10 Rotational Kinematics and Energy

Can you imagine life without rotating objects: vehicles without wheels, machinery without gears, carnivals without merry-go-rounds? The people on this roller coaster certainly know that rotational motion is very different from motion on a straight, linear stretch of track. In this chapter we show that the motion of rotating objects, such as a roller coaster executing a loop-the-loop, can be analyzed using many of the same methods that we applied earlier to linear motion.

It is certainly no exaggeration to say that rotation is a part of everyday life. After all, we live on a planet that rotates about its axis once a day and that revolves about the Sun once a year. The apparent motion of the Sun across the sky, for example, is actually the result of the Earth's rotational motion. In addition, engines that power cars and trucks have moving parts that rotate quite rapidly, as do CDs, CD-ROMs, and DVDs, not to mention the tumbling, rotating molecules in the air we breathe. Thus, a study of rotation yields results that apply to a great variety of natural phenomena.

In this chapter, then, we study various aspects of rotational motion. As we do, we shall make extensive use of the close analogies that exist between rotational and linear motion. In fact, many of the results derived in earlier chapters can be applied to rotation by simply replacing linear quantities with their rotational counterparts.

10–1 Angular Position, Velocity, and Acceleration

To describe the motion of an object moving in a straight line, it is useful to establish a coordinate system with a definite origin and positive direction. In terms of this coordinate system we can measure the object's position, velocity, and acceleration.

Similarly, to describe rotational motion, we define "angular" quantities that are analogous to the linear position, velocity, and acceleration. These angular quantities form the basis of our study of rotation. We begin by defining the most basic angular quantity—the angular position.

Angular Position, θ

Consider a bicycle wheel that is free to rotate about its axle, as shown in **Figure 10–1**. We say that the axle is the **axis of rotation** for the wheel. As the wheel rotates, each and every point on it moves in a circular path centered on the axis of rotation.

Now, suppose there is a small spot of red paint on the tire, and we want to describe the rotational motion of the spot. The **angular position** of the spot is defined to be the angle, θ, that a line from the axle to the spot makes with a reference line, as indicated in Figure 10–1.

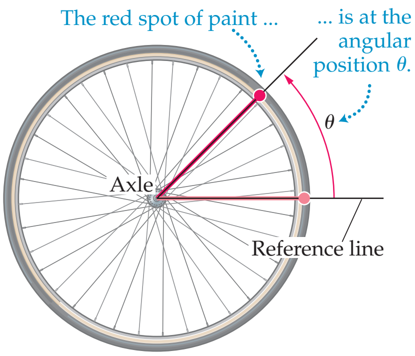


FIGURE 10–1 Angular position
The angular position, θ, of a spot of paint on a bicycle wheel. The reference line, where $\theta = 0$, is drawn horizontal here but can be chosen in any direction.

Definition of Angular Position, θ

θ = angle measured from reference line 10–1

SI unit: radian (rad), which is dimensionless

The reference line simply defines $\theta = 0$; it is analogous to the origin in a linear coordinate system. The reference line begins at the axis of rotation, and may be chosen to point in any direction—just as an origin may be placed anywhere along a coordinate axis. Once chosen, however, the reference line must be used consistently.

Note that the spot of paint in Figure 10–1 is rotated counterclockwise from the reference line by the angle θ. By convention, we say that this angle is positive. Similarly, clockwise rotations correspond to negative angles.

Sign Convention for Angular Position
By convention:

$\theta > 0$ counterclockwise rotation from reference line
$\theta < 0$ clockwise rotation from reference line

Rotational motion is everywhere in our universe, on every scale of length and time. A galaxy like the one at left may take millions of years to complete a single rotation about its center, while the skater in the middle photo spins several times in a second. The bacterium at right moves in a corkscrew path by rapidly twirling its flagella (the fine projections at either end of the cell) like whips.

Now that we have established a reference line (for $\theta = 0$), and a positive direction of rotation (counterclockwise), we must choose units in which to measure angles. Common units are degrees (°) and revolutions (rev), where one revolution—that is, going completely around a circle—corresponds to 360°:

$$1 \text{ rev} = 360°$$

The most convenient units for scientific calculations, however, are radians. A **radian** (rad) is defined as follows:

> A radian is the angle for which the arc length on a circle of radius r is equal to the radius of the circle.

This definition is useful because it establishes a particularly simple relationship between an angle measured in radians and the corresponding arc length, as illustrated in **Figure 10–2**. For example, it follows from our definition that for an angle of one radian, the arc length s is equal to the radius: $s = r$. Similarly, an angle of two radians corresponds to an arc length of two radii, $s = 2r$, and so on. Thus, the arc length s for an arbitrary angle θ measured in radians is given by the following relation:

$$s = r\theta \qquad 10\text{–}2$$

This simple and straightforward relation does not hold for degrees or revolutions—additional conversion factors would be needed.

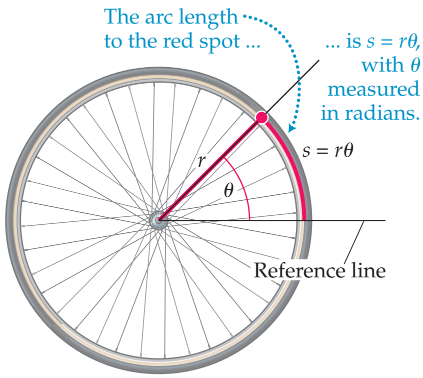
In one complete revolution, the arc length is the circumference of a circle, $C = 2\pi r$. Comparing with $s = r\theta$, we see that a complete revolution corresponds to 2π radians:

$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

Equivalently,

$$1 \text{ rad} = \frac{360°}{2\pi} = 57.3°$$

One final note on the units for angles: Radians, as well as degrees and revolutions, are dimensionless. In the relation $s = r\theta$, for example, the arc length and the radius both have SI units of meters. For the equation to be dimensionally consistent, it is necessary that θ have no dimensions. Still, if an angle θ is, let's say, three radians, we will write it as $\theta = 3$ rad to remind us of the angular units being used.



▲ **FIGURE 10–2 Arc length**
The arc length, s, from the reference line to the spot of paint is given by $s = r\theta$ if the angular position θ is measured in radians.

PROBLEM-SOLVING NOTE

Radians

Remember to measure angles in radians when using the relation $s = r\theta$.

Angular Velocity, ω

As the bicycle wheel in Figure 10–1 rotates, the angular position of the spot of red paint changes. This is illustrated in **Figure 10–3**. The **angular displacement** of the spot, $\Delta\theta$, is

$$\Delta\theta = \theta_f - \theta_i$$

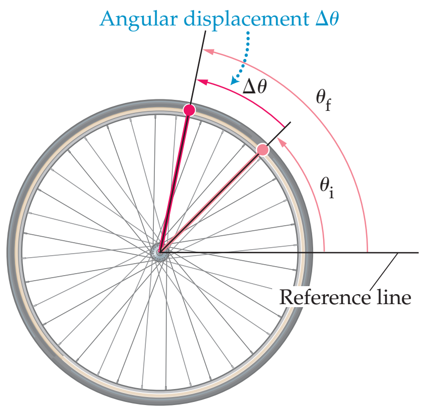
If we divide the angular displacement by the time, Δt, during which the displacement occurs, the result is the **average angular velocity,** ω_{av}.

Definition of Average Angular Velocity, ω_{av}

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad 10\text{–}3$$

SI unit: radian per second (rad/s) $= s^{-1}$

This is analogous to the definition of the average linear velocity $v_{av} = \Delta x/\Delta t$. Note that the units of linear velocity are m/s, whereas the units of angular velocity are rad/s.



▲ **FIGURE 10–3 Angular displacement**
As the wheel rotates, the spot of paint undergoes an angular displacement, $\Delta\theta = \theta_f - \theta_i$.

▲ Star trails provide a clear illustration of the relationship between angle, arc, and radius in circular motion. The stars, of course, do not actually move like this, but because of the Earth's rotation they appear to follow circular paths across the sky each night, with Polaris, the North Star, very near the axis of rotation. This photo was made by opening the camera shutter for an extended period of time. Notice that each star moves through the same angle in the course of the exposure. However, the farther a star is from the axis of rotation, the longer the arc it traces out in a given period of time. (Can you estimate the length of the exposure?)

In addition to the average angular velocity, we can define an **instantaneous angular velocity** as the limit of ω_{av} as the time interval, Δt, approaches zero. The instantaneous angular velocity, then, is

Definition of Instantaneous Angular Velocity, ω

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \text{10–4}$$

SI unit: $\text{rad/s} = \text{s}^{-1}$

Generally, we shall refer to the instantaneous angular velocity simply as the angular velocity.

Note that we call ω the angular velocity, not the angular speed. The reason is that ω can be positive or negative, depending on the sense of rotation. For example, if the red paint spot rotates in the counterclockwise sense, the angular position, θ, increases. As a result, $\Delta\theta$ is positive and therefore, so is ω. Similarly, clockwise rotation corresponds to a negative $\Delta\theta$ and hence a negative ω.

Sign Convention for Angular Velocity

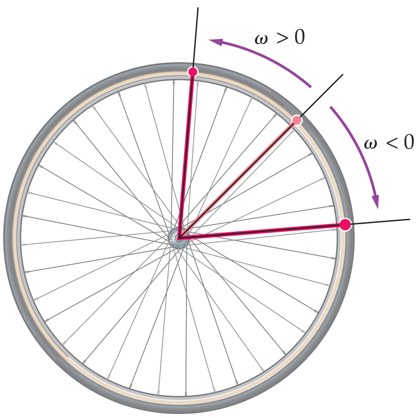
By convention:

$\omega > 0$ counterclockwise rotation

$\omega < 0$ clockwise rotation

The sign convention for angular velocity is illustrated in **Figure 10–4**. In analogy with linear motion, the sign of ω indicates the *direction* of the angular velocity *vector*, as we shall see in detail in Chapter 11. Similarly, the magnitude of the angular velocity is the **angular speed,** just as in the one-dimensional case.

In Exercise 10–1 we utilize the definitions and conversion factors presented so far in this section.



▲ **FIGURE 10–4 Angular speed and velocity**

Counterclockwise rotation is defined to correspond to a positive angular velocity, ω. Similarly, clockwise rotation corresponds to a negative angular velocity. The magnitude of the angular velocity is referred to as the angular speed.

EXERCISE 10–1

(a) An old phonograph record rotates clockwise at $33\frac{1}{3}$ rpm (revolutions per minute). What is its angular velocity in rad/s? **(b)** If a CD rotates at 22.0 rad/s, what is its angular speed in rpm?

SOLUTION

a. Convert from rpm to rad/s, and note that clockwise rotation corresponds to a negative angular velocity:

$$\omega = -33\tfrac{1}{3}\text{ rpm} = \left(-33\tfrac{1}{3}\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = -3.49\text{ rad/s}$$

b. Converting angular speed from rad/s to rpm gives

$$\omega = \left(22.0\frac{\text{rad}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 210\frac{\text{rev}}{\text{min}} = 210\text{ rpm}$$

Note that the same symbol, ω, is used for both angular velocity and angular speed in Exercise 10–1. Which quantity is meant in a given situation will be clear from the context in which it is used.

As a simple application of angular velocity, consider the following question: An object rotates with a constant angular velocity, ω. How much time, T, is required for it to complete one full revolution?

To solve this problem, note that since ω is constant, the instantaneous angular velocity is equal to the average angular velocity. That is,

$$\omega = \omega_{av} = \frac{\Delta\theta}{\Delta t}$$

In one revolution, we know that $\Delta\theta = 2\pi$ and $\Delta t = T$. Therefore,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

Finally, solving for T we find

$$T = \frac{2\pi}{\omega}$$

The time to complete one revolution, T, is referred to as the **period.**

Definition of Period, *T*

$$T = \frac{2\pi}{\omega} \qquad \text{10–5}$$

SI unit: second, s

EXERCISE 10–2

Find the period of a record that is rotating at 45 rpm.

SOLUTION

To apply $T = 2\pi/\omega$ we must first express ω in terms of rad/s:

$$45\text{ rpm} = \left(45\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 4.7\text{ rad/s}$$

Now we can calculate the period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{4.7\text{ rad/s}} = 1.3\text{ s}$$

Angular Acceleration, α

If the angular velocity of the rotating bicycle wheel increases or decreases with time, we say that the wheel experiences an **angular acceleration,** α. The average angular acceleration is the change in angular velocity, $\Delta\omega$, in a given interval of time, Δt:

Definition of Average Angular Acceleration, α_{av}

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \qquad \text{10–6}$$

SI unit: radian per second per second $(\text{rad/s}^2) = \text{s}^{-2}$

Note that the SI units of α are rad/s^2, which, since rad is dimensionless, is simply s^{-2}.

As expected, the instantaneous angular acceleration is the limit of α_{av} as the time interval, Δt, approaches zero:

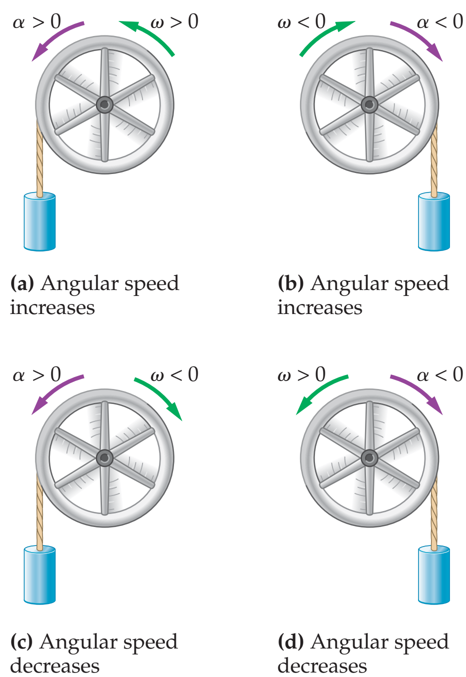
Definition of Instantaneous Angular Acceleration, α

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad \text{10–7}$$

SI unit: rad/s^2 = s^{-2}

When referring to the instantaneous angular acceleration, we will usually just say angular acceleration.

The sign of the angular acceleration is determined by whether the change in angular velocity is positive or negative. For example, if ω is becoming more positive, so that ω_f is greater than ω_i, it follows that α is positive. Similarly, if ω is becoming more negative, so that ω_f is less than ω_i, it follows that α is negative. Therefore, if ω and α have the same sign, the speed of rotation is increasing. If ω and α have opposite signs, the speed of rotation is decreasing. This is illustrated in **Figure 10–5**.



▲ **FIGURE 10–5 Angular acceleration and angular speed**

When angular velocity and acceleration have the same sign, as in **(a)** and **(b)**, the angular speed increases. When angular velocity and angular acceleration have opposite signs, as in **(c)** and **(d)**, the angular speed decreases.

EXERCISE 10–3

As the wind dies, a windmill that was rotating at 2.1 rad/s begins to slow down with a constant angular acceleration of 0.45 rad/s^2. How long does it take for the windmill to come to a complete stop?

SOLUTION

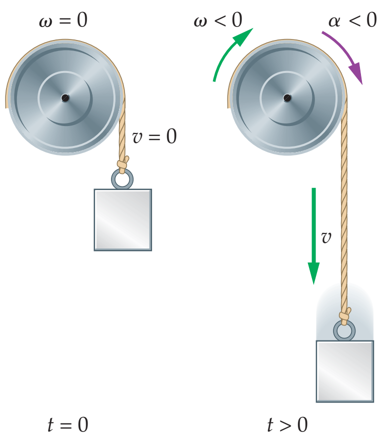
If we choose the initial angular velocity to be positive, the angular acceleration is negative, corresponding to a deceleration. Hence, Equation 10–6 gives

$$\Delta t = \frac{\Delta\omega}{\alpha_{\text{av}}} = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 2.1\ \text{rad/s}}{-0.45\ \text{rad/s}^2} = 4.7\ \text{s}$$

10–2 Rotational Kinematics

Just as the kinematics of Chapter 2 described linear motion, rotational kinematics describes rotational motion. In this section, as in Chapter 2, we concentrate on the important special case of constant acceleration.

As an example of a system with constant angular acceleration, consider the pulley shown in **Figure 10–6**. Wrapped around the circumference of the pulley is a string, with a mass attached to its free end. When the mass is released, the pulley begins to rotate—slowly at first, but then faster and faster. As we shall see in Chapter 11, the pulley is accelerating with constant angular acceleration.



▲ **FIGURE 10–6 A pulley with constant angular acceleration**

A mass is attached to a string wrapped around a pulley. As the mass falls, it causes the pulley to increase its angular speed with a constant angular acceleration.

Since α is constant, it follows that the average and instantaneous angular accelerations are equal. Hence,

$$\alpha = \alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t}$$

Suppose the pulley starts with the initial angular velocity ω_0 at time $t = 0$, and that at the later time t its angular velocity is ω. Substituting these values into the preceding expression for α yields

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{t - 0} = \frac{\omega - \omega_0}{t}$$

Rearranging, we see that the angular velocity, ω, varies with time as follows:

$$\omega = \omega_0 + \alpha t \qquad \text{10–8}$$

EXERCISE 10–4

If the angular velocity of the pulley in Figure 10–6 is −8.4 rad/s at a given time, and its angular acceleration is −2.8 rad/s^2, what is the angular velocity of the pulley 1.5 s later?

SOLUTION

The angular velocity, ω, is found by applying Equation 10–8:

$$\omega = \omega_0 + \alpha t = -8.4\ \text{rad/s} + (-2.8\ \text{rad/s}^2)(1.5\ \text{s}) = -12.6\ \text{rad/s}$$

Note that the angular speed has increased, as expected, since ω and α have the same sign.

Note the close analogy between Equation 10–8 for angular velocity and the corresponding relation for linear velocity, Equation 2–7:

$$v = v_0 + at$$

Clearly, the equation for angular velocity can be obtained from our previous equation for linear velocity by replacing v with ω and replacing a with α. This type of analogy between linear and angular quantities can be most useful both in deriving angular equations—by starting with linear equations and using analogies—and in obtaining a better physical understanding of angular systems. Several linear-to-angular analogs are listed in the adjacent table.

