2}{r} = m\frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 rm}{T^2}$$

Setting the centripetal force equal to the force of gravity yields

$$\frac{4\pi^2 rm}{T^2} = G\frac{mM_s}{r^2}$$

Eliminating $m$ and rearranging, we find

$$T^2 = \frac{4\pi^2}{GM_s}r^3$$

or

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right)r^{3/2} = (\text{constant})r^{3/2} \qquad \text{12–7}$$

As predicted by Kepler, $T$ is proportional to $r^{3/2}$.

Deriving Kepler's third law by using Newton's law of gravitation has allowed us to calculate the constant that multiplies $r^{3/2}$. Note that the constant depends on the mass of the Sun; that is, *T depends on the mass being orbited*. It does not depend on the mass of the planet orbiting the Sun, however, as long as the planet's mass is much less than the mass of the Sun. As a result, Equation 12–7 applies equally to all the planets.

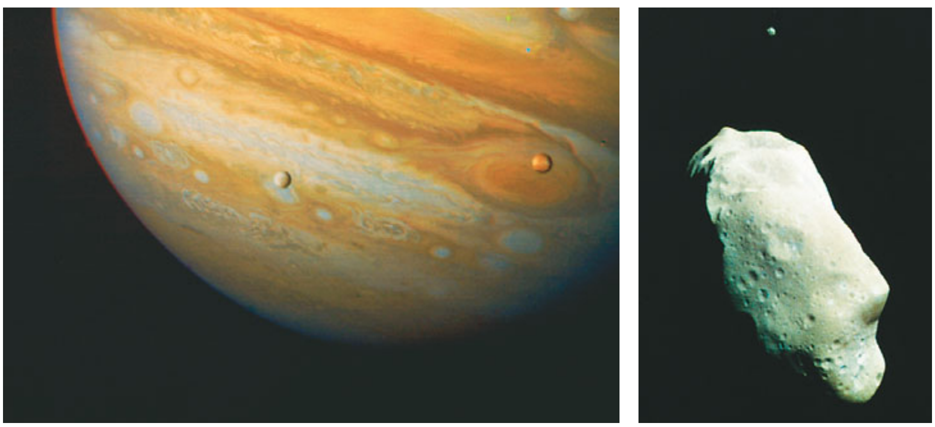
This result can also be applied to the case of a moon or a satellite (an artificial moon) orbiting a planet. To do so, we simply note that it is the planet that is being orbited, not the Sun. Hence, to apply Equation 12–7, we just replace the mass of the Sun, $M_s$, with the mass of the appropriate planet.

As an example, let's calculate the mass of Jupiter. One of the four moons of Jupiter discovered by Galileo is Io, which completes one orbit every 42 h 27 min =

**PROBLEM-SOLVING NOTE**

**The Mass in Kepler's Third Law**

When applying Kepler's third law, recall that the mass in Equation 12–7, $M_s$, refers to the mass of the object being orbited. Thus, the third law can be applied to satellites of any object, as long as $M_s$ is replaced by the orbited mass.



Kepler's laws of orbital motion apply to planetary satellites as well as planets. Jupiter, the largest planet in the solar system, has at least 16 moons, all of which travel in elliptical orbits that obey Kepler's laws. (The moons in the photo at left, passing in front of Jupiter, are Io and Europa, two of the four largest Jovian satellites discovered by Galileo in 1609.) Even some asteroids have been found to have their own satellites. The large cratered object in the photo at right is 243 Ida, an asteroid some 56 km long; its miniature companion at the top of the photo is Dactyl, about 1.5 km in diameter. Like all gravitationally bound bodies, Ida and Dactyl orbit their common center of mass.

$1.53 \times 10^5$ s. Given that the average distance from the center of Jupiter to Io is $4.22 \times 10^8$ m, we can find the mass of Jupiter as follows:

$$T = \left(\frac{2\pi}{\sqrt{GM_J}}\right) r^{3/2}$$

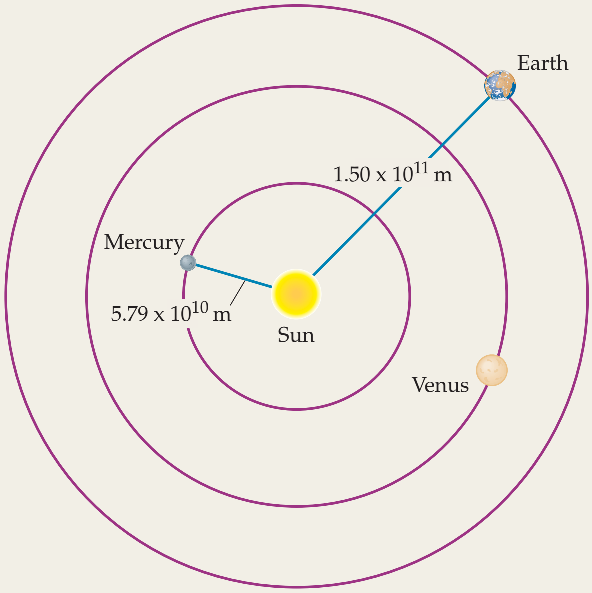
$$M_J = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(4.22 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.53 \times 10^5 \text{ s})^2} = 1.90 \times 10^{27} \text{ kg}$$

## EXAMPLE 12–4 THE SUN AND MERCURY

The Earth revolves around the Sun once a year at an average distance of $1.50 \times 10^{11}$ m. **(a)** Use this information to calculate the mass of the Sun. **(b)** Find the period of revolution for the planet Mercury, whose average distance from the Sun is $5.79 \times 10^{10}$ m.

### PICTURE THE PROBLEM

Our sketch shows the orbits of Mercury, Venus, and the Earth in correct proportion. In addition, each of these orbits is slightly elliptical, though the deviation from circularity is too small for the eye to see. Finally, we indicate that the orbital radius for Mercury is $5.79 \times 10^{10}$ m and the orbital radius for Earth is $1.50 \times 10^{11}$ m.




### STRATEGY

**a.** To find the mass of the Sun, we solve Equation 12–7 for $M_s$. Note that the period $T = 1$ yr must be converted to seconds before we evaluate the formula.

**b.** The period of Mercury is found by substituting $r = 5.79 \times 10^{10}$ m in Equation 12–7.

### SOLUTION

**Part (a)**

**1.** Solve Equation 12–7 for the mass of the Sun:

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right) r^{3/2}$$

$$M_s = \frac{4\pi^2 r^3}{GT^2}$$

**2.** Calculate the period of the Earth in seconds:

$$T = 1 \text{ y}\left(\frac{365.24 \text{ days}}{1 \text{ y}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right) = 3.16 \times 10^7 \text{ s}$$

**3.** Substitute numerical values in the expression for the mass of the Sun obtained in Step 1:

$$M_s = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2(1.50 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(3.16 \times 10^7 \text{ s})^2}$$

$$= 2.00 \times 10^{30} \text{ kg}$$

**Part (b)**

**4.** Substitute $r = 5.79 \times 10^{10}$ m into Equation 12–7. In addition, use the mass of the Sun obtained in part (a):

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right) r^{3/2}$$

$$= \left(\frac{2\pi}{\sqrt{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(2.00 \times 10^{30} \text{ kg})}}\right) \times (5.79 \times 10^{10} \text{ m})^{3/2}$$

$$= 7.58 \times 10^6 \text{ s} = 0.240 \text{ y} = 87.7 \text{ days}$$

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**INSIGHT**

In part (a), notice that the mass of the Sun is almost a million times more than the mass of the Earth, as determined in Exercise 12–3. In fact, the Sun accounts for 99.9% of all the mass in the solar system.

In part (b) we see that Mercury, with its smaller orbital radius, has a shorter year than the Earth.

**PRACTICE PROBLEM**

Venus orbits the Sun with a period of $1.94 \times 10^7$ s. What is its average distance from the Sun? [**Answer:** $r = 1.08 \times 10^{11}$ m]

*Some related homework problems: Problem 28, Problem 32*

▲ Many weather and communications satellites are placed in geosynchronous orbits that allow them to remain "stationary" in the sky—that is, fixed over one point on the Earth's equator. Because the Earth rotates, the period of such a satellite must exactly match that of the Earth. The altitude needed for such an orbit is about 36,000 km (see Active Example 12–1). Other satellites, such as those used in the Global Positioning System (GPS), the Hubble Space Telescope, and the American space shuttles, operate at much lower altitudes—typically just a few hundred miles. The photo at left shows the communications satellite Intelsat VI just prior to its capture by astronauts of the space shuttle *Endeavour*. A launch failure had left the satellite stranded in low orbit. The astronauts snared the satellite (right) and fitted it with a new engine that boosted it to its geosynchronous orbit, where it is still in operation today.

**REAL-WORLD PHYSICS**

**Geosynchronous satellites**

A *geosynchronous satellite* is one that orbits above the equator with a period equal to one day. From the Earth, such a satellite appears to be in the same location in the sky at all times, making it particularly useful for applications such as communications and weather forecasting. From Kepler's third law, we know that a satellite has a period of one day only if its orbital radius has a particular value. We determine this value in the following Active Example.

## ACTIVE EXAMPLE 12–1 FIND THE ALTITUDE OF A GEOSYNCHRONOUS SATELLITE

Find the altitude above the Earth's surface where a satellite orbits with a period of one day ($R_E = 6.37 \times 10^6$ m, $M_E = 5.97 \times 10^{24}$ kg, $T = 1$ day $= 8.64 \times 10^4$ s).

**SOLUTION** *(Test your understanding by performing the calculations indicated in each step.)*

1. Rewrite Equation 12–7, using the mass of the Earth in place of the mass of the Sun: $T = \left(2\pi/\sqrt{GM_E}\right)r^{3/2}$
2. Solve for the radius, $r$: $r = (T/2\pi)^{2/3}(GM_E)^{1/3}$
3. Substitute numerical values: $r = 4.22 \times 10^7$ m
4. Subtract the radius of the Earth to find the altitude: $r - R_E = 3.58 \times 10^7$ m

**INSIGHT**

Thus, *all* geosynchronous satellites orbit $3.58 \times 10^7$ m ≈ 22,300 mi above our heads.

**YOUR TURN**

Find the altitude above the surface of the Moon where a "lunasynchronous" satellite would orbit. [*Note:* The length of a lunar day is one month (27.332 days), which is why we see only one side of the Moon.]

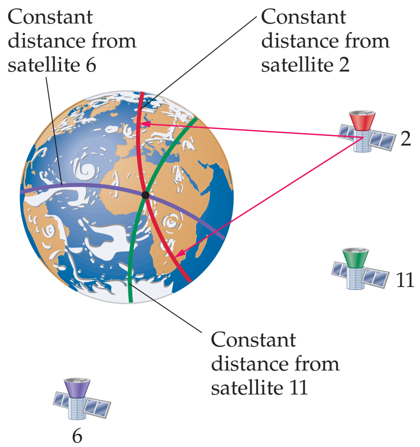
*(Answers to* **Your Turn** *problems are given in the back of the book.)*

Not all spacecraft are placed in geosynchronous orbits, however. The U.S. space shuttle, for example, orbits at an altitude of about 150 mi. At that altitude, it takes less than an hour and a half to complete one orbit. The International Space Station, operational although still under construction, orbits at a similar altitude.

**REAL-WORLD PHYSICS**

**The Global Positioning System (GPS)**

The 24 satellites of the Global Positioning System (GPS) are also in relatively low orbits. These satellites, which have an average altitude of 12,550 mi and orbit the Earth every 12 hours, are used to provide a precise determination of an observer's position anywhere on Earth. The operating principle of the GPS is illustrated in **Figure 12–12**. Imagine, for example, that satellite 2 emits a radio signal at a particular time (all GPS satellites carry atomic clocks on board). This signal travels away from the satellite with the speed of light (see Chapter 25) and is detected a short time later by an observer's GPS receiver. Multiplying the time delay by the speed of light gives the distance of the receiver from satellite 2. Thus, in our example, the observer must lie somewhere on the red circle in Figure 12–12. Similar time delay measurements for signals from satellite 11 show that the observer is also somewhere on the green circle; hence the observer is either at the point shown in Figure 12–12, or at the second intersection of the red and green circles on the other side of the planet. Measurements from satellite 6 can resolve the ambiguity and place the observer at the point shown in the figure. Measurements from additional satellites can even determine the observer's altitude. GPS receivers, which are used by hikers, boaters, and others who need to know their precise location, typically use signals from as many as 12 satellites. As currently operated, the GPS gives positions with a typical accuracy of 2 m to 10 m.




▲ **FIGURE 12–12 The Global Positioning System**

A system of 24 satellites in orbit about the Earth makes it possible to determine a person's location with great accuracy. Measuring the distance of a person from satellite 2 places the person somewhere on the red circle. Similar measurements using satellite 11 place the person's position somewhere on the green circle, and further measurements can pinpoint the person's location.

