UCONN Physics 1201Q : Unit 8 | **Name: ____________________**
Worksheet I : Angular Momentum | **/ 26** | **Date: __________ Period: ____**

Angular momentum and the Principle of Conservation of Angular Momentum

1. A bicycle odometer (which measures distance traveled) is attached near the wheel hub and is designed for 27-inch wheels. What happens if you use it on a bicycle with 24-inch wheels?

2. Suppose a disk rotates at constant angular velocity. Does a point on the rim have radial and/or tangential acceleration?

If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration?

For which cases would the magnitude of either component of linear acceleration change?

3. Can the diver of Fig. 8–29 do a somersault without having any initial rotation when she leaves the board?

4. On the basis of the law of conservation of angular momentum, discuss why a helicopter must have more than one rotor (or propeller). Discuss one or more ways the second propeller can operate to keep the helicopter stable.

1. (I) Express the following angles in radians: (*a*) 30º, (*b*) 57º, (*c*) 90º, (*d*) 360º, and (*e*) 420º. Give as numerical values and as fractions of π.

2. (I) The blades in a blender rotate at a rate of 6500 rpm. When the motor is turned off during operation, the blades slow to rest in 3.0 s. What is the angular acceleration as the blades slow down?

3. (I) Pilots can be tested for the stresses of flying highspeed jets in a whirling "human centrifuge," which takes 1.0 min to turn through 20 complete revolutions before reaching its final speed.

(*a*) What was its angular acceleration (assumed constant), and

(*b*) what was its final angular speed in rpm?

4. (I) Determine the moment of inertia of a 10.8-kg sphere of radius 0.648 m when the axis of rotation is through its center.

5. (I) A centrifuge rotor has a moment of inertia of $3.75\times10^{-2}\ \text{kg}\cdot\text{m}^2$. How much energy is required to bring it from rest to 8250 rpm?

6. (I) What is the angular momentum of a 0.210-kg ball rotating on the end of a thin string in a circle of radius 1.10 m at an angular speed of $10.4\ \text{rad/s}$?

1. The odometer designed for 27-inch wheels increases its reading by the circumference of a 27-inch wheel $(27\pi")$ for every revolution of the wheel. If a 24-inch wheel is used, the odometer will still register $(27\pi")$ for every revolution, but only $24\pi"$ of linear distance will have been traveled. Thus the odometer will read a distance that is further than you actually traveled, by a factor of $27/24 = 1.125$. The odometer will read 12.5% too high.

2. If a disk rotates at constant angular velocity, a point on the rim has radial acceleration only – no tangential acceleration. If the disk's angular velocity increases uniformly, the point will have both radial and tangential acceleration. If the disk rotates at constant angular velocity, neither component of linear acceleration is changing – both radial and tangential acceleration are constant. If the disk rotates with a uniformly increasing angular velocity, then the radial acceleration is changing, but the tangential acceleration is a constant non-zero value.

3. In order to do a somersault, the diver needs some initial angular momentum when she leaves the diving board, because angular momentum will be conserved during the free-fall motion of the dive. She cannot exert a torque on herself in isolation, and so if there is no angular momentum initially, there will be no rotation during the rest of the dive.

4. Consider a helicopter in the air with the rotor spinning. To change the rotor's angular speed, a torque must be applied to the rotor. That torque has to come from the helicopter, and so by Newton's 3rd law, and equal and opposite torque will be applied by the rotor to the helicopter. Any change in rotor speed would therefore cause the body of the helicopter to spin in a direction opposite to the change in the rotor's angular velocity.

Some large helicopters have two rotor systems, spinning in opposite directions. That makes any change in the speed of the rotor pair require a net torque of 0, and so the helicopter body would not tend to spin. Smaller helicopters have a tail rotor which rotates in a vertical plane, causing a force on the tail of the helicopter in the opposite direction of the tendency of the tail to spin.

1. (*a*) $(30^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{\pi/6\,\text{rad}} = \boxed{0.52\ \text{rad}}$

(*b*) $(57^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{19\pi/60\,\text{rad}} = \boxed{0.99\ \text{rad}}$

(*c*) $(90^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{\pi/2\,\text{rad}} = \boxed{1.57\ \text{rad}}$

(*d*) $(360^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{2\pi\ \text{rad}} = \boxed{6.28\ \text{rad}}$

(*e*) $(420^\circ)(2\pi\ \text{rad}/360^\circ) = \boxed{7\pi/3\,\text{rad}} = \boxed{7.33\ \text{rad}}$

2. The initial angular velocity is $\omega_o = \left(6500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = 681\,\text{rad/s}$. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 681\,\text{rad/s}}{3.0\ \text{s}} = -227\,\text{rad/s}^2 \approx \boxed{-2.3\times10^2\ \text{rad/s}^2}$$

3. (*a*) The angular acceleration can be found from $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ with $\omega_o = 0$.

$$\alpha = \frac{2\theta}{t^2} = \frac{2(20\ \text{rev})}{(1.0\ \text{min})^2} = \boxed{4.0\times10^1\ \text{rev/min}^2}$$

(*b*) The final angular speed can be found from $\theta = \frac{1}{2}(\omega_o + \omega)t$, with $\omega_o = 0$.

$$\omega = \frac{2\theta}{t} - \omega_o = \frac{2(20\ \text{rev})}{1.0\ \text{min}} = \boxed{4.0\times10^1\ \text{rpm}}$$

4. For a sphere rotating about an axis through its center, the moment of inertia is given by

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(10.8\ \text{kg})(0.648\ \text{m})^2 = \boxed{1.81\ \text{kg}\cdot\text{m}^2}\ .$$

5. The energy required to bring the rotor up to speed from rest is equal to the final rotational KE of the rotor.

$$KE_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(3.75\times10^{-2}\,\text{kg}\cdot\text{m}^2)\left[8250\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right]^2 = \boxed{1.40\times10^4\,\text{J}}$$

6. The angular momentum is given by.

$$L = I\omega = MR^2\omega = (0.210\ \text{kg})(1.10\ \text{m})^2(10.4\,\text{rad/s}) = \boxed{2.64\,\text{kg}\cdot\text{m}^2/\text{s}}$$