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Using these analogies, we can rewrite all the kinematic equations in Chapter 2 in angular form. The following table gives both the linear kinematic equations and their angular counterparts.

Linear Equation (a = constant)		Angular Equation (α = constant)	
$v = v_0 + at$	2–7	$\omega = \omega_0 + \alpha t$	10–8
$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$	10–9
$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	10–10
$v^2 = v_0{}^2 + 2a(x - x_0)$	2–12	$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$	10–11

In solving kinematic problems involving rotation, we apply these angular equations in the same way that the linear equations were applied in Chapter 2. In a sense, then, this material is a review—since the mathematics is essentially the same. The only difference comes in the physical interpretation of the results. We will emphasize the rotational interpretations throughout the chapter.

PROBLEM-SOLVING NOTE

Rotational Kinematics

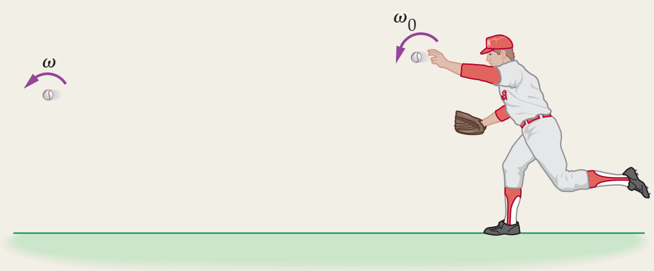
Using analogies between linear and angular quantities often helps when solving problems involving rotational kinematics.

EXAMPLE 10–1 THROWN FOR A CURVE

To throw a curve ball, a pitcher gives the ball an initial angular speed of 36.0 rad/s. When the catcher gloves the ball 0.595 s later, its angular speed has decreased (due to air resistance) to 34.2 rad/s. **(a)** What is the ball's angular acceleration, assuming it to be constant? **(b)** How many revolutions does the ball make before being caught?

PICTURE THE PROBLEM

We choose the ball's initial direction of rotation to be positive. As a result, the angular acceleration will be negative. We can also identify the initial angular velocity to be $\omega_0 = 36.0$ rad/s, and the final angular velocity to be $\omega = 34.2$ rad/s.



STRATEGY

The problem states that the angular acceleration of the ball is constant. It follows that Equations 10–8 to 10–11 apply to its rotation.

a. To relate angular velocity to time, we use $\omega = \omega_0 + \alpha t$. This can be solved for α.

b. To relate angle to time we use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. The angular displacement of the ball is $\theta - \theta_0$.

SOLUTION

Part (a)

1. Solve $\omega = \omega_0 + \alpha t$ for the angular acceleration, α:

$$\omega = \omega_0 + \alpha t$$
$$\alpha = \frac{\omega - \omega_0}{t}$$

2. Substitute numerical values to find α:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{34.2 \text{ rad/s} - 36.0 \text{ rad/s}}{0.595 \text{ s}} = -3.03 \text{ rad/s}^2$$

Part (b)

3. Use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ to calculate the angular displacement of the ball:

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (36.0 \text{ rad/s})(0.595 \text{ s}) + \tfrac{1}{2}(-3.03 \text{ rad/s}^2)(0.595 \text{ s})^2 = 20.9 \text{ rad}$$

2. Convert the angular displacement to revolutions:

$$\theta - \theta_0 = 20.9 \text{ rad} = 20.9 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 3.33 \text{ rev}$$

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INSIGHT

The ball rotates through three-and-one-third revolutions during its time in flight.

An alternative method of solution is to use the kinematic relation given in Equation 10–9. This procedure yields $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t = 20.9$ rad, in agreement with our previous result.

PRACTICE PROBLEM

(a) What is the angular velocity of the ball 0.500 s after it is thrown? **(b)** What is the ball's angular velocity after it completes its first full revolution? [**Answer:** (a) Use $\omega = \omega_0 + \alpha t$ to find $\omega = 34.5$ rad/s. (b) Use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ to find $\omega = 35.5$ rad/s.]

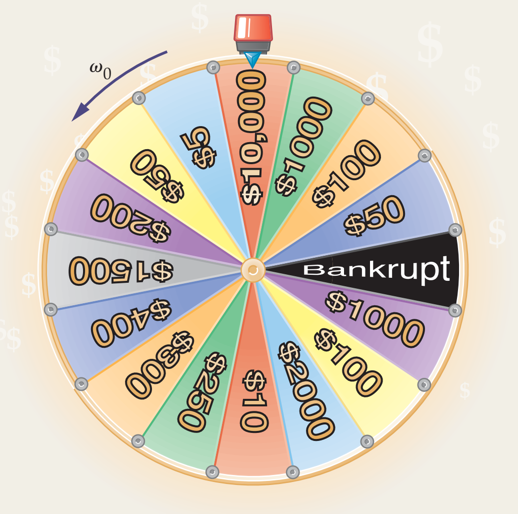
Some related homework problems: Problem 19, Problem 22

EXAMPLE 10–2 WHEEL OF MISFORTUNE

On a certain game show, contestants spin a wheel when it is their turn. One contestant gives the wheel an initial angular speed of 3.40 rad/s. It then rotates through one-and-one-quarter revolutions and comes to rest on the BANKRUPT space. **(a)** Find the angular acceleration of the wheel, assuming it to be constant. **(b)** How long does it take for the wheel to come to rest?

PICTURE THE PROBLEM

We choose the initial angular velocity to be positive, $\omega_0 = +3.40$ rad/s, and indicate it with a counterclockwise rotation in our sketch. Since the wheel slows to a stop, the angular acceleration must be negative; that is, in the clockwise direction. After a rotation of 1.25 rev the wheel will read BANKRUPT.



STRATEGY

As in Example 10–1, we can use the kinematic equations for constant angular acceleration, Equations 10–8 to 10–11.

a. To begin, we are given the initial angular velocity, $\omega_0 = +3.40$ rad/s, the final angular velocity, $\omega = 0$ (the wheel comes to rest), and the angular displacement, $\theta - \theta_0 = 1.25$ rev. We can find the angular acceleration using $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$.

b. Knowing the angular velocity and acceleration, we can find the time with $\omega = \omega_0 + \alpha t$.

SOLUTION

Part (a)

1. Solve $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ for the angular acceleration, α:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)}$$

2. Convert $\theta - \theta_0 = 1.25$ rev to radians:

$$\theta - \theta_0 = 1.25 \text{ rev} = 1.25 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 7.85 \text{ rad}$$

3. Substitute numerical values to find α:

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{0 - (3.40 \text{ rad/s})^2}{2(7.85 \text{ rad})} = -0.736 \text{ rad/s}^2$$

Part (b)

4. Solve $\omega = \omega_0 + \alpha t$ for the time, t:

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{\omega - \omega_0}{\alpha}$$

5. Substitute numerical values to find t:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 3.40 \text{ rad/s}}{(-0.736 \text{ rad/s}^2)} = 4.62 \text{ s}$$

INSIGHT

Note that it was not necessary to define a reference line; that is, a direction for $\theta = 0$. All we need to know is the angular displacement, $\theta - \theta_0$, not the individual angles θ and θ_0. Finally, notice that we can also solve Equation 10–9 for the time in part (b), which yields $t = 2(\theta - \theta_0)/(\omega_0 + \omega) = 4.62$ s, as expected.

PRACTICE PROBLEM

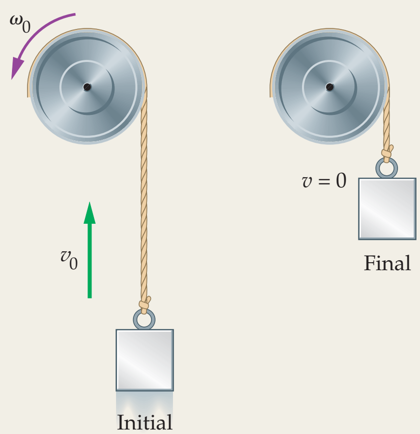
What is the angular speed of the wheel after one complete revolution? [**Answer:** $\omega = 1.52$ rad/s]

Some related homework problems: Problem 18, Problem 20

Finally, we consider a pulley that is rotating in such a way that initially it is lifting a mass with speed v. Gravity acting on the mass causes it and the pulley to slow and momentarily come to rest.

ACTIVE EXAMPLE 10–1 FIND THE TIME TO REST

A pulley rotating in the counterclockwise direction is attached to a mass suspended from a string. The mass causes the pulley's angular velocity to decrease with a constant angular acceleration $\alpha = -2.10\ \text{rad/s}^2$. **(a)** If the pulley's initial angular velocity is $\omega_0 = 5.40$ rad/s, how long does it take for the pulley to come to rest? **(b)** Through what angle does the pulley turn during this time?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. **(a)** Relate angular velocity to time: $\omega = \omega_0 + \alpha t$
2. Solve for the time, t: $t = (\omega - \omega_0)/\alpha$
3. Substitute numerical values: $t = 2.57$ s
4. **(b)** Use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ to solve for $\theta - \theta_0$: $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 = 6.94$ rad
5. Alternatively, use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$: $\theta - \theta_0 = (\omega^2 - \omega_0^2)/2\alpha = 6.94$ rad

INSIGHT

After the pulley comes to rest, it immediately begins to rotate in the clockwise direction as the mass falls. The pulley's angular *acceleration* is constant—it has the same value before the pulley stops, when it stops, and after it begins rotating in the opposite direction. This is analogous to a projectile thrown straight upward, where the linear velocity starts out positive, goes to zero, then changes sign, all while the linear acceleration remains constant in the negative direction.

YOUR TURN

Find the angular displacement of the pulley at the time when its angular velocity is half its initial value.

(Answers to **Your Turn** *problems are given in the back of the book.)*

10–3 Connections Between Linear and Rotational Quantities

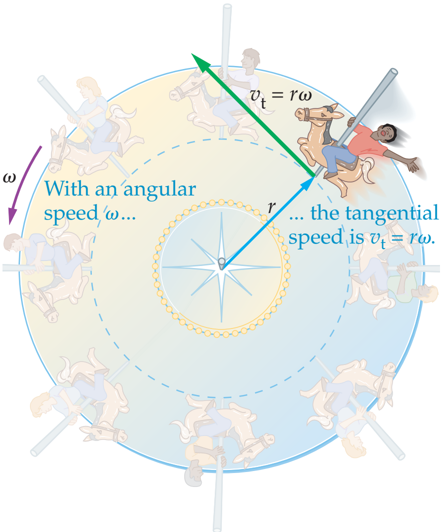
At a local county fair a child rides on a merry-go-round. The ride completes one circuit every $T = 7.50$ s. Therefore, the angular velocity of the child, from Equation 10–5, is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{7.50\ \text{s}} = 0.838\ \text{rad/s}$$

The path followed by the child is circular, with the center of the circle at the axis of rotation of the merry-go-round. In addition, at any instant of time the child is moving in a direction that is *tangential* to the circular path, as **Figure 10–7** shows. What is the tangential speed, v_t, of the child? In other words, what is the speed of the wind in the child's face?

We can find the child's tangential speed by dividing the circumference of the circular path, $2\pi r$, by the time required to complete one circuit, T. Thus,

$$v_t = \frac{2\pi r}{T} = r\left(\frac{2\pi}{T}\right)$$



▲ **FIGURE 10–7 Angular and linear speed**
Overhead view of a child riding on a merry-go-round. The child's path is a circle centered on the merry-go-round's axis of rotation. At any given time the child is moving tangential to the circular path with a speed $v_t = r\omega$.

Because $2\pi/T$ is simply ω, we can express the tangential speed as follows:

Tangential Speed of a Rotating Object

$$v_t = r\omega \quad \text{10–12}$$

SI unit: m/s

Note that ω must be given in rad/s for this relation to be valid.

In the case of the merry-go-round, if the radius of the child's circular path is $r = 4.25$ m, the tangential speed is $v_t = r\omega = (4.25\text{ m})(0.838\text{ rad/s}) = 3.56\text{ m/s}$. When it is clear that we are referring to the tangential speed, we will often drop the subscript t, and simply write $v = r\omega$.

An interesting application of the relation between linear and angular speeds is provided in the operation of a compact disk (CD). As you know, a CD is played by shining a laser beam onto the disk, and then converting the pattern of reflected light into a pattern of sound waves. For proper operation, however, the linear speed of the disk where the laser beam shines on it must be maintained at the constant value of 1.25 m/s. As the CD is played, the laser beam scans the disk in a spiral track from near the center outward to the rim. In order to maintain the required linear speed, the angular speed of the disk must decrease as the beam scans outward. The required angular speeds are determined in the following Exercise.

REAL-WORLD PHYSICS
The operation of a CD

EXERCISE 10–5

Find the angular speed a CD must have to give a linear speed of 1.25 m/s when the laser beam shines on the disk **(a)** 2.50 cm and **(b)** 6.00 cm from its center.

SOLUTION

a. Using $v = 1.25$ m/s and $r = 0.0250$ m in Equation 10–12, we find

$$\omega = \frac{v}{r} = \frac{1.25\text{ m/s}}{0.0250\text{ m}} = 50.0\text{ rad/s} = 477\text{ rpm}$$

b. Similarly, with $r = 0.0600$ m we find

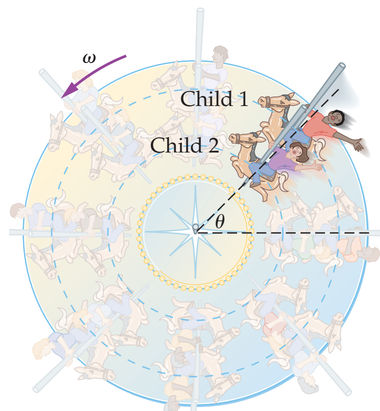
$$\omega = \frac{v}{r} = \frac{1.25\text{ m/s}}{0.0600\text{ m}} = 20.8\text{ rad/s} = 199\text{ rpm}$$

Thus, a CD slows from about 500 rpm to roughly 200 rpm as it plays.

How do the angular and tangential speeds of an object vary from one point to another? We explore this question in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 10–1 COMPARE THE SPEEDS

Two children ride on a merry-go-round, with child 1 at a greater distance from the axis of rotation than child 2. Is the angular speed of child 1 **(a)** greater than, **(b)** less than, or **(c)** the same as the angular speed of child 2?



REASONING AND DISCUSSION

At any given time, the angle θ for child 1 is the same as the angle for child 2, as shown. Therefore, when the angle for child 1 has gone through 2π, for example, so has the angle for child 2. As a result, they have the same angular speed. In fact, *each and every point on the merry-go-round has exactly the same angular speed*.

The tangential speeds are different, however. Child 1 has the greater tangential speed since he travels around a larger circle in the same time that child 2 travels around a smaller circle. This is in agreement with the relation $v = r\omega$, since the radius to child 1 is greater than the radius to child 2. That is, $v_1 = r_1\omega > v_2 = r_2\omega$.

ANSWER

(c) The angular speeds are the same.


◀ In the photo at left, two identical plastic letter "E"s have been placed on a rotating turntable at different distances from the axis of rotation. The stretching and blurring of the image of the outermost letter clearly show that it is moving faster than the letter closer to the axis. Similarly, the boy near the rim of this playground merry-go-round is moving faster than the girl near the hub.

Because the children on the merry-go-round move in a circular path, they experience a centripetal acceleration, a_{cp} (Section 6–5). The centripetal acceleration is always directed toward the axis of rotation and has a magnitude given by

$$a_{cp} = \frac{v^2}{r}$$

Note that the speed v in this expression is the tangential speed, $v = v_t = r\omega$, and therefore the centripetal acceleration in terms of ω is

$$a_{cp} = \frac{(r\omega)^2}{r}$$

Canceling one power of r, we have

Centripetal Acceleration of a Rotating Object

$$a_{cp} = r\omega^2 \qquad 10\text{–}13$$

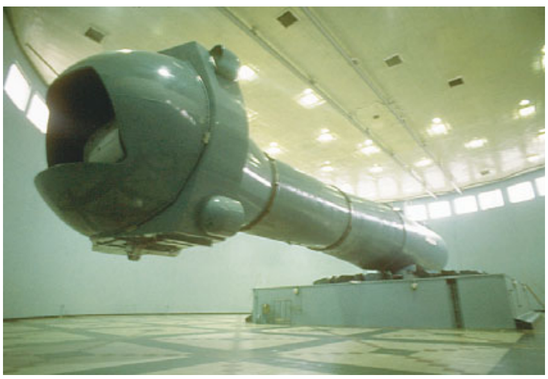
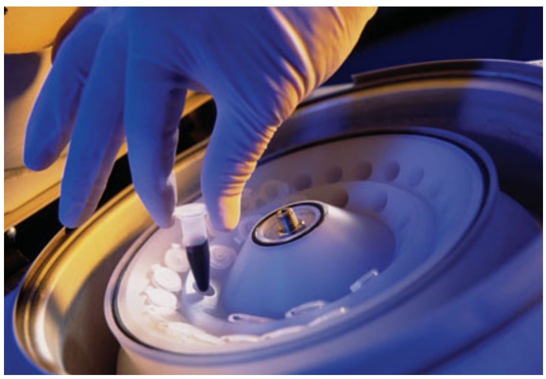
SI unit: m/s^2

If the radius of a child's circular path on the merry-go-round is 4.25 m, and the angular speed of the ride is 0.838 rad/s, the centripetal acceleration of the child is $a_{cp} = r\omega^2 = (4.25\ m)(0.838\ rad/s)^2 = 2.98\ m/s^2$.

Though the centripetal acceleration of a merry-go-round is typically only a fraction of the acceleration of gravity, rotating devices referred to as **centrifuges** can produce centripetal accelerations many times greater than gravity. For example, the world's most powerful research centrifuge, operated by the U.S. Army Corps of Engineers, can subject 2.2-ton payloads to accelerations as high as 350g (350 times greater than the acceleration of gravity). This centrifuge is used to study earthquake engineering and dam erosion. The Air Force uses centrifuges to subject prospective jet pilots to the accelerations they will experience during rapid flight maneuvers, and in the future NASA may even use a human-powered centrifuge for gravity studies aboard the International Space Station.