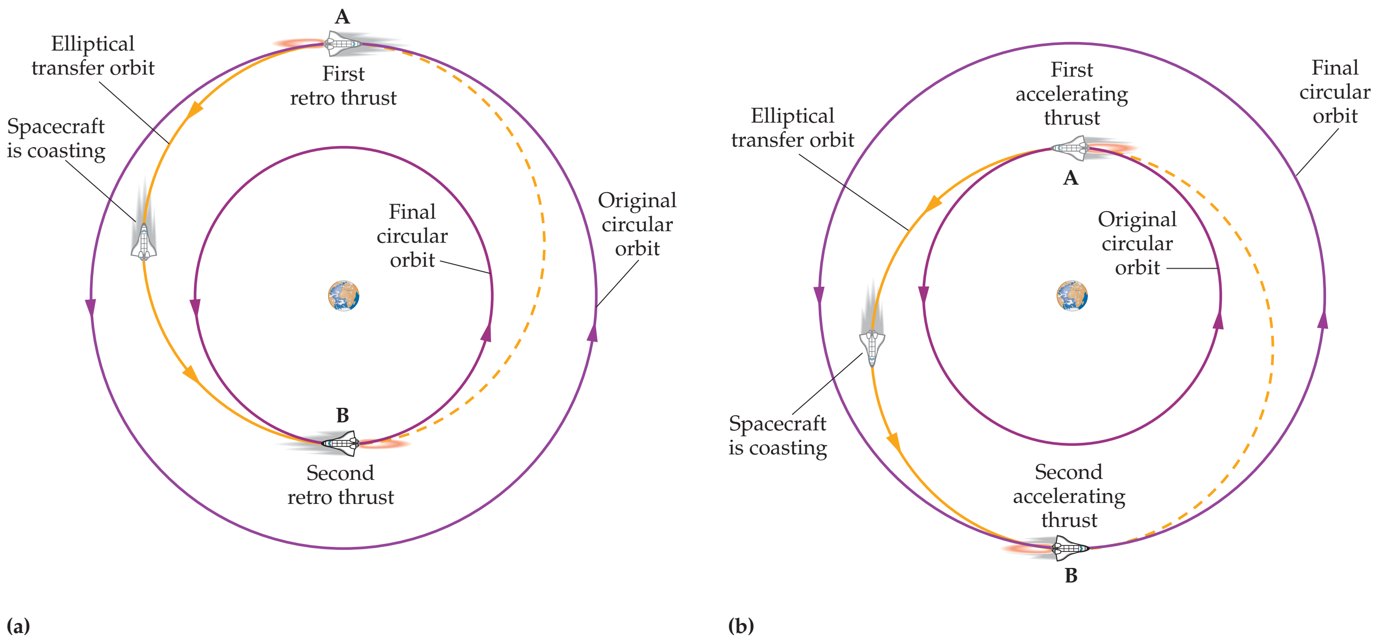
## Orbital Maneuvers

We now show how Kepler's laws can give insight into maneuvering a satellite in orbit. Suppose, for example, that you are piloting a spacecraft in a circular orbit, and you would like to move to a lower circular orbit. As you might expect, you should begin by using your rockets to decrease your speed—that is, fire the rockets that point in the forward direction so that their thrust (Section 9–8) is opposite to your direction of motion. The result of firing the decelerating rockets at a given point A in your original orbit is shown in **Figure 12–13 (a)**. Note that your new orbit is not a circle, as desired, but rather an ellipse. To produce a circular orbit you can simply fire the decelerating rockets once again at point B, on the opposite side of the Earth from point A. The net result of these two firings is that you now move in a circular orbit of smaller radius.

**REAL-WORLD PHYSICS**

**Maneuvering spacecraft**

Similarly, to move to a larger orbit, you must fire your accelerating rockets twice. The first firing puts you into an elliptical orbit that moves farther from the Earth, as **Figure 12–13 (b)** shows. After the second firing you are again in a circular orbit. This simplest type of orbital transfer, requiring just two rocket burns, is referred to as a *Hohmann transfer*. The Hohmann transfer is the basic maneuver used to send spacecraft such as the Mars lander from Earth's orbit about the Sun to the orbit of Mars.




▲ **FIGURE 12–13 Orbital maneuvers**

**(a)** The radius of a satellite's orbit can be decreased by firing the decelerating rockets once at point A and again at point B. Between firings the satellite follows an elliptical orbit. The satellite speeds up as it falls inward toward the Earth during this maneuver. For this reason its final speed in the new circular orbit is greater than its speed in the original orbit, even though the decelerating rockets have slowed it down twice. **(b)** The radius of a satellite's orbit can be increased by firing the accelerating rockets once at point A and again at point B. Between firings the satellite follows an elliptical orbit. The satellite slows down as it moves farther from the Earth during this maneuver. For this reason its final speed in the new circular orbit is less than its speed in the original orbit, even though the accelerating rockets have sped it up twice.

## CONCEPTUAL CHECKPOINT 12–2 WHICH ROCKETS TO USE?

As you pilot your spacecraft in a circular orbit about the Earth, you notice the space station you want to dock with several miles ahead in the same orbit. To catch up with the space station, should you **(a)** fire your accelerating rockets or **(b)** fire your decelerating rockets?

**REASONING AND DISCUSSION**

Since you want to catch up with something miles ahead, you must accelerate, right? Well, not in this case. Accelerating moves you into an elliptical orbit, as in Figure 12–13 (b), and with a second acceleration you can make your new orbit circular with a greater radius. Recall from Kepler's third law, however, that the larger the radius of an orbit the larger the period, as Equation 12–7 shows. Thus, on your new higher path you take longer to complete an orbit, so you fall farther behind the space station. The same is true even if you fire your rockets only once and stay on the elliptical orbit—it also has a longer period than the original orbit.

On the other hand, two decelerating burns will put you into a circular orbit of smaller radius, and thus smaller period. As a result, you complete an orbit in less time than before and catch up with the space station. After catching up, you can perform two accelerating burns to move you back into the original orbit to dock.

**ANSWER**

**(b)** You should fire your decelerating rockets.

## 12–4 Gravitational Potential Energy

In Chapter 8 we saw that the principle of conservation of energy can be used to solve a number of problems that would be difficult to handle with a straightforward application of Newton's laws of mechanics. Before we can apply energy conservation to astronomical situations, however, we must know the gravitational potential energy for a spherical object such as the Earth. Now you may be wondering, "Don't we already know the potential energy of gravity?" Well, in fact, in Chapter 8 we said that the gravitational potential energy a distance $h$ above the Earth's surface is $U = mgh$. As was mentioned at the time, however, this result is valid only near the Earth's surface, where we can say that the acceleration of gravity, $g$, is approximately constant.

As the distance from the Earth increases we know that $g$ decreases, as was shown in Example 12–2. It follows that $mgh$ cannot be valid for arbitrary $h$. Indeed, it can be shown that the gravitational potential energy of a system consisting of a mass $m$ a distance $r$ from the center of the Earth is

$$U = -G\frac{mM_E}{r} \qquad \text{12–8}$$

A plot of $U = -GmM_E/r$ is presented in **Figure 12–14**. Note that $U$ approaches zero as $r$ approaches infinity. This is a common convention in astronomical systems. In fact, since only *differences* in potential energy matter, as was mentioned in Chapter 8, the choice of the reference point ($U = 0$) is completely arbitrary. When we considered systems that were near the Earth's surface, it was natural to let $U = 0$ at ground level. When we consider, instead, distances of astronomical scale, it is generally more convenient to choose the potential energy to be zero when objects are separated by an infinite distance.

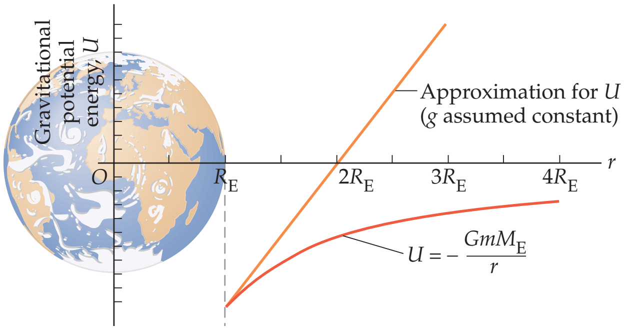



**FIGURE 12–14 Gravitational potential energy as a function of the distance $r$ from the center of the Earth**

The lower curve in this plot shows the gravitational potential energy, $U = -GmM_E/r$, for $r$ greater than $R_E$. Near the Earth's surface, $U$ is approximately linear, corresponding to the result $U = mgh$ given in Chapter 8.

## EXERCISE 12–4

Use Equation 12–8 to find the gravitational potential energy of a 12.0-kg meteorite when it is **(a)** one Earth radius above the surface of the Earth, and **(b)** on the surface of the Earth.

**SOLUTION**

**a.** In this case, the distance from the center of the Earth is $2R_E$, thus

$$U = -G\frac{mM_E}{2R_E}$$

$$= -(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})\frac{(12.0\,\mathrm{kg})(5.97 \times 10^{24}\,\mathrm{kg})}{2(6.37 \times 10^6\,\mathrm{m})} = -3.75 \times 10^8\,\mathrm{J}$$

**b.** Now, the distance from the center of the Earth is $R_E$, therefore

$$U = -\frac{GmM_E}{R_E}$$

$$= -(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})\frac{(12.0\,\mathrm{kg})(5.97 \times 10^{24}\,\mathrm{kg})}{6.37 \times 10^6\,\mathrm{m}} = -7.50 \times 10^8\,\mathrm{J}$$

Note that the potential energy in part (b) is twice what it was in part (a), since the distance from the center of the Earth to the meteorite has been halved.

At first glance, Equation 12–8 doesn't seem to bear any similarity to $mgh$, which we know to be valid near the surface of the Earth. Even so, there is a direct connection between these two expressions. Recall that when we say that the potential energy at a height $h$ is $mgh$, what we mean is that when a mass $m$ is raised from the ground to a height $h$, the potential energy of the system increases by the amount $mgh$. Let's calculate the corresponding difference in potential energy using Equation 12–8.

First, at a height $h$ above the surface of the Earth we have $r = R_E + h$; hence the potential energy there is

$$U = -G\frac{mM_E}{R_E + h}$$

On the surface of the Earth, where $r = R_E$, we have

$$U = -G\frac{mM_E}{R_E}$$

The corresponding difference in potential energy is

$$\begin{aligned}\Delta U &= \left(-G\frac{mM_E}{R_E + h}\right) - \left(-G\frac{mM_E}{R_E}\right)\\ &= \left(-G\frac{mM_E}{R_E}\right)\left(\frac{1}{1 + h/R_E}\right) - \left(-G\frac{mM_E}{R_E}\right)\end{aligned}$$

If $h$ is much smaller than the radius of the Earth, it follows that $h/R_E$ is a small number. In this case, we can apply the useful approximation $1/(1 + x) \approx 1 - x$ [see Figure A–5 (b) in Appendix A] to write $1/(1 + h/R_E) \approx 1 - h/R_E$. As a result, we have

$$\Delta U = \left(-G\frac{mM_E}{R_E}\right)(1 - h/R_E) - \left(-G\frac{mM_E}{R_E}\right) = m\left[\frac{GM_E}{R_E{}^2}\right]h$$

The term in square brackets should look familiar—according to Equation 12–4 it is simply $g$. Hence, the increase in potential energy at the height $h$ is

$$\Delta U = mgh$$

as expected.

The straight line in Figure 12–14 corresponds to the potential energy $mgh$. Near the Earth's surface, it is clear that $mgh$ and $-GmM_E/r$ are in close agreement. For larger $r$, however, the fact that gravity is getting weaker means that the potential energy does not continue rising as rapidly as it would if gravity were of constant strength.

An important distinction between the potential energy, $U$, and the gravitational force, $\vec{\mathbf{F}}$, is that the force is a vector, whereas the potential energy is a scalar—that is, $U$ is simply a number. As a result:

> The total gravitational potential energy of a system of objects is the sum of the gravitational potential energies of each pair of objects separately.

Since $U$ is not a vector, there are no $x$ or $y$ components to consider, as would be the case with a vector. Finally, the potential energy given in Equation 12–8 applies to a mass $m$ and the Earth, with mass $M_E$. More generally, if two point masses, $m_1$ and $m_2$, are separated by a distance $r$, their gravitational potential energy is

**Gravitational Potential Energy, *U***

$$U = -G\frac{m_1 m_2}{r} \qquad 12\text{–}9$$

SI unit: joule, J

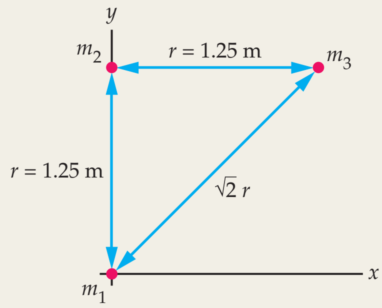
In the next Example we use this result, and the fact that $U$ is a scalar, to find the total gravitational potential energy for a system of three point masses.

**EXAMPLE 12–5** SIMPLE ADDITION

Three masses are positioned as follows: $m_1 = 2.5$ kg is at the origin; $m_2 = 0.75$ kg is at $x = 0$, $y = 1.25$ m; and $m_3 = 0.75$ kg is at $x = 1.25$ m and $y = 1.25$ m. Find the total gravitational potential energy of this system.

**PICTURE THE PROBLEM**

The masses and their positions are shown in our sketch. The horizontal and vertical distances are $r = 1.25$ m; the diagonal distance is $\sqrt{2}r$.




**STRATEGY**

The potential energy associated with each pair of masses is given by Equation 12–9. The total potential energy of the system is the sum of the potential energy for each of the three pairs of masses.

**SOLUTION**

1. Use Equation 12–9 to calculate the potential energy for masses 1 and 2:

$$U_{12} = -G\frac{m_1 m_2}{r_{12}}$$
$$= -(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(2.5\ \mathrm{kg})(0.75\ \mathrm{kg})}{(1.25\ \mathrm{m})}$$
$$= -1.0 \times 10^{-10}\ \mathrm{J}$$

2. Similarly, calculate the potential energy for masses 2 and 3:

$$U_{23} = -G\frac{m_2 m_3}{r_{23}}$$
$$= -(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(0.75\ \mathrm{kg})(0.75\ \mathrm{kg})}{(1.25\ \mathrm{m})}$$
$$= -3.0 \times 10^{-11}\ \mathrm{J}$$

3. Do the same calculation for masses 1 and 3:

$$U_{13} = -G\frac{m_1 m_3}{r_{13}}$$
$$= -(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(2.5\ \mathrm{kg})(0.75\ \mathrm{kg})}{\sqrt{2}(1.25\ \mathrm{m})}$$
$$= -7.1 \times 10^{-11}\ \mathrm{J}$$

4. The total potential energy is the sum of the three contributions calculated above:

$$U_{\text{total}} = U_{12} + U_{23} + U_{13}$$
$$= -1.0 \times 10^{-10}\ \mathrm{J} - 3.0 \times 10^{-11}\ \mathrm{J} - 7.1 \times 10^{-11}\ \mathrm{J}$$
$$= -2.0 \times 10^{-10}\ \mathrm{J}$$

**INSIGHT**

Note that the total gravitational potential energy of this system, $U_{\text{total}} = -2.0 \times 10^{-10}$ J, is less than it would be if the separation of the masses were to approach infinity, in which case $U_{\text{total}} = 0$. The implications of this change in potential energy, in terms of energy conservation, are considered in the next section.

**PRACTICE PROBLEM**

If the distance $r = 1.25$ m is reduced by a factor of two to $r = 0.625$ m, does the potential energy of the system increase, decrease, or stay the same? Verify your answer by calculating the potential energy in this case. [**Answer:** The potential energy decreases; that is, it becomes more negative. We find $U = 2(-2.0 \times 10^{-10}\ \mathrm{J})$.]

*Some related homework problems: Problem 42, Problem 43*

## 12–5 Energy Conservation

Now that we know the gravitational potential energy, $U$, at an arbitrary distance from a spherical object, we can apply energy conservation to astronomical situations in the same way we applied it to systems near the Earth's surface in Chapter 8. To be specific, the mechanical energy, $E$, of an object of mass $m$ a distance $r$ from the Earth is

$$E = K + U = \tfrac{1}{2}mv^2 - G\frac{mM_E}{r} \qquad 12\text{–}10$$

Using energy conservation—that is, setting the initial mechanical energy equal to the final mechanical energy—we can answer questions such as the following: Suppose that an asteroid has zero speed infinitely far from the Earth. If this asteroid were to fall directly toward the Earth, what speed would it have when it strikes the Earth's surface?

**REAL-WORLD PHYSICS**

**The impact of meteorites**

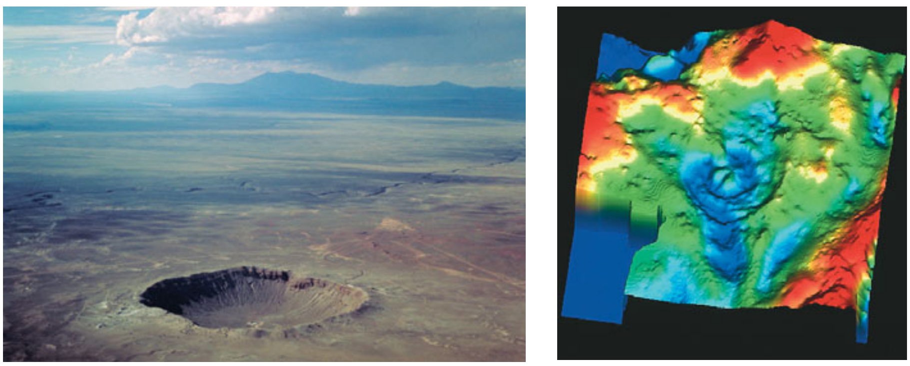
As you probably know, this is not an entirely academic question. Asteroids and comets, both large and small, have struck the Earth innumerable times during its history. In fact, a particularly large object appears to have struck the Earth on the Yucatan Peninsula in Mexico, near the town of Chicxulub, some 65 million years ago. Evidence suggests that this impact may have led to the mass extinctions of the Cretaceous period, during which the dinosaurs disappeared from the Earth. Unfortunately, such events are not limited to the distant past. For example, as recently as 50,000 years ago, an iron asteroid tens of meters in diameter and shining 10,000 times brighter than the Sun (from atmospheric heating) slammed into the ground near Winslow, Arizona, forming the 1.2-km-wide Barringer Meteor Crater. More recently yet, at sunrise on June 30, 1908, a relatively small stony asteroid streaked through the atmosphere and exploded at an altitude of several kilometers near the Tunguska River in Siberia. The energy released by the explosion was comparable to that of an H-bomb, and it flattened the forest for kilometers in all directions. One can only imagine the consequences if an event like this were to occur near a populated area. Finally, an uncomfortably close call occurred in the early evening of December 9, 1994, when an asteroid the size of a mountain passed the Earth at a distance only one-third the distance from the Earth to the Moon. Thus, though extremely unlikely, the scenarios depicted in movies such as *Armageddon* and *Deep Impact* are not completely unrealistic.

**PROBLEM-SOLVING NOTE**

**Energy Conservation in Astronomical Systems**

To apply conservation of energy to an object that moves far from the surface of a planet, one must use $U = -GmM/r$, where $r$ is the distance from the center of the planet.

Returning to our original question, we can use energy conservation to determine the speed such an asteroid or comet might have when it hits the Earth. To begin, we assume the asteroid starts at rest, and hence its initial kinetic energy is zero, $K_i = 0$. In addition, the initial potential energy of the system, $U_i$, is also zero, since $U = -GmM_E/r$ approaches zero as $r$ approaches infinity. As a result, the total initial mechanical energy of the asteroid–Earth system is zero: $E_i = K_i + U_i = 0$. Because gravity is a conservative force (as discussed in Section 8–1), the total mechanical energy remains constant as the asteroid falls toward the Earth. Thus, as the



▲ Bodies from space have struck the Earth countless times in the past and continue to do so on a regular basis. Most such objects are relatively small, ranging in size from grains of dust to fist-sized rocks, and burn up from friction as they pass through the atmosphere, creating the bright streaks that we know as meteors. But larger objects, including the occasional comet or asteroid, also cross our path from time to time, and some of these make it to the surface—often with very dramatic results. The crater above, in Arizona, must be of relatively recent origin (thousands rather than millions of years old), since erosion has not yet erased this scar on the Earth's surface.

The image at right is a false-color gravity anomaly map of the Chicxulub impact crater in Mexico. The object that struck here some 65 million years ago may have produced such far-reaching climatic disruption that the dinosaurs and many other species became extinct as a result. At the center of the crater the strength of gravity is lower than normal (blue) because of the presence of low-density rock: debris from the impact and sediments that have accumulated in the crater.

asteroid moves closer to the Earth and $U$ becomes increasingly negative, the kinetic energy $K$ must become increasingly positive so that their sum, $U + K$, is always zero.

We now set the initial energy equal to the final energy to determine the final speed, $v_f$. Recalling that the final distance $r$ is the radius of the Earth, $R_E$, we have

$$E_i = E_f$$
$$0 = \frac{1}{2}mv_f^2 - G\frac{mM_E}{R_E}$$

Solving for the final speed yields

$$v_f = \sqrt{\frac{2GM_E}{R_E}} \qquad \text{12–11}$$

Substituting numerical values into this expression gives

$$v_f = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\,\text{kg})}{6.37 \times 10^6\,\text{m}}}$$
$$= 11{,}200\,\text{m/s}\ (25{,}000\,\text{mi/h}) \qquad \text{12–12}$$

Thus, a typical asteroid hits the Earth moving at about 7.0 mi/s—about 16 times faster than a rifle bullet! Note that this result is independent of the asteroid's mass.

To help visualize energy conservation in this system, we plot the gravitational potential energy $U$ in **Figure 12–15**. Also indicated in the plot is the total energy, $E = 0$. Since $U + K$ must always equal zero, the value of $K$ goes up as the value of $U$ goes down. This is illustrated graphically in the figure with the help of several histogram bars.

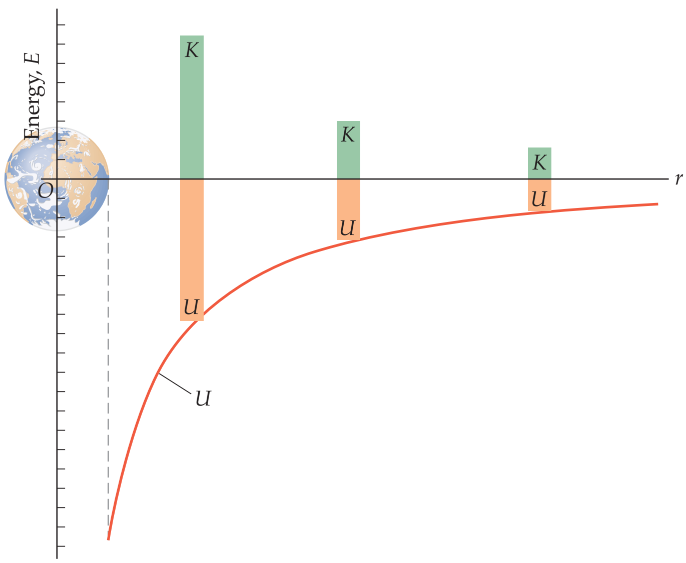



**FIGURE 12–15 Potential and kinetic energies of an object falling toward Earth**
As an object with zero total energy moves closer to the Earth, its gravitational potential energy, $U$, becomes increasingly negative. In order for the total energy to remain zero, $E = U + K = 0$, it is necessary for the kinetic energy to become increasingly positive.

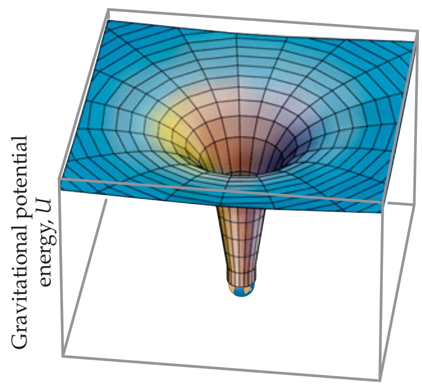



**FIGURE 12–16 A gravitational potential "well"**
The illustration is a three-dimensional plot of the gravitational potential energy near an object such as the Earth. An object approaching the Earth speeds up as it "falls" into the gravitational potential well.

Another way to think about this is to imagine a smooth wooden or plastic surface constructed to have the same shape as the plot of $U$ shown in Figure 12–15. An object placed on this surface has a gravitational potential energy proportional to the height of the surface above a given reference level. Thus, if a small block is allowed to slide without friction on the surface, it will move downhill and speed up as it drops lower in elevation. That is, its kinetic energy will increase as the potential energy of the system decreases. This is completely analogous to the behavior of an asteroid as it "falls" toward the Earth.

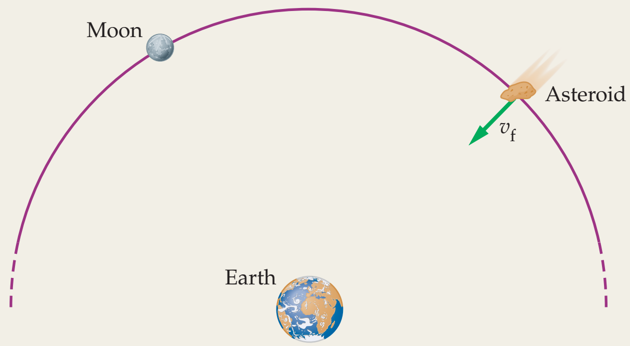
A somewhat more elaborate plot showing the same physics is presented in **Figure 12–16**. The two-dimensional surface in this case represents the potential energy function $U$ as one moves away from the Earth in any direction. In particular, the dependence of $U$ on distance $r$ along any radial line in Figure 12–16 is the same as the shape of $U$ versus $r$ in Figure 12–15. Because the potential energy drops downward in such a plot, this type of situation is often referred to as a "potential well." If a marble is allowed to roll on such a surface, its motion is similar in many ways to the motion of an object near the Earth. In fact, if the marble is started with the right initial velocity, it will roll in a circular or elliptical "orbit" for a long time before falling into the center of the well. (Eventually, of course, the well does swallow up the marble. Though the retarding force of rolling friction is quite small, it still causes the marble to descend into a lower and lower orbit—just as air resistance causes a satellite to descend lower and lower into the Earth's atmosphere until it finally burns up.)

## EXAMPLE 12–6 ARMAGEDDON RENDEZVOUS

In the movie *Armageddon*, a crew of hard-boiled oil drillers rendezvous with a menacing asteroid just as it passes the orbit of the Moon on its way toward Earth. Assuming the asteroid starts at rest infinitely far from the Earth, as in the previous discussion, find its speed when it passes the Moon's orbit. Assume the Moon orbits at a distance of $60R_E$ from the center of the Earth and that its gravitational influence may be neglected.

### PICTURE THE PROBLEM

Our sketch shows the Earth, the Moon, and the asteroid. The initial position of the asteroid is at infinity, where its speed is zero. For the purposes of this problem, its final position is at the Moon's orbit, where its speed is $v_f$. At this point, the asteroid is heading directly for the Earth.




### STRATEGY

The basic strategy is the same as that used to obtain the speed of an asteroid in Equation 12–12; namely, we set the initial energy equal to the final energy and solve for the final speed $v_f$. In this case, the final radius is $r = 60R_E$. As before, the initial energy is zero.