REAL-WORLD PHYSICS

The centrifuge

▲ The large centrifuge shown at left, at the Gagarin Cosmonaut Training Center, is used to train Russian cosmonauts for space missions. This device, which rotates at 36 rpm, can produce a centripetal acceleration of over 290 m/s^2, 30 times the acceleration of gravity. The device at right is a microhematocrit centrifuge, used to separate blood cells from plasma. The volume of red blood cells in a given quantity of whole blood is a major factor in determining the oxygen-carrying capacity of the blood, an important clinical indicator.

The centrifuges most commonly encountered in everyday life are those found in virtually every medical laboratory in the world. These devices, which can produce centripetal accelerations in excess of 13,000g, are used to separate blood cells from blood plasma. They do this by speeding up the natural tendency of cells to settle out of plasma from days to minutes. The ratio of the packed cell volume to the total blood volume gives the *hematocrit value*, which is a useful clinical indicator of blood quality. In the next Example we consider the operation of a *microhematocrit centrifuge*, which measures the hematocrit value of a small (micro) sample of blood.

REAL-WORLD PHYSICS: BIO

The microhematocrit centrifuge

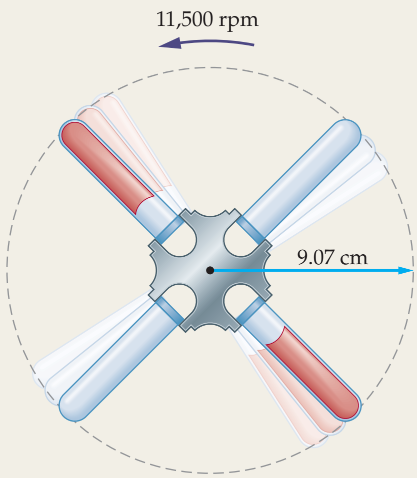
EXAMPLE 10–3 THE MICROHEMATOCRIT

REAL-WORLD PHYSICS: BIO

In a microhematocrit centrifuge, small samples of blood are placed in heparinized capillary tubes (heparin is an anticoagulant). The tubes are rotated at 11,500 rpm, with the bottoms of the tubes 9.07 cm from the axis of rotation. **(a)** Find the linear speed of the bottom of the tubes. **(b)** What is the centripetal acceleration at the bottom of the tubes?

PICTURE THE PROBLEM

Our sketch shows a top view of the centrifuge, with the capillary tubes rotating at 11,500 rpm. Notice that the bottoms of the tubes move in a circular path of radius 9.07 cm.



STRATEGY

a. Linear and angular speeds are related by $v = r\omega$. Once we convert the angular speed to rad/s we can use this relation to determine v.

b. The centripetal acceleration is $a_{cp} = r\omega^2$. Using ω from part (a) yields the desired result.

SOLUTION

Part (a)

1. Convert the angular speed, ω, to radians per second:

$$\omega = (11{,}500\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 1.20 \times 10^3\ \text{rad/s}$$

2. Use $v = r\omega$ to calculate the linear speed:

$$v = r\omega = (0.0907\ \text{m})(1.20 \times 10^3\ \text{rad/s}) = 109\ \text{m/s}$$

Part (b)

3. Calculate the centripetal acceleration using $a_{cp} = r\omega^2$:

$$a_{cp} = r\omega^2 = (0.0907\ \text{m})(1.20 \times 10^3\ \text{rad/s})^2 = 131{,}000\ \text{m/s}^2$$

4. As a check, calculate the centripetal acceleration using $a_{cp} = v^2/r$:

$$a_{cp} = \frac{v^2}{r} = \frac{(109\ \text{m/s})^2}{0.0907\ \text{m}} = 131{,}000\ \text{m/s}^2$$

INSIGHT

Note that every point on a tube has the same angular speed. As a result, points near the top of a tube have smaller linear speeds and centripetal accelerations than do points near the bottom of a tube. In this case, the bottoms of the tubes experience a centripetal acceleration about 13,400 times greater than the acceleration of gravity on the surface of the Earth; that is, $a_{cp} = 131{,}000\ \text{m/s}^2 = 13{,}400g$.

PRACTICE PROBLEM

What angular speed must this centrifuge have if the centripetal acceleration at the bottom of the tubes is to be 98,100 m/s^2 ($\approx 10{,}000g$)? [**Answer:** $\omega = \sqrt{a_{cp}/r} = 1040\ \text{rad/s} = 9930\ \text{rpm}$]

Some related homework problems: Problem 34, Problem 37

If the angular speed of the merry-go-round in Conceptual Checkpoint 10–1 changes, the tangential speed of the children changes as well. It follows, then, that the children will experience a tangential acceleration, a_t. We can determine a_t by considering the relation $v_t = r\omega$. If ω changes by the amount $\Delta\omega$, with r remaining constant, the corresponding change in tangential speed is

$$\Delta v_t = r\Delta\omega$$

If this change in ω occurs in the time Δt, the tangential acceleration is

$$a_t = \frac{\Delta v_t}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

Since $\Delta\omega/\Delta t$ is the angular acceleration, α, we find that

Tangential Acceleration of a Rotating Object

$$a_t = r\alpha \qquad 10\text{–}14$$

SI unit: m/s^2

As with the tangential speed, we will often drop the subscript t in a_t when no confusion will arise.

In general, the children on the merry-go-round may experience both tangential and centripetal accelerations at the same time. Recall that a_t is due to a changing tangential speed, and that a_{cp} is caused by a changing direction of motion, even if the tangential speed remains constant. To summarize:

Tangential Versus Centripetal Acceleration

$a_t = r\alpha$	due to changing angular speed
$a_{cp} = r\omega^2$	due to changing direction of motion

As the names suggest, the tangential acceleration is always tangential to an object's path; the centripetal acceleration is always perpendicular to its path.

In cases in which both the centripetal and tangential accelerations are present, the total acceleration is the vector sum of the two, as indicated in **Figure 10–8**. Note that $\vec{a}_t$ and $\vec{a}_{cp}$ are at right angles to one another, and hence the magnitude of the total acceleration is given by the Pythagorean theorem:

$$a = \sqrt{a_t^2 + a_{cp}^2}$$

The direction of the total acceleration, measured relative to the tangential direction, is

$$\phi = \tan^{-1}\left(\frac{a_{cp}}{a_t}\right)$$

This angle is shown in Figure 10–8.

In the next Active Example, we consider an object that is rotating with a constant angular acceleration, α. In this case, the tangential acceleration, $a_t = r\alpha$, is constant in magnitude. On the other hand, the centripetal acceleration, $a_{cp} = r\omega^2$, changes with time since the angular speed changes.

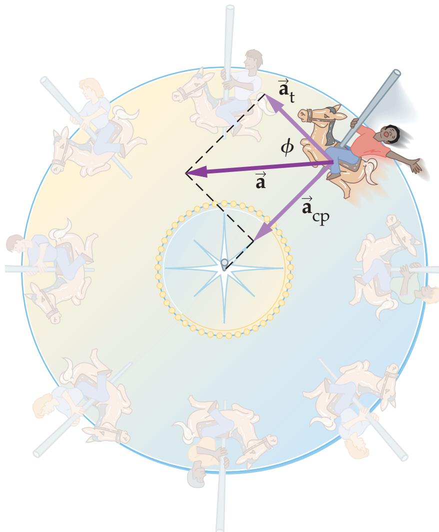


FIGURE 10–8 Centripetal and tangential acceleration

If the angular speed of the merry-go-round is increased, the child will experience two accelerations: (i) a tangential acceleration, $\vec{a}_t$, and (ii) a centripetal acceleration, $\vec{a}_{cp}$. The child's total acceleration, $\vec{a}$, is the vector sum of $\vec{a}_t$ and $\vec{a}_{cp}$.

ACTIVE EXAMPLE 10–2 FIND THE ACCELERATION

Suppose the centrifuge in Example 10–3 is starting up with a constant angular acceleration of 95.0 rad/s^2. **(a)** What are the magnitudes of the centripetal, tangential, and total accelerations of the bottom of a tube when the angular speed is 8.00 rad/s? **(b)** What angle does the total acceleration make with the direction of motion?

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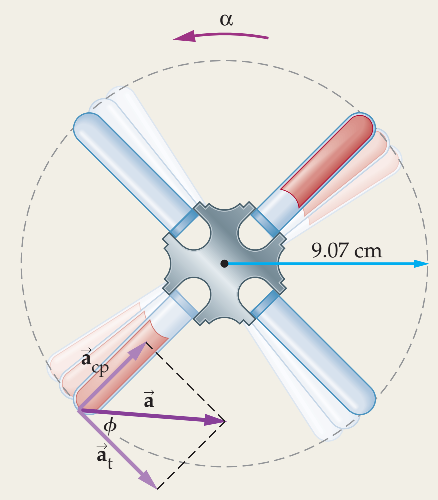

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SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Calculate the centripetal acceleration: $a_{cp} = r\omega^2 = 5.80 \text{ m/s}^2$
2. Calculate the tangential acceleration: $a_t = r\alpha = 8.62 \text{ m/s}^2$
3. Find the magnitude of the total acceleration: $a = \sqrt{a_{cp}^2 + a_t^2} = 10.4 \text{ m/s}^2$

Part (b)

4. Find the angle ϕ for the total acceleration: $\phi = \tan^{-1}(a_{cp}/a_t) = 33.9°$

INSIGHT

Note that all points on a tube have the same angular speed. In addition, all points have the same angular acceleration. In contrast, different points have different centripetal and tangential accelerations, due to their dependence on the distance r from the axis of rotation.

YOUR TURN

Find the magnitude and direction of the total acceleration of a point *halfway* between the top and bottom of a tube.

(Answers to **Your Turn** *problems are given in the back of the book.)*

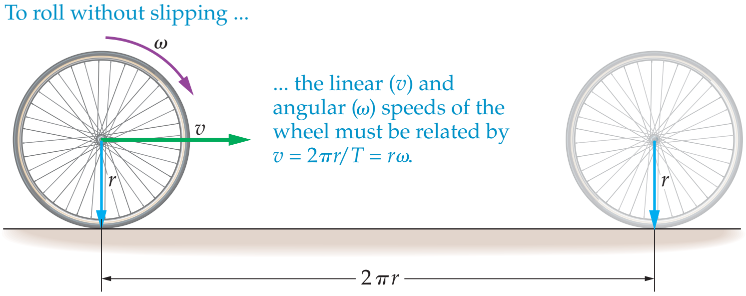


FIGURE 10–9 Rolling without slipping
A wheel of radius r rolling without slipping. During one complete revolution, the center of the wheel moves forward through a distance $2\pi r$.

10–4 Rolling Motion

We began this chapter with a bicycle wheel rotating about its axle. In that case, the axle was at rest and every point on the wheel, such as the spot of red paint, moved in a circular path about the axle. We would like to consider a different situation now. Suppose the bicycle wheel is rolling freely, as indicated in **Figure 10–9**, with no slipping between the tire and the ground. The wheel still rotates about the axle, but the axle itself is moving in a straight line. As a result, the motion of the wheel is a combination of both rotational motion and linear (or **translational**) motion.

To see the connection between the wheel's rotational and translational motions, we show one full rotation of the wheel in Figure 10–9. During this rotation, the axle translates forward through a distance equal to the circumference of the wheel, $2\pi r$. Because the time required for one rotation is the period, T, the translational speed of the axle is

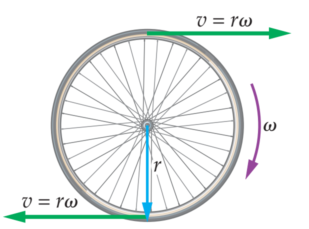
$$v = \frac{2\pi r}{T}$$

Recalling that $\omega = 2\pi/T$, we find

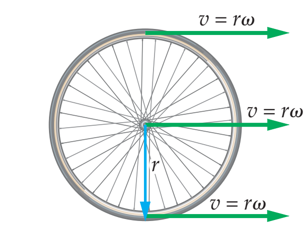
$$v = r\omega = v_t \tag{10–15}$$

Hence, the translational speed of the axle is equal to the tangential speed of a point on the rim of a wheel spinning with angular speed ω.

A rolling object, then, combines rotational motion with angular speed ω, and translational motion with linear speed $v = r\omega$, where r is the radius of the object. Let's consider these two motions one at a time. First, in **Figure 10–10 (a)** we show pure rotational motion with angular speed ω. In this case, the axle is at rest, and points at the top and bottom of the wheel have tangential velocities that are equal in magnitude, $v = r\omega$, but point in opposite directions.



(a) Pure rotational motion



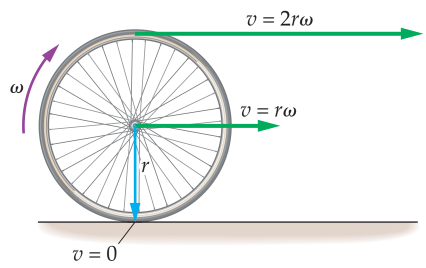
(b) Pure translational motion

FIGURE 10–10 Rotational and translational motions of a wheel
(a) In pure rotational motion, the velocities at the top and bottom of the wheel are in opposite directions. **(b)** In pure translational motion, each point on the wheel moves with the same speed in the same direction.

Next, we consider translational motion with speed $v = r\omega$. This is illustrated in **Figure 10–10 (b)**, where we see that each point on the wheel moves in the same direction with the same speed. If this were the only motion the wheel had, it would be skidding across the ground, instead of rolling without slipping.

Finally, we combine these two motions by simply adding the velocity vectors in Figures 10–10 (a) and (b). The result is shown in **Figure 10–11**. At the top of the wheel the two velocity vectors are in the same direction, so they sum to give a speed of $2r\omega$. At the axle, the velocity vectors sum to give a speed $r\omega$. Finally, at the bottom of the wheel, the velocity vectors from rotation and translation have equal magnitude, but are in opposite directions. As a result, these velocities cancel, giving a speed of zero where the wheel is in contact with the ground.

The fact that the bottom of the wheel is instantaneously at rest, so that it is in static contact with the ground, is precisely what is meant by "rolling without slipping." Thus, a wheel that rolls without slipping is just like the situation when you are walking—even though your body as a whole moves forward, the soles of your shoes are momentarily at rest every time you place them on the ground. This point was discussed in detail in Conceptual Checkpoint 6–1.



▲ **FIGURE 10–11 Velocities in rolling motion**

In a wheel that rolls without slipping, the point in contact with the ground is instantaneously at rest. The center of the wheel moves forward with the speed $v = r\omega$, and the top of the wheel moves forward with twice that speed, $v = 2r\omega$.

EXERCISE 10–6

A car with tires of radius 32 cm drives on the highway at 55 mph. **(a)** What is the angular speed of the tires? **(b)** What is the linear speed of the tops of the tires?

SOLUTION

a. Using Equation 10–15 we find

$$\omega = \frac{v}{r} = \frac{(55\text{ mph})\left(\dfrac{0.447\text{ m/s}}{1\text{ mph}}\right)}{0.32\text{ m}} = 77\text{ rad/s}$$

This is about 12 revolutions per second.

b. The tops of the tires have a speed of $2v = 110$ mph.

▲ This photograph of a rolling wheel gives a visual indication of the speed of its various parts. The bottom of the wheel is at rest at any instant, so the image there is sharp. The top of the wheel has the greatest speed, and the image there shows the most blurring. (Compare Figure 10–11.)

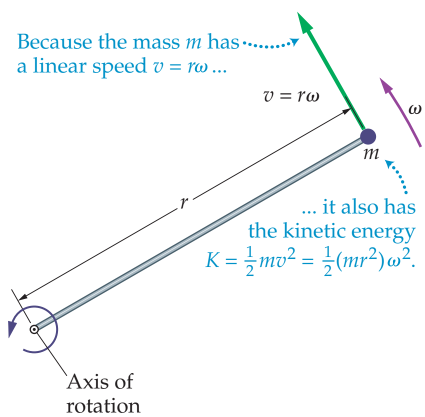
10–5 Rotational Kinetic Energy and the Moment of Inertia

An object in motion has kinetic energy, whether that motion is translational, rotational, or a combination of the two. In translational motion, for example, the kinetic energy of a mass m moving with a speed v is $K = \frac{1}{2}mv^2$. We cannot use this expression for a rotating object, however, because the speed v of each particle within a rotating object varies with its distance r from the axis of rotation, as we have seen in Equation 10–12. Thus, there is no unique value of v for an entire rotating object. On the other hand, there *is* a unique value of ω, the angular speed, that applies to all particles in the object.

To see how the kinetic energy of a rotating object depends on its angular speed, we start with a particularly simple system consisting of a rod of length r and negligible mass rotating about one end with an angular speed ω. Attached to the other end of the rod is a point mass m, as **Figure 10–12** shows. To find the kinetic energy of the mass, recall that its linear speed is $v = r\omega$ (Equation 10–12). Therefore, the translational kinetic energy of the mass m is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(r\omega)^2 = \tfrac{1}{2}(mr^2)\omega^2 \qquad \text{10–16}$$

Notice that the kinetic energy of the mass depends not only on the angular speed squared (analogous to the way the translational kinetic energy depends on the linear speed squared), but also on the radius squared—that is, the kinetic energy depends on the *distribution* of mass in the rotating object. To be specific, mass near the axis of rotation contributes little to the kinetic energy since its speed ($v = r\omega$) is small. On the other hand, the farther a mass is from the axis of rotation, the greater its speed v for a given angular velocity, and thus the greater its kinetic energy.



▲ **FIGURE 10–12 Kinetic energy of a rotating object**

As this rod rotates about the axis of rotation with an angular speed ω, the mass has a speed of $v = r\omega$. It follows that the kinetic energy of the mass is $K = \frac{1}{2}mv^2 = \frac{1}{2}(mr^2)\omega^2$.

You have probably noticed that the kinetic energy in Equation 10–16 is similar in form to the translational kinetic energy. Instead of $\frac{1}{2}(m)v^2$, we now have $\frac{1}{2}(mr^2)\omega^2$. Clearly, then, the quantity mr^2 plays the role of the mass for the rotating object. This "rotational mass" is given a special name in physics: the **moment of inertia,** I. Thus, in general, the kinetic energy of an object rotating with an angular speed ω can be written as:

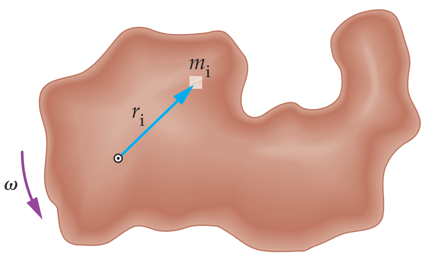
Rotational Kinetic Energy

$$K = \frac{1}{2}I\omega^2 \qquad 10\text{–}17$$

SI unit: J

The greater the moment of inertia—which some books call the rotational inertia—the greater an object's rotational kinetic energy. As we have just seen, in the special case of a point mass m a distance r from the axis of rotation, the moment of inertia is simply $I = mr^2$.

We now show how to find the moment of inertia for an object of arbitrary, fixed shape, as in **Figure 10–13**. Suppose, for example, that this object rotates about the axis indicated in the figure with an angular speed ω. To calculate the kinetic energy of the object, we first imagine dividing it into a collection of small mass elements, m_i. We then calculate the kinetic energy of each element and sum over all elements. This extends to a large number of mass elements what we did for the single mass m.



▲ **FIGURE 10–13 Kinetic energy of a rotating object of arbitrary shape**

To calculate the kinetic energy of an object of arbitrary shape as it rotates about an axis with angular speed ω, imagine dividing it into small mass elements, m_i. The total kinetic energy of the object is the sum of the kinetic energies of all the mass elements.

Following this plan, the total kinetic energy of an arbitrary rotating object is

$$K = \sum\left(\frac{1}{2}m_i v_i^2\right)$$

In this expression, m_i is the mass of one of the small mass elements and v_i is its speed. If m_i is at the radius r_i from the axis of rotation, as indicated in Figure 10–13, its speed is $v_i = r_i\omega$. Note that it is not necessary to write a separate angular speed, ω_i, for each element, because *all* mass elements of the object have exactly the same angular speed, ω. Therefore,

$$K = \sum\left(\frac{1}{2}m_i r_i^2\omega^2\right) = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

Now, in analogy with our results for the single mass, we can define the moment of inertia, I, as follows:

Definition of Moment of Inertia, *I*

$$I = \sum m_i r_i^2 \qquad 10\text{–}18$$

SI unit: $kg \cdot m^2$

The precise value of I for a given object depends on its distribution of mass. A simple example of this dependence is given in the following Exercise.

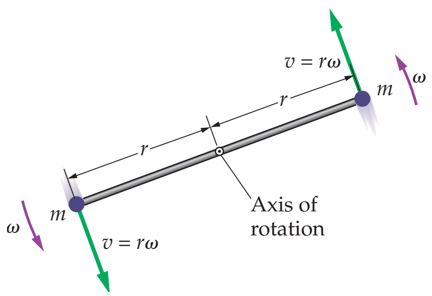
EXERCISE 10–7

Use the general definition of the moment of inertia, as given in Equation 10–18, to find the moment of inertia for the dumbbell-shaped object shown in **Figure 10–14**. Note that the axis of rotation goes through the center of the object and points out of the page. In addition, assume that the masses may be treated as point masses.

SOLUTION

Referring to Figure 10–14, we see that $m_1 = m_2 = m$ and $r_1 = r_2 = r$. Therefore, the moment of inertia is

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = mr^2 + mr^2 = 2mr^2$$



▲ **FIGURE 10–14 A dumbbell-shaped object rotating about its center**

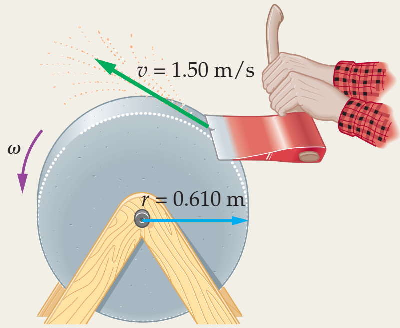
The connection between rotational kinetic energy and the moment of inertia is explored in more detail in the following Example.

EXAMPLE 10–4 NOSE TO THE GRINDSTONE

A grindstone with a radius of 0.610 m is being used to sharpen an ax. **(a)** If the linear speed of the stone relative to the ax is 1.50 m/s, and the stone's rotational kinetic energy is 13.0 J, what is its moment of inertia? **(b)** If the linear speed is doubled to 3.00 m/s, what is the corresponding kinetic energy of the grindstone?

PICTURE THE PROBLEM

Our sketch shows the grindstone spinning with an angular speed ω, which is not given in the problem statement. We do know, however, that the linear speed of the grindstone at its rim is $v = 1.50$ m/s and that its radius is $r = 0.610$ m. At this rate of rotation, the stone has a kinetic energy of 13.0 J.



STRATEGY

a. Recall that rotational kinetic energy and moment of inertia are related by $K = \frac{1}{2}I\omega^2$; thus $I = 2K/\omega^2$. We are not given ω, but we can find it from the connection between linear and angular speed, $v = r\omega$. Thus, we begin by finding ω. We then use ω, along with the kinetic energy K, to find I.

b. Find the new angular speed with $\omega = v/r$. Use I from part (a), along with $K = \frac{1}{2}I\omega^2$, to find the new kinetic energy.

SOLUTION

Part (a)

1. Find the angular speed of the grindstone: $\omega = \dfrac{v}{r} = \dfrac{1.50 \text{ m/s}}{0.610 \text{ m}} = 2.46 \text{ rad/s}$

2. Solve for the moment of inertia in terms of kinetic energy: $K = \frac{1}{2}I\omega^2 \quad \text{or} \quad I = \dfrac{2K}{\omega^2}$

3. Substitute numerical values for K and ω: $I = \dfrac{2K}{\omega^2} = \dfrac{2(13.0 \text{ J})}{(2.46 \text{ rad/s})^2} = 4.30 \text{ J}\cdot\text{s}^2 = 4.30 \text{ kg}\cdot\text{m}^2$

Part (b)

4. Find the angular speed of the grindstone corresponding to $v = 3.00$ m/s: $\omega = \dfrac{v}{r} = \dfrac{3.00 \text{ m/s}}{0.610 \text{ m}} = 4.92 \text{ rad/s}$

5. Determine the kinetic energy, K, using the moment of inertia, I, from part (a): $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(4.30 \text{ kg}\cdot\text{m}^2)(4.92 \text{ rad/s})^2 = 52.0 \text{ J}$

INSIGHT

(a) We found I by relating it to the rotational kinetic energy of the grindstone. Later in this section we show how to calculate the moment of inertia of a disk directly, given its radius and mass. **(b)** Doubling the linear speed, v, results in a doubling of the angular speed, ω. The kinetic energy K depends on ω^2; therefore doubling ω increases K by a factor of 4, from 13.0 J to 4(13.0 J) = 52.0 J.

PRACTICE PROBLEM

When the ax is pressed firmly against the grindstone for sharpening, the angular speed of the grindstone decreases. If the rotational kinetic energy of the grindstone is cut in half to 6.50 J, what is its angular speed? [**Answer:** The moment of inertia is unchanged; it depends only on the size, shape, and mass of the grindstone. Hence, $\omega = \sqrt{2K/I} = 1.74$ rad/s, which is smaller than the original $\omega = 2.46$ rad/s by a factor of $\sqrt{2}$.]

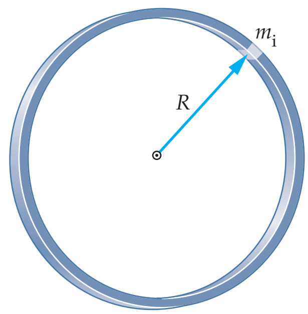
Some related homework problems: Problem 56, Problem 57

We return now to the dependence of the moment of inertia on the particular shape, or mass distribution, of an object. Suppose, for example, that a mass M is formed into the shape of a *hoop* of radius R. In addition, consider the case where the axis of rotation is perpendicular to the plane of the hoop and passes through its center, as shown in **Figure 10–15**. This is similar to a bicycle wheel rotating about its axle, if one ignores the spokes. In terms of small mass elements, we can write the moment of inertia as

$$I = \sum m_i r_i^2$$

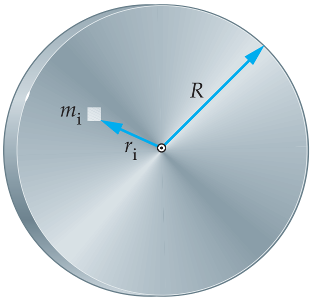
Each mass element of the hoop, however, is at the same radius R from the axis of rotation; that is, $r_i = R$. Hence, the moment of inertia in this case is

$$I = \sum m_i r_i^2 = \sum m_i R^2 = \left(\sum m_i\right) R^2$$



▲ **FIGURE 10–15 The moment of inertia of a hoop**

Consider a hoop of mass M and radius R. Each small mass element is at the same distance, R, from the center of the hoop. The moment of inertia in this case is $I = MR^2$.



▲ **FIGURE 10–16 The moment of inertia of a disk**

Consider a disk of mass M and radius R. Mass elements for the disk are at distances from the center ranging from 0 to R. The moment of inertia in this case is $I = \frac{1}{2}MR^2$.

Clearly, the sum of all the elementary masses is simply the total mass of the hoop, $\Sigma m_i = M$. Therefore, the moment of inertia of a hoop of mass M and radius R is

$$I = MR^2 \text{ (hoop)}$$

In contrast, if the same mass, M, is formed into a uniform *disk* of the same radius, R, the moment of inertia is different. To see this, note that it is no longer true that $r_i = R$ for all mass elements. In fact, most of the mass elements are closer to the axis of rotation than was the case for the hoop, as indicated in **Figure 10–16**. Thus, since the r_i are generally less than R, the moment of inertia will be smaller for the disk than for the hoop. A detailed calculation, summing over all mass elements, yields the following result:

$$I = \tfrac{1}{2}MR^2 \text{ (disk)}$$

As expected, I is less for the disk than for the hoop.

EXERCISE 10–8

If the grindstone in Example 10–4 is a uniform disk, what is its mass?

SOLUTION

Applying the preceding equation yields

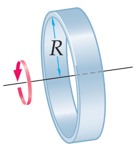
$$M = \frac{2I}{R^2} = \frac{2(4.30 \text{ kg}\cdot\text{m}^2)}{(0.610 \text{ m})^2} = 23.1 \text{ kg}$$

Thus, the grindstone has a weight of roughly 51 lb.

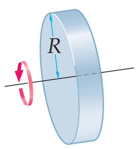
Table 10–1 collects moments of inertia for a variety of objects. Note that in all cases the moment of inertia is of the form $I = (\text{constant})MR^2$. It is only the constant in front of MR^2 that changes from one object to another.

Note also that objects of the same general shape but with different mass distributions—such as solid and hollow spheres—have different moments of inertia. In particular, a hollow sphere has a larger I than a solid sphere of the same mass, for the same reason that a hoop's moment of inertia is greater than a disk's—more of its mass is at a greater distance from the axis of rotation. Thus, I is a measure of both the shape *and* the mass distribution of an object.

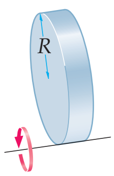
TABLE 10–1 Moments of Inertia for Uniform, Rigid Objects of Various Shapes and Total Mass *M*



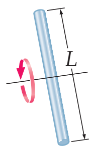
Hoop or cylindrical shell
$I = MR^2$



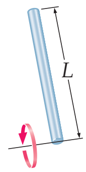
Disk or solid cylinder
$I = \frac{1}{2}MR^2$



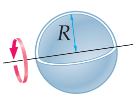
Disk or solid cylinder (axis at rim)
$I = \frac{3}{2}MR^2$



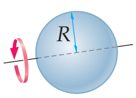
Long thin rod (axis through midpoint)
$I = \frac{1}{12}ML^2$



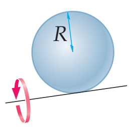
Long thin rod (axis at one end)
$I = \frac{1}{3}ML^2$



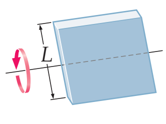
Hollow sphere
$I = \frac{2}{3}MR^2$



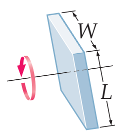
Solid sphere
$I = \frac{2}{5}MR^2$



Solid sphere (axis at rim)
$I = \frac{7}{5}MR^2$

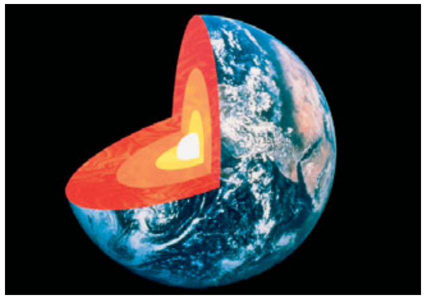


Solid plate (axis through center, in plane of plate)
$I = \frac{1}{12}ML^2$



Solid plate (axis perpendicular to plane of plate)
$I = \frac{1}{12}M(L^2 + W^2)$

Consider, for example, the moment of inertia of the Earth. If the Earth were a uniform sphere of mass M_E and radius R_E, its moment of inertia would be $\frac{2}{5}M_E R_E^2 = 0.4 M_E R_E^2$. In fact, the Earth's moment of inertia is only $0.331 M_E R_E^2$, considerably less than for a uniform sphere. This is due to the fact that the Earth is not homogeneous, but instead has a dense inner core surrounded by a less dense outer core and an even less dense mantle. The resulting concentration of mass near its axis of rotation gives the Earth a much smaller moment of inertia than it would have if its mass were uniformly distributed.



▲ The distribution of mass in the Earth is not uniform. Dense materials, like iron and nickel, have concentrated near the center, while less dense materials, like silicon and aluminum, have risen to the surface. This concentration of mass near the axis of rotation lowers the Earth's moment of inertia.

On the other hand, if the polar ice caps were to melt and release their water into the oceans, the Earth's moment of inertia would increase. This is because mass that had been near the axis of rotation (in the polar ice) would now be distributed more or less uniformly around the Earth (in the oceans). With more of the Earth's mass at greater distances from the axis of rotation, the moment of inertia would increase. If such an event were to occur, not only would the moment of inertia increase, but the length of the day would increase as well. We shall discuss the reasons for this in the next chapter.

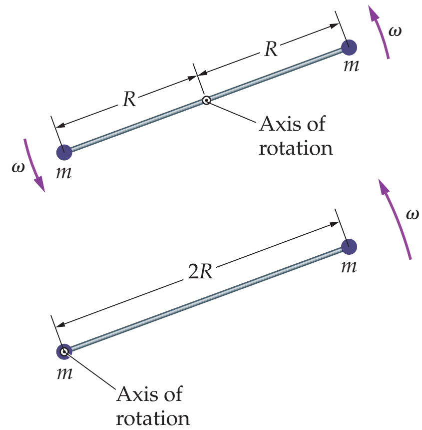
The moment of inertia of an object also depends on the location and orientation of the axis of rotation. If the axis of rotation is moved, all of the r_i change, leading to a different result for I. This is investigated for the dumbbell system in the following Conceptual Checkpoint.

REAL-WORLD PHYSICS

Moment of inertia of the Earth

CONCEPTUAL CHECKPOINT 10–2 COMPARE THE MOMENTS OF INERTIA

If the dumbbell-shaped object in Figure 10–14 is rotated about one end, is its moment of inertia **(a)** more than, **(b)** less than, or **(c)** the same as the moment of inertia about its center? As before, assume that the masses can be treated as point masses.



REASONING AND DISCUSSION

As we saw in Exercise 10–7, the moment of inertia about the center of the dumbbell is $I = 2mR^2$. When the axis is at one end, that mass is at the radius $r = 0$, and the other mass is at $r = 2R$. Therefore, the moment of inertia is

$$I = \sum m_i r_i^2 = m \cdot 0 + m(2R)^2 = 4mR^2$$

Thus, the moment of inertia doubles when the axis of rotation is moved from the center to one end.

The reason I increases is that the moment of inertia depends on the radius squared. Hence, even small increases in r can cause significant increases in I. By moving the axis to one end, the radius to the other mass is increased to its greatest possible value. As a result, I increases.

ANSWER

(a) The moment of inertia is greater about one end than about the center.

Finally, we summarize in the accompanying table the similarities between the translational kinetic energy, $K = \frac{1}{2}mv^2$, and the rotational kinetic energy, $K = \frac{1}{2}I\omega^2$. As expected, we see that the linear speed, v, has been replaced with the angular speed, ω. In addition, note that the mass m has been replaced with the moment of inertia I.

Linear Quantity	Angular Quantity
v	ω
m	I
$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$

As suggested by these analogies, the moment of inertia I plays the same role in rotational motion that mass plays in translational motion. For example, the larger I the more resistant an object is to any change in its angular velocity—an object with a large I is difficult to start rotating, and once it is rotating, it is difficult to stop. We shall see further applications of this analogy in the next chapter when we consider angular momentum.

10–6 Conservation of Energy

In this section, we consider the mechanical energy of objects that roll without slipping, and show how to apply energy conservation to such systems. In addition, we consider objects that rotate as a string or rope unwinds: for example, a pulley with a string wrapped around its circumference, or a yo-yo with a string wrapped around its axle. As long as the unwinding process and the rolling motion occur without slipping, the two situations are basically the same—at least as far as energy considerations are concerned.

To apply energy conservation to rolling objects, we first need to determine the kinetic energy of rolling motion. In Section 10–4 we saw that rolling motion is a combination of rotation and translation. It follows, then, that the kinetic energy of a rolling object is simply the sum of its translational kinetic energy, $\frac{1}{2}mv^2$, and its rotational kinetic energy, $\frac{1}{2}I\omega^2$:

PROBLEM-SOLVING NOTE

Energy Conservation with Rotational Motion

When applying energy conservation to a system with rotational motion, be sure to include the rotational kinetic energy, $\frac{1}{2}I\omega^2$.