### SOLUTION

1. Set the initial energy of the system equal to its final energy:

$$E_i = E_f$$

$$0 = \tfrac{1}{2}mv_f^2 - G\frac{mM_E}{60R_E}$$

2. Solve for the final speed, $v_f$:

$$v_f = \sqrt{\frac{2GM_E}{60R_E}} = \frac{1}{\sqrt{60}}\left(\sqrt{\frac{2GM_E}{R_E}}\right)$$

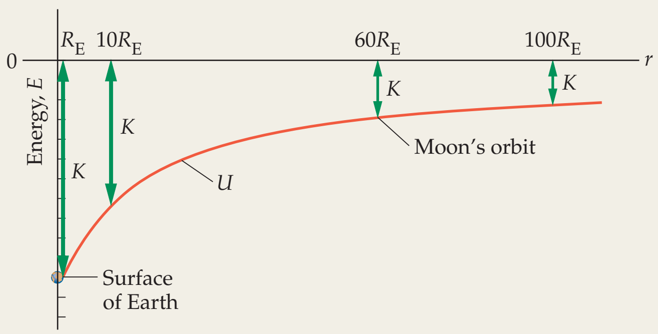
3. Substitute the numerical value given in Equation 12–12 for the quantity in parentheses:

$$v_f = \frac{1}{\sqrt{60}}(11{,}200\ \mathrm{m/s}) = 1450\ \mathrm{m/s} \sim 3200\ \mathrm{mi/h}$$

### INSIGHT

Note that the majority of the asteroid's increase in speed occurs after it passes the Moon. The reason for this can be seen in the accompanying plot of the gravitational potential energy, $U$.

Note that $U$ drops downward more and more rapidly as the Earth is approached. Thus, while there is relatively little increase in $K$ from infinite distance to $r = 60R_E$, there is a substantially larger increase in $K$ from $r = 60R_E$ to $r = R_E$.




### PRACTICE PROBLEM

At what distance from the center of the Earth is the asteroid's speed equal to 3535 m/s? **[Answer:** $r = 6.37 \times 10^7$ m $= 10R_E$]

*Some related homework problems: Problem 50, Problem 52*

## ACTIVE EXAMPLE 12–2 FIND THE DISTANCE TO A SATELLITE

A satellite in an elliptical orbit has a speed of 9.00 km/s when it is at its closest approach to the Earth (perigee). The satellite is $7.00 \times 10^6$ m from the center of the Earth at this time. When the satellite is at its greatest distance from the center of the Earth (apogee), its speed is 3.66 km/s. How far is the satellite from the center of the Earth at apogee? ($R_E = 6.37 \times 10^6$ m, $M_E = 5.97 \times 10^{24}$ kg)

**SOLUTION** *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the energy at perigee, $E_1$, equal to the energy at apogee, $E_2$: $\frac{1}{2}mv_1^2 - GmM_E/r_1 = \frac{1}{2}mv_2^2 - GmM_E/r_2$
2. Solve for $1/r_2$: $1/r_2 = 1/r_1 + (v_2^2 - v_1^2)/(2GM_E)$
3. Substitute numerical values: $1/r_2 = 5.80 \times 10^{-8}\ \text{m}^{-1}$
4. Invert to obtain $r_2$: $r_2 = 1.72 \times 10^7$ m

**INSIGHT**

In this case, apogee is about 2.5 times farther from the center of the Earth than perigee.

**YOUR TURN**

What is the speed of the satellite when it is $8.75 \times 10^6$ m from the center of the Earth?

*(Answers to* **Your Turn** *problems are given in the back of the book.)*

▲ Comet Hale-Bopp, one of the largest and brightest comets to visit our celestial neighborhood in recent decades, photographed in April 1997. While most of the planets and planetary satellites in the solar system have roughly circular orbits, the orbits of many comets are highly elliptical. In accordance with Kepler's second law, these objects spend most of their time moving slowly through cold, distant regions of the solar system (often far beyond the orbit of Pluto). Their visits to the inner solar system are infrequent and relatively brief.

### Escape Speed

Resisting the pull of Earth's gravity has always held a fascination for the human species, from Daedalus and Icarus with their wings of feathers and wax, to Leonardo da Vinci and his flying machine, to the Montgolfier brothers and their hot-air balloons. In his 1865 novel, *From the Earth to the Moon*, Jules Verne imagined launching a spacecraft to the Moon by firing it straight upward from a cannon. Not a bad idea—if you could survive the initial blast. Today, rockets are fired into space using the same basic idea, though they smooth out the initial blast by burning their engines over a period of several minutes.

Suppose, then, that you would like to launch a rocket of mass $m$ with an initial speed sufficient not only to reach the Moon, but to allow it to escape the Earth altogether. If we refer to this speed as the **escape speed** for the Earth, $v_e$, the initial energy of the rocket is

$$E_i = K_i + U_i = \frac{1}{2}mv_e^2 - G\frac{mM_E}{R_E}$$

If the rocket just barely escapes the Earth, its speed will decrease to zero as its distance from the Earth approaches infinity. Therefore, the rocket's final kinetic energy is zero, as is the potential energy of the system, since $U = -GmM_E/r$ goes to zero as $r \to \infty$. It follows that

$$E_f = K_f + U_f = 0 - 0 = 0$$

Equating these energies yields

$$\frac{1}{2}mv_e^2 - G\frac{mM_E}{R_E} = 0$$

Therefore, the escape speed from the Earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = 11{,}200\ \text{m/s} \approx 25{,}000\ \text{mi/h} \qquad \text{12–13}$$

Note that the escape speed is precisely the same as the speed of the asteroid calculated in Equation 12–12. This is not surprising when you consider that an object launched from the Earth to infinity is just the reverse of an object falling from infinity to the Earth.

The result given in Equation 12–13 can be applied to other astronomical objects as well by simply replacing $M_E$ and $R_E$ with the appropriate mass and radius for that object.

## EXERCISE 12–5

Calculate the escape speed for an object launched from the Moon.

**SOLUTION**

For the Moon we use $M_m = 7.35 \times 10^{22}$ kg and $R_m = 1.74 \times 10^{6}$ m. With these values, the escape speed is

$$v_e = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(7.35 \times 10^{22}\ \mathrm{kg})}{1.74 \times 10^{6}\ \mathrm{m}}}$$
$$= 2370\ \mathrm{m/s}\ (5320\ \mathrm{mi/h})$$

The relatively low escape speed of the Moon means that it is much easier to launch a rocket into space from the Moon than from the Earth. For example, the tiny lunar module that blasted off from the Moon to return the astronauts to Earth could not have come close to escaping from the Earth.

**REAL-WORLD PHYSICS**
**Planetary atmospheres**

Similarly, the Moon's low escape speed is the reason it has no atmosphere. Even if you could magically supply the Moon with an atmosphere, it would soon evaporate into space because the individual molecules in the air move with speeds great enough to escape. On the Earth, however, where the escape speed is much higher, gravity can prevent the rapidly moving molecules from moving off into space. Even so, light molecules, like hydrogen and helium, move faster for a given temperature than the heavier molecules like nitrogen and oxygen, as we shall see in Chapter 17. For this reason, the Earth's atmosphere contains virtually no hydrogen or helium. (In fact, helium was first discovered on the Sun, as we point out in Chapter 31; hence its name, which derives from the Greek word for the Sun, "helios.") Since a stable atmosphere is a likely requirement for the development of life, it follows that the escape speed is an important quantity when considering the possibility of life on other planets.

## CONCEPTUAL CHECKPOINT 12–3 COMPARE ESCAPE SPEEDS

Is the escape speed for a 10-N rocket **(a)** equal to, **(b)** less than, or **(c)** greater than the escape speed for a 10,000-N rocket?

**REASONING AND DISCUSSION**

The derivation of the escape speed in Equation 12–13 shows that the mass of the rocket, $m$, cancels. Hence, the escape speed is the same for all objects, regardless of their mass. On the other hand, the kinetic energy required to give the 10,000-N rocket the escape speed is 1000 times greater than the kinetic energy required for the 10-N rocket.

**ANSWER**

**(a)** Equal. The escape speed is independent of the mass that is escaping.

## EXAMPLE 12–7 HALF ESCAPE

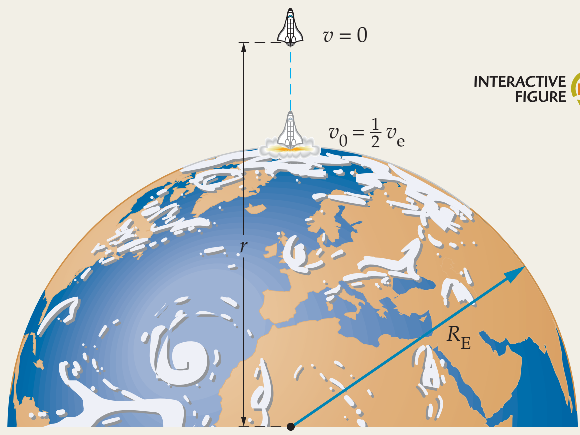
Suppose Jules Verne's cannon launches a rocket straight upward with an initial speed equal to one-half the escape speed. How far from the center of the Earth does this rocket travel before momentarily coming to rest? (Ignore air resistance in the Earth's atmosphere.)

**PICTURE THE PROBLEM**

Our sketch shows the rocket launched vertically from the Earth's surface with an initial speed equal to half the escape speed, $v_0 = \frac{1}{2}v_e$. The rocket moves radially away from the Earth until it comes to rest, $v = 0$, at a distance $r$ from the center of the Earth.

**STRATEGY**

Since we ignore air resistance, the final energy of the rocket, $E_f$, must be equal to its initial energy, $E_0$. Setting these energies equal determines the point where the rocket comes to rest.




**SOLUTION**

1. The initial speed, $v_0$, is one-half the escape speed. Use Equation 12–13 to write an expression for $v_0$:

$$v_0 = \tfrac{1}{2}v_e = \tfrac{1}{2}\sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{GM_E}{2R_E}}$$

2. Write out the initial energy of the rocket, $E_0$:

$$E_0 = K_0 + U_0 = \tfrac{1}{2}mv_0^2 - \frac{GmM_E}{R_E}$$

$$= \tfrac{1}{2}m\left(\sqrt{\frac{GM_E}{2R_E}}\right)^2 - \frac{GmM_E}{R_E} = -\frac{3}{4}\frac{GmM_E}{R_E}$$

3. Write out the final energy of the rocket. Note that the rocket is a distance $r$ from the center of the Earth when it comes to rest:

$$E_f = K_f + U_f = 0 - \frac{GmM_E}{r} = -\frac{GmM_E}{r}$$

4. Equate the initial and final energies:

$$-\frac{3}{4}\frac{GmM_E}{R_E} = -\frac{GmM_E}{r}$$

5. Solve the relation for $r$:

$$r = \tfrac{4}{3}R_E$$

**INSIGHT**

An initial speed of $v_e$ allows the rocket to go to infinity before stopping. If the rocket is launched with half that initial speed, however, it can only rise to a height of $4R_E/3 - R_E = R_E/3$ above the Earth's surface. Quite a dramatic difference.

**PRACTICE PROBLEM**

Find the rocket's maximum distance from the center of the Earth, $r$, if its launch speed is $3v_e/4$. **[Answer:** $r = 16R_E/7 = 2.29R_E$]

*Some related homework problems: Problem 49, Problem 56*

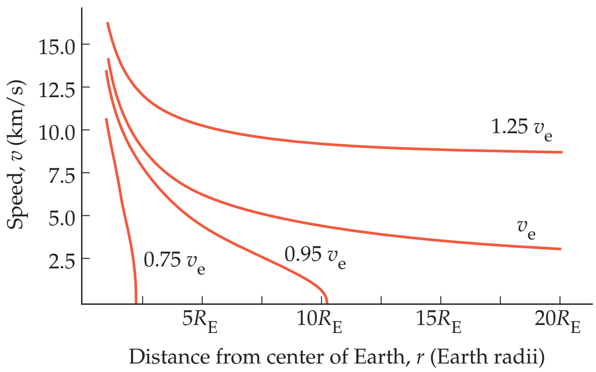



**FIGURE 12–17 Speed of a rocket as a function of distance from the center of the Earth, *r*, for various vertical launch speeds**

The lower two curves show launch speeds that are less than the escape speed, $v_e$. In these cases the rocket comes to rest momentarily at a finite height above the Earth. The next higher curve shows the speed of a rocket launched with the escape speed, $v_e$. In this case, the rocket slows to zero speed as the distance approaches infinity. The top curve corresponds to a launch speed greater than the escape speed—this rocket has a finite speed even at infinite distance.

A plot of the speed of a rocket as a function of its distance from the center of the Earth is presented in **Figure 12–17** for a variety of initial speeds. Note that when the initial speed is less than the escape speed, the rocket comes to rest momentarily at a finite distance, $r$. In particular, if the launch speed is $0.75v_e$, as in the Practice Problem of Example 12–7, the rocket's maximum distance from the center of the Earth is $2.29R_E$.

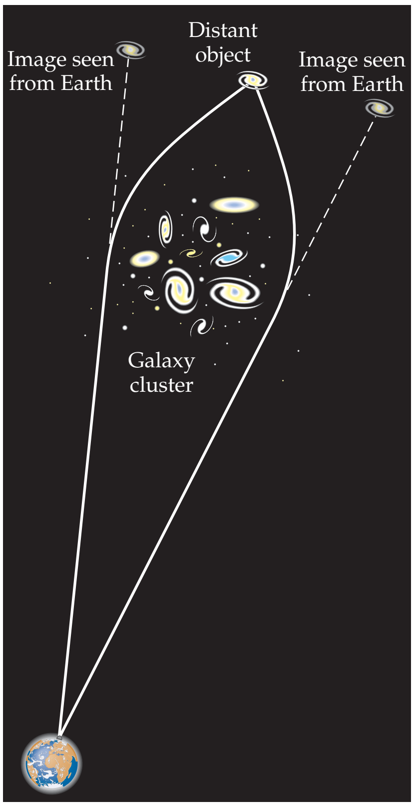
## Black Holes

As we can see from Equation 12–13, the escape speed of an object increases with increasing mass and decreasing radius. Thus, for example, if a massive star were to collapse to a relatively small size, its escape speed would become very large. According to Einstein's theory of general relativity, the escape speed of a compressed, massive star could even exceed the speed of light. In this case nothing—not even light—could escape from the star. For this reason, such objects are referred to as *black holes*. Anything entering a black hole would be making a one-way trip to an unknown destiny.

Since black holes cannot be seen directly, our evidence for their existence is indirect. However, we can predict that as matter is drawn toward a black hole it should become heated to the point where it would emit strong beams of X-rays before disappearing from view. X-ray beams matching these predictions have in fact been observed. These observations, coupled with a variety of others, give astronomers confidence that massive black holes reside at the core of many galaxies—including our own!

Finally, just as a black hole can bend a beam of light back on itself and prevent it from escaping, any massive object can bend light—at least a little. For example, light from distant stars is deflected as it passes by the Sun by 1.75 seconds of an arc (the size of a quarter at a distance of 1.8 miles). Light passing by an entire galaxy of stars or cluster of galaxies can be bent by significant amounts, however, as **Figure 12–18** indicates. This effect is referred to as *gravitational lensing*, since the galaxies act much like the lenses we will study in Chapter 26. Because of gravitational lensing, the images of very distant galaxies or quasars in deep-space astronomical photographs sometimes appear in duplicate, in quadruplicate, or even spread out into circular arcs.

**REAL-WORLD PHYSICS**
**Black holes and gravitational lensing**



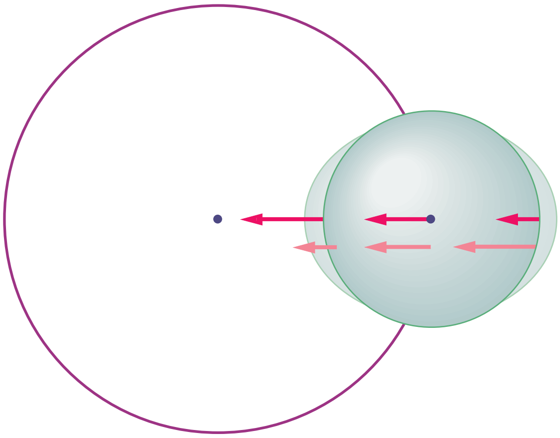

▲ **FIGURE 12–18 Gravitational lensing**
Astronomers often find that very distant objects seem to produce multiple images in their photographs. The cause is the gravitational attraction of intervening galaxies or clusters of galaxies, which are so massive that they can significantly bend the light from remote objects as it passes by them on its way to Earth.

## *12–6 Tides

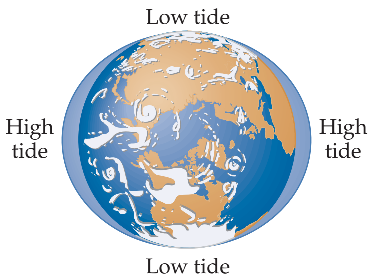
The reason for the ocean tides that rise and fall twice a day was a perplexing and enduring mystery until Newton introduced his law of universal gravitation. Even Galileo, who made so many advances in physics and astronomy, could not explain the tides. However, with the understanding that a force is required to cause an object to move in a circular path, and that the force of gravity becomes weaker with distance, it is possible to describe the tides in detail. In this section we show how it can be done. In addition, we extend the basic idea of tides to several related phenomena.

To begin, consider the idealized situation shown in **Figure 12–19 (a)**. Here we see an object of finite size (a moon or a planet, for example) orbiting a point mass. If all the mass of the object were concentrated at its center, the gravitational force exerted on it by the central mass would be precisely the amount needed to cause it to move in its circular path. Since the object is of finite size, however, the force exerted on various parts of it has different magnitudes. For example, points closer to the central mass experience a greater force than points farther away.

To see the effect of this variation in force, we use a dark red vector in Figure 12–19 (a) to indicate the force exerted by the central mass at three different points on the object. In addition, we use a light red vector to show the force that is required at each of these three points to cause a mass at that distance to orbit the central mass. Comparing these vectors, we see that the forces are identical at the center of the object—as expected. On the near side of the object, however, the force exerted by the central mass is larger than the force needed to hold the object in orbit, and on the far side the force due to the central mass is less than the force needed to hold the object in orbit. The result is that the near side of the object is pulled closer to the central mass and the far side tends to move farther from the central mass. This causes an egg-shaped deformation of the object, as indicated in Figure 12–19 (a).



**(a)** The mechanism responsible for tides




**(b)** Tidal deformations on Earth

▲ **FIGURE 12–19 The reason for two tides a day**

**(a)** Tides are caused by a disparity between the gravitational force exerted at various points on a finite-sized object (dark red arrows) and the centripetal force needed for circular motion (light red arrows). Note that the gravitational force decreases with distance, as expected. On the other hand, the centripetal force required to keep an object moving in a circular path *increases* with distance. On the near side, therefore, the gravitational force is stronger than required, and the object is stretched inward. On the far side, the gravitational force is weaker than required, and the object stretches outward. **(b)** On the Earth, the water in the oceans responds more to the deforming effects of tides than do the solid rocks of the land. The result is two high tides and two low tides daily on opposite sides of the Earth.

**REAL-WORLD PHYSICS**
**Tides**

Any two objects orbiting one another cause deformations of this type. For example, the Earth causes a deformation in the Moon, and the Moon causes a similar deformation in the Earth. In **Figure 12–19 (b)** we show the Earth and the waters of its ocean deformed into an egg shape. Since the waters in the oceans can flow, they deform much more than the underlying rocky surface of the Earth. As a result, the water level relative to the surface of the Earth is greater at the *tidal bulges* shown in the figure. As the Earth rotates about its axis, a person at a given location will observe two high tides and two low tides each day. This is the basic mechanism of the tides on Earth, but the actual situation is complicated by the shape of the coastline at different locations and by the additional tidal effects due to the Sun.

The Moon has no oceans, of course, but the tidal bulges produced in it by the Earth are the reason we see only one side of the Moon. Specifically, the Earth exerts gravitational forces on the tidal bulges of the Moon, causing them to point directly toward the Earth. If the Moon were to rotate slightly away from this alignment, the forces exerted by the Earth would cause a torque that would return the Moon to the original alignment. The net result is that the Moon's period of rotation about its axis is equal to its period of revolution about the Earth. This effect, known as **tidal locking**, is common among the various moons in the solar system.

**REAL-WORLD PHYSICS**
**Tidal locking**

A particularly interesting example of tidal locking is provided by Jupiter's moon Io, a site of intense volcanism (see the photo on p. 107). Io follows an elliptical orbit around Jupiter, and its tidal deformation is larger when it is closer to Jupiter than when it is farther away. As a result, each time Io orbits Jupiter it is squeezed into a greater deformation and then released. This continual flexing of

◀ Tides on Earth are caused chiefly by the Moon's gravitational pull, though at full and new moon, when the Moon and Sun are aligned, the Sun's gravity can enhance the effect. In some places on Earth, such as the Bay of Fundy between Maine and Nova Scotia, local topographic conditions produce abnormally large tides.

Io causes its internal temperature to rise, just as a rubber ball gets warmer if you squeeze and release it in your hand over and over. It is this mechanism that is largely responsible for Io's ongoing volcanic activity.

**REAL-WORLD PHYSICS**
**Roche limit and Saturn's rings**

In extreme cases, tidal deformation can become so large that an object is literally torn apart. Since tidal deformation increases as a moon moves closer to the planet it orbits, there is a limiting orbital radius—known as the **Roche limit**—inside of which this breakup occurs. A most spectacular example of this effect can be seen in the rings of Saturn, all of which exist well within the Roche limit. The small chunks of ice and other materials that make up the rings may be the remains of a moon that moved too close to Saturn and was destroyed by tidal forces. On the other hand, they may represent material that tidal forces prevented from aggregating to form a moon in the first place. In either case, this dramatic debris field will now never coalesce to form a moon—tidal effects will not allow such a process to occur. Similar remarks apply to the smaller, much fainter rings that spacecraft have observed around Jupiter, Uranus, and Neptune.

## THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

### LOOKING BACK

The general force of gravity, as presented in Equation 12–1, is a vector quantity. Therefore, vector calculations (Chapter 3) are important here. See, in particular, Example 12–1. We also use the connection between force and acceleration, $F = ma$ (Chapter 5), in Section 12–2.

The conservation of angular momentum (Chapter 11) plays a key role in gravity, leading to Kepler's second law in Section 12–3.

Just as the force of gravity is generalized in this chapter, so too is the gravitational potential energy. Thus, the expression $U = mgh$ (Chapter 8) is generalized to $U = -Gm_1m_2/r$ in Section 12–4. We then use this new form of the potential energy in situations involving energy conservation in Section 12–5.

### LOOKING AHEAD

In Chapter 19 we shall see that the force between two electric charges, denoted $q_1$ and $q_2$, has exactly the same form as the general force of gravity between two masses, Equation 12–1. The electric force is referred to as Coulomb's law, and is presented in Equation 19–5.

The force between electric charges is conservative, and hence it leads to an electric potential energy that has the same form as the gravitational potential energy in Section 12–4. See Sections 20–1 and 20–2.

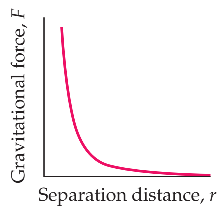
The analysis used to derive Kepler's third law in Section 12–3 is used again when we explore the Bohr model of the hydrogen atom in Chapter 31. The calculation is the same, but in hydrogen the Coulomb force between electric charges (Equation 19–5) is responsible for the orbital motion.

## CHAPTER SUMMARY

### 12–1 NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of gravity between two point masses, $m_1$ and $m_2$, separated by a distance $r$ is attractive and of magnitude

$$F = G\frac{m_1m_2}{r^2} \qquad 12\text{–}1$$




$G$ is the universal gravitation constant:

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2} \qquad 12\text{–}2$$

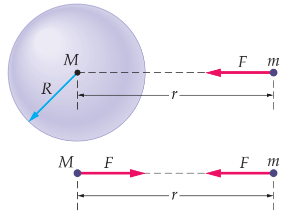
Gravity exerts an action-reaction pair of forces on $m_1$ and $m_2$; that is, the force exerted by gravity on $m_1$ is equal in magnitude but opposite in direction to the force exerted on $m_2$.

**Inverse Square Dependence**
The force of gravity decreases with distance, $r$, as $1/r^2$. This is referred to as an inverse square dependence.

**Superposition**
If more than one mass exerts a gravitational force on a given object, the net force is simply the vector sum of each force individually.

## 12–2 GRAVITATIONAL ATTRACTION OF SPHERICAL BODIES

In calculating gravitational forces, spherical objects can be replaced by point masses.




**Uniform Sphere**
If a mass $m$ is outside a uniform sphere of mass $M$, the gravitational force between $m$ and the sphere is equivalent to the force exerted by a point mass $M$ located at the center of the sphere.

**Acceleration of Gravity**
Replacing the Earth with a point mass at its center, we find that the acceleration of gravity on the surface of the Earth is

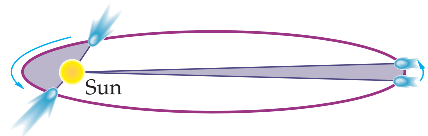
$$g = \frac{GM_E}{R_E^2} \quad 12\text{–}4$$

**Weighing the Earth**
Cavendish was the first to determine the value of the universal gravitation constant $G$ by direct experiment. Knowing $G$ allows one to calculate the mass of the Earth:

$$M_E = \frac{gR_E^2}{G} \quad 12\text{–}5$$

## 12–3 KEPLER'S LAWS OF ORBITAL MOTION

Kepler determined three laws that describe the motion of the planets in our solar system. Newton showed that Kepler's laws are a direct consequence of his law of universal gravitation.




**Kepler's First Law**
The orbits of the planets are ellipses, with the Sun at one focus.

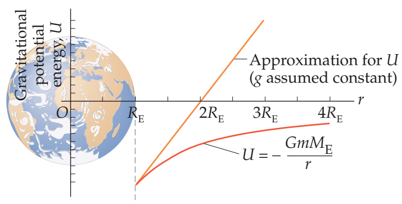
**Kepler's Second Law**
Planets sweep out equal area in equal time.

**Kepler's Third Law**
The period of a planet's orbit, $T$, is proportional to the 3/2 power of its average distance from the Sun, $r$:

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right)r^{3/2} = (\text{constant})r^{3/2} \quad 12\text{–}7$$

## 12–4 GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy, $U$, between two point masses $m_1$ and $m_2$ separated by a distance $r$ is



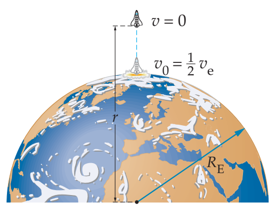

$$U = -G\frac{m_1 m_2}{r} \quad 12\text{–}9$$

**Zero Level**
The zero level of the gravitational potential energy between two point masses is chosen to be at infinite separation of the two masses.