Kinetic Energy of Rolling Motion

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \qquad \text{10–19}$$

Note that I in this expression is the moment of inertia about the center of the rolling object.

We can simplify the expression for the kinetic energy of a rolling object by using the fact that linear and angular speeds are related. In fact, recall that $v = r\omega$ (Equation 10–12), which can be rewritten as $\omega = v/r$. Substituting this into our expression for the rolling kinetic energy yields

Kinetic Energy of Rolling Motion: Alternative Form

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{r}\right)^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) \qquad \text{10–20}$$

Since $I = (\text{constant})mr^2$, the last term in Equation 10–20 is a constant that depends on the shape and mass distribution of the rolling object.

A special case of some interest is the point particle. In this case, by definition, all of the mass is at a single point. Therefore, $r = 0$, and hence $I = 0$. Substituting $I = 0$ in either Equation 10–19 or Equation 10–20 yields $K = \frac{1}{2}mv^2$, as expected.

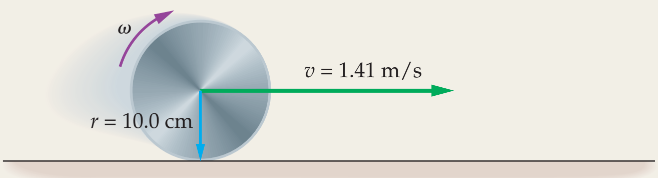
Next, we apply Equations 10–19 and 10–20 to a disk that rolls with no slipping.

EXAMPLE 10–5 LIKE A ROLLING DISK

A 1.20-kg disk with a radius of 10.0 cm rolls without slipping. If the linear speed of the disk is 1.41 m/s, find **(a)** the translational kinetic energy, **(b)** the rotational kinetic energy, and **(c)** the total kinetic energy of the disk.

PICTURE THE PROBLEM

Because the disk rolls without slipping, the angular speed and the linear speed are related by $v = r\omega$. Note that the linear speed is $v = 1.41$ m/s and the radius is $r = 10.0$ cm. Finally, we are given that the mass of the disk is 1.20 kg.



STRATEGY

We calculate each contribution to the kinetic energy separately. The linear kinetic energy, of course, is simply $\frac{1}{2}mv^2$. For the rotational kinetic energy, $\frac{1}{2}I\omega^2$, we must use the fact that the moment of inertia for a disk is $I = \frac{1}{2}mr^2$. Finally, since the disk rolls without slipping, its angular speed is $\omega = v/r$.

SOLUTION

Part (a)

1. Calculate the translational kinetic energy, $\frac{1}{2}mv^2$: $\quad \frac{1}{2}mv^2 = \frac{1}{2}(1.20\text{ kg})(1.41\text{ m/s})^2 = 1.19\text{ J}$

Part (b)

2. Calculate the rotational kinetic energy symbolically, using $I = \frac{1}{2}mr^2$ and $\omega = v/r$: $\quad \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

3. Substitute the numerical value for $\frac{1}{2}mv^2$ (the translational kinetic energy) obtained in Step 1: $\quad \frac{1}{2}I\omega^2 = \frac{1}{2}(1.19\text{ J}) = 0.595\text{ J}$

Part (c)

4. Sum the kinetic energies obtained in parts (a) and (b): $\quad K = 1.19\text{ J} + 0.595\text{ J} = 1.79\text{ J}$

5. Note that the same result is obtained using Equation 10–20:

$$K = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) = \tfrac{1}{2}mv^2\left(1 + \frac{1}{2}\right)$$
$$= \tfrac{3}{2}\left(\tfrac{1}{2}mv^2\right) = \tfrac{3}{2}(1.19\ \mathrm{J}) = 1.79\ \mathrm{J}$$

INSIGHT

The symbolic result in Step 2 shows that the rotational kinetic energy of a uniform disk rolling without slipping is precisely one-half the disk's translational kinetic energy. Thus, 2/3 of the disk's kinetic energy is translational, 1/3 rotational. This result is independent of the disk's radius, as we can see by the cancellation of the radius r in Step 2.

To understand this cancellation, note that a larger disk has a larger moment of inertia, since it has mass farther from the axis of rotation. On the other hand, the larger disk also has a smaller angular speed, since the angular speed is inversely proportional to the radius: $\omega = v/r$. These two effects cancel, giving the same rotational kinetic energy for uniform disks of any radius—provided their linear speed is the same.

PRACTICE PROBLEM

Repeat this problem for the case of a rolling, hollow sphere. [**Answer:** (a) 1.19 J, (b) 0.793 J, (c) 1.98 J]

Some related homework problems: Problem 58, Problem 62

CONCEPTUAL CHECKPOINT 10–3 COMPARE KINETIC ENERGIES

A solid sphere and a hollow sphere of the same mass and radius roll without slipping at the same speed. Is the kinetic energy of the solid sphere **(a)** more than, **(b)** less than, or **(c)** the same as the kinetic energy of the hollow sphere?

REASONING AND DISCUSSION

Both spheres have the same translational kinetic energy since they have the same mass and speed. The rotational kinetic energy, however, is proportional to the moment of inertia. Since the hollow sphere has the greater moment of inertia, it has the greater kinetic energy.

ANSWER

(b) The solid sphere has less kinetic energy than the hollow sphere.

Now that we can calculate the kinetic energy of rolling motion, we show how to apply it to energy conservation. For example, consider an object of mass m, radius r, and moment of inertia I at the top of a ramp, as shown in **Figure 10–17**. The object is released from rest and allowed to roll to the bottom, a vertical height h below the starting point. What is the object's speed on reaching the bottom?

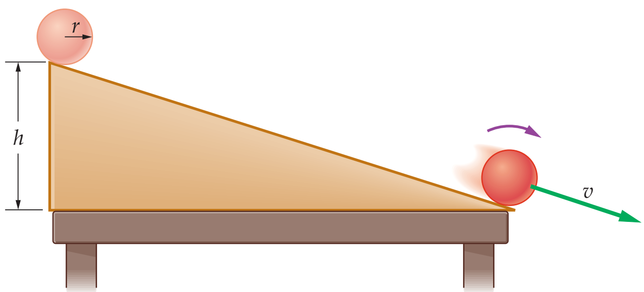


FIGURE 10–17 An object rolls down an incline

An object starts at rest at the top of an inclined plane and rolls without slipping to the bottom. The speed of the object at the bottom depends on its moment of inertia—a larger moment of inertia results in a lower speed.

The simplest way to solve this problem is to use energy conservation. To do so, we set the initial mechanical energy at the top (i) equal to the final mechanical energy at the bottom (f). That is,

$$K_i + U_i = K_f + U_f$$

Since we are dealing with rolling motion, the kinetic energy is

$$K = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

The potential energy is simply that due to the uniform gravitational field. Therefore,

$$U = mgy$$

With $y = h$ at the top of the ramp and the object starting at rest, we have

$$K_i + U_i = 0 + mgh = mgh$$

Similarly, with $y = 0$ at the bottom of the ramp and the object rolling with a speed v, we find

$$K_f + U_f = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) + 0 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

Setting the initial and final energies equal yields

$$mgh = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

Solving for v, we find

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}$$

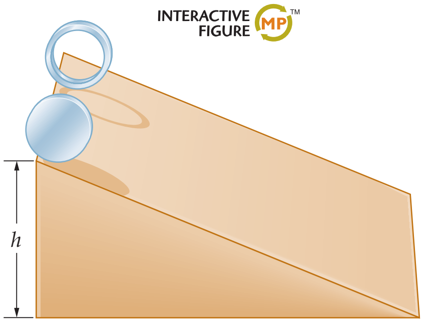
Let's quickly check one special case: namely, $I = 0$. With this substitution, we find

$$v = \sqrt{2gh}$$

This is the speed an object would have after falling straight down with no rotation through a distance h. Thus, setting $I = 0$ means there is no rotational kinetic energy, and hence the result is the same as for a point particle. As I becomes larger, the speed at the bottom of the ramp is smaller.

CONCEPTUAL CHECKPOINT 10–4 WHICH OBJECT WINS THE RACE?

A disk and a hoop of the same mass and radius are released at the same time at the top of an inclined plane. Does the disk reach the bottom of the plane **(a)** before, **(b)** after, or **(c)** at the same time as the hoop?



REASONING AND DISCUSSION

As we have just seen, the larger the moment of inertia, I, the smaller the speed, v. Hence the object with the larger moment of inertia (the hoop in this case) loses the race to the bottom, because its speed is less than the speed of the disk at any given height.

Another way to think about this is to recall that both objects have the same mechanical energy to begin with, namely, mgh. For the hoop, more of this initial potential energy goes into rotational kinetic energy, since the hoop has the larger moment of inertia; therefore, less energy is left for translational motion. As a result, the hoop moves more slowly and loses the race.

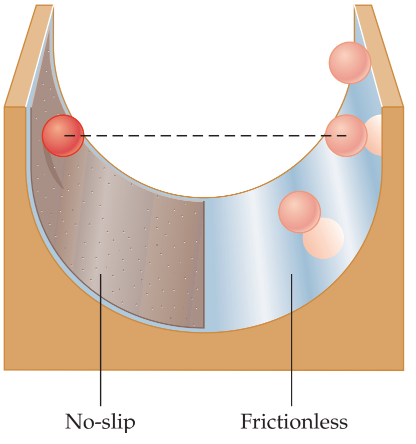
ANSWER

(a) The disk wins the race by reaching the bottom before the hoop.

In the next Conceptual Checkpoint, we consider the effects of a surface that changes from nonslip to frictionless.

CONCEPTUAL CHECKPOINT 10–5 COMPARE HEIGHTS

A ball is released from rest on a no-slip surface, as shown. After reaching its lowest point, the ball begins to rise again, this time on a frictionless surface. When the ball reaches its maximum height on the frictionless surface, is it **(a)** at a greater height, **(b)** at a lesser height, or **(c)** at the same height as when it was released?



REASONING AND DISCUSSION

As the ball descends on the no-slip surface, it begins to rotate, increasing its angular speed until it reaches the lowest point of the surface. When it begins to rise again, there is no friction to slow the rotational motion; thus, the ball continues to rotate with the same angular speed it had at its lowest point. Therefore, some of the ball's initial gravitational potential energy remains in the form of rotational kinetic energy. As a result, less energy is available to be converted back into gravitational potential energy, and the height is less.

ANSWER

(b) The height on the frictionless side is less.

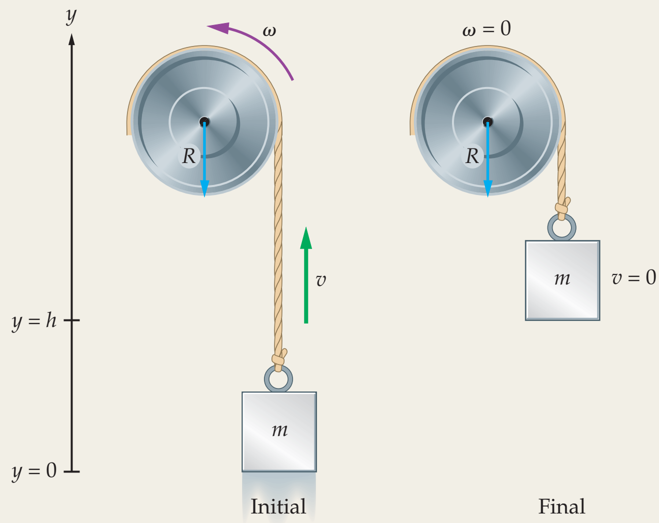
We can also apply energy conservation to the case of a pulley, or similar object, with a string that winds or unwinds without slipping. In such cases, the relation $v = r\omega$ is valid and we can follow the same methods applied to an object that rolls without slipping.

EXAMPLE 10–6 SPINNING WHEEL

A block of mass m is attached to a string that is wrapped around the circumference of a wheel of radius R and moment of inertia I. The wheel rotates freely about its axis and the string wraps around its circumference without slipping. Initially the wheel rotates with an angular speed ω, causing the block to rise with a linear speed v. To what height does the block rise before coming to rest? Give a symbolic answer.

PICTURE THE PROBLEM

Note in our sketch that we choose the origin of the y axis to be at the initial height of the block. The positive y direction, as usual, is chosen to be upward. When the block comes to rest, then, it is at the height $y = h > 0$, where h is to be determined from the initial speed of the block and the properties of the wheel.



STRATEGY

The problem statement gives two key pieces of information. First, the string wraps onto the disk without slipping; therefore, $v = R\omega$. Second, the wheel rotates freely, which means that the mechanical energy of the system is conserved. Thus, at the height h the initial kinetic energy of the system has been converted to gravitational potential energy. This condition can be used to find h.

Before we continue, note that the mechanical energy of the system includes the following contributions: (i) linear kinetic energy for the block, (ii) rotational kinetic energy for the wheel, and (iii) gravitational potential energy for the block. We do not include the gravitational potential energy of the wheel because its height does not change.

SOLUTION

1. Write an expression for the initial mechanical energy of the system, E_i, including all three contributions mentioned in the Strategy:

$$E_i = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{R}\right)^2 + 0$$

2. Write an expression for the final mechanical energy of the system, E_f:

$$E_f = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$
$$= 0 + 0 + mgh$$

3. Set the initial and final mechanical energies equal to one another, $E_i = E_f$:

$$E_i = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{R}\right)^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mR^2}\right)$$
$$= E_f = mgh$$

4. Solve for the height, h:

$$h = \left(\frac{v^2}{2g}\right)\left(1 + \frac{I}{mR^2}\right)$$

INSIGHT

If the block were moving upward with speed v on its own—not attached to anything—it would rise to the height $h = v^2/2g$. We recover this result if $I = 0$, since in that case it is as if the wheel were not there. If the wheel is there, and I is nonzero, the block rises to a height that is *greater* than $v^2/2g$. The reason is that the wheel has kinetic energy, in addition to the kinetic energy of the block, and the sum of these kinetic energies must be converted to gravitational potential energy before the block and the wheel stop moving.

PRACTICE PROBLEM

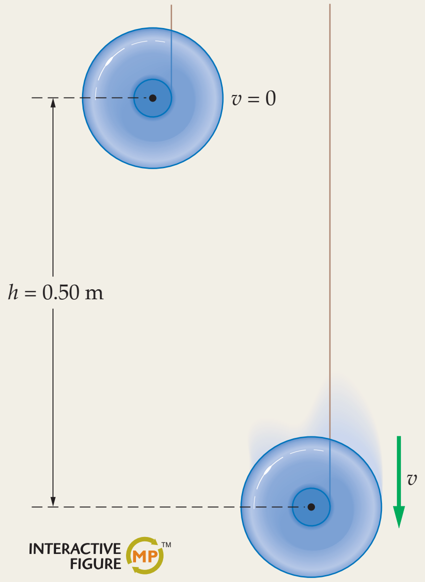
Suppose the wheel is a disk with a mass equal to the mass m of the block. Find an expression for the height h in this case. [**Answer:** The moment of inertia of the wheel is $I = \tfrac{1}{2}mR^2$. Therefore, $h = (3/2)(v^2/2g)$.]

Some related homework problems: Problem 66, Problem 70, Problem 73

The situation with a yo-yo is similar, as we see in the next Active Example.

ACTIVE EXAMPLE 10–3 FIND THE YO-YO'S SPEED

Yo-Yo man releases a yo-yo from rest and allows it to drop, as he keeps the top end of the string stationary. The mass of the yo-yo is 0.056 kg, its moment of inertia is $2.9 \times 10^{-5}\ \mathrm{kg \cdot m^2}$, and the radius, r, of the axle the string wraps around is 0.0064 m. What is the linear speed, v, of the yo-yo after it has dropped through a height $h = 0.50$ m?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial energy of the system: $E_i = mgh$
2. Write the final energy of the system: $E_f = \frac{1}{2}mv^2(1 + I/mr^2)$
3. Set $E_f = E_i$ and solve for v: $v = \sqrt{2gh/(1 + I/mr^2)}$
4. Substitute numerical values: $v = 0.85\ \mathrm{m/s}$

INSIGHT

The linear speed of the yo-yo is $v = r\omega$, where r is the radius of the axle from which the string unwraps without slipping. Therefore, the r in the term I/mr^2 is the radius of the axle. The outer radius of the yo-yo affects its moment of inertia, but since I is given to us in the problem statement, the outer radius is not pertinent.

YOUR TURN

If the yo-yo's moment of inertia is increased, does its final speed increase, decrease, or stay the same? Calculate the final speed for the case $I = 3.9 \times 10^{-5}\ \mathrm{kg \cdot m^2}$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Our definitions of position, velocity, and acceleration from Chapter 2 are generalized in Section 10–1 to apply to rotational motion. We then use the kinematics of Chapters 2 and 4 in Section 10–2 to relate these quantities. The basic equations of motion are the same; only the names have been changed.

The kinetic energy, first defined in Chapter 7, plays a key role in defining the moment of inertia in Section 10–5.

Conservation of energy (Chapter 8) is just as important in rotational motion as it is in linear motion. We apply it to rotational motion in Section 10–6.

LOOKING AHEAD

In Chapter 11 we relate force to angular acceleration, in much the same way that force and acceleration are related in linear motion. This results in the concept of torque in Section 11–1.

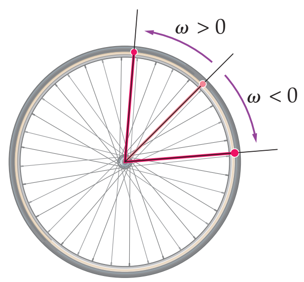
Just as linear speed is related to linear momentum (Chapter 9), angular speed is related to angular momentum. This is discussed in detail in Section 11–6.

Though a bit surprising at first, rotational motion is directly related to the motion of a pendulum swinging back and forth, and to the motion of a mass oscillating up and down on a spring. These connections are established in Section 13–3.

CHAPTER SUMMARY

10–1 ANGULAR POSITION, VELOCITY, AND ACCELERATION

To describe rotational motion, rotational analogues of position, velocity, and acceleration are defined.