***U* Is a Scalar**
The gravitational potential energy, $U$, is a scalar. Therefore, the total potential energy for a group of objects is simply the numerical sum of the potential energy associated with each pair of masses.

## 12–5 ENERGY CONSERVATION

With the gravitational potential energy given in Section 12–4, energy conservation can be applied to astronomical situations.




**Total Mechanical Energy**
An object with mass $m$, speed $v$, and at a distance $r$ from the center of the Earth has a total energy given by

$$E = K + U = \tfrac{1}{2}mv^2 - \frac{GmM_E}{r} \quad 12\text{–}10$$

**Escape Speed**

An object launched from the surface of the Earth with the escape speed $v_e$ can move infinitely far from the Earth. In the limit of infinite separation, the object slows to zero speed.

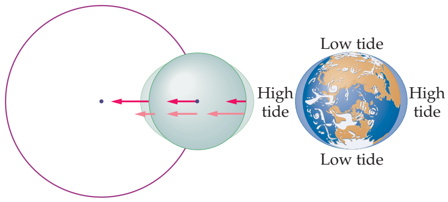
The escape speed for the Earth is given by

$$v_e = \sqrt{\frac{2GM_E}{R_E}} \qquad 12\text{–}13$$

Its numerical value is 11,200 m/s = 25,000 mi/h. A similar expression can be applied to other astronomical bodies.

## *12–6 TIDES

Tides result from the variation of the gravitational force from one side of an astronomical object to the other side.



(a) (b)

**Tidal Locking**

Tidal locking occurs when one astronomical object always points its tidal bulge at the object it orbits.

**Roche Limit**

Tidal deformation increases as an astronomical object moves closer to the body it orbits. At the Roche limit, the tidal deformation is so great that it breaks the object into small pieces.

## PROBLEM-SOLVING SUMMARY

| Type of Problem | Relevant Physical Concepts | Related Examples |
|---|---|---|
| Find the force due to gravity. | The magnitude of the force is given by Newton's law of universal gravitation, $F = Gm_1m_2/r^2$. The direction of the force is attractive and along the line connecting $m_1$ and $m_2$. If more than one force is involved, the net force is the vector sum of the individual forces. | Examples 12–1, 12–2, 12–3 |
| Relate the period of a planet to the radius of its orbit and the mass of the body it orbits. | Use Kepler's third law, $T = \left(2\pi/\sqrt{GM}\right)r^{3/2}$. | Example 12–4<br>Active Example 12–1 |
| Determine the speed of an object at a particular location, given its initial speed and location. | Use energy conservation, with the gravitational potential energy given by $U = -Gm_1m_2/r$. | Examples 12–6, 12–7<br>Active Example 12–2 |

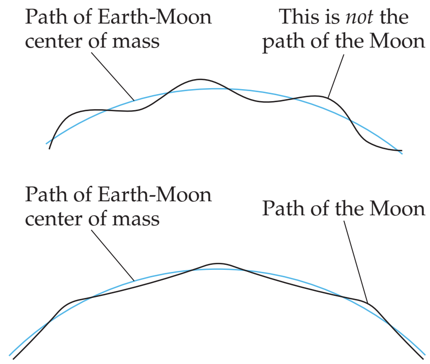
## CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. It is often said that astronauts in orbit experience weightlessness because they are beyond the pull of Earth's gravity. Is this statement correct? Explain.
2. When a person passes you on the street, you do not feel a gravitational tug. Explain.
3. Two objects experience a gravitational attraction. Give a reason why the gravitational force between them does not depend on the sum of their masses.
4. Imagine bringing the tips of your index fingers together. Each finger contains a certain finite mass, and the distance between them goes to zero as they come into contact. From the force law $F = Gm_1m_2/r^2$ one might conclude that the attractive force between the fingers is infinite, and, therefore, that your fingers must remain forever stuck together. What is wrong with this argument?
5. Does the radius vector of Mars sweep out the same amount of area per time as that of the Earth? Why or why not?
6. When a communications satellite is placed in a geosynchronous orbit above the equator, it remains fixed over a given point on the ground. Is it possible to put a satellite into an orbit so that it remains fixed above the North Pole? Explain.
7. **The Mass of Pluto** On June 22, 1978, James Christy made the first observation of a moon orbiting Pluto. Until that time the mass of Pluto was not known, but with the discovery of its moon, Charon, its mass could be calculated with some accuracy. Explain.
8. Rockets are launched into space from Cape Canaveral in an easterly direction. Is there an advantage to launching to the east versus launching to the west? Explain.
9. One day in the future you may take a pleasure cruise to the Moon. While there you might climb a lunar mountain and throw a rock horizontally from its summit. If, in principle, you could throw the rock fast enough, it might end up hitting you in the back. Explain.
10. Apollo astronauts orbiting the Moon at low altitude noticed occasional changes in their orbit that they attributed to localized concentrations of mass below the lunar surface. Just what effect would such "mascons" have on their orbit?

11. If you light a candle on the space shuttle—which would not be a good idea—would it burn the same as on the Earth? Explain.
12. The force exerted by the Sun on the Moon is more than twice the force exerted by the Earth on the Moon. Should the Moon be thought of as orbiting the Earth or the Sun? Explain.
13. **The Path of the Moon** The Earth and Moon exert gravitational forces on one another as they orbit the Sun. As a result, the path they follow is not the simple circular orbit you would expect if either one orbited the Sun alone. Occasionally you will see a suggestion that the Moon follows a path like a sine wave centered on a circular path, as in the upper part of **Figure 12–20**. This is *incorrect*. The Moon's path is qualitatively like that shown in the lower part of Figure 12–20. Explain. (Refer to Conceptual Question 12.)




▲ **FIGURE 12–20** Conceptual Question 13

## PROBLEMS AND CONCEPTUAL EXERCISES

*Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book.* **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

### SECTION 12–1 NEWTON'S LAW OF UNIVERSAL GRAVITATION

1. • **CE** System A has masses $m$ and $m$ separated by a distance $r$; system B has masses $m$ and $2m$ separated by a distance $2r$; system C has masses $2m$ and $3m$ separated by a distance $2r$; and system D has masses $4m$ and $5m$ separated by a distance $3r$. Rank these systems in order of increasing gravitational force. Indicate ties where appropriate.

2. • In each hand you hold a 0.16-kg apple. What is the gravitational force exerted by each apple on the other when their separation is **(a)** 0.25 m and **(b)** 0.50 m?

3. • A 6.1-kg bowling ball and a 7.2-kg bowling ball rest on a rack 0.75 m apart. **(a)** What is the force of gravity exerted on each of the balls by the other ball? **(b)** At what separation is the force of gravity between the balls equal to $2.0 \times 10^{-9}$ N?

4. • A communications satellite with a mass of 480 kg is in a circular orbit about the Earth. The radius of the orbit is 35,000 km as measured from the center of the Earth. Calculate **(a)** the weight of the satellite on the surface of the Earth and **(b)** the gravitational force exerted on the satellite by the Earth when it is in orbit.

5. • **The Attraction of Ceres** Ceres, the largest asteroid known, has a mass of roughly $8.7 \times 10^{20}$ kg. If Ceres passes within 14,000 km of the spaceship in which you are traveling, what force does it exert on you? (Use an approximate value for your mass, and treat yourself and the asteroid as point objects.)

6. • In one hand you hold a 0.11-kg apple, in the other hand a 0.24-kg orange. The apple and orange are separated by 0.85 m. What is the magnitude of the force of gravity that **(a)** the orange exerts on the apple and **(b)** the apple exerts on the orange?

7. •• **IP** A spaceship of mass $m$ travels from the Earth to the Moon along a line that passes through the center of the Earth and the center of the Moon. **(a)** At what distance from the center of the Earth is the force due to the Earth twice the magnitude of the force due to the Moon? **(b)** How does your answer to part (a) depend on the mass of the spaceship? Explain.

8. •• At new moon, the Earth, Moon, and Sun are in a line, as indicated in **Figure 12–21**. Find the direction and magnitude of the net gravitational force exerted on **(a)** the Earth, **(b)** the Moon, and **(c)** the Sun.


▲ **FIGURE 12–21** Problem 8

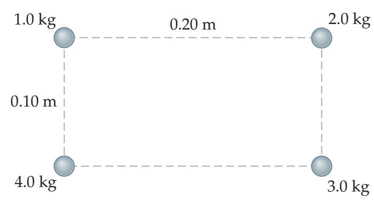
9. •• When the Earth, Moon, and Sun form a right triangle, with the Moon located at the right angle, as shown in **Figure 12–22**, the Moon is in its third-quarter phase. (The Earth is viewed here from above its North Pole.) Find the magnitude and direction of the net force exerted on the Moon. Give the direction relative to the line connecting the Moon and the Sun.


▲ **FIGURE 12–22** Problems 9, 10, and 73

10. •• Repeat the previous problem, this time finding the magnitude and direction of the net force acting on the Sun. Give the direction relative to the line connecting the Sun and the Moon.

11. •• **IP** Three 6.75-kg masses are at the corners of an equilateral triangle and located in space far from any other masses. **(a)** If the sides of the triangle are 1.25 m long, find the magnitude of the net force exerted on each of the three masses. **(b)** How does your answer to part (a) change if the sides of the triangle are doubled in length?

12. •• **IP** Four masses are positioned at the corners of a rectangle, as indicated in **Figure 12–23. (a)** Find the magnitude and direction of the net force acting on the 2.0-kg mass. **(b)** How do your answers to part (a) change (if at all) if all sides of the rectangle are doubled in length?




▲ **FIGURE 12–23** Problems 12 and 42

13. ••• Suppose that three astronomical objects (1, 2, and 3) are observed to lie on a line, and that the distance from object 1 to object 3 is $D$. Given that object 1 has four times the mass of object 3 and seven times the mass of object 2, find the distance between objects 1 and 2 for which the net force on object 2 is zero.

## SECTION 12–2 GRAVITATIONAL ATTRACTION OF SPHERICAL BODIES

14. • Find the acceleration due to gravity on the surface of **(a)** Mercury and **(b)** Venus.

15. • At what altitude above the Earth's surface is the acceleration due to gravity equal to $g/2$?

16. • Two 6.7-kg bowling balls, each with a radius of 0.11 m, are in contact with one another. What is the gravitational attraction between the bowling balls?

17. • What is the acceleration due to Earth's gravity at a distance from the center of the Earth equal to the orbital radius of the Moon?

18. • **Gravity on Titan** Titan is the largest moon of Saturn and the only moon in the solar system known to have a substantial atmosphere. Find the acceleration due to gravity on Titan's surface, given that its mass is $1.35 \times 10^{23}$ kg and its radius is 2570 km.

19. •• **IP** At a certain distance from the center of the Earth, a 4.6-kg object has a weight of 2.2 N. **(a)** Find this distance. **(b)** If the object is released at this location and allowed to fall toward the Earth, what is its initial acceleration? **(c)** If the object is now moved twice as far from the Earth, by what factor does its weight change? Explain. **(d)** By what factor does its initial acceleration change? Explain.

20. •• The acceleration due to gravity on the Moon's surface is known to be about one-sixth the acceleration due to gravity on the Earth. Given that the radius of the Moon is roughly one-quarter that of the Earth, find the mass of the Moon in terms of the mass of the Earth.

21. •• **IP An Extraterrestrial Volcano** Several volcanoes have been observed erupting on the surface of Jupiter's closest Galilean moon, Io. Suppose that material ejected from one of these volcanoes reaches a height of 5.00 km after being projected straight upward with an initial speed of 134 m/s. Given that the radius of Io is 1820 km, **(a)** outline a strategy that allows you to calculate the mass of Io. **(b)** Use your strategy to calculate Io's mass.

22. •• **IP Verne's Trip to the Moon** In his novel *From the Earth to the Moon*, Jules Verne imagined that astronauts inside a spaceship would walk on the floor of the cabin when the force exerted on the ship by the Earth was greater than the force exerted by the Moon. When the force exerted by the Moon was greater, he thought the astronauts would walk on the ceiling of the cabin. **(a)** At what distance from the center of the Earth would the forces exerted on the spaceship by the Earth and the Moon be equal? **(b)** Explain why Verne's description of gravitational effects is incorrect.

23. ••• Consider an asteroid with a radius of 19 km and a mass of $3.35 \times 10^{15}$ kg. Assume the asteroid is roughly spherical. **(a)** What is the acceleration due to gravity on the surface of the asteroid? **(b)** Suppose the asteroid spins about an axis through its center, like the Earth, with a rotational period $T$. What is the smallest value $T$ can have before loose rocks on the asteroid's equator begin to fly off the surface?

## SECTION 12–3 KEPLER'S LAWS OF ORBITAL MOTION

24. • **CE Predict/Explain The Speed of the Earth** The orbital speed of the Earth is greatest around January 4 and least around July 4. **(a)** Is the distance from the Earth to the Sun on January 4 greater than, less than, or equal to its distance from the Sun on July 4? **(b)** Choose the *best explanation* from among the following:
    - **I.** The Earth's orbit is circular, with equal distance from the Sun at all times.
    - **II.** The Earth sweeps out equal area in equal time, thus it must be closer to the Sun when it is moving faster.
    - **III.** The greater the speed of the Earth, the greater its distance from the Sun.