Angular Position

Angular position, θ, is the angle measured from an arbitrary reference line:

$$\theta\ (\text{in radians}) = \text{arc length/radius} = s/r \qquad 10\text{–}2$$

Angular Velocity

Angular velocity, ω, is the rate of change of angular position. The average angular velocity is

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad 10\text{–}3$$

The instantaneous angular velocity is the limit of ω_{av} as Δt approaches zero:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \qquad 10\text{–}4$$

Angular Acceleration

Angular acceleration, α, is the rate of change of angular velocity. The average angular acceleration is

$$\alpha_{av} = \frac{\Delta \omega}{\Delta t} \qquad 10\text{–}6$$

The instantaneous angular acceleration is the limit of α_{av} as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \qquad 10\text{–}7$$

Period of Rotation

The period, T, is the time required to complete one full rotation. If the angular velocity is constant, T is related to ω as follows:

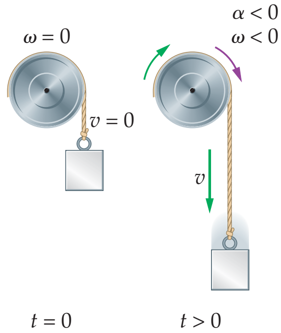
$$T = \frac{2\pi}{\omega} \qquad 10\text{–}5$$

Sign Convention

Counterclockwise rotations are positive; clockwise rotations are negative.

10–2 ROTATIONAL KINEMATICS

Rotational kinematics is the description of angular motion, in the same way that linear kinematics describes linear motion. In both cases, we assume constant acceleration.



Linear–Angular Analogues

Rotational kinematics is related to linear kinematics by the following linear–angular analogies:

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Kinematic Equations (Constant Acceleration)

The equations of rotational kinematics are the same as the equations of linear kinematics, with the substitutions indicated by the linear–angular analogies:

Linear Equation		Angular Equation	
$v = v_0 + at$	2–7	$\omega = \omega_0 + \alpha t$	10–8
$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$	10–9
$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	10–10
$v^2 = v_0^2 + 2a(x - x_0)$	2–12	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	10–11

10–3 CONNECTIONS BETWEEN LINEAR AND ROTATIONAL QUANTITIES

A point on a rotating object follows a circular path. At any instant of time, the point is moving in a direction tangential to the circle, with a linear speed and acceleration. The linear speed and acceleration are related to the angular speed and acceleration.

Tangential Speed

The tangential speed, v_t, of a point on a rotating object is

$$v_t = r\omega \qquad 10\text{–}12$$

Centripetal Acceleration

The centripetal acceleration, a_{cp}, of a point on a rotating object is

$$a_{cp} = r\omega^2 \qquad 10\text{–}13$$

Centripetal acceleration is due to a change in direction of motion.

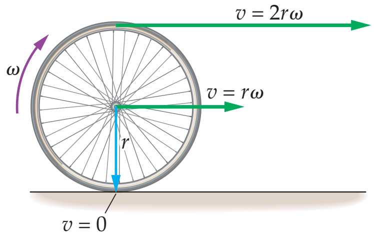
Tangential Acceleration
The tangential acceleration, a_t, of a point on a rotating object is

$$a_t = r\alpha \qquad \text{10–14}$$

Tangential acceleration is due to a change in speed.

Total Acceleration
The total acceleration of a rotating object is the vector sum of its tangential and centripetal accelerations.

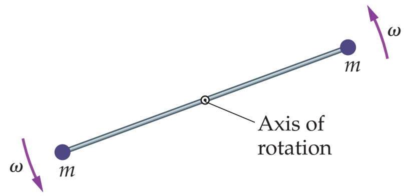
10–4 ROLLING MOTION



Rolling motion is a combination of translational and rotational motions. An object of radius r, rolling without slipping, translates with linear speed v and rotates with angular speed

$$\omega = v/r \qquad \text{10–15}$$

10–5 ROTATIONAL KINETIC ENERGY AND THE MOMENT OF INERTIA



Rotating objects have kinetic energy, just as objects in linear motion have kinetic energy.

Rotational Kinetic Energy
The kinetic energy of a rotating object is

$$K = \tfrac{1}{2}I\omega^2 \qquad \text{10–17}$$

The quantity I is the moment of inertia.

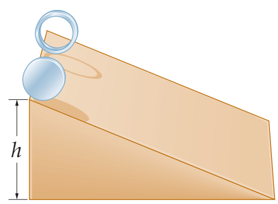
Moment of Inertia, Discrete Masses
The moment of inertia, I, of a collection of masses, m_i, at distances r_i from the axis of rotation is

$$I = \sum m_i r_i^2 \qquad \text{10–18}$$

Moment of Inertia, Continuous Distribution of Mass
In a continuous object, the moment of inertia is calculated by dividing the object into a collection of small mass elements and summing $m_i r_i^2$ for each element. Results for a variety of continuous objects are collected in Table 10–1 on p. 314.

Linear–Angular Analogue
The moment of inertia is the rotational analogue to mass in linear systems. In particular, an object with a large moment of inertia is hard to start rotating and hard to stop rotating.

10–6 CONSERVATION OF ENERGY



Energy conservation can be applied to a variety of rotational systems in the same way that it is applied to translational systems.

Kinetic Energy of Rolling Motion
The kinetic energy of an object that rolls without slipping is

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \qquad \text{10–19}$$

Since rolling without slipping implies that $\omega = v/r$, the kinetic energy can be written as follows:

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{r}\right)^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) \qquad \text{10–20}$$

Energy Conservation
Conservation of mechanical energy is a statement that the initial kinetic plus potential energy is equal to the final kinetic plus potential energy: $K_i + U_i = K_f + U_f$. By taking into account both rotational and translational kinetic energy, energy conservation can be applied in the same way as was done for linear systems.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Apply rotational kinematics with constant angular acceleration.	Rotational kinematics is completely analogous to the linear kinematics studied in Chapter 2. Angular problems are solved in the same way as the corresponding linear problems.	Example 10–1, Example 10–2 Active Example 10–1
Relate linear and angular motion.	Linear speed and angular speed are related by $v = r\omega$. Similarly, linear and angular accelerations are related by $a = r\alpha$. The centripetal acceleration of an object in circular motion is $a_{cp} = r\omega^2$.	Example 10–3 Active Example 10–2
Find the rotational kinetic energy of an object.	Rotational kinetic energy is given by $K = \frac{1}{2}I\omega^2$. The moment of inertia, I, plays the same role in rotational motion as the mass in linear motion.	Example 10–4, Example 10–5
Apply energy conservation to a rotational system.	To use energy conservation in a system with rotational motion, it is necessary to include the kinetic energy of rotation as one of the forms of energy.	Example 10–6 Active Example 10–3

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A rigid object rotates about a fixed axis. Do all points on the object have the same angular speed? Do all points on the object have the same linear speed? Explain.
2. Can you drive your car in such a way that your tangential acceleration is zero while at the same time your centripetal acceleration is nonzero? Give an example if your answer is yes, state why not if your answer is no.
3. Can you drive your car in such a way that your tangential acceleration is nonzero while at the same time your centripetal acceleration is zero? Give an example if your answer is yes, state why not if your answer is no.
4. The fact that the Earth rotates gives people in New York a linear speed of about 750 mi/h. Where should you stand on the Earth to have the smallest possible linear speed?
5. At the local carnival you and a friend decide to take a ride on the Ferris wheel. As the wheel rotates with a constant angular speed, your friend poses the following questions: **(a)** Is my linear velocity constant? **(b)** Is my linear speed constant? **(c)** Is the magnitude of my centripetal acceleration constant? **(d)** Is the direction of my centripetal acceleration constant? What is your answer to each of these questions?
6. Why should changing the axis of rotation of an object change its moment of inertia, given that its shape and mass remain the same?
7. Give a common, everyday example for each of the following: **(a)** An object that has zero rotational kinetic energy but nonzero translational kinetic energy. **(b)** An object that has zero translational kinetic energy but nonzero rotational kinetic energy. **(c)** An object that has nonzero rotational and translational kinetic energies.
8. Two spheres have identical radii and masses. How might you tell which of these spheres is hollow and which is solid?
9. At the grocery store you pick up a can of beef broth and a can of chunky beef stew. The cans are identical in diameter and weight. Rolling both of them down the aisle with the same initial speed, you notice that the can of chunky stew rolls much farther than the can of broth. Why?
10. Suppose we change the race shown in Conceptual Checkpoint 10–4 so that a hoop of radius R and mass M races a hoop of radius R and mass $2M$. **(a)** Does the hoop with mass M finish before, after, or at the same time as the hoop with mass $2M$? Explain. **(b)** How would your answer to part (a) change if the hoops had different radii? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

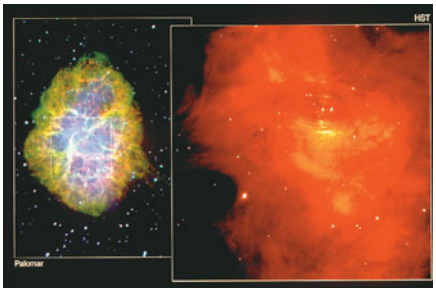
SECTION 10–1 ANGULAR POSITION, VELOCITY, AND ACCELERATION

1. • The following angles are given in degrees. Convert them to radians: 30°, 45°, 90°, 180°.
2. • The following angles are given in radians. Convert them to degrees: $\pi/6$, 0.70π, 1.5π, 5π.
3. • Find the angular speed of **(a)** the minute hand and **(b)** the hour hand of the famous clock in London, England, that rings the bell known as Big Ben.
4. • Express the angular velocity of the second hand on a clock in the following units: **(a)** rev/hr, **(b)** deg/min, and **(c)** rad/s.
5. • Rank the following in order of increasing angular speed: an automobile tire rotating at 2.00×10^3 deg/s, an electric drill rotating at 400.0 rev/min, and an airplane propeller rotating at 40.0 rad/s.
6. • A spot of paint on a bicycle tire moves in a circular path of radius 0.33 m. When the spot has traveled a linear distance of 1.95 m, through what angle has the tire rotated? Give your answer in radians.

7. • What is the angular speed (in rev/min) of the Earth as it orbits about the Sun?

8. • Find the angular speed of the Earth as it spins about its axis. Give your result in rad/s.

9. • **The Crab Nebula** One of the most studied objects in the night sky is the Crab nebula, the remains of a supernova explosion observed by the Chinese in 1054. In 1968 it was discovered that a pulsar—a rapidly rotating neutron star that emits a pulse of radio waves with each revolution—lies near the center of the Crab nebula. The period of this pulsar is 33 ms. What is the angular speed (in rad/s) of the Crab nebula pulsar?



The photo at left is a true-color visible light image of the Crab nebula. In the false-color breakout, the pulsar can be seen as the left member of the pair of stars just above the center of the frame. (Problems 9 and 106)

10. •• **IP** A 3.5-inch floppy disk in a computer rotates with a period of 2.00×10^{-1} s. What are **(a)** the angular speed of the disk and **(b)** the linear speed of a point on the rim of the disk? **(c)** Does a point near the center of the disk have an angular speed that is greater than, less than, or the same as the angular speed found in part (a)? Explain. (*Note:* A 3.5-inch floppy disk is 3.5 inches in diameter.)

11. •• The angle an airplane propeller makes with the horizontal as a function of time is given by $\theta = (125\ \text{rad/s})t + (42.5\ \text{rad/s}^2)t^2$. **(a)** Estimate the instantaneous angular velocity at $t = 0.00$ s by calculating the average angular velocity from $t = 0.00$ s to $t = 0.010$ s. **(b)** Estimate the instantaneous angular velocity at $t = 1.000$ s by calculating the average angular velocity from $t = 1.000$ s to $t = 1.010$ s. **(c)** Estimate the instantaneous angular velocity at $t = 2.000$ s by calculating the average angular velocity from $t = 2.000$ s to $t = 2.010$ s. **(d)** Based on your results from parts (a), (b), and (c), is the angular acceleration of the propeller positive, negative, or zero? Explain. **(e)** Calculate the average angular acceleration from $t = 0.00$ s to $t = 1.00$ s and from $t = 1.00$ s to $t = 2.00$ s.

SECTION 10–2 ROTATIONAL KINEMATICS

12. • **CE** An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle θ in the time t, through what angle did it rotate in the time $t/2$?

13. • **CE** An object at rest begins to rotate with a constant angular acceleration. If the angular speed of the object is ω after the time t, what was its angular speed at the time $t/2$?

14. • In Active Example 10–1, how long does it take before the angular velocity of the pulley is equal to -5.0 rad/s?

15. • In Example 10–2, through what angle has the wheel turned when its angular speed is 2.45 rad/s?

16. • The angular speed of a propeller on a boat increases with constant acceleration from 12 rad/s to 26 rad/s in 2.5 revolutions. What is the acceleration of the propeller?

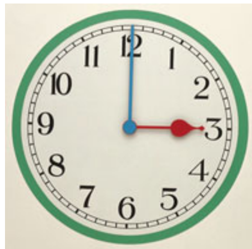
17. • The angular speed of a propeller on a boat increases with constant acceleration from 11 rad/s to 28 rad/s in 2.4 seconds. Through what angle did the propeller turn during this time?

18. •• After fixing a flat tire on a bicycle you give the wheel a spin. **(a)** If its initial angular speed was 6.35 rad/s and it rotated 14.2 revolutions before coming to rest, what was its average angular acceleration? **(b)** For what length of time did the wheel rotate?

19. •• **IP** A ceiling fan is rotating at 0.96 rev/s. When turned off, it slows uniformly to a stop in 2.4 min. **(a)** How many revolutions does the fan make in this time? **(b)** Using the result from part (a), find the number of revolutions the fan must make for its speed to decrease from 0.96 rev/s to 0.48 rev/s.

20. •• A discus thrower starts from rest and begins to rotate with a constant angular acceleration of 2.2 rad/s^2. **(a)** How many revolutions does it take for the discus thrower's angular speed to reach 6.3 rad/s? **(b)** How much time does this take?

21. •• **Half Time** At 3:00 the hour hand and the minute hand of a clock point in directions that are 90.0° apart. What is the first time after 3:00 that the angle between the two hands has decreased by half to 45.0°?



When the little hand is on the 3 and the big hand is on the 12 . . . . (Problem 21)

22. •• **BIO** A centrifuge is a common laboratory instrument that separates components of differing densities in solution. This is accomplished by spinning a sample around in a circle with a large angular speed. Suppose that after a centrifuge in a medical laboratory is turned off, it continues to rotate with a constant angular deceleration for 10.2 s before coming to rest. **(a)** If its initial angular speed was 3850 rpm, what is the magnitude of its angular deceleration? **(b)** How many revolutions did the centrifuge complete after being turned off?

23. •• **The Slowing Earth** The Earth's rate of rotation is constantly decreasing, causing the day to increase in duration. In the year 2006 the Earth took about 0.840 s longer to complete 365 revolutions than it did in the year 1906. What was the average angular acceleration of the Earth during this time? Give your answer in rad/s^2.

24. •• **IP** A compact disk (CD) speeds up uniformly from rest to 310 rpm in 3.3 s. **(a)** Describe a strategy that allows you to calculate the number of revolutions the CD makes in this time. **(b)** Use your strategy to find the number of revolutions.

25. •• When a carpenter shuts off his circular saw, the 10.0-inch-diameter blade slows from 4440 rpm to 0.00 rpm in 2.50 s. **(a)** What is the angular acceleration of the blade? **(b)** What is the distance traveled by a point on the rim of the blade during the deceleration? **(c)** What is the magnitude of the net displacement of a point on the rim of the blade during the deceleration?

26. •• **The World's Fastest Turbine** The drill used by most dentists today is powered by a small air turbine that can operate at angular speeds of 350,000 rpm. These drills, along with ultrasonic dental drills, are the fastest turbines in the world—far exceeding the angular speeds of jet engines. Suppose a drill starts at rest and comes up to operating speed in 2.1 s. **(a)** Find the angular acceler-

ation produced by the drill, assuming it to be constant. **(b)** How many revolutions does the drill bit make as it comes up to speed?

An air-turbine dentist drill—faster than a jet engine. (Problem 26)

SECTION 10–3 CONNECTIONS BETWEEN LINEAR AND ROTATIONAL QUANTITIES

27. • **CE Predict/Explain** Two children, Jason and Betsy, ride on the same merry-go-round. Jason is a distance R from the axis of rotation; Betsy is a distance $2R$ from the axis. Is the rotational period of Jason greater than, less than, or equal to the rotational period of Betsy? **(b)** Choose the *best explanation* from among the following:
I. The period is greater for Jason because he moves more slowly than Betsy.
II. The period is greater for Betsy since she must go around a circle with a larger circumference.
III. It takes the same amount of time for the merry-go-round to complete a revolution for all points on the merry-go-round.

28. • **CE** Referring to the previous problem, what are **(a)** the ratio of Jason's angular speed to Betsy's angular speed, **(b)** the ratio of Jason's linear speed to Betsy's linear speed, and **(c)** the ratio of Jason's centripetal acceleration to Betsy's centripetal acceleration?

29. • **CE Predict/Explain A Tall Building** The world's tallest building is the Taipei 101 Tower in Taiwan, which rises to a height of 508 m (1667 ft). **(a)** When standing on the top floor of the building, is your angular speed due to the Earth's rotation greater than, less than, or equal to your angular speed when you stand on the ground floor? **(b)** Choose the *best explanation* from among the following:
I. The angular speed is the same at all distances from the axis of rotation.
II. At the top of the building you are farther from the axis of rotation and hence you have a greater angular speed.
III. You are spinning faster when you are closer to the axis of rotation.

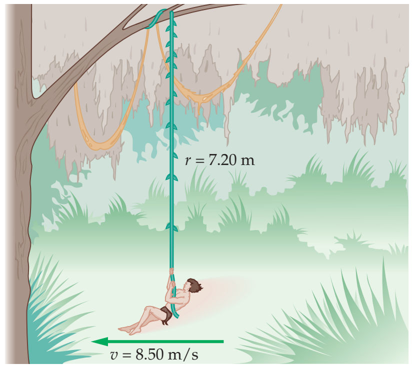
30. • The hour hand on a certain clock is 8.2 cm long. Find the tangential speed of the tip of this hand.

31. • Two children ride on the merry-go-round shown in Conceptual Checkpoint 10–1. Child 1 is 2.0 m from the axis of rotation, and child 2 is 1.5 m from the axis. If the merry-go-round completes one revolution every 4.5 s, find **(a)** the angular speed and **(b)** the linear speed of each child.

32. • The outer edge of a rotating Frisbee with a diameter of 29 cm has a linear speed of 3.7 m/s. What is the angular speed of the Frisbee?

33. • A carousel at the local carnival rotates once every 45 seconds. **(a)** What is the linear speed of an outer horse on the carousel, which is 2.75 m from the axis of rotation? **(b)** What is the linear speed of an inner horse that is 1.75 m from the axis of rotation?

34. •• **IP** Jeff of the Jungle swings on a vine that is 7.20 m long (**Figure 10–18**). At the bottom of the swing, just before hitting the tree, Jeff's linear speed is 8.50 m/s. **(a)** Find Jeff's angular speed at this time. **(b)** What centripetal acceleration does Jeff experience at the bottom of his swing? **(c)** What exerts the force that is responsible for Jeff's centripetal acceleration?



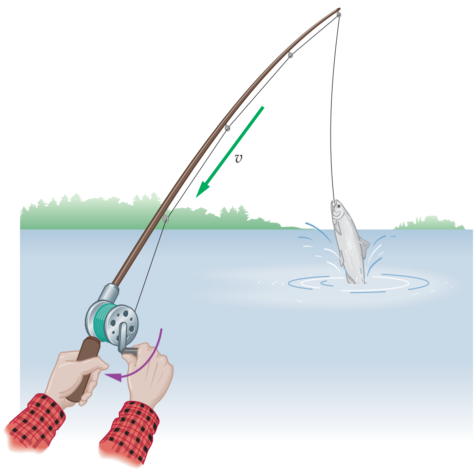
▲ **FIGURE 10–18** Problems 34 and 35

35. •• Suppose, in Problem 34, that at some point in his swing Jeff of the Jungle has an angular speed of 0.850 rad/s and an angular acceleration of 0.620 rad/s^2. Find the magnitude of his centripetal, tangential, and total accelerations, and the angle his total acceleration makes with respect to the tangential direction of motion.

36. •• A compact disk, which has a diameter of 12.0 cm, speeds up uniformly from 0.00 to 4.00 rev/s in 3.00 s. What is the tangential acceleration of a point on the outer rim of the disk at the moment when its angular speed is **(a)** 2.00 rev/s and **(b)** 3.00 rev/s?

37. •• **IP** When a compact disk with a 12.0-cm diameter is rotating at 5.05 rad/s, what are **(a)** the linear speed and **(b)** the centripetal acceleration of a point on its outer rim? **(c)** Consider a point on the CD that is halfway between its center and its outer rim. Without repeating all of the calculations required for parts (a) and (b), determine the linear speed and the centripetal acceleration of this point.

38. •• **IP** As Tony the fisherman reels in a "big one," he turns the spool on his fishing reel at the rate of 3.0 complete revolutions every second (**Figure 10–19**). **(a)** If the radius of the reel is 3.7 cm, what is the linear speed of the fishing line as it is reeled in? **(b)** How would your answer to part (a) change if the radius of the reel were doubled?



▲ **FIGURE 10–19** Problem 38

39. •• A Ferris wheel with a radius of 9.5 m rotates at a constant rate, completing one revolution every 36 s. Find the direction and magnitude of a passenger's acceleration when **(a)** at the top and **(b)** at the bottom of the wheel.

40. •• Suppose the Ferris wheel in the previous problem begins to decelerate at the rate of 0.22 rad/s^2 when the passenger is at the top of the wheel. Find the direction and magnitude of the passenger's acceleration at that time.

41. •• **IP** A person swings a 0.52-kg tether ball tied to a 4.5-m rope in an approximately horizontal circle. **(a)** If the maximum tension the rope can withstand before breaking is 11 N, what is the maximum angular speed of the ball? **(b)** If the rope is shortened, does the maximum angular speed found in part (a) increase, decrease, or stay the same? Explain.

42. •• To polish a filling, a dentist attaches a sanding disk with a radius of 3.20 mm to the drill. **(a)** When the drill is operated at 2.15×10^4 rad/s, what is the tangential speed of the rim of the disk? **(b)** What period of rotation must the disk have if the tangential speed of its rim is to be 275 m/s?

43. •• In the previous problem, suppose the disk has an angular acceleration of 232 rad/s^2 when its angular speed is 640 rad/s. Find both the tangential and centripetal accelerations of a point on the rim of the disk.

44. •• **The Bohr Atom** The Bohr model of the hydrogen atom pictures the electron as a tiny particle moving in a circular orbit about a stationary proton. In the lowest-energy orbit the distance from the proton to the electron is 5.29×10^{-11} m, and the linear speed of the electron is 2.18×10^6 m/s. **(a)** What is the angular speed of the electron? **(b)** How many orbits about the proton does it make each second? **(c)** What is the electron's centripetal acceleration?

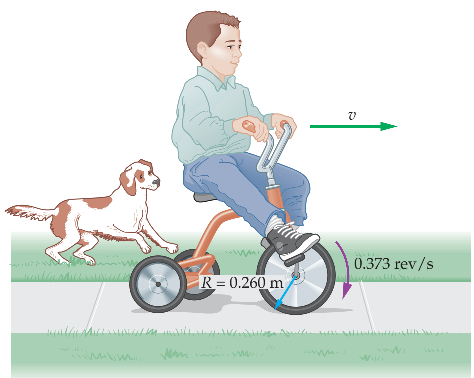
45. ••• A wheel of radius R starts from rest and accelerates with a constant angular acceleration α about a fixed axis. At what time t will the centripetal and tangential accelerations of a point on the rim have the same magnitude?

SECTION 10–4 ROLLING MOTION

46. • **CE** As you drive down the highway, the top of your tires are moving with a speed v. What is the reading on your speedometer?

47. •• The tires on a car have a radius of 31 cm. What is the angular speed of these tires when the car is driven at 15 m/s?

48. • A child pedals a tricycle, giving the driving wheel an angular speed of 0.373 rev/s (**Figure 10–20**). If the radius of the wheel is 0.260 m, what is the child's linear speed?



▲ **FIGURE 10–20** Problem 48

49. • A soccer ball, which has a circumference of 70.0 cm, rolls 14.0 yards in 3.35 s. What was the average angular speed of the ball during this time?

50. •• As you drive down the road at 17 m/s, you press on the gas pedal and speed up with a uniform acceleration of 1.12 m/s^2 for 0.65 s. If the tires on your car have a radius of 33 cm, what is their angular displacement during this period of acceleration?

51. •• **IP** A bicycle coasts downhill and accelerates from rest to a linear speed of 8.90 m/s in 12.2 s. **(a)** If the bicycle's tires have a radius of 36.0 cm, what is their angular acceleration? **(b)** If the radius of the tires had been smaller, would their angular acceleration be greater than or less than the result found in part (a)?

SECTION 10–5 ROTATIONAL KINETIC ENERGY AND THE MOMENT OF INERTIA

52. • **CE Predict/Explain** The minute and hour hands of a clock have a common axis of rotation and equal mass. The minute hand is long, thin, and uniform; the hour hand is short, thick, and uniform. **(a)** Is the moment of inertia of the minute hand greater than, less than, or equal to the moment of inertia of the hour hand? **(b)** Choose the *best explanation* from among the following:

I. The hands have equal mass, and hence equal moments of inertia.
II. Having mass farther from the axis of rotation results in a greater moment of inertia.
III. The more compact hour hand concentrates its mass and has the greater moment of inertia.

53. • **CE Predict/Explain** Tons of dust and small particles rain down onto the Earth from space every day. As a result, does the Earth's moment of inertia increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The dust adds mass to the Earth and increases its radius slightly.
II. As the dust moves closer to the axis of rotation, the moment of inertia decreases.
III. The moment of inertia is a conserved quantity and cannot change.

54. **CE** • **Predict/Explain** Suppose a bicycle wheel is rotated about an axis through its rim and parallel to its axle. **(a)** Is its moment of inertia about this axis greater than, less than, or equal to its moment of inertia about its axle? **(b)** Choose the *best explanation* from among the following:

I. The moment of inertia is greatest when an object is rotated about its center.
II. The mass and shape of the wheel remain the same.
III. Mass is farther from the axis when the wheel is rotated about the rim.

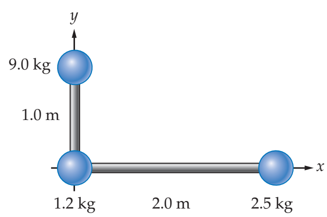
55. • The moment of inertia of a 0.98-kg bicycle wheel rotating about its center is 0.13 kg · m^2. What is the radius of this wheel, assuming the weight of the spokes can be ignored?

56. • What is the kinetic energy of the grindstone in Example 10–4 if it completes one revolution every 4.20 s?

57. • An electric fan spinning with an angular speed of 13 rad/s has a kinetic energy of 4.6 J. What is the moment of inertia of the fan?

58. • Repeat Example 10–5 for the case of a rolling hoop of the same mass and radius.

59. •• **CE** The L-shaped object in **Figure 10–21** can be rotated in one of the following three ways: case 1, rotation about the x axis; case 2, rotation about the y axis; and case 3, rotation about the



▲ **FIGURE 10–21** Problem 59

z axis (which passes through the origin perpendicular to the plane of the figure). Rank these three cases in order of increasing moment of inertia. Indicate ties where appropriate.

60. •• **IP** A 12-g CD with a radius of 6.0 cm rotates with an angular speed of 34 rad/s. **(a)** What is its kinetic energy? **(b)** What angular speed must the CD have if its kinetic energy is to be doubled?

61. •• When a pitcher throws a curve ball, the ball is given a fairly rapid spin. If a 0.15-kg baseball with a radius of 3.7 cm is thrown with a linear speed of 48 m/s and an angular speed of 42 rad/s, how much of its kinetic energy is translational and how much is rotational? Assume the ball is a uniform, solid sphere.

62. •• **IP** A basketball rolls along the floor with a constant linear speed v. **(a)** Find the fraction of its total kinetic energy that is in the form of rotational kinetic energy about the center of the ball. **(b)** If the linear speed of the ball is doubled to $2v$, does your answer to part (a) increase, decrease, or stay the same? Explain.

63. •• Find the rate at which the rotational kinetic energy of the Earth is decreasing. The Earth has a moment of inertia of $0.331 M_E R_E^2$, where $R_E = 6.38 \times 10^6$ m and $M_E = 5.97 \times 10^{24}$ kg, and its rotational period increases by 2.3 ms with each passing century. Give your answer in watts.

64. •• A lawn mower has a flat, rod-shaped steel blade that rotates about its center. The mass of the blade is 0.65 kg and its length is 0.55 m. **(a)** What is the rotational energy of the blade at its operating angular speed of 3500 rpm? **(b)** If all of the rotational kinetic energy of the blade could be converted to gravitational potential energy, to what height would the blade rise?

SECTION 10–6 CONSERVATION OF ENERGY

65. • **CE** Consider the physical situation shown in Conceptual Checkpoint 10–5. Suppose this time a ball is released from rest on the frictionless surface. When the ball comes to rest on the no-slip surface, is its height greater than, less than, or equal to the height from which it was released?

66. • Suppose the block in Example 10–6 has a mass of 2.1 kg and an initial upward speed of 0.33 m/s. Find the moment of inertia of the wheel if its radius is 8.0 cm and the block rises to a height of 7.4 cm before momentarily coming to rest.

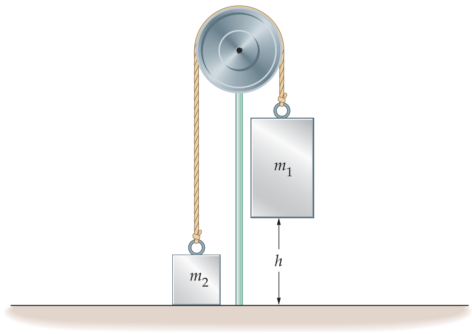
67. • Through what height must the yo-yo in Active Example 10–3 fall for its linear speed to be 0.65 m/s?

68. •• **CE** Suppose we change the race shown in Conceptual Checkpoint 10–4 to a race between three different disks. Let disk 1 have a mass M and a radius R, disk 2 have a mass M and a radius $2R$, and disk 3 have a mass $2M$ and a radius R. Rank the three disks in the order in which they finish the race. Indicate ties where appropriate.

69. •• Calculate the speeds of **(a)** the disk and **(b)** the hoop at the bottom of the inclined plane in Conceptual Checkpoint 10–4 if the height of the incline is 0.82 m.

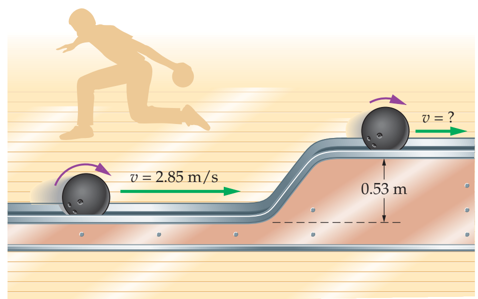
70. •• **IP** **Atwood's Machine** The two masses ($m_1 = 5.0$ kg and $m_2 = 3.0$ kg) in the Atwood's machine shown in **Figure 10–22** are released from rest, with m_1 at a height of 0.75 m above the floor. When m_1 hits the ground its speed is 1.8 m/s. Assuming that the pulley is a uniform disk with a radius of 12 cm, **(a)** outline a strategy that allows you to find the mass of the pulley. **(b)** Implement the strategy given in part (a) and determine the pulley's mass.

71. •• In Conceptual Checkpoint 10–5, assume the ball is a solid sphere of radius 2.9 cm and mass 0.14 kg. If the ball is released from rest at a height of 0.78 m above the bottom of the track on the no-slip side, **(a)** what is its angular speed when it is on the frictionless side of the track? **(b)** How high does the ball rise on the frictionless side?



▲ **FIGURE 10–22** Problem 70

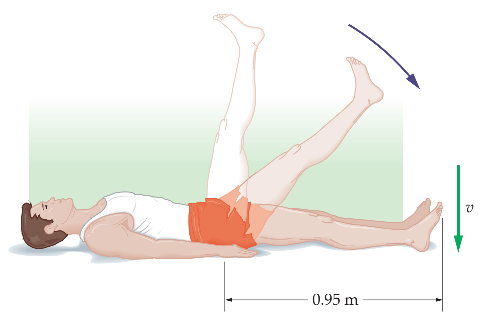
72. •• **IP** After you pick up a spare, your bowling ball rolls without slipping back toward the ball rack with a linear speed of 2.85 m/s (**Figure 10–23**). To reach the rack, the ball rolls up a ramp that rises through a vertical distance of 0.53 m. **(a)** What is the linear speed of the ball when it reaches the top of the ramp? **(b)** If the radius of the ball were increased, would the speed found in part (a) increase, decrease, or stay the same? Explain.



▲ **FIGURE 10–23** Problem 72

73. •• **IP** A 1.3-kg block is tied to a string that is wrapped around the rim of a pulley of radius 7.2 cm. The block is released from rest. **(a)** Assuming the pulley is a uniform disk with a mass of 0.31 kg, find the speed of the block after it has fallen through a height of 0.50 m. **(b)** If a small lead weight is attached near the rim of the pulley and this experiment is repeated, will the speed of the block increase, decrease, or stay the same? Explain.

74. •• After doing some exercises on the floor, you are lying on your back with one leg pointing straight up. If you allow your leg to fall freely until it hits the floor (**Figure 10–24**), what is the tangential speed of your foot just before it lands? Assume the leg can be treated as a uniform rod 0.95 m long that pivots freely about the hip.



▲ **FIGURE 10–24** Problem 74

75. ••• A 2.0-kg solid cylinder (radius = 0.10 m, length = 0.50 m) is released from rest at the top of a ramp and allowed to roll without slipping. The ramp is 0.75 m high and 5.0 m long. When the cylinder reaches the bottom of the ramp, what are

(a) its total kinetic energy, **(b)** its rotational kinetic energy, and **(c)** its translational kinetic energy?

76. ••• A 2.5-kg solid sphere (radius = 0.10 m) is released from rest at the top of a ramp and allowed to roll without slipping. The ramp is 0.75 m high and 5.6 m long. When the sphere reaches the bottom of the ramp, what are **(a)** its total kinetic energy, **(b)** its rotational kinetic energy, and **(c)** its translational kinetic energy?

GENERAL PROBLEMS

77. • **CE** When you stand on the observation deck of the Empire State Building in New York, is your linear speed due to the Earth's rotation greater than, less than, or the same as when you were waiting for the elevators on the ground floor?

78. • **CE Hard-Boiled Versus Raw Eggs** One way to tell whether an egg is raw or hard boiled—without cracking it open—is to place it on a kitchen counter and give it a spin. If you do this to two eggs, one raw the other hard boiled, you will find that one spins considerably longer than the other. Is the raw egg the one that spins a long time, or the one that stops spinning in a short time?

79. • **CE** When the Hoover Dam was completed and the reservoir behind it filled with water, did the moment of inertia of the Earth increase, decrease, or stay the same?

80. • **Weightless on the Equator** In Quito, Ecuador, near the equator, you weigh about half a pound less than in Barrow, Alaska, near the pole. Find the rotational period of the Earth that would make you feel weightless at the equator. (With this rotational period, your centripetal acceleration would be equal to the acceleration due to gravity, g.)

81. • A diver completes $2\frac{1}{2}$ somersaults during a 2.3-s dive. What was the diver's average angular speed during the dive?