25. • **CE** A satellite orbits the Earth in a circular orbit of radius $r$. At some point its rocket engine is fired in such a way that its speed increases rapidly by a small amount. As a result, do the **(a)** apogee distance and **(b)** perigee distance increase, decrease, or stay the same?

26. • **CE** Repeat the previous problem, only this time with the rocket engine of the satellite fired in such a way as to slow the satellite.

27. • **CE Predict/Explain The Earth–Moon Distance Is Increasing** Laser reflectors left on the surface of the Moon by the Apollo astronauts show that the average distance from the Earth to the Moon is increasing at the rate of 3.8 cm per year. **(a)** As a result, will the length of the month increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:
    - **I.** The greater the radius of an orbit, the greater the period, which implies a longer month.
    - **II.** The length of the month will remain the same due to conservation of angular momentum.
    - **III.** The speed of the Moon is greater with increasing radius; therefore, the length of the month will be less.

28. • **Apollo Missions** On Apollo missions to the Moon, the command module orbited at an altitude of 110 km above the lunar surface. How long did it take for the command module to complete one orbit?

29. • Find the orbital speed of a satellite in a geosynchronous circular orbit $3.58 \times 10^{7}$ m above the surface of the Earth.

30. • **An Extrasolar Planet** In July of 1999 a planet was reported to be orbiting the Sun-like star Iota Horologii with a period of 320 days. Find the radius of the planet's orbit, assuming that Iota Horologii has the same mass as the Sun. (This planet is presumably similar to Jupiter, but it may have large, rocky moons that enjoy a relatively pleasant climate.)

31. • Phobos, one of the moons of Mars, orbits at a distance of 9378 km from the center of the red planet. What is the orbital period of Phobos?

32. • The largest moon in the solar system is Ganymede, a moon of Jupiter. Ganymede orbits at a distance of $1.07 \times 10^{9}$ m from the center of Jupiter with an orbital period of about $6.18 \times 10^{5}$ s. Using this information, find the mass of Jupiter.

33. •• **IP An Asteroid with Its Own Moon** The asteroid 243 Ida has its own small moon, Dactyl. (See the photo on p. 390) **(a)** Outline a strategy to find the mass of 243 Ida, given that the orbital radius of Dactyl is 89 km and its period is 19 hr. **(b)** Use your strategy to calculate the mass of 243 Ida.

34. •• **GPS Satellites** GPS (Global Positioning System) satellites orbit at an altitude of $2.0 \times 10^7$ m. Find **(a)** the orbital period, and **(b)** the orbital speed of such a satellite.

35. •• **IP** Two satellites orbit the Earth, with satellite 1 at a greater altitude than satellite 2. **(a)** Which satellite has the greater orbital speed? Explain. **(b)** Calculate the orbital speed of a satellite that orbits at an altitude of one Earth radius above the surface of the Earth. **(c)** Calculate the orbital speed of a satellite that orbits at an altitude of two Earth radii above the surface of the Earth.

36. •• **IP** Calculate the orbital periods of satellites that orbit **(a)** one Earth radius above the surface of the Earth and **(b)** two Earth radii above the surface of the Earth. **(c)** How do your answers to parts (a) and (b) depend on the mass of the satellites? Explain. **(d)** How do your answers to parts (a) and (b) depend on the mass of the Earth? Explain.

37. •• **IP** The Martian moon Deimos has an orbital period that is greater than the other Martian moon, Phobos. Both moons have approximately circular orbits. **(a)** Is Deimos closer to or farther from Mars than Phobos? Explain. **(b)** Calculate the distance from the center of Mars to Deimos given that its orbital period is $1.10 \times 10^5$ s.

38. ••• **Binary Stars** Centauri A and Centauri B are binary stars with a separation of $3.45 \times 10^{12}$ m and an orbital period of $2.52 \times 10^9$ s. Assuming the two stars are equally massive (which is approximately the case), determine their mass.

39. ••• Find the speed of Centauri A and Centauri B, using the information given in the previous problem.

## SECTION 12–4 GRAVITATIONAL POTENTIAL ENERGY

40. • **Sputnik** The first artificial satellite to orbit the Earth was Sputnik I, launched October 4, 1957. The mass of Sputnik I was 83.5 kg, and its distances from the center of the Earth at apogee and perigee were 7330 km and 6610 km, respectively. Find the difference in gravitational potential energy for Sputnik I as it moved from apogee to perigee.

41. •• **CE Predict/Explain** **(a)** Is the amount of energy required to get a spacecraft from the Earth to the Moon greater than, less than, or equal to the energy required to get the same spacecraft from the Moon to the Earth? **(b)** Choose the *best explanation* from among the following:
   **I.** The escape speed of the Moon is less than that of the Earth; therefore, less energy is required to leave the Moon.
   **II.** The situation is symmetric, and hence the same amount of energy is required to travel in either direction.
   **III.** It takes more energy to go from the Moon to the Earth because the Moon is orbiting the Earth.

42. •• **IP** Consider the four masses shown in Figure 12–23. **(a)** Find the total gravitational potential energy of this system. **(b)** How does your answer to part (a) change if all the masses in the system are doubled? **(c)** How does your answer to part (a) change if, instead, all the sides of the rectangle are halved in length?

43. •• Calculate the gravitational potential energy of a 8.8-kg mass **(a)** on the surface of the Earth and **(b)** at an altitude of 350 km. **(c)** Take the difference between the results for parts (b) and (a), and compare with $mgh$, where $h = 350$ km.

44. •• Two 0.59-kg basketballs, each with a radius of 12 cm, are just touching. How much energy is required to change the separation between the centers of the basketballs to **(a)** 1.0 m and **(b)** 10.0 m? (Ignore any other gravitational interactions.)

45. •• Find the minimum kinetic energy needed for a 39,000-kg rocket to escape **(a)** the Moon or **(b)** the Earth.

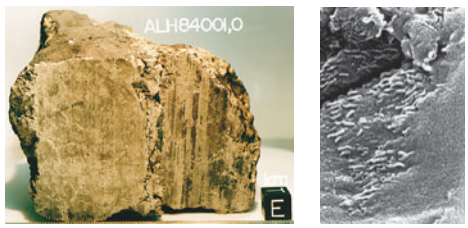
## SECTION 12–5 ENERGY CONSERVATION

46. • **CE Predict/Explain** Suppose the Earth were to suddenly shrink to half its current diameter, with its mass remaining constant. **(a)** Would the escape speed of the Earth increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   **I.** Since the radius of the Earth would be smaller, the escape speed would also be smaller.
   **II.** The Earth would have the same amount of mass, and hence its escape speed would be unchanged.
   **III.** The force of gravity would be much stronger on the surface of the compressed Earth, leading to a greater escape speed.

47. • **CE** Is the energy required to launch a rocket vertically to a height $h$ greater than, less than, or equal to the energy required to put the same rocket into orbit at the height $h$? Explain.

48. • Suppose one of the Global Positioning System satellites has a speed of 4.46 km/s at perigee and a speed of 3.64 km/s at apogee. If the distance from the center of the Earth to the satellite at perigee is $2.00 \times 10^4$ km, what is the corresponding distance at apogee?

49. • **Meteorites from Mars** Several meteorites found in Antarctica are believed to have come from Mars, including the famous ALH84001 meteorite that some believe contains fossils of ancient life on Mars. Meteorites from Mars are thought to get to Earth by being blasted off the Martian surface when a large object (such as an asteroid or a comet) crashes into the planet. What speed must a rock have to escape Mars?




The meteorite ALH84001 (left), dislodged from the Martian surface by a tremendous impact, drifted through space for millions of years before falling to Earth in Antarctica about 13,000 years ago. The electron micrograph at right shows tubular structures within the meteorite; some scientists think they are traces of primitive, bacteria-like organisms that may have lived on Mars billions of years ago. (Problem 49)

50. • Referring to Example 12–1, if the *Millennium Eagle* is at rest at point A, what is its speed at point B?

51. • What is the launch speed of a projectile that rises vertically above the Earth to an altitude equal to one Earth radius before coming to rest momentarily?

52. • A projectile launched vertically from the surface of the Moon rises to an altitude of 365 km. What was the projectile's initial speed?

53. • Find the escape velocity for **(a)** Mercury and **(b)** Venus.

54. •• **IP** **Halley's Comet** Halley's comet, which passes around the Sun every 76 years, has an elliptical orbit. When closest to the Sun (perihelion) it is at a distance of $8.823 \times 10^{10}$ m and moves with a speed of 54.6 km/s. The greatest distance between Halley's comet and the Sun (aphelion) is $6.152 \times 10^{12}$ m. **(a)** Is the speed of Halley's comet greater than or less than 54.6 km/s

when it is at aphelion? Explain. **(b)** Calculate its speed at aphelion.

55. •• **The End of the Lunar Module** On Apollo Moon missions, the lunar module would blast off from the Moon's surface and dock with the command module in lunar orbit. After docking, the lunar module would be jettisoned and allowed to crash back onto the lunar surface. Seismometers placed on the Moon's surface by the astronauts would then pick up the resulting seismic waves. Find the impact speed of the lunar module, given that it is jettisoned from an orbit 110 km above the lunar surface moving with a speed of 1630 m/s.

56. •• If a projectile is launched vertically from the Earth with a speed equal to the escape speed, how high above the Earth's surface is it when its speed is half the escape speed?

57. •• Suppose a planet is discovered orbiting a distant star. If the mass of the planet is 10 times the mass of the Earth, and its radius is one-tenth the Earth's radius, how does the escape speed of this planet compare with that of the Earth?

58. •• A projectile is launched vertically from the surface of the Moon with an initial speed of 1050 m/s. At what altitude is the projectile's speed one-half its initial value?

59. •• To what radius would the Sun have to be contracted for its escape speed to equal the speed of light? (Black holes have escape speeds greater than the speed of light; hence we see no light from them.)

60. •• **IP** Two baseballs, each with a mass of 0.148 kg, are separated by a distance of 395 m in outer space, far from any other objects. **(a)** If the balls are released from rest, what speed do they have when their separation has decreased to 145 m? **(b)** Suppose the mass of the balls is doubled. Would the speed found in part (a) increase, decrease, or stay the same? Explain.

61. ••• On Earth, a person can jump vertically and rise to a height $h$. What is the radius of the largest spherical asteroid from which this person could escape by jumping straight upward? Assume that each cubic meter of the asteroid has a mass of 3500 kg.

## *SECTION 12–6 TIDES

62. •• As will be shown in Problem 63, the magnitude of the tidal force exerted on an object of mass $m$ and length $a$ is approximately $4GmMa/r^3$. In this expression, $M$ is the mass of the body causing the tidal force and $r$ is the distance from the center of $m$ to the center of $M$. Suppose you are 1 million miles away from a black hole whose mass is a million times that of the Sun. **(a)** Estimate the tidal force exerted on your body by the black hole. **(b)** At what distance will the tidal force be approximately 10 times greater than your weight?

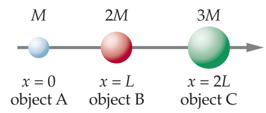
63. ••• A dumbbell has a mass $m$ on either end of a rod of length $2a$. The center of the dumbbell is a distance $r$ from the center of the Earth, and the dumbbell is aligned radially. If $r \gg a$, show that the difference in the gravitational force exerted on the two masses by the Earth is approximately $4GmM_Ea/r^3$. (*Note:* The difference in force causes a tension in the rod connecting the masses. We refer to this as a *tidal force*.) [*Hint:* Use the fact that $1/(r-a)^2 - 1/(r+a)^2 \sim 4a/r^3$ for $r \gg a$.]

64. ••• Referring to the previous problem, suppose the rod connecting the two masses $m$ is removed. In this case, the only force between the two masses is their mutual gravitational attraction. In addition, suppose the masses are spheres of radius $a$ and mass $m = \frac{4}{3}\pi a^3 \rho$ that touch each other. (The Greek letter $\rho$ stands for the density of the masses.) **(a)** Write an expression for the gravitational force between the masses $m$. **(b)** Find the distance from the center of the Earth, $r$, for which the gravitational force found in part (a) is equal to the tidal force found in Problem 63. This distance is known as the *Roche limit*. **(c)** Calculate the Roche limit for Saturn, assuming $\rho = 3330\ \mathrm{kg/m^3}$. (The famous rings of Saturn are within the Roche limit for that planet. Thus, the innumerable small objects, composed mostly of ice, that make up the rings will never coalesce to form a moon.)

## GENERAL PROBLEMS

65. • **CE** You weigh yourself on a scale inside an airplane flying due east above the equator. If the airplane now turns around and heads due west with the same speed, will the reading on the scale increase, decrease, or stay the same? Explain.

66. • **CE** Rank objects A, B, and C in **Figure 12–24** in order of increasing net gravitational force experienced by the object. Indicate ties where appropriate.




**▲ FIGURE 12–24**
Problems 66 and 67

67. • **CE** Referring to Figure 12–24, rank objects A, B, and C in order of increasing initial acceleration each would experience if it alone were allowed to move. Indicate ties where appropriate.

68. • **CE** When the Moon is in its new-moon position (directly between the Earth and the Sun), does the net force exerted on it by the Sun and the Earth point toward the Sun, or point toward the Earth? Explain. (Refer to Conceptual Questions 12 and 13 as well as Figure 12–20.)