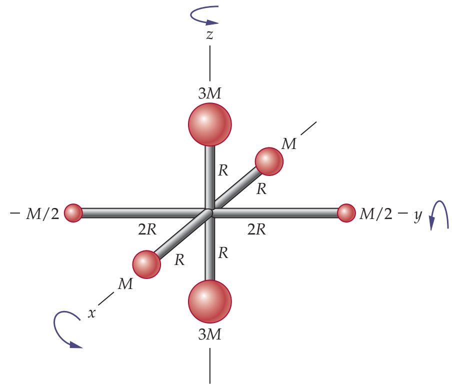
82. • What linear speed must a 0.065-kg hula hoop have if its total kinetic energy is to be 0.12 J? Assume the hoop rolls on the ground without slipping.

83. • **BIO Losing Consciousness** A pilot performing a horizontal turn will lose consciousness if she experiences a centripetal acceleration greater than 7.00 times the acceleration of gravity. What is the minimum radius turn she can make without losing consciousness if her plane is flying with a constant speed of 245 m/s?

84. •• **CE** Place two quarters on a table with their rims touching, as shown in **Figure 10–25**. While holding one quarter fixed, roll the other one—without slipping—around the circumference of the fixed quarter until it has completed one round trip. How many revolutions has the rolling quarter made about its center?

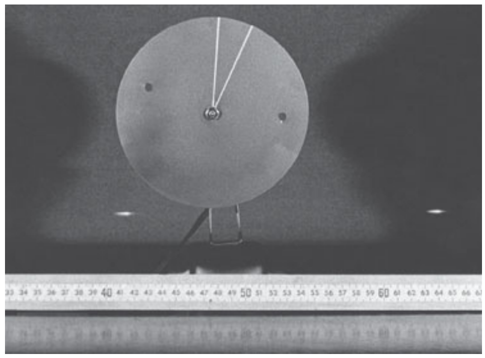
▲ **FIGURE 10–25** Problem 84

85. • **CE** The object shown in **Figure 10–26** can be rotated in three different ways: case 1, rotation about the x axis; case 2, rotation about the y axis; and case 3, rotation about the z axis. Rank these three cases in order of increasing moment of inertia. Indicate ties where appropriate.



▲ **FIGURE 10–26** Problem 85

86. •• The accompanying double-exposure photograph illustrates a method for determining the speed of a BB. The circular disk in the upper part of the photo rotates with a constant angular speed of 50.4 revolutions per second. A single white radial line drawn on the disk is seen in two locations in the double exposure. Below the disk are two bright images of a BB taken during the two exposures. Use the information given here and in the photo to estimate the speed of the BB.



Speeding BB and spinning wheel. (Problems 86 and 87)

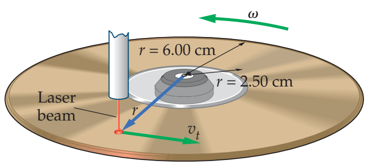
87. •• Referring to the previous problem, **(a)** estimate the linear speed of a point on the rim of the rotating disk. **(b)** By comparing the arc length between the two white lines to the distance covered by the BB, estimate the speed of the BB. **(c)** What radius must the disk have for the linear speed of a point on its rim to be the same as the speed of the BB? **(d)** Suppose a 1.0-g lump of putty is stuck to the rim of the disk. What centripetal force is required to hold the putty in place?

88. •• **IP When the Hands Align** A mathematically inclined friend e-mails you the following instructions: "Meet me in the cafeteria the first time after 2:00 P.M. today that the hands of a clock point in the same direction." **(a)** Is the desired meeting time before, after, or equal to 2:10 P.M.? Explain. **(b)** Is the desired meeting time before, after, or equal to 2:15 P.M.? Explain. **(c)** When should you meet your friend?

89. •• **IP** A diver runs horizontally off the end of a diving tower 3.0 m above the surface of the water with an initial speed of 2.6 m/s. During her fall she rotates with an average angular speed of 2.2 rad/s. **(a)** How many revolutions has she made when she hits the water? **(b)** How does your answer to part (a) depend on the diver's initial speed? Explain.

90. •• **IP** A potter's wheel of radius 6.8 cm rotates with a period of 0.52 s. What are **(a)** the linear speed and **(b)** the centripetal acceleration of a small lump of clay on the rim of the wheel? **(c)** How do your answers to parts (a) and (b) change if the period of rotation is doubled?

91. •• **IP Playing a CD** The record in an old-fashioned record player always rotates at the same angular speed. With CDs, the situation is different. For a CD to play properly, the point on the CD where the laser beam shines must have a linear speed $v_t = 1.25\ \mathrm{m/s}$, as indicated in **Figure 10–27**. **(a)** As the CD plays from the center outward, does its angular speed increase, decrease, or stay the same? Explain. **(b)** Find the angular speed of a CD when the laser beam is 2.50 cm from its center. **(c)** Repeat part (b) for the laser beam 6.00 cm from the center. **(d)** If the CD plays for 66.5 min, and the laser beam moves from 2.50 cm to 6.00 cm during this time, what is the CD's average angular acceleration?



▲ **FIGURE 10–27** Problem 91

92. •• **BIO Roller Pigeons** Pigeons are bred to display a number of interesting characteristics. One breed of pigeon, the "roller," is remarkable for the fact that it does a number of backward somersaults as it drops straight down toward the ground. Suppose a roller pigeon drops from rest and free falls downward for a distance of 14 m. If the pigeon somersaults at the rate of 12 rad/s, how many revolutions has it completed by the end of its fall?

93. •• As a marble with a diameter of 1.6 cm rolls down an incline, its center moves with a linear acceleration of $3.3\ \mathrm{m/s^2}$. **(a)** What is the angular acceleration of the marble? **(b)** What is the angular speed of the marble after it rolls for 1.5 s from rest?

94. •• A rubber ball with a radius of 3.2 cm rolls along the horizontal surface of a table with a constant linear speed v. When the ball rolls off the edge of the table, it falls 0.66 m to the floor below. If the ball completes 0.37 revolution during its fall, what was its linear speed, v?

95. •• A college campus features a large fountain surrounded by a circular pool. Two students start at the northernmost point of the pool and walk slowly around it in opposite directions. **(a)** If the angular speed of the student walking in the clockwise direction (as viewed from above) is 0.045 rad/s and the angular speed of the other student is 0.023 rad/s, how long does it take before they meet? **(b)** At what angle, measured clockwise from due north, do the students meet? **(c)** If the difference in linear speed between the students is 0.23 m/s, what is the radius of the fountain?

96. •• **IP** A yo-yo moves downward until it reaches the end of its string, where it "sleeps." As it sleeps—that is, spins in place—its angular speed decreases from 35 rad/s to 25 rad/s. During this time it completes 120 revolutions. **(a)** How long did it take for the yo-yo to slow from 35 rad/s to 25 rad/s? **(b)** How long does it take for the yo-yo to slow from 25 rad/s to 15 rad/s? Assume a constant angular acceleration as the yo-yo sleeps.

97. •• **IP (a)** An automobile with tires of radius 32 cm accelerates from 0 to 45 mph in 9.1 s. Find the angular acceleration of the tires. **(b)** How does your answer to part (a) change if the radius of the tires is halved?

98. •• **IP** In Problems 75 and 76 we considered a cylinder and a solid sphere, respectively, rolling down a ramp. **(a)** Which object do you expect to have the greater speed at the bottom of the ramp? **(b)** Verify your answer to part (a) by calculating the speed of the cylinder and of the sphere when they reach the bottom of the ramp.

99. •• A centrifuge (Problem 22) with an angular speed of 6050 rpm produces a maximum centripetal acceleration equal to $6840g$ (that is, 6840 times the acceleration of gravity). **(a)** What is the diameter of this centrifuge? **(b)** What force must the bottom of the sample holder exert on a 15.0-g sample under these conditions?

100. •• **A Yo-Yo with a Brain** Yomega ("The yo-yo with a brain") is constructed with a clever clutch mechanism in its axle that allows it to rotate freely and "sleep" when its angular speed is greater than a certain critical value. When the yo-yo's angular speed falls below this value, the clutch engages, causing the yo-yo to climb the string to the user's hand. If the moment of inertia of the yo-yo is $7.4 \times 10^{-5}\ \mathrm{kg \cdot m^2}$, its mass is 0.11 kg, and the string is 1.0 m long, what is the smallest angular speed that will allow the yo-yo to return to the user's hand?

A brain or just a clutch? (Problem 100)

101. •• The rotor in a centrifuge has an initial angular speed of 430 rad/s. After 8.2 s of constant angular acceleration, its angular speed has increased to 550 rad/s. During this time, what were **(a)** the angular acceleration of the rotor and **(b)** the angle through which it turned?

102. •• **BIO** A honey bee has two pairs of wings that can beat 250 times a second. Estimate **(a)** the maximum angular speed of the wings and **(b)** the maximum linear speed of a wing tip.

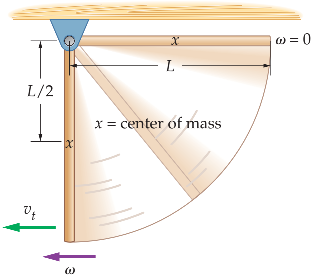
103. •• The Sun, with Earth in tow, orbits about the center of the Milky Way galaxy at a speed of 137 miles per second, completing one revolution every 240 million years. **(a)** Find the angular speed of the Sun relative to the center of the Milky Way. **(b)** Find the distance from the Sun to the center of the Milky Way.

104. •• A person walks into a room and switches on the ceiling fan. The fan accelerates with constant angular acceleration for 15 s until it reaches its operating angular speed of 1.9 rotations/s—after that its speed remains constant as long as the switch is "on." The person stays in the room for a short time; then, 5.5 minutes after turning the fan on, she switches it off again and leaves the room. The fan now decelerates with constant angular acceleration, taking 2.4 minutes to come to rest. What is the total number of revolutions made by the fan, from the time it was turned on until the time it stopped?

105. •• **BIO Preventing Bone Loss in Space** When astronauts return from prolonged space flights, they often suffer from bone loss, resulting in brittle bones that may take weeks for their bodies to rebuild. One solution may be to expose astronauts to periods of substantial "g forces" in a centrifuge carried aboard their spaceship. To test this approach, NASA conducted a study in which four people spent 22 hours each in a compartment attached to the end of a 28-foot arm that rotated with an angular speed of 10.0 rpm. **(a)** What centripetal acceleration did these volunteers experience? Express your answer in terms of g. **(b)** What was their linear speed?

106. ••• **Angular Acceleration of the Crab Nebula** The pulsar in the Crab nebula (Problem 9) was created by a supernova explosion that was observed on Earth in A.D. 1054. Its current period of rotation (33.0 ms) is observed to be increasing by 1.26×10^{-5} seconds per year. **(a)** What is the angular acceleration of the pulsar in $\mathrm{rad/s^2}$? **(b)** Assuming the angular acceleration of the pulsar to be constant, how many years will it take for the pulsar to slow to a stop? **(c)** Under the same assumption, what was the period of the pulsar when it was created?

107. ••• A thin, uniform rod of length L and mass M is pivoted about one end, as shown in **Figure 10–28**. The rod is released from rest in a horizontal position, and allowed to swing downward without friction or air resistance. When the rod is vertical, what are **(a)** its angular speed ω and **(b)** the tangential speed v_t of its free end?



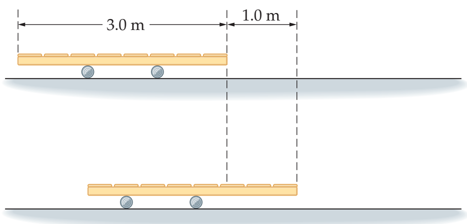
▲ **FIGURE 10–28** Problem 107

108. ••• **Center of Percussion** In the previous problem, suppose a small metal ball of mass $m = 2M$ is attached to the rod a distance d from the pivot. The rod and ball are released from rest in the horizontal position. **(a)** Show that when the rod reaches the vertical position, the speed of its tip is

$$v_t = \sqrt{3gL}\sqrt{\frac{1 + 4(d/L)}{1 + 6(d/L)^2}}$$

(b) At what finite value of d/L is the speed of the rod the same as it is for $d = 0$? (This value of d/L is the **center of percussion,** or "sweet spot," of the rod.)

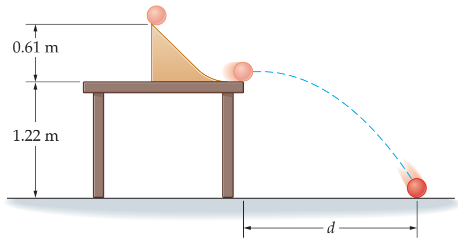
109. ••• A wooden plank rests on two soup cans laid on their sides. Each can has a diameter of 6.5 cm, and the plank is 3.0 m long. Initially, one can is placed 1.0 m inward from either end of the plank, as **Figure 10–29** shows. The plank is now pulled 1.0 m to the right, and the cans roll without slipping. **(a)** How far does the center of each can move? **(b)** How many rotations does each can make?



▲ **FIGURE 10–29** Problem 109

110. ••• A person rides on a 12-m-diameter Ferris wheel that rotates at the constant rate of 8.1 rpm. Calculate the magnitude and direction of the force that the seat exerts on a 65-kg person when he is **(a)** at the top of the wheel, **(b)** at the bottom of the wheel, and **(c)** halfway up the wheel.

111. ••• **IP** A solid sphere with a diameter of 0.17 m is released from rest; it then rolls without slipping down a ramp, dropping through a vertical height of 0.61 m. The ball leaves the bottom of the ramp, which is 1.22 m above the floor, moving horizontally (**Figure 10–30**). **(a)** Through what horizontal distance d does the ball move before landing? **(b)** How many revolutions does the ball make during its fall? **(c)** If the ramp were to be made frictionless, would the distance d increase, decrease, or stay the same? Explain.



▲ **FIGURE 10–30** Problem 111

PASSAGE PROBLEMS

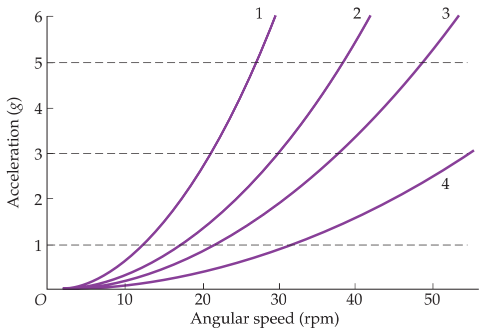
BIO Human-Powered Centrifuge

Space travel is fraught with hazards, not the least of which are the many side effects of prolonged weightlessness, including weakened muscles, bone loss, decreased coordination, and unsteady balance. If you are fortunate enough to go on a trip to Mars, which could take more than a year each way, you might be a bit "weak in the knees" by the time you arrive. This could lead to problems when you try to take your first "small step" on the surface.

To counteract these effects, NASA is looking into ways to provide astronauts with "portable gravity" on long space flights. One method under consideration is the human-powered centrifuge, which not only subjects the astronauts to artificial gravity, but also gives them aerobic exercise. The device is basically a rotating, circular platform on which two astronauts lie supine along a diameter, head-to-head at the center, with their feet at opposite rims, as shown in the accompanying photo. The radius of the platform in this test model is 6.25 ft. As one astronaut pedals to rotate the platform, the astronaut facing the other direction can exercise in the artificial gravity. Alternatively, a third astronaut on a stationary bicycle can provide the rotation for the other two.

Human-powered centrifuge, designed to give astronauts exercise and artificial gravity during long space flights.

Figure 10–31 shows the centripetal acceleration (in gs) produced by a rotating platform at four different radii. Notice that the acceleration increases as the square of the angular speed. Also indicated in Figure 10–31 are acceleration levels corresponding to 1, 3, and 5 gs. It is thought that enhanced gravitational effects may be desirable since the astronauts will experience the artificial gravity for only relatively brief periods of time during the flight.



▲ **FIGURE 10–31** Problems 112, 113, 114, and 115

112. • Rank the four curves shown in Figure 10–31 in order of increasing radius. Indicate ties where appropriate.

113. • What angular speed (in rpm) must the platform in this test model have to give a centripetal acceleration of 5.00 *g*s at the rim?

A. 5.07 rpm **B.** 26.1 rpm

C. 36.2 rpm **D.** 48.5 rpm

114. • Which of the curves shown in Figure 10–31 corresponds to the test model?

A. 1 **B.** 2

C. 3 **D.** 4

115. •• Estimate the radius corresponding to curve 4 in Figure 10–31.

A. 0.03 ft **B.** 0.3 ft

C. 3 ft **D.** 6 ft

INTERACTIVE PROBLEMS

116. •• **Referring to Conceptual Checkpoint 10–4** Suppose we race a disk and a hollow spherical shell, like a basketball. The spherical shell has a mass M and a radius R; the disk has a mass $2M$ and a radius $2R$. **(a)** Which object wins the race? If the two objects are released at rest, and the height of the ramp is $h = 0.75$ m, find the speed of **(b)** the disk and **(c)** the spherical shell when they reach the bottom of the ramp.

117. •• **Referring to Conceptual Checkpoint 10–4** Consider a race between the following three objects: object 1, a disk; object 2, a solid sphere; and object 3, a hollow spherical shell. All objects have the same mass and radius. **(a)** Rank the three objects in the order in which they finish the race. Indicate a tie where appropriate. **(b)** Rank the objects in order of increasing kinetic energy at the bottom of the ramp. Indicate a tie where appropriate.

118. •• **Referring to Active Example 10–3** **(a)** Suppose the radius of the axle the string wraps around is increased. Does the speed of the yo-yo after falling through a given height increase, decrease, or stay the same? **(b)** Find the speed of the yo-yo after falling from rest through a height $h = 0.50$ m if the radius of the axle is 0.0075 m. Everything else in Active Example 10–3 remains the same.

119. •• **Referring to Active Example 10–3** Suppose we use a new yo-yo that has the same mass as the original yo-yo and an axle of the same radius. The new yo-yo has a different mass distribution—most of its mass is concentrated near the rim. **(a)** Is the moment of inertia of the new yo-yo greater than, less than, or the same as that of the original yo-yo? **(b)** Find the moment of inertia of the new yo-yo if its speed after dropping from rest through a height $h = 0.50$ m is $v = 0.64$ m/s.