69. • **CE** A satellite goes through one complete orbit of the Earth. **(a)** Is the net work done on it by the Earth's gravitational force positive, negative, or zero? Explain. **(b)** Does your answer to part (a) depend on whether the orbit is circular or elliptical?

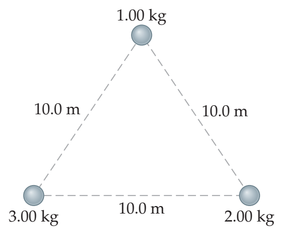
70. • **CE** **The Crash of Skylab** Skylab, the largest spacecraft ever to fall back to the Earth, met its fiery end on July 11, 1979, after flying directly over Everett, WA, on its last orbit. On the *CBS Evening News* the night before the crash, anchorman Walter Cronkite, in his rich baritone voice, made the following statement: "NASA says there is a little chance that Skylab will land in a populated area." After the commercial, he immediately corrected himself by saying, "I meant to say '*there is little chance*' Skylab will hit a populated area." In fact, it landed primarily in the Indian Ocean off the west coast of Australia, though several pieces were recovered near the town of Esperance, Australia, which later sent the U.S. State Department a $400 bill for littering. The cause of Skylab's crash was the friction it experienced in the upper reaches of the Earth's atmosphere. As the radius of Skylab's orbit decreased, did its speed increase, decrease, or stay the same? Explain.

71. • Consider a system consisting of three masses on the $x$ axis. Mass $m_1 = 1.00$ kg is at $x = 1.00$ m; mass $m_2 = 2.00$ kg is at $x = 2.00$ m; and mass $m_3 = 3.00$ kg is at $x = 3.00$ m. What is the total gravitational potential energy of this system?

72. •• An astronaut exploring a distant solar system lands on an unnamed planet with a radius of 3860 km. When the astronaut jumps upward with an initial speed of 3.10 m/s, she rises to a height of 0.580 m. What is the mass of the planet?

73. •• **IP** When the Moon is in its third-quarter phase, the Earth, Moon, and Sun form a right triangle, as shown in Figure 12–22. Calculate the magnitude of the force exerted on the Moon by **(a)** the Earth and **(b)** the Sun. **(c)** Does it make more sense to think of the Moon as orbiting the Sun, with a small effect due to the Earth, or as orbiting the Earth, with a small effect due to the Sun?

74. •• An equilateral triangle 10.0 m on a side has a 1.00-kg mass at one corner, a 2.00-kg mass at another corner, and a 3.00-kg mass at the third corner (**Figure 12–25**). Find the magnitude and direction of the net force acting on the 1.00-kg mass.




▲ **FIGURE 12–25**
Problems 74 and 75

75. •• Suppose that each of the three masses in Figure 12–25 is replaced by a mass of 5.95 kg and radius 0.0714 m. If the balls are released from rest, what speed will they have when they collide at the center of the triangle? Ignore gravitational effects from any other objects.

76. •• **A Near Miss!** In the early morning hours of June 14, 2002, the Earth had a remarkably close encounter with an asteroid the size of a small city. The previously unknown asteroid, now designated 2002 MN, remained undetected until three days after it had passed the Earth. At its closest approach, the asteroid was 73,600 miles from the center of the Earth—about a third of the distance to the Moon. **(a)** Find the speed of the asteroid at closest approach, assuming its speed at infinite distance to be zero and considering only its interaction with the Earth. **(b)** Observations indicate the asteroid to have a diameter of about 2.0 km. Estimate the kinetic energy of the asteroid at closest approach, assuming it has an average density of $3.33\ \mathrm{g/cm^3}$. (For comparison, a 1-megaton nuclear weapon releases about $5.6 \times 10^{15}$ J of energy.)

77. •• **IP** Suppose a planet is discovered that has the same amount of mass in a given volume as the Earth, but has half its radius. **(a)** Is the acceleration due to gravity on this planet more than, less than, or the same as the acceleration due to gravity on the Earth? Explain. **(b)** Calculate the acceleration due to gravity on this planet.

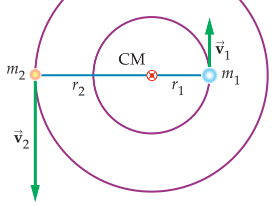
78. •• **IP** Suppose a planet is discovered that has the same total mass as the Earth, but half its radius. **(a)** Is the acceleration due to gravity on this planet more than, less than, or the same as the acceleration due to gravity on the Earth? Explain. **(b)** Calculate the acceleration due to gravity on this planet.

79. •• Show that the speed of a satellite in a circular orbit a height $h$ above the surface of the Earth is

$$v = \sqrt{\frac{GM_E}{R_E + h}}$$

80. •• In a binary star system, two stars orbit about their common center of mass, as shown in **Figure 12–26**. If $r_2 = 2r_1$, what is the ratio of the masses $m_2/m_1$ of the two stars?

81. •• Find the orbital period of the binary star system described in the previous problem.




▲ **FIGURE 12–26**
Problems 80 and 81

82. •• Using the results from Problem 54, find the angular momentum of Halley's comet **(a)** at perihelion and **(b)** at aphelion. (Take the mass of Halley's comet to be $9.8 \times 10^{14}$ kg.)

83. •• **Exploring Mars** In the not-too-distant future astronauts will travel to Mars to carry out scientific explorations. As part of their mission, it is likely that a "geosynchronous" satellite will be placed above a given point on the Martian equator to facilitate communications. At what altitude above the surface of Mars should such a satellite orbit? (*Note:* The Martian "day" is 24.6229 hours. Other relevant information can be found in Appendix C.)

84. •• **IP** A satellite is placed in Earth orbit 1000 miles higher than the altitude of a geosynchronous satellite. Referring to Active Example 12–1, we see that the altitude of the satellite is 23,300 mi. **(a)** Is the period of this satellite greater than or less than 24 hours? **(b)** As viewed from the surface of the Earth, does the satellite move eastward or westward? Explain. **(c)** Find the orbital period of this satellite.

85. •• Find the speed of the *Millennium Eagle* at point A in Example 12–1 if its speed at point B is 0.905 m/s.

86. •• Show that the force of gravity between the Moon and the Sun is always greater than the force of gravity between the Moon and the Earth.

87. •• The astronomical unit AU is defined as the mean distance from the Sun to the Earth ($1\ \mathrm{AU} = 1.50 \times 10^{11}$ m). Apply Kepler's third law (Equation 12–7) to the solar system, and show that it can be written as

$$T = Cr^{3/2}$$

In this expression, the period $T$ is measured in years, the distance $r$ is measured in astronomical units, and the constant $C$ has a magnitude that you must determine.

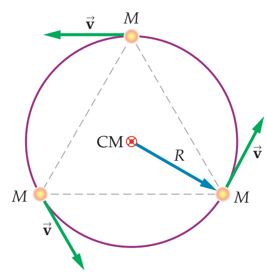
88. •• **(a)** Find the kinetic energy of a 1720-kg satellite in a circular orbit about the Earth, given that the radius of the orbit is 12,600 miles. **(b)** How much energy is required to move this satellite to a circular orbit with a radius of 25,200 miles?

89. •• **IP** **Space Shuttle Orbit** On a typical mission, the space shuttle ($m = 2.00 \times 10^6$ kg) orbits at an altitude of 250 km above the Earth's surface. **(a)** Does the orbital speed of the shuttle depend on its mass? Explain. **(b)** Find the speed of the shuttle in its orbit. **(c)** How long does it take for the shuttle to complete one orbit of the Earth?

90. ••• **IP** Consider an object of mass $m$ orbiting the Earth at a radius $r$. **(a)** Find the speed of the object. **(b)** Show that the total mechanical energy of this object is equal to $(-1)$ times its kinetic energy. **(c)** Does the result of part (b) apply to an object orbiting the Sun? Explain.

91. ••• In a binary star system two stars orbit about their common center of mass. Find the orbital period of such a system, given that the stars are separated by a distance $d$ and have masses $m$ and $2m$.

92. ••• Three identical stars, at the vertices of an equilateral triangle, orbit about their common center of mass (**Figure 12–27**). Find




▲ **FIGURE 12–27**
Problem 92

the period of this orbital motion in terms of the orbital radius, $R$, and the mass of each star, $M$.

**93.** ••• Find an expression for the kinetic energy of a satellite of mass $m$ in an orbit of radius $r$ about a planet of mass $M$.

**94.** ••• Referring to Example 12–1, find the $x$ component of the net force acting on the *Millennium Eagle* as a function of $x$. Plot your result, showing both negative and positive values of $x$.

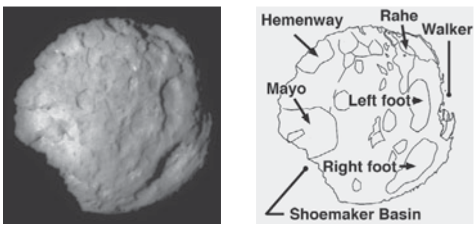
**95.** ••• A satellite orbits the Earth in an elliptical orbit. At perigee its distance from the center of the Earth is 22,500 km and its speed is 4280 m/s. At apogee its distance from the center of the Earth is 24,100 km and its speed is 3990 m/s. Using this information, calculate the mass of the Earth.

## PASSAGE PROBLEMS

### Exploring Comets with the *Stardust* Spacecraft

On February 7, 1999, NASA launched a spacecraft with the ambitious mission of making a close encounter with a comet, collecting samples from its tail, and returning the samples to Earth for analysis. This spacecraft, appropriately named *Stardust*, took almost five years to rendezvous with its objective—comet Wild 2 (pronounced "Vilt 2")—and another two years to return its samples. The reason for the long round trip is that the spacecraft had to make three orbits around the Sun, and also an Earth Gravity Assist (EGA) flyby, to increase its speed enough to put it in an orbit appropriate for the encounter.

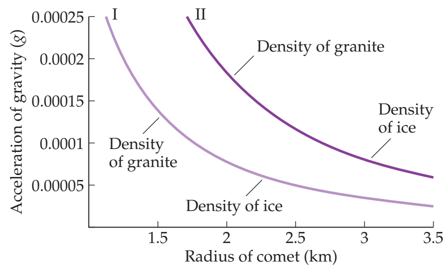
When *Stardust* finally reached comet Wild 2 on January 2, 2004, it flew within 147 miles of the comet's nucleus, snapping pictures and collecting tiny specks of dust in the glistening coma. The approach speed between the spacecraft and the comet at the encounter was a relatively "slow" 6200 m/s, so that dust particles could be collected safely without destroying the vehicle. Note that "slow" is put in quotation marks; after all, 6200 m/s is still about six times the speed of a rifle bullet!




Comet Wild 2 and some of its surface features, including the Walker basin, the site of unusual jets of outward-flowing dust and rocks.

The roughly spherical comet Wild 2 has a radius of 2.7 km, and the acceleration due to gravity on its surface is $0.00010g$. The two curves in **Figure 12–28** show the surface acceleration as a function of radius for a spherical comet with two different masses, one of which corresponds to comet Wild 2. Also indicated are radii at which these two hypothetical comets have densities equal to that of ice and granite.

The *Stardust* spacecraft is still in space; only its small return capsule came back to Earth. It has now been given a new assignment—to visit and photograph comet Tempel 1, the object of the Deep Impact collision on July 4, 2005. This mission, called *N*ew *Ex*ploration of *T*empel 1 (NExT), is scheduled to make its close encounter with comet Tempel 1 on February 14, 2011.




▲ **FIGURE 12–28** Problems 96, 97, 98, and 99

**96.** • Which of the two curves in Figure 12–28 corresponds to comet Wild 2?

**A.** Curve I **B.** Curve II

**97.** • What is the mass of comet Wild 2?

**A.** $1.1 \times 10^{8}$ kg **B.** $1.1 \times 10^{12}$ kg

**C.** $1.1 \times 10^{14}$ kg **D.** $1.1 \times 10^{18}$ kg

**98.** • Find the speed needed to escape from the surface of comet Wild 2. (*Note:* It is easy for a person to jump upward with a speed of 3 m/s.)

**A.** 1.6 m/s **B.** 2.3 m/s

**C.** 72 m/s **D.** 230 m/s

**99.** •• Suppose comet Wild 2 had a small satellite in orbit around it, just as Dactyl orbits asteroid 243 Ida (see page 390). If this satellite were to orbit at twice the radius of the comet, what would be its period of revolution?

**A.** 0.93 h **B.** 2.9 h

**C.** 5.8 h **D.** 8.2 h

## INTERACTIVE PROBLEMS

**100.** •• Find the orbital radius that corresponds to a "year" of 150 days.

**101.** •• **IP** Suppose the mass of the Sun is suddenly doubled, but the Earth's orbital radius remains the same. **(a)** Would the length of an Earth year increase, decrease, or stay the same? **(b)** Find the length of a year for the case of a Sun with twice the mass. **(c)** Suppose the Sun retains its present mass, but the mass of the Earth is doubled instead. Would the length of the year increase, decrease, or stay the same?

**102.** •• **IP Referring to Example 12–7 (a)** If the mass of the Earth were doubled, would the escape speed of a rocket increase, decrease, or stay the same? **(b)** Calculate the escape speed of a rocket for the case of an Earth with twice its present mass. **(c)** If the mass of the Earth retains its present value, but the mass of the rocket is doubled, does the escape speed increase, decrease, or stay the same?

**103.** •• **IP Referring to Example 12–7** Suppose the Earth is suddenly shrunk to half its present radius without losing any of its mass. **(a)** Would the escape speed of a rocket increase, decrease, or stay the same? **(b)** Find the escape speed for an Earth with half its present radius.